Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.2.1-a+b-cos-^m-c+d-cos-^n

Nasser M. Abbasi

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3.221 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx$ 943

3.222 $\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx$ 946

3.223 $\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx$ 949

3.224 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$ 952

3.225 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$ 956

3.226 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$ 960

3.227 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$ 963

3.228 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$ 966

3.229 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$ 969

3.230 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$ 973

3.231 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$ 977

3.232 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$ 981

3.233 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$ 984

3.234 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx$ 987

3.235 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$ 990

3.236 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$ 993

3.237 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$ 997

3.238 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$ 1001

3.239 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$ 1005

3.240 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$ 1009

3.241 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$ 1012

3.242 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$ 1015

3.243 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$ 1019

3.244 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$ 1023

3.245 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$ 1028

3.246 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$ 1032

3.247 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$ 1036

3.248 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$ 1040

3.249 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$ 1044

3.250 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$ 1048

3.251 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$ 1053

3.252 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$ 1058

3.253 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$ 1063

3.254 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$ 1067

3.255 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$ 1071

3.256 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx$ 1075

3.257 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$ 1079

3.258 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$ 1083

3.259 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$ 1088

3.260 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$ 1092

3.261 $\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx$ 1096

3.262 $\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx$ 1099

3.263 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx$ 1102

3.264 $\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx$ 1106

3.265 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$ 1110

3.266 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 1113

3.267 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 1116

3.268 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 1119

3.269 $\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx$ 1122

3.270 $\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$ 1126

3.271 $\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$ 1130

3.272 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 1133

3.273 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 1136

3.274 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 1139

3.275 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$ 1142

3.276 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$ 1146

3.277 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx$ 1150

3.278 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx$ 1153

3.279 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$ 1156

3.280 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$ 1159

3.281 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$ 1163

3.282 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$ . . . 1167

3.283 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$ . . . 1171

3.284 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx$ . . . 1175

3.285 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$ . . . 1178

3.286 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$ . . . 1181

3.287 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$ . . . 1185

3.288 $\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx$ . . . 1189

3.289 $\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$ . . . 1192

3.290 $\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$ . . . 1195

3.291 $\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx$ . . . 1198

3.292 $\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$ . . . 1202

3.293 $\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$ . . . 1205

3.294 $\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$ . . . 1208

3.295 $\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$ . . . 1211

3.296 $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ . . . 1214

3.297 $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$ . . . 1218

3.298 $\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$ . . . 1222

3.299 $\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx$ . . . 1226

3.300 $\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx$ . . . 1230

3.301 $\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$ . . . 1233

3.302 $\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$ . . . 1236

3.303 $\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ . . . 1240

3.304 $\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx$ . . . 1244

3.305 $\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx$ . . . 1248

3.306 $\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$ . . . 1252

3.307 $\int (a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx$ . . . 1256

3.308 $\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$ . . . 1260

3.309 $\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$ . . . 1263

3.310 $\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ . . . 1267

3.311 $\int (a+a\cos(c+dx))^4\sec^{\frac{9}{2}}(c+dx)\,dx$ . . . 1271

3.312 $\int (a+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx$ . . . 1275

3.313 $\int (a+a\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\,dx$ . . . 1279

3.314 $\int (a+a\cos(c+dx))^4\sec^{\frac{3}{2}}(c+dx)\,dx$ . . . 1283

3.315 $\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx$ . . . 1287

3.316 $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$ . . . 1291

3.317 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$ . . . 1295

3.318 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$ . . . 1299

3.319 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$ . . . 1303

3.320 $\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$ 1306

3.321 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1309

3.322 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1313

3.323 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1317

3.324 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$ 1321

3.325 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$ 1325

3.326 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$ 1329

3.327 $\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$ 1333

3.328 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1336

3.329 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1340

3.330 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1344

3.331 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx$ 1348

3.332 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$ 1353

3.333 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$ 1358

3.334 $\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$ 1362

3.335 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1367

3.336 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1372

3.337 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1376

3.338 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx$ 1380

3.339 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx$ 1385

3.340 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$ 1388

3.341 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx$ 1391

3.342 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$ 1394

3.343 $\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$ 1397

3.344 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$ 1400

3.345 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1404

3.346 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$ 1408

3.347 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$ 1412

3.348 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx$ 1415

3.349 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$ 1418

3.350 $\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$ 1422

3.351 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$ 1426

3.352 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1430

3.353 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$ 1435

3.354 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$ 1439

3.355 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$ 1443

3.356 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$ 1446

3.357 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$ 1450

3.358 $\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$ 1454

3.359 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$ 1458

3.360 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1463

3.361 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$ 1470

3.362 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$ 1474

3.363 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$ 1478

3.364 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$ 1481

3.365 $\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$ 1484

3.366 $\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1487

3.367 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$ 1491

3.368 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$ 1495

3.369 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$ 1499

3.370 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$ 1502

3.371 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$ 1505

3.372 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1508

3.373 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$ 1512

3.374 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$ 1516

3.375 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$ 1520

3.376 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$ 1523

3.377 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1526

3.378 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1530

3.379 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$ 1535

3.380 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$ 1540

3.381 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$ 1545

3.382 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$ 1549

3.383 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$ 1553

3.384 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$ 1557

3.385 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$ 1562

3.386 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$ 1567

3.387 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$ 1572

3.388 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$ 1577

3.389 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$ . . . 1581
3.390 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$ . . . 1585
3.391 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$ . . . 1589
3.392 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$ . . . 1594
3.393 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$ . . . 1599
3.394 $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$ . . . 1604
3.395 $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$ . . . 1609
3.396 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx$ . . . 1614
3.397 $\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$ . . . 1617
3.398 $\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx$ . . . 1621
3.399 $\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx$ . . . 1624
3.400 $\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx$ . . . 1627
3.401 $\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx$ . . . 1630
3.402 $\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx$ . . . 1633
3.403 $\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx$ . . . 1636
3.404 $\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$ . . . 1639
3.405 $\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx$ . . . 1642
3.406 $\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$ . . . 1645
3.407 $\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx$ . . . 1648
3.408 $\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx$ . . . 1651
3.409 $\int \cos(c+dx)(a+b\cos(c+dx))\,dx$ . . . 1654
3.410 $\int (a+b\cos(c+dx))\,dx$ . . . 1656
3.411 $\int (a+b\cos(c+dx))\sec(c+dx)\,dx$ . . . 1658
3.412 $\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx$ . . . 1661
3.413 $\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx$ . . . 1664
3.414 $\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx$ . . . 1667
3.415 $\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx$ . . . 1670
3.416 $\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$ . . . 1673
3.417 $\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$ . . . 1676
3.418 $\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$ . . . 1679
3.419 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx$ . . . 1682
3.420 $\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx$ . . . 1685
3.421 $\int (a+b\cos(c+dx))^2\,dx$ . . . 1688
3.422 $\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx$ . . . 1691
3.423 $\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx$ . . . 1694
3.424 $\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx$ . . . 1697
3.425 $\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx$ . . . 1700
3.426 $\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx$ . . . 1703
3.427 $\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$ . . . 1706
3.428 $\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$ . . . 1709
3.429 $\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$ . . . 1713
3.430 $\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx$ . . . 1716
3.431 $\int (a+b\cos(c+dx))^3\,dx$ . . . 1719
3.432 $\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx$ . . . 1722
3.433 $\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$ . . . 1725
3.434 $\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx$ . . . 1728
3.435 $\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$ . . . 1731
3.436 $\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$ . . . 1735
3.437 $\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx$ . . . 1739

3.438 $\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$ 1743
3.439 $\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$ 1747
3.440 $\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$ 1751
3.441 $\int (a+b\cos(c+dx))^4\,dx$ 1754
3.442 $\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx$ 1757
3.443 $\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx$ 1761
3.444 $\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$ 1764
3.445 $\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$ 1768
3.446 $\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$ 1772
3.447 $\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx$ 1776
3.448 $\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$ 1780
3.449 $\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$ 1784
3.450 $\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx$ 1789
3.451 $\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$ 1793
3.452 $\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$ 1797
3.453 $\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx$ 1800
3.454 $\int \frac{1}{a+b\cos(c+dx)}\,dx$ 1803
3.455 $\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$ 1806
3.456 $\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$ 1809
3.457 $\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$ 1813
3.458 $\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$ 1817
3.459 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1821
3.460 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1826
3.461 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1830
3.462 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1834
3.463 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1838
3.464 $\int \frac{1}{(a+b\cos(c+dx))^2}\,dx$ 1841
3.465 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1844
3.466 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1848
3.467 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1852
3.468 $\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 1857
3.469 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 1862
3.470 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 1867
3.471 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 1872
3.472 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 1876
3.473 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 1880
3.474 $\int \frac{1}{(a+b\cos(c+dx))^3}\,dx$ 1884
3.475 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 1888
3.476 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 1893
3.477 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 1898
3.478 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$ 1904
3.479 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$ 1910

3.480 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx$ 1915
3.481 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$ 1920
3.482 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$ 1924
3.483 $\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$ 1928
3.484 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$ 1932
3.485 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$ 1938
3.486 $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$ 1944
3.487 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$ 1949
3.488 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$ 1953
3.489 $\int \sqrt{a+b\cos(c+dx)}\,dx$ 1957
3.490 $\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx$ 1960
3.491 $\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$ 1963
3.492 $\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$ 1967
3.493 $\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$ 1972
3.494 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$ 1977
3.495 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$ 1981
3.496 $\int (a+b\cos(c+dx))^{3/2}\,dx$ 1985
3.497 $\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$ 1989
3.498 $\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$ 1993
3.499 $\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$ 1998
3.500 $\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$ 2003
3.501 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$ 2008
3.502 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$ 2012
3.503 $\int (a+b\cos(c+dx))^{5/2}\,dx$ 2016
3.504 $\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$ 2020
3.505 $\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$ 2024
3.506 $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$ 2029
3.507 $\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$ 2034
3.508 $\int (a+b\cos(c+dx))^{7/2}\,dx$ 2039
3.509 $\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$ 2043
3.510 $\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$ 2047
3.511 $\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$ 2050
3.512 $\int \sqrt{3+4\cos(c+dx)}\,dx$ 2053
3.513 $\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx$ 2055
3.514 $\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx$ 2058
3.515 $\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx$ 2062
3.516 $\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$ 2066
3.517 $\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx$ 2070
3.518 $\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx$ 2073
3.519 $\int \sqrt{3-4\cos(c+dx)}\,dx$ 2076
3.520 $\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx$ 2078
3.521 $\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx$ 2081
3.522 $\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$ 2085
3.523 $\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ 2089
3.524 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ 2093
3.525 $\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ 2097
3.526 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$ 2100
3.527 $\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ 2103
3.528 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ 2106

3.529 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ 2110
3.530 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2115
3.531 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2120
3.532 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2124
3.533 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2128
3.534 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2132
3.535 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2135
3.536 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2139
3.537 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2144
3.538 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2149
3.539 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2154
3.540 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2159
3.541 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2164
3.542 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2168
3.543 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2172
3.544 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2176
3.545 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2181
3.546 $\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx$ 2187
3.547 $\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$ 2191
3.548 $\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$ 2194
3.549 $\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$ 2197
3.550 $\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx$ 2200
3.551 $\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$ 2202
3.552 $\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$ 2205
3.553 $\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$ 2209
3.554 $\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$ 2213
3.555 $\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$ 2216
3.556 $\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$ 2219
3.557 $\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx$ 2222
3.558 $\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$ 2225
3.559 $\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$ 2228
3.560 $\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$ 2232
3.561 $\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$ 2236
3.562 $\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx$ 2239
3.563 $\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx$ 2242
3.564 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$ 2245
3.565 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 2248
3.566 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 2251

3.567 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 2254
3.568 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$ 2257
3.569 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$ 2260
3.570 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx$ 2263
3.571 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$ 2266
3.572 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 2269
3.573 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 2272
3.574 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 2275
3.575 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$ 2279
3.576 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$ 2283
3.577 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$ 2287
3.578 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 2290
3.579 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 2294
3.580 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 2298
3.581 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ 2302
3.582 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$ 2306
3.583 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$ 2310
3.584 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$ 2313
3.585 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$ 2316
3.586 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$ 2319
3.587 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$ 2323
3.588 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 2327
3.589 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 2332
3.590 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ 2336
3.591 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$ 2340
3.592 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$ 2344
3.593 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$ 2348
3.594 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$ 2353
3.595 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 2358
3.596 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 2363
3.597 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 2368
3.598 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$ 2373
3.599 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$ 2378

3.600 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$ . . . 2383
3.601 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$ . . . 2388
3.602 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$ . . . 2393
3.603 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$ . . . 2398
3.604 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$ . . . 2403
3.605 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$ . . . 2408
3.606 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ . . . 2411
3.607 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ . . . 2414
3.608 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ . . . 2418
3.609 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ . . . 2423
3.610 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$ . . . 2428
3.611 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$ . . . 2434
3.612 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$ . . . 2439
3.613 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ . . . 2444
3.614 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ . . . 2448
3.615 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ . . . 2452
3.616 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ . . . 2457
3.617 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$ . . . 2462
3.618 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$ . . . 2468
3.619 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$ . . . 2474
3.620 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ . . . 2479
3.621 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ . . . 2484
3.622 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ . . . 2489
3.623 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ . . . 2494
3.624 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$ . . . 2499
3.625 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$ . . . 2505
3.626 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2511
3.627 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2516
3.628 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2519
3.629 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2522
3.630 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2525
3.631 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ . . . 2529
3.632 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ . . . 2534

3.633 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$ 2539
3.634 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$ 2543
3.635 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$ 2547
3.636 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$ 2551
3.637 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$ 2556
3.638 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2562
3.639 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2568
3.640 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$ 2573
3.641 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$ 2578
3.642 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$ 2583
3.643 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$ 2589
3.644 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$ 2595
3.645 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$ 2598
3.646 $\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$ 2601
3.647 $\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$ 2604
3.648 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx$ 2607
3.649 $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$ 2610
3.650 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx$ 2613
3.651 $\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$ 2616
3.652 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$ 2619
3.653 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$ 2622
3.654 $\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$ 2625
3.655 $\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$ 2628
3.656 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx$ 2631
3.657 $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$ 2634
3.658 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx$ 2637
3.659 $\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$ 2640
3.660 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx$ 2643
3.661 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx$ 2646
3.662 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$ 2649
3.663 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx$ 2652
3.664 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$ 2655
3.665 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$ 2658
3.666 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$ 2661
3.667 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx$ 2664
3.668 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx$ 2667
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3.671 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx$ . . . 2676
3.672 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$ . . . 2679
3.673 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$ . . . 2682
3.674 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$ . . . 2685
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3.677 $\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$ . . . 2696
3.678 $\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$ . . . 2701
3.679 $\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx$ . . . 2706
3.680 $\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2711
3.681 $\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2713
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3.683 $\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2717
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3.698 $\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx$ . . . 2757
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3.718 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ . . . 2833
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3.728 $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$ . . . 2885
3.729 $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$ . . . 2890
3.730 $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$ . . . 2895
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3.743 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$ . . . 2956
3.744 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$ . . . 2961
3.745 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$ . . . 2966
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3.751 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 2999
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3.753 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$ . . . 3007
3.754 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$ . . . 3010
3.755 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$ . . . 3013
3.756 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$ . . . 3018
3.757 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ . . . 3023
3.758 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$ . . . 3028
3.759 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$ . . . 3032
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3.761 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$ . . . 3040
3.762 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$ . . . 3045
3.763 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ . . . 3051
3.764 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$ . . . 3057
3.765 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$ . . . 3063
3.766 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$ . . . 3068
3.767 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$ . . . 3073
3.768 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$ . . . 3078
3.769 $\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$ . . . 3085
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3.771 $\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx$ . . . 3092
3.772 $\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx$ . . . 3095
3.773 $\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx$ . . . 3098
3.774 $\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx$ . . . 3101
3.775 $\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx$ . . . 3104
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3.780 $\int (a+a\cos(c+dx))\left(-\frac{B}{2}+B\cos(c+dx)\right) dx$ . . . 3120
3.781 $\int (a+a\cos(c+dx))^4\left(-\frac{4B}{5}+B\cos(c+dx)\right) dx$ . . . 3122
3.782 $\int (a+a\cos(c+dx))^n\left(-\frac{Bn}{1+n}+B\cos(c+dx)\right) dx$ . . . 3125
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3.793 $\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)} \, dx$ . . . 3158
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3.819 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx) \, dx$ . . . 3246
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3.834 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 3300
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3.841 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$ . . . 3324
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3.843 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$ . . . 3331
3.844 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$ . . . 3334
3.845 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$ . . . 3337
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3.848 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ . . . 3346
3.849 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ . . . 3350
3.850 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$ . . . 3354
3.851 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$ . . . 3357
3.852 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$ . . . 3360
3.853 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ . . . 3363
3.854 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ . . . 3366
3.855 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ . . . 3369
3.856 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ . . . 3372
3.857 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$ . . . 3376
3.858 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$ . . . 3380
3.859 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$ . . . 3383

3.860 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 3386
3.861 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 3389
3.862 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 3392
3.863 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ 3395
3.864 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$ 3398
3.865 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$ 3402
3.866 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$ 3406
3.867 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$ 3409
3.868 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$ 3412
3.869 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$ 3415
3.870 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$ 3418
3.871 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$ 3421
3.872 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$ 3425
3.873 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$ 3429
3.874 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$ 3432
3.875 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$ 3435
3.876 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$ 3438
3.877 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$ 3441
3.878 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$ 3444
3.879 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$ 3448
3.880 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$ 3452
3.881 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$ 3455
3.882 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$ 3458
3.883 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$ 3461
3.884 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$ 3464
3.885 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$ 3467
3.886 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$ 3471
3.887 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$ 3475
3.888 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$ 3478
3.889 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$ 3481
3.890 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$ 3484
3.891 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$ 3487
3.892 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$ 3490
3.893 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$ 3493
3.894 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$ 3496

3.895 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$ 3499
3.896 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx$ 3502
3.897 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$ 3505
3.898 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$ 3508
3.899 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$ 3511
3.900 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$ 3514
3.901 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ 3517
3.902 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ 3520
3.903 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ 3523
3.904 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ 3526
3.905 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$ 3529
3.906 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$ 3532
3.907 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ 3535
3.908 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ 3538
3.909 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ 3541
3.910 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ 3544
3.911 $\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$ 3547
3.912 $\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$ 3550
3.913 $\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$ 3553
3.914 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$ 3556
3.915 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx$ 3559
3.916 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx$ 3562
3.917 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx$ 3565
3.918 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx$ 3568
3.919 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$ 3571
3.920 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$ 3574
3.921 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$ 3577
3.922 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$ 3580
3.923 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 3583
3.924 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 3586
3.925 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 3589
3.926 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ 3592
3.927 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$ 3595
3.928 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$ 3598
3.929 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$ 3601
3.930 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$ 3604
3.931 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$ 3607
3.932 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$ 3610

4 Listing of Grading functions **3613**

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [932]. This is test number [89].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (932)	% 0. (0)
Mathematica	% 99.03 (923)	% 0.97 (9)
Maple	% 91.63 (854)	% 8.37 (78)
Maxima	% 30.58 (285)	% 69.42 (647)
Fricas	% 47.53 (443)	% 52.47 (489)
Sympy	% 8.26 (77)	% 91.74 (855)
Giac	% 27.25 (254)	% 72.75 (678)

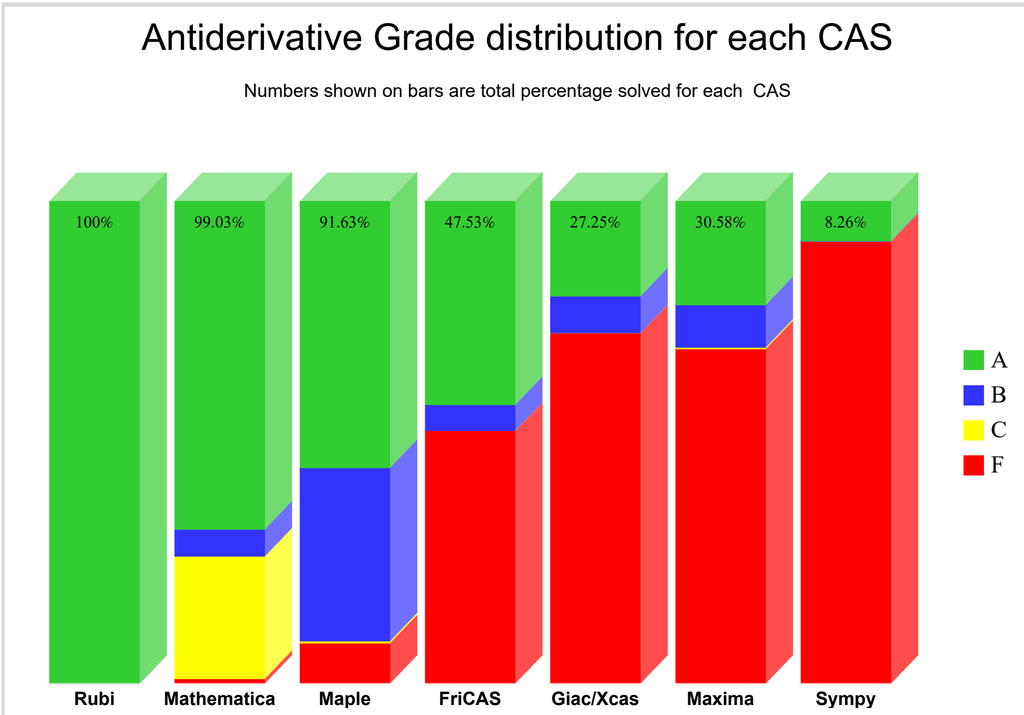
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

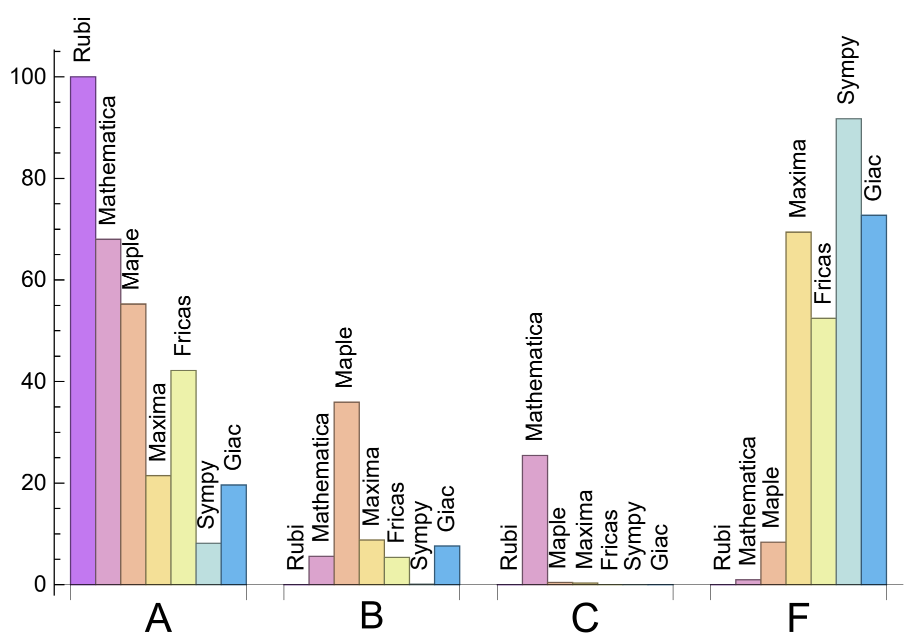
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	68.03	5.58	25.43	0.97
Maple	55.26	35.94	0.43	8.37
Maxima	21.46	8.8	0.32	69.42
Fricas	42.17	5.36	0.	52.47
Sympy	8.15	0.11	0.	91.74
Giac	19.64	7.62	0.	72.75

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	153.82	0.99	131.	1.
Mathematica	3.09	277.81	1.73	140.	1.
Maple	1.79	415.18	2.22	215.	1.79
Maxima	1.65	450.86	4.06	155.	1.69
Fricas	2.06	474.68	4.	362.	3.14
Sympy	5.97	179.75	2.04	121.	1.83
Giac	1.9	212.98	2.13	152.	1.67

1.4 list of integrals that has no closed form antiderivative

{680, 681, 682, 683, 684, 685, 686, 687, 688, 689}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {22, 31, 117, 118, 119, 120, 128, 129, 130, 136, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 213, 214, 215, 216, 217, 228, 229, 230, 231, 232, 235, 236, 237, 242, 243, 244, 249, 250, 252, 253, 254, 256, 257, 260, 269, 286, 311, 356, 357, 358, 359, 360, 361, 362, 363, 367, 368, 369, 374, 379, 380, 382, 386, 387, 390, 392, 394, 400, 607, 610, 614, 618, 619, 620, 621, 622, 625, 636, 638, 639, 643, 676, 677, 678, 679, 718, 720, 721, 724, 725, 726, 728, 730, 732, 734, 735, 736, 737, 743, 744, 745, 747, 748, 749, 750, 751, 754, 755, 756, 757, 758, 761, 762, 763, 764, 765, 766, 768, 773, 774, 795, 796, 797, 798}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
```

'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

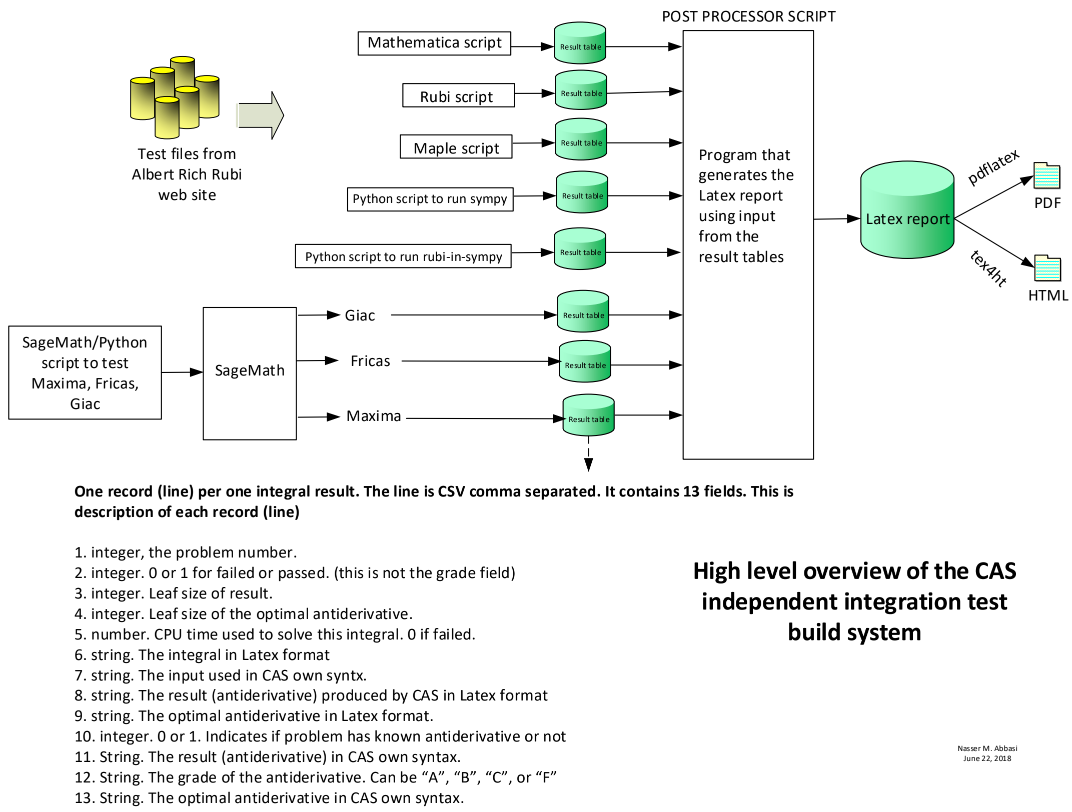
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 29, 33, 34, 35, 36, 39, 42, 43, 44, 45, 47, 48, 53, 54, 55, 56, 57, 58, 63, 64, 65, 66, 67, 68, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 125, 126, 127, 131, 132, 133, 134, 135, 137, 140, 141, 142, 143, 157, 171, 185, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 218, 219, 220, 221, 222, 227, 234, 240, 241, 246, 247, 248, 253, 254, 255, 256, 259, 260, 261, 262, 266, 267, 268, 272, 273, 274, 300, 313, 327, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 364, 370, 375, 376, 381, 382, 383, 388, 389, 390, 391, 394, 395, 396, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 493, 494, 495, 496, 497, 500, 501, 502, 503, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 530, 531, 532, 533, 534, 538, 539, 540, 541, 542, 543, 546, 547, 548, 549, 550, 551, 554, 555, 556, 557, 558, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 604, 605, 606, 607, 608, 611, 612, 613, 614, 615, 619, 621, 622, 627, 628, 629, 630, 633, 634, 639, 650, 651, 656, 657, 661, 662, 663, 664, 665, 667, 668, 669, 670, 672, 673, 675, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 727, 729, 730, 731, 732, 733, 734, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 753, 754, 757, 758, 759, 760, 762, 763, 764, 765, 766, 767, 769, 770, 771, 772, 775, 776, 777, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

B grade: { 21, 22, 28, 30, 31, 32, 37, 38, 40, 41, 46, 49, 50, 51, 52, 59, 60, 61, 62, 69, 70, 71, 80, 81, 91, 92, 139, 586, 644, 645, 646, 647, 648, 649, 652, 653, 654, 655, 658, 659, 660, 671, 676, 677, 678, 679, 720, 721, 726, 728, 773, 774 }

C grade: { 117, 118, 119, 120, 128, 129, 130, 136, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 213, 214, 215, 216, 217, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 242, 243, 244, 245, 249, 250, 251, 252, 257, 258, 263, 264, 265, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 291, 292, 293, 294, 295, 296, 297, 298, 299, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 356, 357, 358, 359, 360, 361, 362, 363, 365, 366, 367, 368, 369, 371, 372, 374, 377, 378, 379, 380, 384, 385, 386, 387, 392, 393, 400, 459, 491, 492, 498, 499, 504, 505, 506, 507, 514, 515, 521, 522, 528, 529, 535, 536, 537, 544, 545, 552, 553, 559, 560, 603, 609, 610, 616, 617, 618, 620, 623, 624, 625, 626, 631, 632, 635, 636, 637, 638, 640, 641, 642, 643, 666, 674, 735, 750, 755, 756, 761, 768, 778, 788, 790 }

F grade: { 288, 289, 290, 373, 397, 398, 399, 401, 402}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 112, 113, 114, 115, 121, 122, 123, 124, 125, 131, 132, 133, 139, 140, 141, 142, 143, 146, 147, 149, 150, 153, 154, 155, 157, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 215, 218, 219, 220, 224, 225, 226, 227, 231, 232, 238, 239, 240, 244, 246, 247, 253, 254, 255, 259, 260, 262, 263, 264, 266, 267, 268, 269, 270, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 283, 284, 287, 293, 294, 295, 296, 297, 300, 301, 302, 303, 307, 308, 309, 310, 314, 315, 316, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 452, 453, 454, 455, 456, 459, 461, 463, 464, 466, 467, 490, 497, 504, 509, 510, 511, 513, 516, 517, 518, 520, 525, 527, 528, 533, 534, 535, 543, 546, 547, 548, 549, 554, 555, 556, 561, 564, 565, 572, 578, 583, 584, 605, 626, 627, 628, 650, 651, 656, 657, 658, 662, 663, 666, 667, 668, 669, 672, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 694, 695, 696, 700, 701, 702, 703, 704, 708, 709, 710, 711, 712, 715, 716, 733, 753, 754, 755, 779, 780, 783, 784, 785, 786, 793, 799, 800, 801, 802, 803, 806, 807, 808, 809, 810, 813, 814, 815, 816, 817, 820, 821, 822, 823, 824, 827, 828, 829, 830, 831, 834, 835, 836, 837, 838, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886 }

B grade: { 100, 101, 102, 103, 108, 109, 110, 111, 116, 117, 118, 119, 120, 127, 128, 129, 130, 134, 135, 136, 137, 138, 144, 145, 148, 151, 152, 156, 158, 159, 164, 165, 166, 171, 172, 173, 180, 185, 187, 188, 196, 201, 208, 209, 216, 217, 222, 223, 228, 229, 230, 233, 234, 235, 236, 237, 241, 242, 243, 245, 248, 249, 250, 251, 252, 256, 257, 258, 261, 265, 271, 281, 285, 286, 291, 292, 298, 299, 304, 305, 306, 311, 312, 313, 317, 327, 332, 343, 349, 356, 361, 362, 392, 449, 450, 451, 457, 458, 460, 462, 465, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 512, 514, 515, 519, 521, 522, 523, 524, 529, 530, 531, 532, 536, 537, 538, 539, 540, 541, 542, 544, 545, 551, 552, 553, 558, 559, 560, 562, 563, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 579, 580, 581, 582, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 652, 653, 654, 655, 659, 660, 661, 664, 665, 670, 671, 673, 674, 675, 690, 691, 697, 698, 699, 705, 706, 707, 713, 714, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 778, 781, 782, 791, 792, 794, 804, 805, 811, 812, 818, 819, 825, 826, 832, 833, 839, 840, 841 }

C grade: { 126, 526, 550, 557 }

F grade: { 221, 288, 289, 290, 396, 397, 398, 399, 400, 401, 402, 676, 677, 678, 679, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 795, 796, 797, 798, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 44, 45, 47, 48, 49, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 112, 113, 114, 115, 121, 210, 212, 218, 219, 220, 346, 348, 353, 354, 355, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 780, 793, 842, 843, 844, 845, 846, 847, 850, 851, 852, 853, 854, 855, 858, 859, 860, 861, 862, 863, 866, 867, 868, 869, 873, 874, 875, 876, 880, 881, 882, 883

B grade: { 40, 42, 46, 50, 51, 52, 101, 102, 109, 110, 118, 126, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 221, 222, 223, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 339, 340, 341, 342, 343, 344, 345, 347, 349, 350, 351, 352, 356, 357, 358, 359, 360, 396, 781, 782, 783, 784, 848, 849, 856, 857, 864, 865, 870, 871, 872, 877, 878, 879, 884, 885, 886 }

C grade: { 285, 286, 287 }

F grade: { 100, 103, 108, 111, 116, 117, 119, 120, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 138, 139, 140, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 403, 404, 405, 406, 407, 408, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 457, 458, 459, 460, 461, 463, 464, 468, 469, 472, 480, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 778, 779, 780, 782, 784, 785, 791, 792, 793, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886 }

B grade: { 7, 8, 19, 101, 108, 109, 127, 128, 129, 134, 135, 136, 137, 141, 142, 143, 144, 145, 234, 235, 261, 283, 284, 285, 411, 412, 423, 456, 462, 465, 466, 467, 470, 471, 473, 474, 475, 476, 477, 478, 479, 481, 482, 483, 484, 485, 781, 783, 786, 869 }

C grade: { }

F grade: { 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 397, 398, 399, 400, 401, 402, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 787, 788, 789, 790, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, 24, 25, 26, 33, 34, 35, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 64, 65, 66, 67, 68, 75, 76, 77, 78, 87, 88, 89, 403, 404, 405, 406, 407, 408, 409, 410, 417, 418, 419, 420, 421, 428, 429, 430, 431, 438, 439, 440, 441, 454, 683, 684, 685, 686, 780, 781, 783, 793, 844, 845, 868 }

B grade: { 411 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 49, 50, 51, 52, 59, 60, 61, 62, 63, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 412, 413, 414, 415, 416, 422, 423, 424, 425, 426, 427, 432, 433, 434, 435, 436, 437, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 782, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 122, 123, 124, 125, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 267, 268, 272, 273, 274, 275, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 403, 404, 405, 406, 407, 408, 409, 410, 416, 417, 418, 419, 420, 421, 428, 429, 430, 431, 433, 434, 438, 439, 440, 441, 443, 444, 450, 451, 452, 453, 454, 457, 459, 460, 462, 463, 464, 465, 467, 468, 470, 471, 472, 476, 478, 480, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 780, 783, 785, 791, 793 }

B grade: { 7, 8, 18, 19, 100, 101, 102, 103, 108, 109, 110, 111, 116, 117, 118, 119, 120, 127, 128, 129, 130, 136, 137, 138, 145, 223, 265, 266, 271, 278, 411, 412, 413, 414, 415, 422, 423, 424, 425, 426, 427, 432, 435, 436, 437, 442, 445, 446, 447, 448, 449, 455, 456, 458, 461, 466, 469, 473, 474, 475, 477, 479, 481, 482, 483, 484, 485, 781, 782, 786, 792 }

C grade: { }

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2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	75	80	113	204	216	124
normalized size	1	1.	0.66	0.7	0.99	1.79	1.89	1.09
time (sec)	N/A	0.07	0.188	0.044	1.177	1.941	4.763	1.281

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	65	70	93	173	168	104
normalized size	1	1.	0.71	0.76	1.01	1.88	1.83	1.13
time (sec)	N/A	0.058	0.108	0.043	1.11	1.956	2.39	1.388

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	77	136	144	84
normalized size	1	1.	0.96	0.79	1.01	1.79	1.89	1.11
time (sec)	N/A	0.052	0.084	0.043	1.184	1.828	1.353	1.289

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	62	105	92	63
normalized size	1	1.	1.06	0.91	1.15	1.94	1.7	1.17
time (sec)	N/A	0.042	0.066	0.04	1.088	1.907	0.611	1.275

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	38	46	72	66	42
normalized size	1	1.	0.84	1.	1.21	1.89	1.74	1.11
time (sec)	N/A	0.014	0.046	0.036	1.089	1.656	0.282	1.33

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	38	17	20
normalized size	1	1.	1.73	1.07	1.33	2.53	1.13	1.33
time (sec)	N/A	0.007	0.005	0.025	1.125	1.64	0.133	1.324

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	30	38	95	49	58
normalized size	1	1.	1.	1.88	2.38	5.94	3.06	3.62
time (sec)	N/A	0.02	0.007	0.055	1.191	1.723	4.942	1.468

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	51	162	0	85
normalized size	1	1.	1.	1.33	2.12	6.75	0.	3.54
time (sec)	N/A	0.034	0.011	0.061	1.129	1.698	0.	1.484

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	78	198	0	108
normalized size	1	1.	1.	1.09	1.66	4.21	0.	2.3
time (sec)	N/A	0.048	0.014	0.067	1.159	1.662	0.	1.442

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	95	236	0	130
normalized size	1	1.	0.95	1.14	1.51	3.75	0.	2.06
time (sec)	N/A	0.048	0.128	0.071	1.103	1.649	0.	1.447

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	128	266	0	149
normalized size	1	1.	0.89	1.08	1.51	3.13	0.	1.75
time (sec)	N/A	0.062	0.131	0.079	1.16	1.739	0.	1.454

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	65	112	144	304	0	167
normalized size	1	1.	0.64	1.11	1.43	3.01	0.	1.65
time (sec)	N/A	0.066	0.228	0.079	1.158	1.732	0.	1.417

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	73	121	163	228	343	143
normalized size	1	1.	0.57	0.94	1.26	1.77	2.66	1.11
time (sec)	N/A	0.13	0.183	0.046	1.142	1.619	4.58	1.362

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	61	96	128	186	221	120
normalized size	1	1.	0.59	0.93	1.24	1.81	2.15	1.17
time (sec)	N/A	0.104	0.118	0.043	1.14	1.728	2.359	1.327

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	53	90	112	154	211	97
normalized size	1	1.	0.61	1.03	1.29	1.77	2.43	1.11
time (sec)	N/A	0.097	0.12	0.043	1.11	1.684	1.271	1.325

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	69	41	64	82	113	107	73
normalized size	1	1.21	0.72	1.12	1.44	1.98	1.88	1.28
time (sec)	N/A	0.041	0.079	0.039	1.124	1.703	0.609	1.345

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	61	82	78	51
normalized size	1	1.	0.76	1.16	1.36	1.82	1.73	1.13
time (sec)	N/A	0.014	0.042	0.039	1.133	1.605	0.253	1.369

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	47	51	58	131	0	107
normalized size	1	1.	1.38	1.5	1.71	3.85	0.	3.15
time (sec)	N/A	0.058	0.012	0.059	1.156	1.648	0.	1.385

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	50	66	193	0	107
normalized size	1	1.	0.82	1.47	1.94	5.68	0.	3.15
time (sec)	N/A	0.058	0.012	0.065	1.075	1.654	0.	1.336

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	58	119	215	0	122
normalized size	1	1.	1.	1.07	2.2	3.98	0.	2.26
time (sec)	N/A	0.079	0.012	0.076	1.102	1.662	0.	1.486

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	162	78	115	246	0	143
normalized size	1	1.	2.45	1.18	1.74	3.73	0.	2.17
time (sec)	N/A	0.088	5.541	0.077	1.136	1.678	0.	1.41

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	797	102	196	288	0	165
normalized size	1	1.	8.3	1.06	2.04	3.	0.	1.72
time (sec)	N/A	0.108	6.37	0.082	1.123	1.726	0.	1.47

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	73	143	193	230	379	143
normalized size	1	1.	0.57	1.11	1.5	1.78	2.94	1.11
time (sec)	N/A	0.147	0.166	0.044	1.1	1.675	4.706	1.334

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	63	121	158	194	272	119
normalized size	1	1.	0.6	1.15	1.5	1.85	2.59	1.13
time (sec)	N/A	0.117	0.127	0.043	1.12	1.647	2.466	1.399

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	88	51	100	127	151	224	96
normalized size	1	1.04	0.6	1.18	1.49	1.78	2.64	1.13
time (sec)	N/A	0.078	0.118	0.04	1.143	1.611	1.291	1.353

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	95	119	121	74
normalized size	1	1.	0.7	1.17	1.51	1.89	1.92	1.17
time (sec)	N/A	0.053	0.065	0.039	1.199	1.573	0.622	1.309

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	81	72	90	159	0	135
normalized size	1	1.	1.37	1.22	1.53	2.69	0.	2.29
time (sec)	N/A	0.062	0.068	0.063	1.2	1.702	0.	1.466

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	211	65	86	238	0	108
normalized size	1	1.	4.4	1.35	1.79	4.96	0.	2.25
time (sec)	N/A	0.067	0.681	0.077	1.076	1.764	0.	1.483

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	50	71	134	251	0	135
normalized size	1	1.	0.85	1.2	2.27	4.25	0.	2.29
time (sec)	N/A	0.083	0.024	0.079	1.212	1.803	0.	1.424

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	154	80	150	254	0	143
normalized size	1	1.	2.14	1.11	2.08	3.53	0.	1.99
time (sec)	N/A	0.095	5.177	0.078	1.27	1.695	0.	1.354

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	797	101	211	286	0	165
normalized size	1	1.	8.57	1.09	2.27	3.08	0.	1.77
time (sec)	N/A	0.117	6.335	0.086	1.21	1.696	0.	1.4

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	487	124	242	329	0	186
normalized size	1	1.	4.27	1.09	2.12	2.89	0.	1.63
time (sec)	N/A	0.127	1.311	0.086	1.147	1.69	0.	1.366

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	73	169	223	231	434	143
normalized size	1	1.	0.57	1.33	1.76	1.82	3.42	1.13
time (sec)	N/A	0.157	0.174	0.046	1.116	1.655	4.896	1.388

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	114	63	133	173	190	280	120
normalized size	1	1.12	0.62	1.3	1.7	1.86	2.75	1.18
time (sec)	N/A	0.107	0.142	0.042	1.144	1.647	2.507	1.242

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	56	111	143	157	224	97
normalized size	1	1.	0.64	1.28	1.64	1.8	2.57	1.11
time (sec)	N/A	0.081	0.105	0.039	1.113	1.624	1.289	1.268

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	91	94	120	201	0	157
normalized size	1	1.	1.25	1.29	1.64	2.75	0.	2.15
time (sec)	N/A	0.081	0.104	0.066	1.145	1.706	0.	1.48

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	241	86	115	270	0	174
normalized size	1	1.	3.3	1.18	1.58	3.7	0.	2.38
time (sec)	N/A	0.083	1.21	0.077	1.211	1.809	0.	1.341

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	272	86	149	286	0	174
normalized size	1	1.	3.73	1.18	2.04	3.92	0.	2.38
time (sec)	N/A	0.087	1.077	0.085	1.158	2.108	0.	1.479

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	61	93	162	281	0	157
normalized size	1	1.	0.84	1.27	2.22	3.85	0.	2.15
time (sec)	N/A	0.096	0.033	0.09	1.134	2.013	0.	1.458

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	797	102	246	292	0	165
normalized size	1	1.	8.3	1.06	2.56	3.04	0.	1.72
time (sec)	N/A	0.126	6.308	0.087	1.158	1.952	0.	1.521

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	498	123	257	325	0	186
normalized size	1	1.	4.49	1.11	2.32	2.93	0.	1.68
time (sec)	N/A	0.144	1.368	0.096	1.13	1.876	0.	1.49

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	211	146	365	366	0	208
normalized size	1	1.	1.55	1.07	2.68	2.69	0.	1.53
time (sec)	N/A	0.182	0.781	0.093	1.138	1.706	0.	1.506

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	173	171	293	213	882	136
normalized size	1	1.	1.47	1.45	2.48	1.81	7.47	1.15
time (sec)	N/A	0.107	0.318	0.047	1.732	1.653	11.541	1.358

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	143	136	238	180	570	119
normalized size	1	1.	1.52	1.45	2.53	1.91	6.06	1.27
time (sec)	N/A	0.092	0.271	0.046	1.729	1.617	6.55	1.361

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	117	103	180	149	325	99
normalized size	1	1.	1.54	1.36	2.37	1.96	4.28	1.3
time (sec)	N/A	0.061	0.234	0.044	1.656	1.589	3.691	1.376

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	89	68	124	116	129	78
normalized size	1	1.	2.07	1.58	2.88	2.7	3.	1.81
time (sec)	N/A	0.08	0.201	0.044	1.667	1.583	1.97	1.362

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	57	37	66	89	27	38
normalized size	1	1.	1.97	1.28	2.28	3.07	0.93	1.31
time (sec)	N/A	0.034	0.073	0.038	1.736	1.553	1.001	1.399

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	17	17	31	53	20	22
normalized size	1	1.	0.77	0.77	1.41	2.41	0.91	1.
time (sec)	N/A	0.012	0.014	0.032	1.208	1.564	0.597	1.387

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	103	58	101	181	0	73
normalized size	1	1.	2.71	1.53	2.66	4.76	0.	1.92
time (sec)	N/A	0.048	0.143	0.05	1.189	1.603	0.	1.448

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	188	99	161	266	0	113
normalized size	1	1.	3.55	1.87	3.04	5.02	0.	2.13
time (sec)	N/A	0.076	0.6	0.057	1.167	1.617	0.	1.376

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	244	143	219	301	0	136
normalized size	1	1.	2.94	1.72	2.64	3.63	0.	1.64
time (sec)	N/A	0.093	1.166	0.066	1.158	1.646	0.	1.522

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	368	183	277	331	0	154
normalized size	1	1.	3.57	1.78	2.69	3.21	0.	1.5
time (sec)	N/A	0.095	4.257	0.069	1.175	1.637	0.	1.388

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	199	156	279	278	700	146
normalized size	1	1.	1.6	1.26	2.25	2.24	5.65	1.18
time (sec)	N/A	0.185	0.44	0.049	1.818	1.642	26.509	1.265

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	177	122	221	257	413	128
normalized size	1	1.	1.55	1.07	1.94	2.25	3.62	1.12
time (sec)	N/A	0.152	0.303	0.046	1.691	1.646	10.703	1.374

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	114	88	159	230	201	107
normalized size	1	1.	1.42	1.1	1.99	2.88	2.51	1.34
time (sec)	N/A	0.17	0.364	0.046	1.752	1.627	5.976	1.379

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	105	56	97	198	56	68
normalized size	1	1.	1.84	0.98	1.7	3.47	0.98	1.19
time (sec)	N/A	0.085	0.228	0.04	1.741	1.807	3.12	1.401

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	32	63	126	48	42
normalized size	1	1.	1.09	0.58	1.15	2.29	0.87	0.76
time (sec)	N/A	0.038	0.113	0.036	1.172	1.699	2.073	1.196

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	53	32	62	123	44	42
normalized size	1	1.	0.96	0.58	1.13	2.24	0.8	0.76
time (sec)	N/A	0.027	0.049	0.033	1.113	1.624	1.229	1.191

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	152	77	132	305	0	104
normalized size	1	1.	2.3	1.17	2.	4.62	0.	1.58
time (sec)	N/A	0.112	0.288	0.053	1.098	1.68	0.	1.395

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	239	120	196	387	0	143
normalized size	1	1.	2.95	1.48	2.42	4.78	0.	1.77
time (sec)	N/A	0.173	1.013	0.064	1.088	1.703	0.	1.324

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	292	162	257	427	0	165
normalized size	1	1.	2.45	1.36	2.16	3.59	0.	1.39
time (sec)	N/A	0.19	1.726	0.072	1.527	1.685	0.	1.261

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	343	204	316	450	0	182
normalized size	1	1.	2.58	1.53	2.38	3.38	0.	1.37
time (sec)	N/A	0.199	3.957	0.072	1.103	1.679	0.	1.427

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	173	141	248	363	0	153
normalized size	1	1.	1.13	0.92	1.62	2.37	0.	1.
time (sec)	N/A	0.265	0.639	0.047	1.712	1.652	0.	1.398

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	161	107	185	325	240	130
normalized size	1	1.	1.35	0.9	1.55	2.73	2.02	1.09
time (sec)	N/A	0.273	0.546	0.048	1.665	1.628	18.245	1.234

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	154	75	124	300	75	92
normalized size	1	1.	1.6	0.78	1.29	3.12	0.78	0.96
time (sec)	N/A	0.184	0.233	0.043	1.655	1.593	8.806	1.412

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	86	45	90	186	68	62
normalized size	1	1.	1.04	0.54	1.08	2.24	0.82	0.75
time (sec)	N/A	0.094	0.186	0.037	1.221	1.502	5.317	1.216

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	71	32	63	182	48	42
normalized size	1	1.	0.86	0.39	0.76	2.19	0.58	0.51
time (sec)	N/A	0.058	0.136	0.036	1.142	1.519	3.412	1.143

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	45	90	186	63	62
normalized size	1	1.	0.78	0.54	1.08	2.24	0.76	0.75
time (sec)	N/A	0.046	0.077	0.034	1.169	1.512	2.384	1.144

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	201	96	161	424	0	127
normalized size	1	1.	2.07	0.99	1.66	4.37	0.	1.31
time (sec)	N/A	0.201	0.466	0.059	1.191	1.67	0.	1.409

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	286	139	223	506	0	165
normalized size	1	1.	2.55	1.24	1.99	4.52	0.	1.47
time (sec)	N/A	0.279	1.116	0.067	1.163	1.698	0.	1.223

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	343	181	285	548	0	188
normalized size	1	1.	2.2	1.16	1.83	3.51	0.	1.21
time (sec)	N/A	0.305	3.833	0.077	1.351	1.725	0.	1.328

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	289	160	275	470	0	173
normalized size	1	1.	1.57	0.87	1.49	2.55	0.	0.94
time (sec)	N/A	0.382	0.539	0.046	1.751	1.763	0.	1.279

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	263	126	213	444	0	151
normalized size	1	1.	1.75	0.84	1.42	2.96	0.	1.01
time (sec)	N/A	0.373	0.399	0.046	1.757	1.714	0.	1.435

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	224	94	151	397	0	112
normalized size	1	1.	1.76	0.74	1.19	3.13	0.	0.88
time (sec)	N/A	0.283	0.333	0.046	1.717	1.58	0.	1.462

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	112	58	117	248	88	80
normalized size	1	1.	0.98	0.51	1.03	2.18	0.77	0.7
time (sec)	N/A	0.199	0.257	0.04	1.424	1.543	22.524	1.423

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	99	58	117	251	87	80
normalized size	1	1.	0.88	0.52	1.04	2.24	0.78	0.71
time (sec)	N/A	0.112	0.248	0.037	1.155	1.54	10.993	1.344

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	58	117	251	85	80
normalized size	1	1.	0.78	0.52	1.04	2.24	0.76	0.71
time (sec)	N/A	0.078	0.22	0.037	1.164	1.533	7.785	1.356

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	77	56	117	248	83	80
normalized size	1	1.	0.69	0.5	1.04	2.21	0.74	0.71
time (sec)	N/A	0.07	0.16	0.035	1.15	1.549	5.563	1.309

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	185	115	188	539	0	149
normalized size	1	1.	1.54	0.96	1.57	4.49	0.	1.24
time (sec)	N/A	0.287	0.829	0.059	1.14	1.678	0.	1.391

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	341	158	251	632	0	188
normalized size	1	1.	2.53	1.17	1.86	4.68	0.	1.39
time (sec)	N/A	0.391	4.087	0.072	1.174	1.728	0.	1.534

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	455	200	312	670	0	209
normalized size	1	1.	2.46	1.08	1.69	3.62	0.	1.13
time (sec)	N/A	0.432	6.246	0.083	1.183	1.708	0.	1.409

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	345	179	302	591	0	196
normalized size	1	1.	1.53	0.8	1.34	2.63	0.	0.87
time (sec)	N/A	0.516	0.725	0.05	1.725	1.738	0.	1.425

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	319	145	240	541	0	174
normalized size	1	1.	1.67	0.76	1.26	2.83	0.	0.91
time (sec)	N/A	0.49	0.771	0.046	1.654	1.715	0.	1.386

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	280	113	178	514	0	135
normalized size	1	1.	1.67	0.67	1.06	3.06	0.	0.8
time (sec)	N/A	0.392	0.488	0.042	1.701	1.654	0.	1.333

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	138	71	144	316	0	97
normalized size	1	1.	0.89	0.46	0.93	2.04	0.	0.63
time (sec)	N/A	0.3	0.264	0.039	1.138	1.574	0.	1.408

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	125	58	117	312	0	80
normalized size	1	1.	0.85	0.39	0.8	2.12	0.	0.54
time (sec)	N/A	0.228	0.229	0.042	1.138	1.527	0.	1.324

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	110	45	90	312	68	62
normalized size	1	1.	0.79	0.32	0.65	2.24	0.49	0.45
time (sec)	N/A	0.144	0.223	0.039	1.178	1.537	23.392	1.305

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	97	58	117	312	85	80
normalized size	1	1.	0.68	0.41	0.82	2.18	0.59	0.56
time (sec)	N/A	0.105	0.172	0.037	1.167	1.563	19.246	1.371

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	89	71	144	316	102	97
normalized size	1	1.	0.62	0.5	1.01	2.21	0.71	0.68
time (sec)	N/A	0.091	0.154	0.034	1.134	1.547	15.268	1.432

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	211	134	215	671	0	170
normalized size	1	1.	1.38	0.88	1.41	4.39	0.	1.11
time (sec)	N/A	0.377	1.828	0.068	1.151	1.691	0.	1.512

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	453	177	278	755	0	209
normalized size	1	1.	2.7	1.05	1.65	4.49	0.	1.24
time (sec)	N/A	0.533	6.34	0.068	1.178	1.768	0.	1.335

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	507	219	339	807	0	231
normalized size	1	1.	2.26	0.98	1.51	3.6	0.	1.03
time (sec)	N/A	0.54	6.32	0.076	1.135	1.746	0.	1.432

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	164	84	171	382	0	115
normalized size	1	1.	0.89	0.46	0.93	2.08	0.	0.62
time (sec)	N/A	0.41	0.343	0.041	1.14	1.537	0.	1.355

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	151	84	171	383	0	115
normalized size	1	1.	0.86	0.48	0.97	2.18	0.	0.65
time (sec)	N/A	0.318	0.297	0.04	1.149	1.593	0.	1.439

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	92	97	107	203	0	0
normalized size	1	1.	0.58	0.61	0.68	1.28	0.	0.
time (sec)	N/A	0.241	0.262	0.932	1.983	1.562	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	84	88	169	0	0
normalized size	1	1.	0.66	0.69	0.72	1.39	0.	0.
time (sec)	N/A	0.176	0.156	0.694	1.928	1.556	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	68	71	69	142	0	0
normalized size	1	1.	0.79	0.83	0.8	1.65	0.	0.
time (sec)	N/A	0.115	0.099	0.818	1.922	1.529	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	58	49	112	0	0
normalized size	1	1.	0.96	1.04	0.88	2.	0.	0.
time (sec)	N/A	0.046	0.071	0.661	1.89	1.553	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	27	84	0	0
normalized size	1	1.	1.12	1.65	1.04	3.23	0.	0.
time (sec)	N/A	0.013	0.03	0.513	1.738	1.567	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	180	0	383	0	153
normalized size	1	1.	1.35	4.86	0.	10.35	0.	4.14
time (sec)	N/A	0.051	0.047	2.346	0.	1.666	0.	3.634

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	79	379	1580	377	0	336
normalized size	1	1.	1.27	6.11	25.48	6.08	0.	5.42
time (sec)	N/A	0.104	0.094	2.484	1.978	1.683	0.	3.634

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	94	545	3567	414	0	443
normalized size	1	1.	0.92	5.34	34.97	4.06	0.	4.34
time (sec)	N/A	0.162	0.187	2.656	22.507	1.722	0.	3.852

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	109	709	0	444	0	554
normalized size	1	1.	0.79	5.14	0.	3.22	0.	4.01
time (sec)	N/A	0.218	0.334	2.813	0.	1.691	0.	3.838

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	93	99	113	219	0	0
normalized size	1	1.	0.57	0.61	0.7	1.35	0.	0.
time (sec)	N/A	0.247	0.243	0.759	2.032	1.583	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	81	86	93	186	0	0
normalized size	1	1.	0.7	0.74	0.8	1.6	0.	0.
time (sec)	N/A	0.141	0.172	0.848	1.952	1.543	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	67	71	72	146	0	0
normalized size	1	1.	0.78	0.83	0.84	1.7	0.	0.
time (sec)	N/A	0.065	0.101	0.744	1.948	1.553	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	51	117	0	0
normalized size	1	1.	0.93	0.98	0.86	1.98	0.	0.
time (sec)	N/A	0.029	0.064	0.83	1.93	1.542	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	207	0	344	0	201
normalized size	1	1.	0.98	3.14	0.	5.21	0.	3.05
time (sec)	N/A	0.107	0.071	2.353	0.	1.63	0.	6.063

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	81	381	1774	387	0	339
normalized size	1	1.	1.25	5.86	27.29	5.95	0.	5.22
time (sec)	N/A	0.118	0.115	2.273	2.134	1.651	0.	4.204

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	97	545	4342	425	0	433
normalized size	1	1.	0.92	5.14	40.96	4.01	0.	4.08
time (sec)	N/A	0.176	0.22	2.506	5.757	1.691	0.	3.111

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	110	710	0	458	0	541
normalized size	1	1.	0.76	4.93	0.	3.18	0.	3.76
time (sec)	N/A	0.237	0.342	2.765	0.	1.745	0.	2.882

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	107	112	150	269	0	0
normalized size	1	1.	0.53	0.55	0.74	1.33	0.	0.
time (sec)	N/A	0.363	0.481	0.794	1.909	1.564	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	95	99	127	234	0	0
normalized size	1	1.	0.65	0.68	0.87	1.6	0.	0.
time (sec)	N/A	0.16	0.284	0.766	2.007	1.514	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	84	86	104	193	0	0
normalized size	1	1.	0.72	0.74	0.9	1.66	0.	0.
time (sec)	N/A	0.087	0.225	0.694	1.949	1.56	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	71	73	81	161	0	0
normalized size	1	1.	0.8	0.82	0.91	1.81	0.	0.
time (sec)	N/A	0.05	0.108	0.764	1.891	1.527	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	89	244	0	386	0	234
normalized size	1	1.	0.91	2.49	0.	3.94	0.	2.39
time (sec)	N/A	0.202	0.434	2.497	0.	1.701	0.	7.025

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	92	92	1547	408	0	427	0	371
normalized size	1	1.	16.82	4.43	0.	4.64	0.	4.03
time (sec)	N/A	0.198	34.294	2.658	0.	1.724	0.	4.604

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	1693	545	4950	439	0	447
normalized size	1	1.	15.97	5.14	46.7	4.14	0.	4.22
time (sec)	N/A	0.222	33.854	2.701	24.981	1.662	0.	5.207

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	1825	709	0	471	0	541
normalized size	1	1.	12.67	4.92	0.	3.27	0.	3.76
time (sec)	N/A	0.282	33.568	2.53	0.	1.734	0.	3.458

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	182	2069	872	0	512	0	649
normalized size	1	1.	11.37	4.79	0.	2.81	0.	3.57
time (sec)	N/A	0.345	34.934	3.157	0.	1.751	0.	3.33

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	83	86	104	194	0	0
normalized size	1	1.	0.7	0.72	0.87	1.63	0.	0.
time (sec)	N/A	0.068	0.252	0.746	1.922	1.586	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	130	194	0	431	0	159
normalized size	1	1.	0.75	1.11	0.	2.48	0.	0.91
time (sec)	N/A	0.372	0.19	1.724	0.	1.682	0.	2.638

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	118	183	0	397	0	157
normalized size	1	1.	0.84	1.31	0.	2.84	0.	1.12
time (sec)	N/A	0.239	0.164	1.327	0.	1.633	0.	2.627

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	135	0	367	0	107
normalized size	1	1.	1.	1.3	0.	3.53	0.	1.03
time (sec)	N/A	0.125	0.112	1.491	0.	1.648	0.	2.775

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	53	120	0	339	0	100
normalized size	1	1.	0.73	1.64	0.	4.64	0.	1.37
time (sec)	N/A	0.051	0.043	1.441	0.	1.621	0.	2.083

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	54	122	347	0	0
normalized size	1	1.	0.87	1.17	2.65	7.54	0.	0.
time (sec)	N/A	0.022	0.012	0.16	1.894	1.586	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	224	0	450	0	219
normalized size	1	1.	0.76	2.64	0.	5.29	0.	2.58
time (sec)	N/A	0.114	0.052	2.721	0.	1.698	0.	4.894

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	1540	462	0	647	0	392
normalized size	1	1.	14.26	4.28	0.	5.99	0.	3.63
time (sec)	N/A	0.213	26.077	3.109	0.	1.749	0.	5.077

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	1791	671	0	684	0	501
normalized size	1	1.	12.18	4.56	0.	4.65	0.	3.41
time (sec)	N/A	0.34	29.705	3.047	0.	1.713	0.	4.716

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	181	181	1921	875	0	717	0	609
normalized size	1	1.	10.61	4.83	0.	3.96	0.	3.36
time (sec)	N/A	0.489	28.422	3.372	0.	1.781	0.	4.62

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	226	265	0	497	0	185
normalized size	1	1.	1.23	1.45	0.	2.72	0.	1.01
time (sec)	N/A	0.4	1.411	1.523	0.	1.648	0.	2.079

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	196	234	0	471	0	155
normalized size	1	1.	1.35	1.61	0.	3.25	0.	1.07
time (sec)	N/A	0.261	0.952	1.602	0.	1.639	0.	2.122

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	164	174	0	440	0	138
normalized size	1	1.	1.56	1.66	0.	4.19	0.	1.31
time (sec)	N/A	0.134	0.459	1.456	0.	1.649	0.	2.057

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	54	140	0	412	0	109
normalized size	1	1.	0.7	1.82	0.	5.35	0.	1.42
time (sec)	N/A	0.059	0.095	1.43	0.	1.597	0.	2.804

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	63	138	0	409	0	109
normalized size	1	1.	0.82	1.79	0.	5.31	0.	1.42
time (sec)	N/A	0.039	0.066	1.49	0.	1.687	0.	2.1

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	1787	290	0	687	0	255
normalized size	1	1.	15.68	2.54	0.	6.03	0.	2.24
time (sec)	N/A	0.221	22.799	3.078	0.	1.782	0.	3.487

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	103	567	0	771	0	435
normalized size	1	1.	0.72	3.94	0.	5.35	0.	3.02
time (sec)	N/A	0.374	0.466	3.164	0.	1.796	0.	3.378

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	185	185	1941	807	0	809	0	543
normalized size	1	1.	10.49	4.36	0.	4.37	0.	2.94
time (sec)	N/A	0.499	27.011	3.483	0.	1.805	0.	3.885

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	587	242	0	566	0	197
normalized size	1	1.	3.21	1.32	0.	3.09	0.	1.08
time (sec)	N/A	0.412	6.354	1.463	0.	1.721	0.	2.446

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	216	208	0	532	0	167
normalized size	1	1.	1.49	1.43	0.	3.67	0.	1.15
time (sec)	N/A	0.27	4.382	1.507	0.	1.688	0.	2.754

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	103	174	0	504	0	139
normalized size	1	1.	0.96	1.63	0.	4.71	0.	1.3
time (sec)	N/A	0.137	1.129	1.465	0.	1.655	0.	2.402

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	174	0	501	0	139
normalized size	1	1.	0.61	1.63	0.	4.68	0.	1.3
time (sec)	N/A	0.08	0.248	1.579	0.	1.682	0.	2.62

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	174	0	501	0	139
normalized size	1	1.	0.61	1.63	0.	4.68	0.	1.3
time (sec)	N/A	0.063	0.157	1.468	0.	1.626	0.	2.05

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	1919	325	0	807	0	285
normalized size	1	1.	13.33	2.26	0.	5.6	0.	1.98
time (sec)	N/A	0.335	23.27	3.241	0.	1.775	0.	3.185

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	174	174	2051	601	0	891	0	464
normalized size	1	1.	11.79	3.45	0.	5.12	0.	2.67
time (sec)	N/A	0.519	23.217	3.185	0.	1.814	0.	3.154

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	490	270	0	0	0	0
normalized size	1	1.	4.41	2.43	0.	0.	0.	0.
time (sec)	N/A	0.078	6.164	2.169	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	232	219	0	0	0	0
normalized size	1	1.	2.67	2.52	0.	0.	0.	0.
time (sec)	N/A	0.067	5.358	2.177	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	61	61	222	225	0	0	0	0
normalized size	1	1.	3.64	3.69	0.	0.	0.	0.
time (sec)	N/A	0.051	4.45	2.171	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	35	35	155	150	0	0	0	0
normalized size	1	1.	4.43	4.29	0.	0.	0.	0.
time (sec)	N/A	0.039	23.407	1.904	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	209	146	0	0	0	0
normalized size	1	1.	3.67	2.56	0.	0.	0.	0.
time (sec)	N/A	0.05	9.221	2.306	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	444	369	0	0	0	0
normalized size	1	1.	5.35	4.45	0.	0.	0.	0.
time (sec)	N/A	0.059	6.147	3.569	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	477	384	0	0	0	0
normalized size	1	1.	4.3	3.46	0.	0.	0.	0.
time (sec)	N/A	0.074	6.198	3.729	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	532	260	0	0	0	0
normalized size	1	1.	3.62	1.77	0.	0.	0.	0.
time (sec)	N/A	0.138	6.131	2.44	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	500	272	0	0	0	0
normalized size	1	1.	4.13	2.25	0.	0.	0.	0.
time (sec)	N/A	0.119	6.115	2.161	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	235	250	0	0	0	0
normalized size	1	1.	2.47	2.63	0.	0.	0.	0.
time (sec)	N/A	0.093	5.647	2.284	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	224	228	0	0	0	0
normalized size	1	1.	3.34	3.4	0.	0.	0.	0.
time (sec)	N/A	0.081	5.026	2.049	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	39	104	0	0	0	0
normalized size	1	1.	0.89	2.36	0.	0.	0.	0.
time (sec)	N/A	0.08	0.16	2.259	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	454	371	0	0	0	0
normalized size	1	1.	4.99	4.08	0.	0.	0.	0.
time (sec)	N/A	0.093	6.155	3.694	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	487	386	0	0	0	0
normalized size	1	1.	4.02	3.19	0.	0.	0.	0.
time (sec)	N/A	0.119	6.195	3.846	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	532	260	0	0	0	0
normalized size	1	1.	3.62	1.77	0.	0.	0.	0.
time (sec)	N/A	0.153	6.134	2.115	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	500	272	0	0	0	0
normalized size	1	1.	4.13	2.25	0.	0.	0.	0.
time (sec)	N/A	0.128	6.117	2.362	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	233	250	0	0	0	0
normalized size	1	1.	2.56	2.75	0.	0.	0.	0.
time (sec)	N/A	0.109	5.634	2.391	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	240	172	0	0	0	0
normalized size	1	1.	2.64	1.89	0.	0.	0.	0.
time (sec)	N/A	0.11	4.683	2.25	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	463	371	0	0	0	0
normalized size	1	1.	5.09	4.08	0.	0.	0.	0.
time (sec)	N/A	0.105	6.187	3.625	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	485	386	0	0	0	0
normalized size	1	1.	4.15	3.3	0.	0.	0.	0.
time (sec)	N/A	0.127	6.202	3.726	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	515	439	0	0	0	0
normalized size	1	1.	3.5	2.99	0.	0.	0.	0.
time (sec)	N/A	0.148	6.239	3.825	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	271	273	0	0	0	0
normalized size	1	1.	1.57	1.58	0.	0.	0.	0.
time (sec)	N/A	0.205	3.634	2.187	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	532	260	0	0	0	0
normalized size	1	1.	3.62	1.77	0.	0.	0.	0.
time (sec)	N/A	0.164	6.145	2.234	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	500	272	0	0	0	0
normalized size	1	1.	4.13	2.25	0.	0.	0.	0.
time (sec)	N/A	0.14	6.168	1.958	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	119	119	245	194	0	0	0	0
normalized size	1	1.	2.06	1.63	0.	0.	0.	0.
time (sec)	N/A	0.122	6.145	2.33	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	70	292	0	0	0	0
normalized size	1	1.	0.71	2.98	0.	0.	0.	0.
time (sec)	N/A	0.122	0.313	3.161	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	283	386	0	0	0	0
normalized size	1	1.	2.34	3.19	0.	0.	0.	0.
time (sec)	N/A	0.146	4.35	4.058	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	298	439	0	0	0	0
normalized size	1	1.	2.03	2.99	0.	0.	0.	0.
time (sec)	N/A	0.167	5.124	4.355	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	315	229	0	0	0	0
normalized size	1	1.	2.46	1.79	0.	0.	0.	0.
time (sec)	N/A	0.11	1.799	2.244	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	289	215	0	0	0	0
normalized size	1	1.	2.89	2.15	0.	0.	0.	0.
time (sec)	N/A	0.101	1.258	2.333	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	264	199	0	0	0	0
normalized size	1	1.	3.67	2.76	0.	0.	0.	0.
time (sec)	N/A	0.086	2.594	2.398	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	256	198	0	0	0	0
normalized size	1	1.	3.66	2.83	0.	0.	0.	0.
time (sec)	N/A	0.083	1.003	1.99	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	257	200	0	0	0	0
normalized size	1	1.	3.67	2.86	0.	0.	0.	0.
time (sec)	N/A	0.085	1.004	2.057	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	297	253	0	0	0	0
normalized size	1	1.	3.09	2.64	0.	0.	0.	0.
time (sec)	N/A	0.099	2.067	2.2	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	332	413	0	0	0	0
normalized size	1	1.	2.68	3.33	0.	0.	0.	0.
time (sec)	N/A	0.112	3.576	4.016	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	367	283	0	0	0	0
normalized size	1	1.	2.29	1.77	0.	0.	0.	0.
time (sec)	N/A	0.219	2.51	2.619	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	337	270	0	0	0	0
normalized size	1	1.	2.44	1.96	0.	0.	0.	0.
time (sec)	N/A	0.202	1.852	2.396	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	319	257	0	0	0	0
normalized size	1	1.	2.85	2.29	0.	0.	0.	0.
time (sec)	N/A	0.184	2.366	2.42	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	640	257	0	0	0	0
normalized size	1	1.	5.87	2.36	0.	0.	0.	0.
time (sec)	N/A	0.188	6.304	2.409	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	63	188	0	0	0	0
normalized size	1	1.	1.11	3.3	0.	0.	0.	0.
time (sec)	N/A	0.054	0.204	2.062	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	304	257	0	0	0	0
normalized size	1	1.	2.79	2.36	0.	0.	0.	0.
time (sec)	N/A	0.184	1.999	2.322	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	334	405	0	0	0	0
normalized size	1	1.	2.46	2.98	0.	0.	0.	0.
time (sec)	N/A	0.208	1.875	2.519	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	364	413	0	0	0	0
normalized size	1	1.	2.25	2.55	0.	0.	0.	0.
time (sec)	N/A	0.239	5.493	4.146	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	388	296	0	0	0	0
normalized size	1	1.	1.87	1.43	0.	0.	0.	0.
time (sec)	N/A	0.335	2.679	2.644	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	369	283	0	0	0	0
normalized size	1	1.	2.04	1.56	0.	0.	0.	0.
time (sec)	N/A	0.323	1.867	2.313	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	349	270	0	0	0	0
normalized size	1	1.	2.25	1.74	0.	0.	0.	0.
time (sec)	N/A	0.299	4.129	2.372	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	705	270	0	0	0	0
normalized size	1	1.	4.55	1.74	0.	0.	0.	0.
time (sec)	N/A	0.308	6.347	2.378	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	334	270	0	0	0	0
normalized size	1	1.	2.15	1.74	0.	0.	0.	0.
time (sec)	N/A	0.304	3.598	2.277	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	334	270	0	0	0	0
normalized size	1	1.	2.15	1.74	0.	0.	0.	0.
time (sec)	N/A	0.305	3.067	2.42	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	705	268	0	0	0	0
normalized size	1	1.	4.55	1.73	0.	0.	0.	0.
time (sec)	N/A	0.312	6.323	2.48	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	364	555	0	0	0	0
normalized size	1	1.	2.01	3.07	0.	0.	0.	0.
time (sec)	N/A	0.347	1.9	2.584	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	394	453	0	0	0	0
normalized size	1	1.	1.9	2.19	0.	0.	0.	0.
time (sec)	N/A	0.359	2.459	4.527	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	105	196	2593	317	0	0
normalized size	1	1.	0.68	1.27	16.84	2.06	0.	0.
time (sec)	N/A	0.233	0.327	0.497	2.4	1.73	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	91	161	1430	286	0	0
normalized size	1	1.	0.78	1.39	12.33	2.47	0.	0.
time (sec)	N/A	0.174	0.2	0.45	2.052	1.748	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	77	123	1068	251	0	0
normalized size	1	1.	1.07	1.71	14.83	3.49	0.	0.
time (sec)	N/A	0.116	0.092	0.432	2.016	1.703	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	80	197	325	0	0
normalized size	1	1.	1.35	2.16	5.32	8.78	0.	0.
time (sec)	N/A	0.058	0.043	0.356	1.782	1.734	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	42	132	130	0	0
normalized size	1	1.	1.08	1.17	3.67	3.61	0.	0.
time (sec)	N/A	0.057	0.048	0.415	1.528	1.632	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	54	257	163	0	0
normalized size	1	1.	0.66	0.7	3.34	2.12	0.	0.
time (sec)	N/A	0.109	0.084	0.408	1.565	1.616	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	66	64	320	190	0	0
normalized size	1	1.	0.57	0.56	2.78	1.65	0.	0.
time (sec)	N/A	0.165	0.092	0.434	1.581	1.64	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	66	74	382	217	0	0
normalized size	1	1.	0.43	0.48	2.5	1.42	0.	0.
time (sec)	N/A	0.228	0.121	0.422	1.598	1.66	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	106	197	2622	328	0	0
normalized size	1	1.	0.66	1.23	16.39	2.05	0.	0.
time (sec)	N/A	0.248	0.359	0.426	2.538	1.714	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	92	160	1458	294	0	0
normalized size	1	1.	0.77	1.33	12.15	2.45	0.	0.
time (sec)	N/A	0.185	0.219	0.378	2.044	1.703	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	79	168	1084	258	0	0
normalized size	1	1.	1.05	2.24	14.45	3.44	0.	0.
time (sec)	N/A	0.12	0.104	0.362	1.971	1.658	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	85	249	1346	298	0	0
normalized size	1	1.	1.12	3.28	17.71	3.92	0.	0.
time (sec)	N/A	0.124	0.147	0.354	1.967	1.733	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	52	55	169	166	0	0
normalized size	1	1.	0.64	0.68	2.09	2.05	0.	0.
time (sec)	N/A	0.118	0.112	0.331	1.583	1.659	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	62	65	293	194	0	0
normalized size	1	1.	0.51	0.54	2.42	1.6	0.	0.
time (sec)	N/A	0.173	0.151	0.338	1.578	1.619	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	72	75	355	235	0	0
normalized size	1	1.	0.45	0.47	2.2	1.46	0.	0.
time (sec)	N/A	0.235	0.221	0.336	1.617	1.657	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	182	234	10058	382	0	0
normalized size	1	1.	0.91	1.17	50.29	1.91	0.	0.
time (sec)	N/A	0.358	4.3	0.441	3.415	1.792	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	160	160	182	197	2651	342	0	0
normalized size	1	1.	1.14	1.23	16.57	2.14	0.	0.
time (sec)	N/A	0.295	4.137	0.395	2.479	1.751	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	182	188	1493	308	0	0
normalized size	1	1.	1.52	1.57	12.44	2.57	0.	0.
time (sec)	N/A	0.232	3.934	0.382	2.064	1.71	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	182	269	1314	338	0	0
normalized size	1	1.	1.6	2.36	11.53	2.96	0.	0.
time (sec)	N/A	0.225	3.955	0.384	2.015	1.764	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	356	333	1883	348	0	0
normalized size	1	1.	3.02	2.82	15.96	2.95	0.	0.
time (sec)	N/A	0.226	9.816	0.368	2.051	1.776	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	64	67	204	209	0	0
normalized size	1	1.	0.53	0.55	1.69	1.73	0.	0.
time (sec)	N/A	0.225	0.171	0.342	1.559	1.764	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	74	77	328	242	0	0
normalized size	1	1.	0.46	0.48	2.04	1.5	0.	0.
time (sec)	N/A	0.286	5.246	0.36	1.622	1.97	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	84	87	390	282	0	0
normalized size	1	1.	0.42	0.43	1.94	1.4	0.	0.
time (sec)	N/A	0.351	5.328	0.368	1.595	1.967	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	163	132	0	0
normalized size	1	1.	1.34	0.	4.29	3.47	0.	0.
time (sec)	N/A	0.056	0.085	0.312	1.513	2.049	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	50	80	197	325	0	0
normalized size	1	1.	1.35	2.16	5.32	8.78	0.	0.
time (sec)	N/A	0.06	0.064	0.392	1.799	2.095	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	188	91	567	435	0	188
normalized size	1	1.	4.95	2.39	14.92	11.45	0.	4.95
time (sec)	N/A	0.072	3.47	0.336	1.849	2.262	0.	2.235

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	289	196	0	459	0	0
normalized size	1	1.	1.69	1.15	0.	2.68	0.	0.
time (sec)	N/A	0.42	1.193	0.432	0.	2.641	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	227	159	0	420	0	0
normalized size	1	1.	1.77	1.24	0.	3.28	0.	0.
time (sec)	N/A	0.28	1.227	0.405	0.	2.393	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	161	125	0	263	0	0
normalized size	1	1.	1.69	1.32	0.	2.77	0.	0.
time (sec)	N/A	0.17	0.398	0.405	0.	2.373	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	69	0	451	0	0
normalized size	1	1.	0.91	1.23	0.	8.05	0.	0.
time (sec)	N/A	0.062	0.051	0.283	0.	2.168	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	180	206	0	373	0	0
normalized size	1	1.	1.94	2.22	0.	4.01	0.	0.
time (sec)	N/A	0.128	2.235	0.382	0.	2.327	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	473	274	0	412	0	0
normalized size	1	1.	3.61	2.09	0.	3.15	0.	0.
time (sec)	N/A	0.236	7.514	0.391	0.	2.288	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	1540	341	0	440	0	0
normalized size	1	1.	9.11	2.02	0.	2.6	0.	0.
time (sec)	N/A	0.365	9.816	0.404	0.	2.228	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	286	187	0	402	0	0
normalized size	1	1.	2.27	1.48	0.	3.19	0.	0.
time (sec)	N/A	0.272	0.824	0.344	0.	2.114	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	224	151	0	365	0	0
normalized size	1	1.	2.64	1.78	0.	4.29	0.	0.
time (sec)	N/A	0.187	0.785	0.305	0.	1.997	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	135	124	0	204	0	0
normalized size	1	1.	2.5	2.3	0.	3.78	0.	0.
time (sec)	N/A	0.117	0.247	0.296	0.	1.967	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	49	63	0	159	0	0
normalized size	1	1.	1.81	2.33	0.	5.89	0.	0.
time (sec)	N/A	0.043	0.039	0.178	0.	2.051	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	178	210	0	343	0	0
normalized size	1	1.	2.87	3.39	0.	5.53	0.	0.
time (sec)	N/A	0.085	1.773	0.289	0.	2.131	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	98	98	471	278	0	382	0	0
normalized size	1	1.	4.81	2.84	0.	3.9	0.	0.
time (sec)	N/A	0.165	6.621	0.306	0.	2.159	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	1538	344	0	410	0	0
normalized size	1	1.	11.48	2.57	0.	3.06	0.	0.
time (sec)	N/A	0.242	7.951	0.316	0.	2.138	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	229	227	0	552	0	0
normalized size	1	1.	1.32	1.3	0.	3.17	0.	0.
time (sec)	N/A	0.425	5.248	0.414	0.	3.045	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	215	195	0	521	0	0
normalized size	1	1.	1.6	1.46	0.	3.89	0.	0.
time (sec)	N/A	0.292	3.599	0.374	0.	3.072	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	118	146	0	401	0	0
normalized size	1	1.	1.22	1.51	0.	4.13	0.	0.
time (sec)	N/A	0.13	0.325	0.373	0.	2.269	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	106	170	0	404	0	0
normalized size	1	1.	1.09	1.75	0.	4.16	0.	0.
time (sec)	N/A	0.129	0.551	0.3	0.	2.318	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	137	137	456	245	0	471	0	0
normalized size	1	1.	3.33	1.79	0.	3.44	0.	0.
time (sec)	N/A	0.247	7.161	0.382	0.	2.502	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	589	313	0	506	0	0
normalized size	1	1.	3.33	1.77	0.	2.86	0.	0.
time (sec)	N/A	0.388	9.124	0.432	0.	2.569	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	385	344	0	672	0	0
normalized size	1	1.	1.8	1.61	0.	3.14	0.	0.
time (sec)	N/A	0.577	6.665	0.418	0.	4.609	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	349	312	0	641	0	0
normalized size	1	1.	2.01	1.79	0.	3.68	0.	0.
time (sec)	N/A	0.44	6.583	0.369	0.	4.356	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	149	214	0	493	0	0
normalized size	1	1.	1.09	1.56	0.	3.6	0.	0.
time (sec)	N/A	0.259	0.798	0.349	0.	2.184	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	122	213	0	490	0	0
normalized size	1	1.	0.89	1.55	0.	3.58	0.	0.
time (sec)	N/A	0.25	0.991	0.344	0.	2.242	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	134	245	0	495	0	0
normalized size	1	1.	0.98	1.79	0.	3.61	0.	0.
time (sec)	N/A	0.261	1.262	0.332	0.	2.292	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	506	303	0	563	0	0
normalized size	1	1.	2.86	1.71	0.	3.18	0.	0.
time (sec)	N/A	0.403	7.684	0.345	0.	2.302	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	639	377	0	599	0	0
normalized size	1	1.	2.94	1.74	0.	2.76	0.	0.
time (sec)	N/A	0.546	10.696	0.355	0.	2.38	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	448	464	0	797	0	0
normalized size	1	1.	1.76	1.83	0.	3.14	0.	0.
time (sec)	N/A	0.749	6.722	0.408	0.	6.255	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	214	214	412	432	0	761	0	0
normalized size	1	1.	1.93	2.02	0.	3.56	0.	0.
time (sec)	N/A	0.601	6.708	0.379	0.	5.096	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	176	280	0	585	0	0
normalized size	1	1.	0.99	1.58	0.	3.31	0.	0.
time (sec)	N/A	0.404	2.688	0.378	0.	2.328	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	148	280	0	585	0	0
normalized size	1	1.	0.84	1.58	0.	3.31	0.	0.
time (sec)	N/A	0.403	1.834	0.379	0.	2.269	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	149	280	0	582	0	0
normalized size	1	1.	0.84	1.58	0.	3.29	0.	0.
time (sec)	N/A	0.403	2.683	0.397	0.	2.117	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	148	313	0	590	0	0
normalized size	1	1.	0.84	1.77	0.	3.33	0.	0.
time (sec)	N/A	0.41	2.135	0.382	0.	2.361	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	559	377	0	662	0	0
normalized size	1	1.	2.58	1.74	0.	3.05	0.	0.
time (sec)	N/A	0.547	8.286	0.388	0.	2.307	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	273	435	0	695	0	0
normalized size	1	1.	1.06	1.69	0.	2.7	0.	0.
time (sec)	N/A	0.703	8.301	0.408	0.	2.432	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	347	346	0	684	0	0
normalized size	1	1.	1.6	1.59	0.	3.15	0.	0.
time (sec)	N/A	0.557	6.036	0.386	0.	2.272	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	158	346	0	680	0	0
normalized size	1	1.	0.73	1.59	0.	3.13	0.	0.
time (sec)	N/A	0.571	2.137	0.387	0.	2.232	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	30	36	0	116	0	0
normalized size	1	1.	1.88	2.25	0.	7.25	0.	0.
time (sec)	N/A	0.043	0.025	0.096	0.	1.931	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	32	42	0	346	0	0
normalized size	1	1.	0.78	1.02	0.	8.44	0.	0.
time (sec)	N/A	0.061	0.019	0.13	0.	2.074	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	289	165	1435	417	0	0
normalized size	1	1.	2.24	1.28	11.12	3.23	0.	0.
time (sec)	N/A	0.192	4.046	0.404	2.065	2.22	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	264	95	1073	389	0	0
normalized size	1	1.	3.11	1.12	12.62	4.58	0.	0.
time (sec)	N/A	0.122	0.721	0.354	2.016	2.254	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	278	84	200	417	0	193
normalized size	1	1.	5.79	1.75	4.17	8.69	0.	4.02
time (sec)	N/A	0.066	0.514	0.252	1.794	2.286	0.	2.289

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	46	111	113	0	126
normalized size	1	1.	1.08	1.24	3.	3.05	0.	3.41
time (sec)	N/A	0.057	0.044	0.332	1.576	1.847	0.	2.413

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	52	56	235	143	0	177
normalized size	1	1.	0.66	0.71	2.97	1.81	0.	2.24
time (sec)	N/A	0.116	0.112	0.323	1.558	1.901	0.	2.398

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	62	66	298	169	0	219
normalized size	1	1.	0.53	0.56	2.53	1.43	0.	1.86
time (sec)	N/A	0.173	0.148	0.34	1.546	1.871	0.	2.204

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	284	164	1762	332	0	0
normalized size	1	1.	2.49	1.44	15.46	2.91	0.	0.
time (sec)	N/A	0.154	0.477	0.338	2.006	2.125	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	252	94	1304	304	0	0
normalized size	1	1.	3.5	1.31	18.11	4.22	0.	0.
time (sec)	N/A	0.086	0.59	0.312	1.88	2.171	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	277	83	298	167	0	149
normalized size	1	1.	7.49	2.24	8.05	4.51	0.	4.03
time (sec)	N/A	0.043	0.489	0.201	1.745	2.129	0.	2.077

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	39	45	101	111	0	84
normalized size	1	1.	1.11	1.29	2.89	3.17	0.	2.4
time (sec)	N/A	0.042	0.044	0.279	1.536	1.852	0.	2.125

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	55	221	140	0	126
normalized size	1	1.	0.68	0.73	2.95	1.87	0.	1.68
time (sec)	N/A	0.085	0.095	0.275	1.594	1.896	0.	2.111

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	61	65	282	166	0	154
normalized size	1	1.	0.54	0.58	2.52	1.48	0.	1.38
time (sec)	N/A	0.132	0.117	0.293	1.566	1.839	0.	2.163

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	256	195	0	626	0	203
normalized size	1	1.	1.38	1.05	0.	3.38	0.	1.1
time (sec)	N/A	0.445	1.187	0.385	0.	2.111	0.	2.215

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	228	162	0	593	0	171
normalized size	1	1.	1.62	1.15	0.	4.21	0.	1.21
time (sec)	N/A	0.295	0.869	0.364	0.	2.003	0.	2.195

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	161	116	0	451	0	113
normalized size	1	1.	1.5	1.08	0.	4.21	0.	1.06
time (sec)	N/A	0.18	0.374	0.262	0.	2.025	0.	2.382

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	118	77	0	400	0	185
normalized size	1	1.	2.03	1.33	0.	6.9	0.	3.19
time (sec)	N/A	0.067	0.328	0.329	0.	2.253	0.	2.158

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	157	160	0	417	0	92
normalized size	1	1.	1.65	1.68	0.	4.39	0.	0.97
time (sec)	N/A	0.133	0.375	0.346	0.	2.227	0.	2.357

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	171	171	0	451	0	122
normalized size	1	1.	1.27	1.27	0.	3.34	0.	0.9
time (sec)	N/A	0.251	0.315	0.358	0.	2.338	0.	2.164

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	218	305	0	485	0	184
normalized size	1	1.	1.26	1.76	0.	2.8	0.	1.06
time (sec)	N/A	0.401	0.653	0.366	0.	2.377	0.	2.257

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	255	194	0	653	0	219
normalized size	1	1.	1.58	1.2	0.	4.06	0.	1.36
time (sec)	N/A	0.302	0.412	0.329	0.	2.006	0.	2.088

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	227	161	0	617	0	190
normalized size	1	1.	1.92	1.36	0.	5.23	0.	1.61
time (sec)	N/A	0.214	0.158	0.31	0.	2.027	0.	2.144

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	160	117	0	460	0	142
normalized size	1	1.	1.88	1.38	0.	5.41	0.	1.67
time (sec)	N/A	0.129	0.105	0.217	0.	1.932	0.	2.169

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	110	84	335	231	0	107
normalized size	1	1.	2.34	1.79	7.13	4.91	0.	2.28
time (sec)	N/A	0.048	0.138	0.289	1.916	2.131	0.	2.043

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	C	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	152	159	540	396	0	97
normalized size	1	1.	1.83	1.92	6.51	4.77	0.	1.17
time (sec)	N/A	0.093	0.138	0.305	1.953	2.222	0.	1.942

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	170	170	760	429	0	120
normalized size	1	1.	1.39	1.39	6.23	3.52	0.	0.98
time (sec)	N/A	0.182	0.286	0.321	1.904	2.141	0.	1.814

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	15.306	0.159	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	3.283	0.168	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	2.654	0.165	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	268	384	0	0	0	0
normalized size	1	1.	1.77	2.54	0.	0.	0.	0.
time (sec)	N/A	0.111	1.606	4.081	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	255	369	0	0	0	0
normalized size	1	1.	2.07	3.	0.	0.	0.	0.
time (sec)	N/A	0.097	1.108	3.5	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	124	146	0	0	0	0
normalized size	1	1.	1.28	1.51	0.	0.	0.	0.
time (sec)	N/A	0.087	1.213	2.437	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	141	150	0	0	0	0
normalized size	1	1.	1.88	2.	0.	0.	0.	0.
time (sec)	N/A	0.076	1.017	1.89	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	140	225	0	0	0	0
normalized size	1	1.	1.39	2.23	0.	0.	0.	0.
time (sec)	N/A	0.09	1.226	1.986	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	224	219	0	0	0	0
normalized size	1	1.	1.76	1.72	0.	0.	0.	0.
time (sec)	N/A	0.1	0.894	2.144	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	198	270	0	0	0	0
normalized size	1	1.	1.31	1.79	0.	0.	0.	0.
time (sec)	N/A	0.113	2.151	2.421	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	261	386	0	0	0	0
normalized size	1	1.	1.62	2.4	0.	0.	0.	0.
time (sec)	N/A	0.147	1.853	4.154	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	250	371	0	0	0	0
normalized size	1	1.	1.91	2.83	0.	0.	0.	0.
time (sec)	N/A	0.134	1.309	3.681	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	48	104	0	0	0	0
normalized size	1	1.	0.75	1.62	0.	0.	0.	0.
time (sec)	N/A	0.108	0.142	2.31	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	127	228	0	0	0	0
normalized size	1	1.	1.19	2.13	0.	0.	0.	0.
time (sec)	N/A	0.122	1.002	2.292	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	136	250	0	0	0	0
normalized size	1	1.	1.01	1.85	0.	0.	0.	0.
time (sec)	N/A	0.138	1.466	2.125	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	149	272	0	0	0	0
normalized size	1	1.	0.93	1.69	0.	0.	0.	0.
time (sec)	N/A	0.151	1.669	2.04	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	279	439	0	0	0	0
normalized size	1	1.	1.49	2.35	0.	0.	0.	0.
time (sec)	N/A	0.233	2.712	4.434	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	259	386	0	0	0	0
normalized size	1	1.	1.65	2.46	0.	0.	0.	0.
time (sec)	N/A	0.206	1.762	3.892	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	157	371	0	0	0	0
normalized size	1	1.	1.2	2.83	0.	0.	0.	0.
time (sec)	N/A	0.18	0.953	3.743	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	135	172	0	0	0	0
normalized size	1	1.	1.03	1.31	0.	0.	0.	0.
time (sec)	N/A	0.176	1.289	2.476	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	137	250	0	0	0	0
normalized size	1	1.	1.05	1.91	0.	0.	0.	0.
time (sec)	N/A	0.178	1.253	2.009	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	0	0	0
normalized size	1	1.	0.91	1.69	0.	0.	0.	0.
time (sec)	N/A	0.209	1.769	2.339	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	0	0	0
normalized size	1	1.	0.83	1.39	0.	0.	0.	0.
time (sec)	N/A	0.235	2.187	2.276	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	271	439	0	0	0	0
normalized size	1	1.	1.45	2.35	0.	0.	0.	0.
time (sec)	N/A	0.253	1.844	4.226	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	278	386	0	0	0	0
normalized size	1	1.	1.73	2.4	0.	0.	0.	0.
time (sec)	N/A	0.227	2.411	3.829	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	70	292	0	0	0	0
normalized size	1	1.	0.59	2.47	0.	0.	0.	0.
time (sec)	N/A	0.211	0.334	3.509	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	150	194	0	0	0	0
normalized size	1	1.	0.94	1.22	0.	0.	0.	0.
time (sec)	N/A	0.212	1.455	2.5	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	146	272	0	0	0	0
normalized size	1	1.	0.91	1.69	0.	0.	0.	0.
time (sec)	N/A	0.227	1.552	2.192	0.	0.	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	156	260	0	0	0	0
normalized size	1	1.	0.83	1.39	0.	0.	0.	0.
time (sec)	N/A	0.255	2.126	2.44	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	285	413	0	0	0	0
normalized size	1	1.	1.74	2.52	0.	0.	0.	0.
time (sec)	N/A	0.168	3.173	3.859	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	256	253	0	0	0	0
normalized size	1	1.	1.88	1.86	0.	0.	0.	0.
time (sec)	N/A	0.154	1.907	2.668	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	180	200	0	0	0	0
normalized size	1	1.	1.64	1.82	0.	0.	0.	0.
time (sec)	N/A	0.144	0.981	2.394	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	181	198	0	0	0	0
normalized size	1	1.	1.65	1.8	0.	0.	0.	0.
time (sec)	N/A	0.139	0.952	2.007	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	311	199	0	0	0	0
normalized size	1	1.	2.78	1.78	0.	0.	0.	0.
time (sec)	N/A	0.141	1.648	2.356	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	312	215	0	0	0	0
normalized size	1	1.	2.23	1.54	0.	0.	0.	0.
time (sec)	N/A	0.162	4.002	2.188	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	341	229	0	0	0	0
normalized size	1	1.	2.03	1.36	0.	0.	0.	0.
time (sec)	N/A	0.17	2.69	2.067	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	287	413	0	0	0	0
normalized size	1	1.	1.42	2.04	0.	0.	0.	0.
time (sec)	N/A	0.273	2.383	4.327	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	252	405	0	0	0	0
normalized size	1	1.	1.43	2.3	0.	0.	0.	0.
time (sec)	N/A	0.251	1.265	2.704	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	242	257	0	0	0	0
normalized size	1	1.	1.62	1.72	0.	0.	0.	0.
time (sec)	N/A	0.24	1.19	2.355	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	188	0	0	0	0
normalized size	1	1.	1.27	2.44	0.	0.	0.	0.
time (sec)	N/A	0.097	0.363	2.233	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	239	257	0	0	0	0
normalized size	1	1.	1.6	1.72	0.	0.	0.	0.
time (sec)	N/A	0.238	1.349	2.415	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	259	257	0	0	0	0
normalized size	1	1.	1.7	1.69	0.	0.	0.	0.
time (sec)	N/A	0.24	1.955	2.485	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	257	270	0	0	0	0
normalized size	1	1.	1.44	1.52	0.	0.	0.	0.
time (sec)	N/A	0.264	1.688	2.381	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	271	283	0	0	0	0
normalized size	1	1.	1.36	1.42	0.	0.	0.	0.
time (sec)	N/A	0.281	1.797	2.322	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	363	555	0	0	0	0
normalized size	1	1.	1.64	2.51	0.	0.	0.	0.
time (sec)	N/A	0.372	2.183	3.016	0.	0.	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	274	268	0	0	0	0
normalized size	1	1.	1.41	1.37	0.	0.	0.	0.
time (sec)	N/A	0.36	2.302	2.56	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	363	270	0	0	0	0
normalized size	1	1.	1.86	1.38	0.	0.	0.	0.
time (sec)	N/A	0.35	2.018	2.411	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	363	270	0	0	0	0
normalized size	1	1.	1.86	1.38	0.	0.	0.	0.
time (sec)	N/A	0.345	1.895	2.447	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	272	270	0	0	0	0
normalized size	1	1.	1.39	1.38	0.	0.	0.	0.
time (sec)	N/A	0.356	2.824	2.234	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	378	270	0	0	0	0
normalized size	1	1.	1.94	1.38	0.	0.	0.	0.
time (sec)	N/A	0.359	2.156	2.625	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	285	283	0	0	0	0
normalized size	1	1.	1.29	1.28	0.	0.	0.	0.
time (sec)	N/A	0.381	2.3	2.506	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	71	82	382	220	0	0
normalized size	1	1.	0.46	0.54	2.5	1.44	0.	0.
time (sec)	N/A	0.282	0.216	0.471	1.628	1.592	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	61	72	320	193	0	0
normalized size	1	1.	0.53	0.63	2.78	1.68	0.	0.
time (sec)	N/A	0.221	0.129	0.46	1.602	1.616	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	62	257	163	0	0
normalized size	1	1.	0.66	0.81	3.34	2.12	0.	0.
time (sec)	N/A	0.159	0.101	0.463	1.605	1.585	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	50	132	112	0	0
normalized size	1	1.	1.08	1.39	3.67	3.11	0.	0.
time (sec)	N/A	0.103	0.063	0.455	1.611	1.563	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	70	100	197	325	0	0
normalized size	1	1.	1.23	1.75	3.46	5.7	0.	0.
time (sec)	N/A	0.108	0.087	0.514	1.792	1.72	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	97	132	1068	251	0	0
normalized size	1	1.	1.05	1.43	11.61	2.73	0.	0.
time (sec)	N/A	0.163	0.122	0.528	1.955	1.707	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	111	169	1430	308	0	0
normalized size	1	1.	0.82	1.24	10.51	2.26	0.	0.
time (sec)	N/A	0.227	0.257	0.536	2.03	1.71	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	72	83	355	238	0	0
normalized size	1	1.	0.45	0.52	2.2	1.48	0.	0.
time (sec)	N/A	0.308	0.277	0.428	1.624	1.651	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	62	73	293	197	0	0
normalized size	1	1.	0.51	0.6	2.42	1.63	0.	0.
time (sec)	N/A	0.239	0.186	0.382	1.623	1.625	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	52	63	169	166	0	0
normalized size	1	1.	0.64	0.78	2.09	2.05	0.	0.
time (sec)	N/A	0.175	0.134	0.387	1.58	1.602	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	85	168	1346	259	0	0
normalized size	1	1.	0.89	1.75	14.02	2.7	0.	0.
time (sec)	N/A	0.186	0.179	0.4	2.039	1.714	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	99	130	1084	258	0	0
normalized size	1	1.	1.04	1.37	11.41	2.72	0.	0.
time (sec)	N/A	0.178	0.15	0.463	1.978	1.713	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	111	170	1458	316	0	0
normalized size	1	1.	0.79	1.21	10.41	2.26	0.	0.
time (sec)	N/A	0.238	0.253	0.463	2.037	1.734	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	126	205	2622	350	0	0
normalized size	1	1.	0.7	1.14	14.57	1.94	0.	0.
time (sec)	N/A	0.305	0.54	0.474	2.445	1.748	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	84	95	390	285	0	0
normalized size	1	1.	0.42	0.47	1.94	1.42	0.	0.
time (sec)	N/A	0.414	5.377	0.405	1.639	1.662	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	74	85	328	244	0	0
normalized size	1	1.	0.46	0.53	2.04	1.52	0.	0.
time (sec)	N/A	0.347	5.345	0.392	1.578	1.645	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	64	75	204	212	0	0
normalized size	1	1.	0.53	0.62	1.69	1.75	0.	0.
time (sec)	N/A	0.285	0.276	0.375	1.566	1.635	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	404	268	1883	344	0	0
normalized size	1	1.	2.93	1.94	13.64	2.49	0.	0.
time (sec)	N/A	0.288	6.295	0.405	2.114	1.712	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	202	186	1314	298	0	0
normalized size	1	1.	1.51	1.39	9.81	2.22	0.	0.
time (sec)	N/A	0.282	3.079	0.414	2.054	1.758	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	202	166	1493	329	0	0
normalized size	1	1.	1.44	1.19	10.66	2.35	0.	0.
time (sec)	N/A	0.292	3.035	0.489	2.086	1.739	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	202	207	2651	363	0	0
normalized size	1	1.	1.12	1.15	14.73	2.02	0.	0.
time (sec)	N/A	0.361	3.128	0.47	2.432	1.758	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	220	220	202	242	10058	404	0	0
normalized size	1	1.	0.92	1.1	45.72	1.84	0.	0.
time (sec)	N/A	0.419	3.111	0.493	3.389	1.84	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	1540	294	0	362	0	0
normalized size	1	1.	10.	1.91	0.	2.35	0.	0.
time (sec)	N/A	0.284	7.769	0.356	0.	1.818	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	118	118	473	228	0	328	0	0
normalized size	1	1.	4.01	1.93	0.	2.78	0.	0.
time (sec)	N/A	0.201	6.634	0.332	0.	1.849	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	82	82	178	144	0	252	0	0
normalized size	1	1.	2.17	1.76	0.	3.07	0.	0.
time (sec)	N/A	0.119	1.851	0.318	0.	1.799	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	68	82	0	112	0	0
normalized size	1	1.	1.45	1.74	0.	2.38	0.	0.
time (sec)	N/A	0.079	0.107	0.374	0.	1.785	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	171	134	0	204	0	0
normalized size	1	1.	1.82	1.43	0.	2.17	0.	0.
time (sec)	N/A	0.154	0.568	0.36	0.	1.815	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	257	159	0	365	0	0
normalized size	1	1.	2.06	1.27	0.	2.92	0.	0.
time (sec)	N/A	0.227	0.827	0.386	0.	1.785	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	189	189	1542	294	0	392	0	0
normalized size	1	1.	8.16	1.56	0.	2.07	0.	0.
time (sec)	N/A	0.421	7.76	0.436	0.	1.883	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	475	227	0	358	0	0
normalized size	1	1.	3.15	1.5	0.	2.37	0.	0.
time (sec)	N/A	0.291	6.563	0.44	0.	1.851	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	113	113	180	142	0	282	0	0
normalized size	1	1.	1.59	1.26	0.	2.5	0.	0.
time (sec)	N/A	0.18	1.823	0.434	0.	1.79	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	76	71	88	0	404	0	0
normalized size	1	1.36	1.27	1.57	0.	7.21	0.	0.
time (sec)	N/A	0.115	0.076	0.44	0.	1.814	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	135	173	134	0	263	0	0
normalized size	1	1.29	1.65	1.28	0.	2.5	0.	0.
time (sec)	N/A	0.252	0.239	0.422	0.	2.047	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	259	167	0	420	0	0
normalized size	1	1.	1.54	0.99	0.	2.5	0.	0.
time (sec)	N/A	0.386	0.422	0.449	0.	2.023	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	A	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	197	197	0	258	0	451	0	0
normalized size	1	1.	0.	1.31	0.	2.29	0.	0.
time (sec)	N/A	0.511	0.	0.441	0.	1.922	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	458	183	0	381	0	0
normalized size	1	1.	2.92	1.17	0.	2.43	0.	0.
time (sec)	N/A	0.353	6.489	0.398	0.	1.905	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	99	151	0	354	0	0
normalized size	1	1.	0.85	1.29	0.	3.03	0.	0.
time (sec)	N/A	0.219	0.477	0.408	0.	1.871	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	140	156	0	351	0	0
normalized size	1	1.	1.2	1.33	0.	3.	0.	0.
time (sec)	N/A	0.216	0.426	0.39	0.	1.897	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	316	203	0	521	0	0
normalized size	1	1.	1.82	1.17	0.	2.99	0.	0.
time (sec)	N/A	0.396	6.523	0.393	0.	2.656	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	316	235	0	572	0	0
normalized size	1	1.	1.48	1.1	0.	2.67	0.	0.
time (sec)	N/A	0.548	6.55	0.42	0.	2.671	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	641	316	0	544	0	0
normalized size	1	1.	2.7	1.33	0.	2.3	0.	0.
time (sec)	N/A	0.64	7.982	0.446	0.	1.906	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	508	258	0	473	0	0
normalized size	1	1.	2.58	1.31	0.	2.4	0.	0.
time (sec)	N/A	0.495	6.752	0.428	0.	1.947	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	131	222	0	466	0	0
normalized size	1	1.	0.83	1.41	0.	2.97	0.	0.
time (sec)	N/A	0.352	0.943	0.444	0.	1.882	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	122	221	0	460	0	0
normalized size	1	1.	0.78	1.41	0.	2.93	0.	0.
time (sec)	N/A	0.35	0.666	0.423	0.	1.893	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	164	222	0	463	0	0
normalized size	1	1.	1.04	1.41	0.	2.95	0.	0.
time (sec)	N/A	0.352	0.714	0.415	0.	1.911	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	373	320	0	662	0	0
normalized size	1	1.	1.74	1.5	0.	3.09	0.	0.
time (sec)	N/A	0.528	2.245	0.424	0.	3.754	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	412	352	0	693	0	0
normalized size	1	1.	1.62	1.39	0.	2.73	0.	0.
time (sec)	N/A	0.666	3.176	0.463	0.	3.767	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	696	390	0	640	0	0
normalized size	1	1.	2.51	1.41	0.	2.31	0.	0.
time (sec)	N/A	0.78	8.375	0.457	0.	2.015	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	561	326	0	571	0	0
normalized size	1	1.	2.37	1.38	0.	2.41	0.	0.
time (sec)	N/A	0.631	6.879	0.435	0.	1.962	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	153	288	0	560	0	0
normalized size	1	1.	0.78	1.46	0.	2.84	0.	0.
time (sec)	N/A	0.481	3.692	0.461	0.	1.921	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	125	288	0	552	0	0
normalized size	1	1.	0.63	1.46	0.	2.8	0.	0.
time (sec)	N/A	0.481	0.843	0.427	0.	1.971	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	197	197	153	288	0	555	0	0
normalized size	1	1.	0.78	1.46	0.	2.82	0.	0.
time (sec)	N/A	0.497	3.644	0.424	0.	2.168	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	196	288	0	555	0	0
normalized size	1	1.	0.99	1.46	0.	2.82	0.	0.
time (sec)	N/A	0.485	4.557	0.426	0.	1.897	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	254	254	454	440	0	782	0	0
normalized size	1	1.	1.79	1.73	0.	3.08	0.	0.
time (sec)	N/A	0.668	6.796	0.434	0.	4.384	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	460	472	0	817	0	0
normalized size	1	1.	1.56	1.61	0.	2.78	0.	0.
time (sec)	N/A	0.817	3.515	0.458	0.	5.249	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	237	237	163	354	0	651	0	0
normalized size	1	1.	0.69	1.49	0.	2.75	0.	0.
time (sec)	N/A	0.631	4.391	0.434	0.	1.998	0.	0.

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	395	354	0	655	0	0
normalized size	1	1.	1.67	1.49	0.	2.76	0.	0.
time (sec)	N/A	0.636	6.092	0.431	0.	2.361	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	51	0	163	115	0	0
normalized size	1	1.	1.34	0.	4.29	3.03	0.	0.
time (sec)	N/A	0.118	0.115	0.362	1.5	2.097	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.531	3.231	2.855	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.308	1.354	2.316	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.535	2.961	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	208	0	0	0	0	0
normalized size	1	1.	1.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.998	1.062	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	156	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.816	0.845	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.305	1.058	0.492	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	135	100	142	289	286	165
normalized size	1	1.	0.9	0.67	0.95	1.93	1.91	1.1
time (sec)	N/A	0.102	0.212	0.035	0.984	2.27	13.777	1.369

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	89	90	127	244	238	144
normalized size	1	1.	0.7	0.7	0.99	1.91	1.86	1.12
time (sec)	N/A	0.086	0.181	0.033	0.965	2.289	8.763	1.395

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	78	80	113	204	216	124
normalized size	1	1.	0.68	0.7	0.99	1.79	1.89	1.09
time (sec)	N/A	0.082	0.102	0.036	0.972	2.205	4.374	1.361

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	89	70	93	173	168	104
normalized size	1	1.	0.97	0.76	1.01	1.88	1.83	1.13
time (sec)	N/A	0.064	0.13	0.033	0.963	1.922	2.327	1.361

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	60	77	136	144	84
normalized size	1	1.	0.96	0.79	1.01	1.79	1.89	1.11
time (sec)	N/A	0.059	0.097	0.033	0.955	1.87	1.188	1.324

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	57	49	62	105	92	63
normalized size	1	1.	1.06	0.91	1.15	1.94	1.7	1.17
time (sec)	N/A	0.047	0.065	0.033	0.946	1.906	0.601	1.309

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	35	38	46	72	66	42
normalized size	1	1.	0.92	1.	1.21	1.89	1.74	1.11
time (sec)	N/A	0.015	0.063	0.036	0.955	1.799	0.237	1.402

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	20	38	17	20
normalized size	1	1.	1.73	1.07	1.33	2.53	1.13	1.33
time (sec)	N/A	0.009	0.006	0.023	0.947	1.862	0.125	1.366

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	30	38	95	49	58
normalized size	1	1.	1.	1.88	2.38	5.94	3.06	3.62
time (sec)	N/A	0.023	0.007	0.046	0.977	1.871	4.843	1.382

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	51	162	0	85
normalized size	1	1.	1.	1.33	2.12	6.75	0.	3.54
time (sec)	N/A	0.037	0.007	0.05	0.97	1.885	0.	1.341

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	51	78	198	0	142
normalized size	1	1.	1.	1.09	1.66	4.21	0.	3.02
time (sec)	N/A	0.049	0.015	0.052	0.968	1.892	0.	1.456

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	72	95	236	0	165
normalized size	1	1.	0.95	1.14	1.51	3.75	0.	2.62
time (sec)	N/A	0.053	0.16	0.056	1.018	1.92	0.	1.445

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	92	128	266	0	221
normalized size	1	1.	0.89	1.08	1.51	3.13	0.	2.6
time (sec)	N/A	0.065	0.226	0.056	0.989	1.91	0.	1.445

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	88	112	144	304	0	240
normalized size	1	1.	0.87	1.11	1.43	3.01	0.	2.38
time (sec)	N/A	0.07	0.325	0.053	0.988	1.983	0.	1.437

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	123	120	162	273	343	171
normalized size	1	1.	0.82	0.8	1.08	1.82	2.29	1.14
time (sec)	N/A	0.106	0.304	0.034	0.99	1.982	4.493	1.379

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	85	95	127	215	221	138
normalized size	1	1.	0.77	0.86	1.14	1.94	1.99	1.24
time (sec)	N/A	0.108	0.134	0.034	0.996	1.907	2.352	1.345

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	86	89	111	184	211	111
normalized size	1	1.	0.85	0.88	1.1	1.82	2.09	1.1
time (sec)	N/A	0.092	0.156	0.034	0.985	1.857	1.24	1.742

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	63	81	124	107	81
normalized size	1	1.	0.83	0.89	1.14	1.75	1.51	1.14
time (sec)	N/A	0.05	0.153	0.033	0.966	1.867	0.611	1.391

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	46	51	59	93	78	58
normalized size	1	1.	0.92	1.02	1.18	1.86	1.56	1.16
time (sec)	N/A	0.015	0.072	0.031	0.955	1.805	0.281	1.521

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	49	57	131	0	105
normalized size	1	1.	1.39	1.48	1.73	3.97	0.	3.18
time (sec)	N/A	0.062	0.013	0.049	0.983	2.004	0.	1.376

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	49	65	193	0	104
normalized size	1	1.	0.97	1.48	1.97	5.85	0.	3.15
time (sec)	N/A	0.066	0.085	0.053	0.975	1.966	0.	1.573

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	67	78	117	236	0	171
normalized size	1	1.	1.14	1.32	1.98	4.	0.	2.9
time (sec)	N/A	0.078	0.013	0.059	0.958	1.998	0.	1.424

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	71	89	113	259	0	240
normalized size	1	1.	0.89	1.11	1.41	3.24	0.	3.
time (sec)	N/A	0.089	0.218	0.06	0.985	2.046	0.	1.486

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	82	142	194	332	0	348
normalized size	1	1.	0.75	1.29	1.76	3.02	0.	3.16
time (sec)	N/A	0.095	0.275	0.062	0.984	2.104	0.	1.432

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	118	157	178	352	0	367
normalized size	1	1.	0.87	1.16	1.32	2.61	0.	2.72
time (sec)	N/A	0.108	0.561	0.061	0.971	2.092	0.	1.396

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	193	159	145	196	324	393	203
normalized size	1	1.14	0.94	0.85	1.15	1.91	2.31	1.19
time (sec)	N/A	0.215	0.329	0.038	0.991	2.03	4.895	1.328

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	130	123	161	265	284	167
normalized size	1	1.	0.72	0.68	0.89	1.47	1.58	0.93
time (sec)	N/A	0.221	0.306	0.034	0.953	1.894	2.595	1.868

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	100	102	128	197	233	130
normalized size	1	1.	0.83	0.84	1.06	1.63	1.93	1.07
time (sec)	N/A	0.116	0.264	0.033	0.965	1.896	1.353	2.186

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	90	80	76	97	153	128	97
normalized size	1	1.18	1.05	1.	1.28	2.01	1.68	1.28
time (sec)	N/A	0.069	0.124	0.032	0.945	1.951	0.667	1.341

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	105	90	93	176	0	185
normalized size	1	1.	1.44	1.23	1.27	2.41	0.	2.53
time (sec)	N/A	0.115	0.14	0.055	0.965	1.989	0.	1.432

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	88	68	89	246	0	174
normalized size	1	1.	1.29	1.	1.31	3.62	0.	2.56
time (sec)	N/A	0.122	0.333	0.059	0.983	2.069	0.	1.427

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	55	95	136	281	0	193
normalized size	1	1.	0.7	1.2	1.72	3.56	0.	2.44
time (sec)	N/A	0.134	0.176	0.06	0.966	1.996	0.	1.801

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	70	118	153	309	0	277
normalized size	1	1.	0.64	1.08	1.4	2.83	0.	2.54
time (sec)	N/A	0.182	0.268	0.065	0.972	1.958	0.	2.579

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	90	160	213	348	0	446
normalized size	1	1.	0.68	1.2	1.6	2.62	0.	3.35
time (sec)	N/A	0.203	0.414	0.066	0.967	2.031	0.	1.697

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	120	206	244	421	0	495
normalized size	1	1.	0.71	1.22	1.44	2.49	0.	2.93
time (sec)	N/A	0.217	0.84	0.064	0.977	2.004	0.	1.388

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	181	190	259	416	495	266
normalized size	1	1.	0.73	0.77	1.05	1.68	2.	1.08
time (sec)	N/A	0.401	0.408	0.052	0.978	1.976	8.572	1.366

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	156	174	230	358	459	227
normalized size	1	1.	0.66	0.74	0.98	1.52	1.95	0.97
time (sec)	N/A	0.32	0.446	0.036	0.98	1.982	5.043	1.3

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	133	138	180	285	301	181
normalized size	1	1.	0.78	0.81	1.06	1.68	1.77	1.06
time (sec)	N/A	0.204	0.489	0.034	1.002	2.016	2.544	1.378

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	104	116	150	224	240	144
normalized size	1	1.	0.76	0.85	1.09	1.64	1.75	1.05
time (sec)	N/A	0.147	0.209	0.033	0.993	1.961	1.285	1.329

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	128	131	128	239	0	286
normalized size	1	1.	1.2	1.22	1.2	2.23	0.	2.67
time (sec)	N/A	0.228	0.159	0.058	0.962	2.051	0.	1.366

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	119	109	122	294	0	230
normalized size	1	1.	1.04	0.96	1.07	2.58	0.	2.02
time (sec)	N/A	0.234	0.633	0.064	0.985	1.96	0.	1.394

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	174	114	155	324	0	239
normalized size	1	1.	1.61	1.06	1.44	3.	0.	2.21
time (sec)	N/A	0.252	2.435	0.067	0.976	2.02	0.	1.475

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	77	135	169	339	0	298
normalized size	1	1.	0.67	1.17	1.47	2.95	0.	2.59
time (sec)	N/A	0.252	0.388	0.069	0.976	2.015	0.	1.343

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	101	188	252	394	0	486
normalized size	1	1.	0.66	1.22	1.64	2.56	0.	3.16
time (sec)	N/A	0.336	0.498	0.074	0.989	1.898	0.	1.432

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	125	225	263	443	0	622
normalized size	1	1.	0.66	1.2	1.4	2.36	0.	3.31
time (sec)	N/A	0.361	0.733	0.068	1.001	1.997	0.	1.434

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	154	302	371	528	0	799
normalized size	1	1.	0.69	1.36	1.67	2.38	0.	3.6
time (sec)	N/A	0.38	0.982	0.072	0.992	2.11	0.	1.495

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	153	672	0	1042	0	531
normalized size	1	1.	0.79	3.48	0.	5.4	0.	2.75
time (sec)	N/A	0.545	0.645	0.115	0.	2.252	0.	1.385

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	122	367	0	860	0	336
normalized size	1	1.	0.82	2.48	0.	5.81	0.	2.27
time (sec)	N/A	0.327	0.33	0.086	0.	2.231	0.	1.393

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	97	222	0	716	0	239
normalized size	1	1.	0.88	2.02	0.	6.51	0.	2.17
time (sec)	N/A	0.184	0.232	0.087	0.	2.12	0.	1.362

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	72	102	0	585	0	170
normalized size	1	1.	0.95	1.34	0.	7.7	0.	2.24
time (sec)	N/A	0.119	0.141	0.083	0.	2.08	0.	1.366

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	67	0	486	0	126
normalized size	1	1.	0.98	1.14	0.	8.24	0.	2.14
time (sec)	N/A	0.056	0.087	0.079	0.	2.091	0.	1.397

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	404	172	105
normalized size	1	1.	0.98	0.9	0.	8.24	3.51	2.14
time (sec)	N/A	0.031	0.036	0.066	0.	2.004	10.272	1.345

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	102	88	0	643	0	161
normalized size	1	1.	1.5	1.29	0.	9.46	0.	2.37
time (sec)	N/A	0.072	0.085	0.09	0.	2.582	0.	1.402

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	115	134	0	887	0	207
normalized size	1	1.	1.35	1.58	0.	10.44	0.	2.44
time (sec)	N/A	0.13	0.374	0.097	0.	2.616	0.	1.454

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	238	262	0	1061	0	285
normalized size	1	1.	2.	2.2	0.	8.92	0.	2.39
time (sec)	N/A	0.324	1.003	0.112	0.	3.537	0.	1.447

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	258	400	0	1229	0	386
normalized size	1	1.	1.64	2.55	0.	7.83	0.	2.46
time (sec)	N/A	0.522	2.37	0.111	0.	3.575	0.	1.467

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	176	504	0	1644	0	450
normalized size	1	1.	0.66	1.89	0.	6.18	0.	1.69
time (sec)	N/A	0.723	0.88	0.098	0.	2.639	0.	1.317

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	213	144	358	0	1428	0	354
normalized size	1	1.28	0.87	2.16	0.	8.6	0.	2.13
time (sec)	N/A	0.429	0.732	0.091	0.	2.392	0.	1.483

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	113	238	0	1220	0	412
normalized size	1	1.	0.73	1.54	0.	7.87	0.	2.66
time (sec)	N/A	0.256	0.665	0.096	0.	2.175	0.	1.42

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	103	200	0	1037	0	236
normalized size	1	1.	0.95	1.85	0.	9.6	0.	2.19
time (sec)	N/A	0.143	0.372	0.089	0.	2.297	0.	1.403

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	83	116	0	728	0	182
normalized size	1	1.	0.98	1.36	0.	8.56	0.	2.14
time (sec)	N/A	0.069	0.229	0.089	0.	2.091	0.	1.31

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	84	116	0	726	0	182
normalized size	1	1.	0.98	1.35	0.	8.44	0.	2.12
time (sec)	N/A	0.055	0.186	0.078	0.	1.987	0.	1.228

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	146	221	0	1331	0	267
normalized size	1	1.	1.24	1.87	0.	11.28	0.	2.26
time (sec)	N/A	0.219	0.333	0.118	0.	5.153	0.	1.364

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	163	271	0	1689	0	448
normalized size	1	1.	1.05	1.75	0.	10.9	0.	2.89
time (sec)	N/A	0.407	0.891	0.115	0.	4.833	0.	1.276

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	285	401	0	2009	0	396
normalized size	1	1.	1.31	1.85	0.	9.26	0.	1.82
time (sec)	N/A	0.681	5.533	0.123	0.	8.46	0.	1.455

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	499	535	0	2218	0	497
normalized size	1	1.	1.85	1.98	0.	8.21	0.	1.84
time (sec)	N/A	0.967	6.152	0.13	0.	8.085	0.	1.393

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	199	802	0	2525	0	1030
normalized size	1	1.	0.66	2.67	0.	8.42	0.	3.43
time (sec)	N/A	0.783	1.997	0.101	0.	2.487	0.	1.39

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	177	679	0	2209	0	478
normalized size	1	1.	0.8	3.07	0.	10.	0.	2.16
time (sec)	N/A	0.488	1.522	0.094	0.	2.42	0.	1.452

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	149	639	0	1944	0	431
normalized size	1	1.	0.83	3.57	0.	10.86	0.	2.41
time (sec)	N/A	0.304	1.099	0.094	0.	2.216	0.	1.46

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	115	400	0	1281	0	338
normalized size	1	1.	0.77	2.68	0.	8.6	0.	2.27
time (sec)	N/A	0.173	0.536	0.085	0.	2.237	0.	1.354

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	115	475	0	1207	0	366
normalized size	1	1.	0.86	3.54	0.	9.01	0.	2.73
time (sec)	N/A	0.123	0.351	0.082	0.	2.271	0.	1.526

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	113	400	0	1276	0	339
normalized size	1	1.	0.85	3.01	0.	9.59	0.	2.55
time (sec)	N/A	0.11	0.392	0.082	0.	2.271	0.	1.22

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	192	660	0	2452	0	464
normalized size	1	1.	1.05	3.63	0.	13.47	0.	2.55
time (sec)	N/A	0.456	1.077	0.115	0.	8.599	0.	1.424

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	205	712	0	2907	0	513
normalized size	1	1.	0.88	3.07	0.	12.53	0.	2.21
time (sec)	N/A	0.784	4.136	0.121	0.	8.666	0.	1.292

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	427	845	0	3380	0	1081
normalized size	1	1.	1.4	2.77	0.	11.08	0.	3.54
time (sec)	N/A	1.078	6.192	0.133	0.	16.412	0.	1.371

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	240	1396	0	3557	0	760
normalized size	1	1.	0.78	4.55	0.	11.59	0.	2.48
time (sec)	N/A	0.9	5.658	0.101	0.	2.856	0.	1.29

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	227	1356	0	3158	0	717
normalized size	1	1.	0.91	5.42	0.	12.63	0.	2.87
time (sec)	N/A	0.571	2.671	0.092	0.	2.59	0.	1.312

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	158	776	0	1958	0	539
normalized size	1	1.	0.71	3.5	0.	8.82	0.	2.43
time (sec)	N/A	0.337	1.17	0.089	0.	2.331	0.	1.365

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	162	930	0	1959	0	576
normalized size	1	1.	0.79	4.51	0.	9.51	0.	2.8
time (sec)	N/A	0.281	1.138	0.091	0.	2.302	0.	1.36

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	164	931	0	1960	0	576
normalized size	1	1.	0.85	4.85	0.	10.21	0.	3.
time (sec)	N/A	0.226	1.047	0.089	0.	2.234	0.	1.358

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	159	776	0	1956	0	539
normalized size	1	1.	0.86	4.22	0.	10.63	0.	2.93
time (sec)	N/A	0.217	0.886	0.087	0.	2.2	0.	1.319

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	274	1377	0	3962	0	748
normalized size	1	1.	1.09	5.49	0.	15.78	0.	2.98
time (sec)	N/A	0.788	2.952	0.121	0.	19.734	0.	1.477

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	416	1429	0	4566	0	792
normalized size	1	1.	1.35	4.64	0.	14.82	0.	2.57
time (sec)	N/A	1.27	6.256	0.129	0.	19.275	0.	1.469

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	214	827	0	0	0	0
normalized size	1	1.	0.81	3.13	0.	0.	0.	0.
time (sec)	N/A	0.412	1.127	2.851	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	180	665	0	0	0	0
normalized size	1	1.	0.87	3.21	0.	0.	0.	0.
time (sec)	N/A	0.284	0.791	2.934	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	137	452	0	0	0	0
normalized size	1	1.	0.85	2.79	0.	0.	0.	0.
time (sec)	N/A	0.173	0.552	3.043	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	170	0	0	0	0
normalized size	1	1.	1.	2.98	0.	0.	0.	0.
time (sec)	N/A	0.039	0.069	2.441	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	81	194	0	0	0	0
normalized size	1	1.	0.69	1.64	0.	0.	0.	0.
time (sec)	N/A	0.226	2.184	2.727	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	307	622	0	0	0	0
normalized size	1	1.	1.56	3.16	0.	0.	0.	0.
time (sec)	N/A	0.498	7.242	2.977	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	515	977	0	0	0	0
normalized size	1	1.	1.97	3.73	0.	0.	0.	0.
time (sec)	N/A	0.731	6.51	3.535	0.	0.	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	262	995	0	0	0	0
normalized size	1	1.	0.83	3.17	0.	0.	0.	0.
time (sec)	N/A	0.518	1.381	3.608	0.	0.	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	214	827	0	0	0	0
normalized size	1	1.	0.83	3.21	0.	0.	0.	0.
time (sec)	N/A	0.391	1.146	3.057	0.	0.	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	174	663	0	0	0	0
normalized size	1	1.	0.87	3.33	0.	0.	0.	0.
time (sec)	N/A	0.248	0.778	2.925	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	450	0	0	0	0
normalized size	1	1.	0.85	2.87	0.	0.	0.	0.
time (sec)	N/A	0.169	0.548	3.059	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	107	249	0	0	0	0
normalized size	1	1.	0.6	1.39	0.	0.	0.	0.
time (sec)	N/A	0.314	2.31	2.746	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	363	740	0	0	0	0
normalized size	1	1.	1.74	3.54	0.	0.	0.	0.
time (sec)	N/A	0.542	10.933	3.527	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	508	980	0	0	0	0
normalized size	1	1.	1.99	3.84	0.	0.	0.	0.
time (sec)	N/A	0.763	6.385	3.326	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	268	1140	0	0	0	0
normalized size	1	1.	0.72	3.07	0.	0.	0.	0.
time (sec)	N/A	0.629	1.189	3.718	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	308	308	263	995	0	0	0	0
normalized size	1	1.	0.85	3.23	0.	0.	0.	0.
time (sec)	N/A	0.509	1.387	3.139	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	214	827	0	0	0	0
normalized size	1	1.	0.86	3.32	0.	0.	0.	0.
time (sec)	N/A	0.36	0.887	2.899	0.	0.	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	177	662	0	0	0	0
normalized size	1	1.	0.9	3.36	0.	0.	0.	0.
time (sec)	N/A	0.261	0.784	3.25	0.	0.	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	379	528	0	0	0	0
normalized size	1	1.	1.71	2.38	0.	0.	0.	0.
time (sec)	N/A	0.586	1.64	3.405	0.	0.	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	390	960	0	0	0	0
normalized size	1	1.	1.76	4.32	0.	0.	0.	0.
time (sec)	N/A	0.592	2.198	3.174	0.	0.	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	395	1134	0	0	0	0
normalized size	1	1.	1.46	4.2	0.	0.	0.	0.
time (sec)	N/A	0.877	2.676	3.832	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	434	1742	0	0	0	0
normalized size	1	1.	1.34	5.39	0.	0.	0.	0.
time (sec)	N/A	1.169	4.16	4.29	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	211	824	0	0	0	0
normalized size	1	1.	0.86	3.35	0.	0.	0.	0.
time (sec)	N/A	0.375	1.082	3.351	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	92	275	0	0	0	0
normalized size	1	1.	0.67	1.99	0.	0.	0.	0.
time (sec)	N/A	0.185	0.245	2.513	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	81	253	0	0	0	0
normalized size	1	1.	0.77	2.41	0.	0.	0.	0.
time (sec)	N/A	0.138	0.139	2.188	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	231	0	0	0	0
normalized size	1	1.	0.88	2.96	0.	0.	0.	0.
time (sec)	N/A	0.084	0.06	2.113	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	137	0	0	0	0
normalized size	1	1.	1.	5.96	0.	0.	0.	0.
time (sec)	N/A	0.012	0.024	1.635	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	41	158	0	0	0	0
normalized size	1	1.	0.85	3.29	0.	0.	0.	0.
time (sec)	N/A	0.087	0.049	2.178	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	157	350	0	0	0	0
normalized size	1	1.	1.65	3.68	0.	0.	0.	0.
time (sec)	N/A	0.246	1.098	3.312	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	194	408	0	0	0	0
normalized size	1	1.	1.44	3.02	0.	0.	0.	0.
time (sec)	N/A	0.364	1.247	3.497	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	114	276	0	0	0	0
normalized size	1	1.	0.81	1.97	0.	0.	0.	0.
time (sec)	N/A	0.187	0.203	2.907	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	104	253	0	0	0	0
normalized size	1	1.	0.97	2.36	0.	0.	0.	0.
time (sec)	N/A	0.138	0.205	2.527	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	94	231	0	0	0	0
normalized size	1	1.	1.18	2.89	0.	0.	0.	0.
time (sec)	N/A	0.084	0.096	2.482	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	44	138	0	0	0	0
normalized size	1	1.	1.83	5.75	0.	0.	0.	0.
time (sec)	N/A	0.011	0.032	1.914	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	61	159	0	0	0	0
normalized size	1	1.	1.22	3.18	0.	0.	0.	0.
time (sec)	N/A	0.088	0.06	2.533	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	178	351	0	0	0	0
normalized size	1	1.	1.82	3.58	0.	0.	0.	0.
time (sec)	N/A	0.249	1.411	4.174	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	237	408	0	0	0	0
normalized size	1	1.	1.72	2.96	0.	0.	0.	0.
time (sec)	N/A	0.374	1.829	3.997	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	182	665	0	0	0	0
normalized size	1	1.	0.85	3.09	0.	0.	0.	0.
time (sec)	N/A	0.287	0.932	3.274	0.	0.	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	137	453	0	0	0	0
normalized size	1	1.	0.83	2.75	0.	0.	0.	0.
time (sec)	N/A	0.188	0.633	4.932	0.	0.	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	86	220	0	0	0	0
normalized size	1	1.	0.7	1.8	0.	0.	0.	0.
time (sec)	N/A	0.108	2.283	3.016	0.	0.	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	75	0	0	0	0
normalized size	1	1.	1.	1.32	0.	0.	0.	0.
time (sec)	N/A	0.037	0.05	0.417	0.	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	166	0	0	0	0
normalized size	1	1.	1.	2.86	0.	0.	0.	0.
time (sec)	N/A	0.126	0.084	3.099	0.	0.	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	310	532	0	0	0	0
normalized size	1	1.	1.5	2.58	0.	0.	0.	0.
time (sec)	N/A	0.492	8.312	4.924	0.	0.	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	518	710	0	0	0	0
normalized size	1	1.	1.93	2.65	0.	0.	0.	0.
time (sec)	N/A	0.713	6.385	5.033	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	242	1285	0	0	0	0
normalized size	1	1.	0.74	3.94	0.	0.	0.	0.
time (sec)	N/A	0.513	1.272	4.046	0.	0.	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	197	984	0	0	0	0
normalized size	1	1.	0.77	3.83	0.	0.	0.	0.
time (sec)	N/A	0.344	0.883	3.94	0.	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	159	530	0	0	0	0
normalized size	1	1.	0.85	2.85	0.	0.	0.	0.
time (sec)	N/A	0.232	0.67	3.902	0.	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	137	373	0	0	0	0
normalized size	1	1.	0.81	2.19	0.	0.	0.	0.
time (sec)	N/A	0.182	0.508	3.72	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	83	217	0	0	0	0
normalized size	1	1.	0.78	2.05	0.	0.	0.	0.
time (sec)	N/A	0.065	0.206	3.772	0.	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	402	376	0	0	0	0
normalized size	1	1.	2.28	2.14	0.	0.	0.	0.
time (sec)	N/A	0.394	5.064	3.403	0.	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	441	894	0	0	0	0
normalized size	1	1.	1.59	3.23	0.	0.	0.	0.
time (sec)	N/A	0.78	4.003	8.279	0.	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	597	1542	0	0	0	0
normalized size	1	1.	1.73	4.47	0.	0.	0.	0.
time (sec)	N/A	1.082	6.476	10.874	0.	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	272	1684	0	0	0	0
normalized size	1	1.	0.62	3.86	0.	0.	0.	0.
time (sec)	N/A	0.862	1.907	17.893	0.	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	237	1291	0	0	0	0
normalized size	1	1.	0.69	3.74	0.	0.	0.	0.
time (sec)	N/A	0.57	1.567	13.386	0.	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	188	907	0	0	0	0
normalized size	1	1.	0.67	3.23	0.	0.	0.	0.
time (sec)	N/A	0.379	1.223	12.308	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	175	846	0	0	0	0
normalized size	1	1.	0.67	3.22	0.	0.	0.	0.
time (sec)	N/A	0.337	1.081	10.696	0.	0.	0.	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	154	742	0	0	0	0
normalized size	1	1.	0.63	3.05	0.	0.	0.	0.
time (sec)	N/A	0.272	0.944	10.819	0.	0.	0.	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	158	489	0	0	0	0
normalized size	1	1.	0.71	2.21	0.	0.	0.	0.
time (sec)	N/A	0.232	0.902	6.918	0.	0.	0.	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	464	845	0	0	0	0
normalized size	1	1.	1.45	2.64	0.	0.	0.	0.
time (sec)	N/A	0.871	4.42	10.338	0.	0.	0.	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	638	1320	0	0	0	0
normalized size	1	1.	1.68	3.47	0.	0.	0.	0.
time (sec)	N/A	1.1	6.525	14.145	0.	0.	0.	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	189	616	0	0	0	0
normalized size	1	1.	0.67	2.18	0.	0.	0.	0.
time (sec)	N/A	0.358	1.283	8.799	0.	0.	0.	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	81	231	0	0	0	0
normalized size	1	1.	0.73	2.08	0.	0.	0.	0.
time (sec)	N/A	0.15	0.17	2.559	0.	0.	0.	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	231	0	0	0	0
normalized size	1	1.	0.9	2.96	0.	0.	0.	0.
time (sec)	N/A	0.102	0.079	2.297	0.	0.	0.	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	155	0	0	0	0
normalized size	1	1.	0.84	3.04	0.	0.	0.	0.
time (sec)	N/A	0.052	0.054	1.96	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	0	0	0	0
normalized size	1	1.	1.	1.	0.	0.	0.	0.
time (sec)	N/A	0.012	0.028	0.046	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	138	0	0	0	0
normalized size	1	1.	1.	5.75	0.	0.	0.	0.
time (sec)	N/A	0.039	0.05	1.839	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	158	350	0	0	0	0
normalized size	1	1.	1.56	3.47	0.	0.	0.	0.
time (sec)	N/A	0.256	1.104	2.809	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	195	408	0	0	0	0
normalized size	1	1.	1.42	2.98	0.	0.	0.	0.
time (sec)	N/A	0.371	1.214	3.541	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	102	254	0	0	0	0
normalized size	1	1.	0.9	2.25	0.	0.	0.	0.
time (sec)	N/A	0.15	0.157	3.38	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	94	232	0	0	0	0
normalized size	1	1.	1.18	2.9	0.	0.	0.	0.
time (sec)	N/A	0.102	0.117	3.379	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	60	158	0	0	0	0
normalized size	1	1.	1.13	2.98	0.	0.	0.	0.
time (sec)	N/A	0.05	0.067	2.565	0.	0.	0.	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	44	54	0	0	0	0
normalized size	1	1.	1.83	2.25	0.	0.	0.	0.
time (sec)	N/A	0.012	0.034	0.163	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	45	139	0	0	0	0
normalized size	1	1.	1.8	5.56	0.	0.	0.	0.
time (sec)	N/A	0.039	0.059	2.503	0.	0.	0.	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	179	351	0	0	0	0
normalized size	1	1.	1.72	3.38	0.	0.	0.	0.
time (sec)	N/A	0.251	1.419	3.337	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	236	408	0	0	0	0
normalized size	1	1.	1.69	2.91	0.	0.	0.	0.
time (sec)	N/A	0.366	1.749	3.833	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	77	290	0	0	0	0
normalized size	1	1.	0.69	2.61	0.	0.	0.	0.
time (sec)	N/A	0.075	0.494	2.775	0.	0.	0.	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	66	262	0	0	0	0
normalized size	1	1.	0.76	3.01	0.	0.	0.	0.
time (sec)	N/A	0.06	0.223	3.26	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	53	229	0	0	0	0
normalized size	1	1.	0.87	3.75	0.	0.	0.	0.
time (sec)	N/A	0.051	0.103	2.297	0.	0.	0.	0.

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	152	0	0	0	0
normalized size	1	1.	1.	4.34	0.	0.	0.	0.
time (sec)	N/A	0.039	0.062	2.808	0.	0.	0.	0.

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	51	148	0	0	0	0
normalized size	1	1.	0.89	2.6	0.	0.	0.	0.
time (sec)	N/A	0.048	0.134	2.744	0.	0.	0.	0.

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	65	397	0	0	0	0
normalized size	1	1.	0.78	4.78	0.	0.	0.	0.
time (sec)	N/A	0.059	0.393	6.086	0.	0.	0.	0.

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	502	0	0	0	0
normalized size	1	1.	0.86	4.52	0.	0.	0.	0.
time (sec)	N/A	0.074	0.298	7.04	0.	0.	0.	0.

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	113	398	0	0	0	0
normalized size	1	1.	0.71	2.49	0.	0.	0.	0.
time (sec)	N/A	0.12	0.754	2.832	0.	0.	0.	0.

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	98	362	0	0	0	0
normalized size	1	1.	0.73	2.68	0.	0.	0.	0.
time (sec)	N/A	0.104	0.579	2.5	0.	0.	0.	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	79	321	0	0	0	0
normalized size	1	1.	0.78	3.18	0.	0.	0.	0.
time (sec)	N/A	0.09	0.299	2.661	0.	0.	0.	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	283	0	0	0	0
normalized size	1	1.	0.89	3.93	0.	0.	0.	0.
time (sec)	N/A	0.084	0.155	2.885	0.	0.	0.	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	202	0	0	0	0
normalized size	1	1.	0.91	2.97	0.	0.	0.	0.
time (sec)	N/A	0.086	0.277	3.197	0.	0.	0.	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	73	514	0	0	0	0
normalized size	1	1.	0.77	5.41	0.	0.	0.	0.
time (sec)	N/A	0.095	0.565	6.148	0.	0.	0.	0.

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	660	0	0	0	0
normalized size	1	1.	0.92	4.89	0.	0.	0.	0.
time (sec)	N/A	0.106	0.362	7.757	0.	0.	0.	0.

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	137	470	0	0	0	0
normalized size	1	1.	0.71	2.42	0.	0.	0.	0.
time (sec)	N/A	0.22	0.915	3.132	0.	0.	0.	0.

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	110	421	0	0	0	0
normalized size	1	1.	0.69	2.65	0.	0.	0.	0.
time (sec)	N/A	0.202	0.711	3.802	0.	0.	0.	0.

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	84	376	0	0	0	0
normalized size	1	1.	0.72	3.24	0.	0.	0.	0.
time (sec)	N/A	0.177	0.353	3.181	0.	0.	0.	0.

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	86	303	0	0	0	0
normalized size	1	1.	0.69	2.44	0.	0.	0.	0.
time (sec)	N/A	0.188	0.508	3.388	0.	0.	0.	0.

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	85	631	0	0	0	0
normalized size	1	1.	0.71	5.26	0.	0.	0.	0.
time (sec)	N/A	0.19	1.222	6.43	0.	0.	0.	0.

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	738	0	0	0	0
normalized size	1	1.	0.84	4.95	0.	0.	0.	0.
time (sec)	N/A	0.211	0.86	8.242	0.	0.	0.	0.

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	177	847	0	0	0	0
normalized size	1	1.	0.91	4.37	0.	0.	0.	0.
time (sec)	N/A	0.233	0.726	9.352	0.	0.	0.	0.

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	160	516	0	0	0	0
normalized size	1	1.	1.43	4.61	0.	0.	0.	0.
time (sec)	N/A	0.391	1.896	3.279	0.	0.	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	84	227	0	0	0	0
normalized size	1	1.	1.12	3.03	0.	0.	0.	0.
time (sec)	N/A	0.163	0.291	3.318	0.	0.	0.	0.

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	188	0	0	0	0
normalized size	1	1.	0.91	3.55	0.	0.	0.	0.
time (sec)	N/A	0.101	0.068	2.827	0.	0.	0.	0.

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	150	0	0	0	0
normalized size	1	1.	1.	5.17	0.	0.	0.	0.
time (sec)	N/A	0.045	0.073	2.258	0.	0.	0.	0.

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	199	354	0	0	0	0
normalized size	1	1.	2.58	4.6	0.	0.	0.	0.
time (sec)	N/A	0.238	2.929	3.508	0.	0.	0.	0.

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	214	452	0	0	0	0
normalized size	1	1.	1.67	3.53	0.	0.	0.	0.
time (sec)	N/A	0.546	4.396	7.319	0.	0.	0.	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	270	1070	0	0	0	0
normalized size	1	1.	1.1	4.37	0.	0.	0.	0.
time (sec)	N/A	0.705	1.819	8.191	0.	0.	0.	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	255	815	0	0	0	0
normalized size	1	1.	1.38	4.41	0.	0.	0.	0.
time (sec)	N/A	0.459	1.747	8.118	0.	0.	0.	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	198	794	0	0	0	0
normalized size	1	1.	1.21	4.87	0.	0.	0.	0.
time (sec)	N/A	0.384	3.089	7.099	0.	0.	0.	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	233	713	0	0	0	0
normalized size	1	1.	1.57	4.82	0.	0.	0.	0.
time (sec)	N/A	0.397	3.443	6.307	0.	0.	0.	0.

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	242	612	0	0	0	0
normalized size	1	1.	1.54	3.9	0.	0.	0.	0.
time (sec)	N/A	0.436	3.27	4.931	0.	0.	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	282	874	0	0	0	0
normalized size	1	1.	1.3	4.03	0.	0.	0.	0.
time (sec)	N/A	0.683	2.93	9.122	0.	0.	0.	0.

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	298	1008	0	0	0	0
normalized size	1	1.	1.06	3.59	0.	0.	0.	0.
time (sec)	N/A	0.995	3.637	11.783	0.	0.	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	358	2194	0	0	0	0
normalized size	1	1.	1.03	6.34	0.	0.	0.	0.
time (sec)	N/A	1.039	3.066	14.392	0.	0.	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	313	1935	0	0	0	0
normalized size	1	1.	1.11	6.86	0.	0.	0.	0.
time (sec)	N/A	0.774	2.714	12.78	0.	0.	0.	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	288	1914	0	0	0	0
normalized size	1	1.	1.09	7.25	0.	0.	0.	0.
time (sec)	N/A	0.777	1.994	11.211	0.	0.	0.	0.

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	276	1836	0	0	0	0
normalized size	1	1.	1.13	7.52	0.	0.	0.	0.
time (sec)	N/A	0.657	1.829	10.702	0.	0.	0.	0.

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	295	1736	0	0	0	0
normalized size	1	1.	1.18	6.94	0.	0.	0.	0.
time (sec)	N/A	0.682	2.807	10.494	0.	0.	0.	0.

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	305	1176	0	0	0	0
normalized size	1	1.	1.17	4.51	0.	0.	0.	0.
time (sec)	N/A	0.775	2.635	6.707	0.	0.	0.	0.

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	338	1992	0	0	0	0
normalized size	1	1.	1.03	6.07	0.	0.	0.	0.
time (sec)	N/A	1.071	3.451	13.674	0.	0.	0.	0.

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	353	2128	0	0	0	0
normalized size	1	1.	0.89	5.39	0.	0.	0.	0.
time (sec)	N/A	1.35	5.33	18.621	0.	0.	0.	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	1152	1233	0	0	0	0
normalized size	1	1.	2.63	2.82	0.	0.	0.	0.
time (sec)	N/A	0.922	17.284	0.377	0.	0.	0.	0.

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	371	371	316	801	0	0	0	0
normalized size	1	1.	0.85	2.16	0.	0.	0.	0.
time (sec)	N/A	0.575	7.19	0.491	0.	0.	0.	0.

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	139	197	0	0	0	0
normalized size	1	1.	1.03	1.46	0.	0.	0.	0.
time (sec)	N/A	0.071	1.151	0.445	0.	0.	0.	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	203	789	0	0	0	0
normalized size	1	1.	0.89	3.45	0.	0.	0.	0.
time (sec)	N/A	0.263	3.192	0.483	0.	0.	0.	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	247	880	0	0	0	0
normalized size	1	1.	0.91	3.25	0.	0.	0.	0.
time (sec)	N/A	0.402	7.426	0.542	0.	0.	0.	0.

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	453	1557	0	0	0	0
normalized size	1	1.	1.38	4.73	0.	0.	0.	0.
time (sec)	N/A	0.646	12.904	0.4	0.	0.	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	1304	1826	0	0	0	0
normalized size	1	1.	3.35	4.69	0.	0.	0.	0.
time (sec)	N/A	0.924	6.197	0.492	0.	0.	0.	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	508	508	1189	1683	0	0	0	0
normalized size	1	1.	2.34	3.31	0.	0.	0.	0.
time (sec)	N/A	1.257	18.646	0.384	0.	0.	0.	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	441	1421	0	0	0	0
normalized size	1	1.	1.02	3.28	0.	0.	0.	0.
time (sec)	N/A	1.17	12.34	0.343	0.	0.	0.	0.

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	341	1003	0	0	0	0
normalized size	1	1.	0.91	2.67	0.	0.	0.	0.
time (sec)	N/A	0.642	7.054	0.463	0.	0.	0.	0.

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	359	1183	0	0	0	0
normalized size	1	1.	1.07	3.51	0.	0.	0.	0.
time (sec)	N/A	0.471	12.808	0.457	0.	0.	0.	0.

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	256	1075	0	0	0	0
normalized size	1	1.	0.92	3.88	0.	0.	0.	0.
time (sec)	N/A	0.435	4.512	0.553	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	443	1539	0	0	0	0
normalized size	1	1.	1.36	4.74	0.	0.	0.	0.
time (sec)	N/A	0.661	12.923	0.408	0.	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	1302	1827	0	0	0	0
normalized size	1	1.	3.36	4.72	0.	0.	0.	0.
time (sec)	N/A	0.949	6.22	0.681	0.	0.	0.	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	1368	2504	0	0	0	0
normalized size	1	1.	3.01	5.52	0.	0.	0.	0.
time (sec)	N/A	1.318	6.267	0.682	0.	0.	0.	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	506	506	1203	1866	0	0	0	0
normalized size	1	1.	2.38	3.69	0.	0.	0.	0.
time (sec)	N/A	1.356	18.47	0.417	0.	0.	0.	0.

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	443	443	331	1629	0	0	0	0
normalized size	1	1.	0.75	3.68	0.	0.	0.	0.
time (sec)	N/A	1.003	6.31	0.519	0.	0.	0.	0.

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	445	445	1185	1623	0	0	0	0
normalized size	1	1.	2.66	3.65	0.	0.	0.	0.
time (sec)	N/A	1.01	17.78	0.404	0.	0.	0.	0.

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	392	392	330	1487	0	0	0	0
normalized size	1	1.	0.84	3.79	0.	0.	0.	0.
time (sec)	N/A	0.739	6.583	0.569	0.	0.	0.	0.

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	338	338	386	1750	0	0	0	0
normalized size	1	1.	1.14	5.18	0.	0.	0.	0.
time (sec)	N/A	0.765	11.461	0.486	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	1302	1827	0	0	0	0
normalized size	1	1.	3.36	4.72	0.	0.	0.	0.
time (sec)	N/A	1.044	6.243	0.51	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	1368	2504	0	0	0	0
normalized size	1	1.	3.01	5.52	0.	0.	0.	0.
time (sec)	N/A	1.409	6.286	0.659	0.	0.	0.	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	522	522	1431	2789	0	0	0	0
normalized size	1	1.	2.74	5.34	0.	0.	0.	0.
time (sec)	N/A	1.775	6.318	0.859	0.	0.	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	414	479	622	0	0	0	0
normalized size	1	1.09	1.26	1.64	0.	0.	0.	0.
time (sec)	N/A	0.736	4.187	0.59	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	132	159	0	0	0	0
normalized size	1	1.	1.14	1.37	0.	0.	0.	0.
time (sec)	N/A	0.068	0.819	0.499	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	170	123	0	0	0	0
normalized size	1	1.	1.56	1.13	0.	0.	0.	0.
time (sec)	N/A	0.071	1.337	0.453	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	211	612	0	0	0	0
normalized size	1	1.	0.94	2.73	0.	0.	0.	0.
time (sec)	N/A	0.234	4.653	0.494	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	371	883	0	0	0	0
normalized size	1	1.	1.35	3.22	0.	0.	0.	0.
time (sec)	N/A	0.404	13.608	0.56	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	465	465	1201	1665	0	0	0	0
normalized size	1	1.	2.58	3.58	0.	0.	0.	0.
time (sec)	N/A	0.986	6.216	0.396	0.	0.	0.	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	985	1206	0	0	0	0
normalized size	1	1.	2.55	3.12	0.	0.	0.	0.
time (sec)	N/A	0.503	16.435	0.515	0.	0.	0.	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	196	809	0	0	0	0
normalized size	1	1.	0.74	3.04	0.	0.	0.	0.
time (sec)	N/A	0.335	4.609	0.471	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	202	830	0	0	0	0
normalized size	1	1.	0.76	3.11	0.	0.	0.	0.
time (sec)	N/A	0.388	5.28	0.563	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	1233	1452	0	0	0	0
normalized size	1	1.	4.33	5.09	0.	0.	0.	0.
time (sec)	N/A	0.461	6.238	0.609	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	357	357	1269	1781	0	0	0	0
normalized size	1	1.	3.55	4.99	0.	0.	0.	0.
time (sec)	N/A	0.724	6.316	0.414	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	1314	2478	0	0	0	0
normalized size	1	1.	3.03	5.72	0.	0.	0.	0.
time (sec)	N/A	1.057	6.361	0.599	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	497	497	1282	3911	0	0	0	0
normalized size	1	1.	2.58	7.87	0.	0.	0.	0.
time (sec)	N/A	1.064	6.281	0.424	0.	0.	0.	0.

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	342	342	277	1782	0	0	0	0
normalized size	1	1.	0.81	5.21	0.	0.	0.	0.
time (sec)	N/A	0.61	6.125	0.557	0.	0.	0.	0.

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	1273	2417	0	0	0	0
normalized size	1	1.	3.55	6.73	0.	0.	0.	0.
time (sec)	N/A	0.642	6.23	0.425	0.	0.	0.	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	1296	2741	0	0	0	0
normalized size	1	1.	3.4	7.19	0.	0.	0.	0.
time (sec)	N/A	0.732	6.251	0.638	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	1321	3693	0	0	0	0
normalized size	1	1.	3.32	9.28	0.	0.	0.	0.
time (sec)	N/A	0.813	6.372	0.525	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	473	473	1351	4189	0	0	0	0
normalized size	1	1.	2.86	8.86	0.	0.	0.	0.
time (sec)	N/A	1.173	6.463	0.512	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	131	115	0	0	0	0
normalized size	1	1.	4.09	3.59	0.	0.	0.	0.
time (sec)	N/A	0.059	3.632	0.462	0.	0.	0.	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	156	107	0	0	0	0
normalized size	1	1.	6.24	4.28	0.	0.	0.	0.
time (sec)	N/A	0.053	0.925	0.497	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	143	119	0	0	0	0
normalized size	1	1.	2.55	2.12	0.	0.	0.	0.
time (sec)	N/A	0.117	1.051	0.499	0.	0.	0.	0.

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	153	132	0	0	0	0
normalized size	1	1.	3.12	2.69	0.	0.	0.	0.
time (sec)	N/A	0.11	1.456	0.447	0.	0.	0.	0.

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	140	116	0	0	0	0
normalized size	1	1.	2.41	2.	0.	0.	0.	0.
time (sec)	N/A	0.053	1.083	0.458	0.	0.	0.	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	144	125	0	0	0	0
normalized size	1	1.	2.4	2.08	0.	0.	0.	0.
time (sec)	N/A	0.066	1.029	0.494	0.	0.	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	144	123	0	0	0	0
normalized size	1	1.	1.71	1.46	0.	0.	0.	0.
time (sec)	N/A	0.124	1.229	0.477	0.	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	153	137	0	0	0	0
normalized size	1	1.	1.87	1.67	0.	0.	0.	0.
time (sec)	N/A	0.115	1.031	0.458	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	150	122	0	0	0	0
normalized size	1	1.	2.78	2.26	0.	0.	0.	0.
time (sec)	N/A	0.101	0.51	0.468	0.	0.	0.	0.

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	158	109	0	0	0	0
normalized size	1	1.	3.36	2.32	0.	0.	0.	0.
time (sec)	N/A	0.104	0.39	0.381	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	145	121	0	0	0	0
normalized size	1	1.	4.26	3.56	0.	0.	0.	0.
time (sec)	N/A	0.055	0.529	0.376	0.	0.	0.	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	155	129	0	0	0	0
normalized size	1	1.	5.74	4.78	0.	0.	0.	0.
time (sec)	N/A	0.055	0.471	0.467	0.	0.	0.	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	154	127	0	0	0	0
normalized size	1	1.	1.92	1.59	0.	0.	0.	0.
time (sec)	N/A	0.105	0.6	0.481	0.	0.	0.	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	146	107	0	0	0	0
normalized size	1	1.	1.78	1.3	0.	0.	0.	0.
time (sec)	N/A	0.106	0.468	0.279	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	160	98	0	0	0	0
normalized size	1	1.	2.58	1.58	0.	0.	0.	0.
time (sec)	N/A	0.056	0.634	0.256	0.	0.	0.	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	155	128	0	0	0	0
normalized size	1	1.	2.58	2.13	0.	0.	0.	0.
time (sec)	N/A	0.056	0.441	0.429	0.	0.	0.	0.

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	175	142	0	0	0	0
normalized size	1	1.	2.27	1.84	0.	0.	0.	0.
time (sec)	N/A	0.049	2.841	0.388	0.	0.	0.	0.

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	142	132	0	0	0	0
normalized size	1	1.	1.89	1.76	0.	0.	0.	0.
time (sec)	N/A	0.059	0.662	0.428	0.	0.	0.	0.

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	147	144	0	0	0	0
normalized size	1	1.	1.48	1.45	0.	0.	0.	0.
time (sec)	N/A	0.116	1.691	0.399	0.	0.	0.	0.

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	175	161	0	0	0	0
normalized size	1	1.	1.73	1.59	0.	0.	0.	0.
time (sec)	N/A	0.102	2.174	0.428	0.	0.	0.	0.

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	117	144	0	0	0	0
normalized size	1	1.	1.6	1.97	0.	0.	0.	0.
time (sec)	N/A	0.047	1.365	0.374	0.	0.	0.	0.

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	119	153	0	0	0	0
normalized size	1	1.	1.59	2.04	0.	0.	0.	0.
time (sec)	N/A	0.049	0.814	0.4	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	135	158	0	0	0	0
normalized size	1	1.	1.36	1.6	0.	0.	0.	0.
time (sec)	N/A	0.1	0.961	0.435	0.	0.	0.	0.

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	115	168	0	0	0	0
normalized size	1	1.	1.19	1.73	0.	0.	0.	0.
time (sec)	N/A	0.103	0.669	0.436	0.	0.	0.	0.

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	194	159	0	0	0	0
normalized size	1	1.	1.96	1.61	0.	0.	0.	0.
time (sec)	N/A	0.099	0.769	0.519	0.	0.	0.	0.

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	144	142	0	0	0	0
normalized size	1	1.	1.48	1.46	0.	0.	0.	0.
time (sec)	N/A	0.098	0.221	0.394	0.	0.	0.	0.

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	149	154	0	0	0	0
normalized size	1	1.	1.94	2.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.501	0.386	0.	0.	0.	0.

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	194	164	0	0	0	0
normalized size	1	1.	2.46	2.08	0.	0.	0.	0.
time (sec)	N/A	0.054	0.623	0.408	0.	0.	0.	0.

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	119	168	0	0	0	0
normalized size	1	1.	1.25	1.77	0.	0.	0.	0.
time (sec)	N/A	0.1	0.501	0.385	0.	0.	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	121	178	0	0	0	0
normalized size	1	1.	1.25	1.84	0.	0.	0.	0.
time (sec)	N/A	0.101	0.193	0.426	0.	0.	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	140	152	0	0	0	0
normalized size	1	1.	1.82	1.97	0.	0.	0.	0.
time (sec)	N/A	0.053	0.15	0.365	0.	0.	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	117	164	0	0	0	0
normalized size	1	1.	1.56	2.19	0.	0.	0.	0.
time (sec)	N/A	0.053	0.295	0.405	0.	0.	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	4614	0	0	0	0	0
normalized size	1	1.	26.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	21.921	0.139	0.	0.	0.	0.

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	4613	0	0	0	0	0
normalized size	1	1.	26.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	21.532	0.155	0.	0.	0.	0.

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	4605	0	0	0	0	0
normalized size	1	1.	26.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	21.476	0.201	0.	0.	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	176	176	4608	0	0	0	0	0
normalized size	1	1.	26.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	21.352	0.188	0.	0.	0.	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	31.372	0.365	0.	0.	0.	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	78.186	0.378	0.	0.	0.	0.

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	18.327	0.333	0.	0.	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	8.511	0.32	0.	0.	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	2.375	0.372	0.	0.	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	1.742	0.315	0.	0.	0.	0.

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.509	0.372	0.	0.	0.	0.

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	79.162	0.32	0.	0.	0.	0.

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	26.707	0.335	0.	0.	0.	0.

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.054	82.658	0.395	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	97	502	0	0	0	0
normalized size	1	1.	0.64	3.32	0.	0.	0.	0.
time (sec)	N/A	0.112	0.322	6.905	0.	0.	0.	0.

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	85	397	0	0	0	0
normalized size	1	1.	0.69	3.23	0.	0.	0.	0.
time (sec)	N/A	0.096	0.217	6.014	0.	0.	0.	0.

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	71	148	0	0	0	0
normalized size	1	1.	0.73	1.53	0.	0.	0.	0.
time (sec)	N/A	0.084	0.103	2.783	0.	0.	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	52	152	0	0	0	0
normalized size	1	1.	0.69	2.03	0.	0.	0.	0.
time (sec)	N/A	0.076	0.065	2.369	0.	0.	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	76	229	0	0	0	0
normalized size	1	1.	0.75	2.27	0.	0.	0.	0.
time (sec)	N/A	0.087	0.127	2.537	0.	0.	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	88	262	0	0	0	0
normalized size	1	1.	0.69	2.06	0.	0.	0.	0.
time (sec)	N/A	0.103	0.301	2.684	0.	0.	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	99	290	0	0	0	0
normalized size	1	1.	0.66	1.92	0.	0.	0.	0.
time (sec)	N/A	0.114	0.523	2.556	0.	0.	0.	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	139	689	0	0	0	0
normalized size	1	1.	0.7	3.44	0.	0.	0.	0.
time (sec)	N/A	0.166	0.781	8.171	0.	0.	0.	0.

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	126	660	0	0	0	0
normalized size	1	1.	0.72	3.77	0.	0.	0.	0.
time (sec)	N/A	0.157	1.217	7.854	0.	0.	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	93	514	0	0	0	0
normalized size	1	1.	0.69	3.81	0.	0.	0.	0.
time (sec)	N/A	0.136	0.312	7.66	0.	0.	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	83	202	0	0	0	0
normalized size	1	1.	0.77	1.87	0.	0.	0.	0.
time (sec)	N/A	0.13	0.186	3.987	0.	0.	0.	0.

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	283	0	0	0	0
normalized size	1	1.	0.78	2.53	0.	0.	0.	0.
time (sec)	N/A	0.132	0.17	2.734	0.	0.	0.	0.

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	100	321	0	0	0	0
normalized size	1	1.	0.71	2.28	0.	0.	0.	0.
time (sec)	N/A	0.144	0.42	3.155	0.	0.	0.	0.

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	120	362	0	0	0	0
normalized size	1	1.	0.69	2.07	0.	0.	0.	0.
time (sec)	N/A	0.164	0.636	2.925	0.	0.	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	135	398	0	0	0	0
normalized size	1	1.	0.68	1.99	0.	0.	0.	0.
time (sec)	N/A	0.174	1.025	3.279	0.	0.	0.	0.

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	191	847	0	0	0	0
normalized size	1	1.	0.82	3.62	0.	0.	0.	0.
time (sec)	N/A	0.269	1.079	10.832	0.	0.	0.	0.

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	134	738	0	0	0	0
normalized size	1	1.	0.71	3.9	0.	0.	0.	0.
time (sec)	N/A	0.238	1.519	9.477	0.	0.	0.	0.

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	106	631	0	0	0	0
normalized size	1	1.	0.66	3.94	0.	0.	0.	0.
time (sec)	N/A	0.231	0.444	6.875	0.	0.	0.	0.

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	108	303	0	0	0	0
normalized size	1	1.	0.65	1.83	0.	0.	0.	0.
time (sec)	N/A	0.229	0.552	3.428	0.	0.	0.	0.

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	106	376	0	0	0	0
normalized size	1	1.	0.68	2.41	0.	0.	0.	0.
time (sec)	N/A	0.223	0.431	3.22	0.	0.	0.	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	132	421	0	0	0	0
normalized size	1	1.	0.66	2.12	0.	0.	0.	0.
time (sec)	N/A	0.248	0.876	3.052	0.	0.	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	159	470	0	0	0	0
normalized size	1	1.	0.68	2.01	0.	0.	0.	0.
time (sec)	N/A	0.277	1.121	2.982	0.	0.	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	167	452	0	0	0	0
normalized size	1	1.	0.89	2.4	0.	0.	0.	0.
time (sec)	N/A	0.547	2.796	7.937	0.	0.	0.	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	86	354	0	0	0	0
normalized size	1	1.	0.74	3.03	0.	0.	0.	0.
time (sec)	N/A	0.209	4.256	3.911	0.	0.	0.	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	63	150	0	0	0	0
normalized size	1	1.	1.29	3.06	0.	0.	0.	0.
time (sec)	N/A	0.131	0.292	2.789	0.	0.	0.	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	49	188	0	0	0	0
normalized size	1	1.	0.53	2.02	0.	0.	0.	0.
time (sec)	N/A	0.187	0.219	2.5	0.	0.	0.	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	178	227	0	0	0	0
normalized size	1	1.	1.32	1.68	0.	0.	0.	0.
time (sec)	N/A	0.236	5.794	3.253	0.	0.	0.	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	198	516	0	0	0	0
normalized size	1	1.	1.15	3.	0.	0.	0.	0.
time (sec)	N/A	0.391	6.125	3.504	0.	0.	0.	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	661	1008	0	0	0	0
normalized size	1	1.	1.94	2.96	0.	0.	0.	0.
time (sec)	N/A	0.966	6.712	14.516	0.	0.	0.	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	355	874	0	0	0	0
normalized size	1	1.	1.28	3.16	0.	0.	0.	0.
time (sec)	N/A	0.711	4.264	9.095	0.	0.	0.	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	217	590	612	0	0	0	0
normalized size	1	1.	2.72	2.82	0.	0.	0.	0.
time (sec)	N/A	0.435	6.604	5.45	0.	0.	0.	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	208	208	580	713	0	0	0	0
normalized size	1	1.	2.79	3.43	0.	0.	0.	0.
time (sec)	N/A	0.42	6.598	7.491	0.	0.	0.	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	251	794	0	0	0	0
normalized size	1	1.	1.13	3.56	0.	0.	0.	0.
time (sec)	N/A	0.412	5.146	6.832	0.	0.	0.	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	323	815	0	0	0	0
normalized size	1	1.	1.32	3.33	0.	0.	0.	0.
time (sec)	N/A	0.474	6.635	9.097	0.	0.	0.	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	455	455	753	2128	0	0	0	0
normalized size	1	1.	1.65	4.68	0.	0.	0.	0.
time (sec)	N/A	1.451	6.78	20.158	0.	0.	0.	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	388	388	729	1992	0	0	0	0
normalized size	1	1.	1.88	5.13	0.	0.	0.	0.
time (sec)	N/A	1.032	6.821	14.863	0.	0.	0.	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	321	700	1176	0	0	0	0
normalized size	1	1.	2.18	3.66	0.	0.	0.	0.
time (sec)	N/A	0.77	6.662	7.448	0.	0.	0.	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	401	1736	0	0	0	0
normalized size	1	1.	1.26	5.48	0.	0.	0.	0.
time (sec)	N/A	0.752	6.612	11.668	0.	0.	0.	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	671	1836	0	0	0	0
normalized size	1	1.	2.22	6.08	0.	0.	0.	0.
time (sec)	N/A	0.696	6.64	11.98	0.	0.	0.	0.

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	282	1914	0	0	0	0
normalized size	1	1.	0.88	6.	0.	0.	0.	0.
time (sec)	N/A	0.685	6.302	12.25	0.	0.	0.	0.

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	369	369	353	1563	0	0	0	0
normalized size	1	1.	0.96	4.24	0.	0.	0.	0.
time (sec)	N/A	0.753	12.021	0.586	0.	0.	0.	0.

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	311	311	301	888	0	0	0	0
normalized size	1	1.	0.97	2.86	0.	0.	0.	0.
time (sec)	N/A	0.502	10.498	0.626	0.	0.	0.	0.

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	215	797	0	0	0	0
normalized size	1	1.	0.8	2.96	0.	0.	0.	0.
time (sec)	N/A	0.36	5.685	0.546	0.	0.	0.	0.

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	148	199	0	0	0	0
normalized size	1	1.	0.95	1.28	0.	0.	0.	0.
time (sec)	N/A	0.143	1.317	0.616	0.	0.	0.	0.

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	431	431	403	806	0	0	0	0
normalized size	1	1.	0.94	1.87	0.	0.	0.	0.
time (sec)	N/A	0.65	13.535	0.753	0.	0.	0.	0.

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	498	498	1113	1241	0	0	0	0
normalized size	1	1.	2.23	2.49	0.	0.	0.	0.
time (sec)	N/A	0.991	18.046	0.546	0.	0.	0.	0.

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	441	1835	0	0	0	0
normalized size	1	1.	1.03	4.3	0.	0.	0.	0.
time (sec)	N/A	1.05	13.645	0.522	0.	0.	0.	0.

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	345	1547	0	0	0	0
normalized size	1	1.	0.95	4.24	0.	0.	0.	0.
time (sec)	N/A	0.759	11.49	0.503	0.	0.	0.	0.

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	291	1083	0	0	0	0
normalized size	1	1.	0.92	3.42	0.	0.	0.	0.
time (sec)	N/A	0.526	7.154	0.615	0.	0.	0.	0.

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	397	397	643	1191	0	0	0	0
normalized size	1	1.	1.62	3.	0.	0.	0.	0.
time (sec)	N/A	0.556	17.449	0.581	0.	0.	0.	0.

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	324	1005	0	0	0	0
normalized size	1	1.	0.74	2.31	0.	0.	0.	0.
time (sec)	N/A	0.715	12.239	0.632	0.	0.	0.	0.

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	493	853	1423	0	0	0	0
normalized size	1	1.	1.73	2.89	0.	0.	0.	0.
time (sec)	N/A	1.27	17.729	0.557	0.	0.	0.	0.

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	568	568	969	1691	0	0	0	0
normalized size	1	1.	1.71	2.98	0.	0.	0.	0.
time (sec)	N/A	1.356	17.591	0.488	0.	0.	0.	0.

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	494	494	521	2512	0	0	0	0
normalized size	1	1.	1.05	5.09	0.	0.	0.	0.
time (sec)	N/A	1.518	16.362	0.662	0.	0.	0.	0.

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	427	427	443	1835	0	0	0	0
normalized size	1	1.	1.04	4.3	0.	0.	0.	0.
time (sec)	N/A	1.167	13.973	0.569	0.	0.	0.	0.

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	378	378	376	1758	0	0	0	0
normalized size	1	1.	0.99	4.65	0.	0.	0.	0.
time (sec)	N/A	0.865	15.373	0.533	0.	0.	0.	0.

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	401	1493	0	0	0	0
normalized size	1	1.	0.89	3.3	0.	0.	0.	0.
time (sec)	N/A	0.824	11.895	0.606	0.	0.	0.	0.

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	505	505	740	1631	0	0	0	0
normalized size	1	1.	1.47	3.23	0.	0.	0.	0.
time (sec)	N/A	1.107	16.583	0.415	0.	0.	0.	0.

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	503	503	423	1631	0	0	0	0
normalized size	1	1.	0.84	3.24	0.	0.	0.	0.
time (sec)	N/A	1.101	14.494	0.557	0.	0.	0.	0.

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	566	566	978	1868	0	0	0	0
normalized size	1	1.	1.73	3.3	0.	0.	0.	0.
time (sec)	N/A	1.451	17.367	0.638	0.	0.	0.	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	638	638	1642	2327	0	0	0	0
normalized size	1	1.	2.57	3.65	0.	0.	0.	0.
time (sec)	N/A	1.883	16.754	0.665	0.	0.	0.	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	314	314	322	891	0	0	0	0
normalized size	1	1.	1.03	2.84	0.	0.	0.	0.
time (sec)	N/A	0.485	13.299	0.551	0.	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	296	620	0	0	0	0
normalized size	1	1.	1.12	2.35	0.	0.	0.	0.
time (sec)	N/A	0.309	12.108	0.519	0.	0.	0.	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	103	125	0	0	0	0
normalized size	1	1.	0.8	0.97	0.	0.	0.	0.
time (sec)	N/A	0.131	0.78	0.625	0.	0.	0.	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	148	143	0	0	0	0
normalized size	1	1.	1.09	1.05	0.	0.	0.	0.
time (sec)	N/A	0.132	1.675	0.648	0.	0.	0.	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	474	474	5017	630	0	0	0	0
normalized size	1	1.	10.58	1.33	0.	0.	0.	0.
time (sec)	N/A	0.796	23.816	0.753	0.	0.	0.	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	505	505	1153	1248	0	0	0	0
normalized size	1	1.	2.28	2.47	0.	0.	0.	0.
time (sec)	N/A	0.954	18.617	0.533	0.	0.	0.	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	397	397	440	1789	0	0	0	0
normalized size	1	1.	1.11	4.51	0.	0.	0.	0.
time (sec)	N/A	0.823	14.006	0.487	0.	0.	0.	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	369	1460	0	0	0	0
normalized size	1	1.	1.14	4.49	0.	0.	0.	0.
time (sec)	N/A	0.549	11.042	0.541	0.	0.	0.	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	237	832	0	0	0	0
normalized size	1	1.	0.77	2.71	0.	0.	0.	0.
time (sec)	N/A	0.479	7.635	0.636	0.	0.	0.	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	235	811	0	0	0	0
normalized size	1	1.	0.77	2.65	0.	0.	0.	0.
time (sec)	N/A	0.419	3.649	0.657	0.	0.	0.	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	447	447	1175	1214	0	0	0	0
normalized size	1	1.	2.63	2.72	0.	0.	0.	0.
time (sec)	N/A	0.595	17.735	0.608	0.	0.	0.	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	525	525	1033	1675	0	0	0	0
normalized size	1	1.	1.97	3.19	0.	0.	0.	0.
time (sec)	N/A	1.091	15.284	0.528	0.	0.	0.	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	513	513	546	4197	0	0	0	0
normalized size	1	1.	1.06	8.18	0.	0.	0.	0.
time (sec)	N/A	1.302	17.232	0.725	0.	0.	0.	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	438	438	525	3701	0	0	0	0
normalized size	1	1.	1.2	8.45	0.	0.	0.	0.
time (sec)	N/A	0.925	16.352	0.48	0.	0.	0.	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	421	421	471	2743	0	0	0	0
normalized size	1	1.	1.12	6.52	0.	0.	0.	0.
time (sec)	N/A	0.844	13.923	0.667	0.	0.	0.	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	399	399	455	2419	0	0	0	0
normalized size	1	1.	1.14	6.06	0.	0.	0.	0.
time (sec)	N/A	0.758	13.232	0.645	0.	0.	0.	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	359	1793	0	0	0	0
normalized size	1	1.	0.94	4.69	0.	0.	0.	0.
time (sec)	N/A	0.729	7.815	0.61	0.	0.	0.	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	557	557	1716	3921	0	0	0	0
normalized size	1	1.	3.08	7.04	0.	0.	0.	0.
time (sec)	N/A	1.213	13.955	0.648	0.	0.	0.	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	242	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.674	1.643	1.476	0.	0.	0.	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	197	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.317	0.808	1.426	0.	0.	0.	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	168	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.316	1.218	0.	0.	0.	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	112	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.153	1.27	0.	0.	0.	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	190	190	6703	0	0	0	0	0
normalized size	1	1.	35.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	24.495	1.111	0.	0.	0.	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	294	294	7214	0	0	0	0	0
normalized size	1	1.	24.54	0.	0.	0.	0.	0.
time (sec)	N/A	0.354	26.08	0.606	0.	0.	0.	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	222	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.424	0.793	1.984	0.	0.	0.	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	159	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	0.309	1.473	0.	0.	0.	0.

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	107	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.183	1.573	0.	0.	0.	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	47	67	0	103	0	0
normalized size	1	1.	1.81	2.58	0.	3.96	0.	0.
time (sec)	N/A	0.078	0.067	0.346	0.	1.686	0.	0.

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	64	67	0	100	0	0
normalized size	1	1.	0.98	1.03	0.	1.54	0.	0.
time (sec)	N/A	0.1	0.068	0.5	0.	1.552	0.	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	51	61	61	87	41
normalized size	1	1.	0.78	1.38	1.65	1.65	2.35	1.11
time (sec)	N/A	0.02	0.051	0.044	1.051	1.389	0.396	1.748

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	31	150	194	167	333	119
normalized size	1	1.	1.19	5.77	7.46	6.42	12.81	4.58
time (sec)	N/A	0.031	0.185	0.048	1.056	1.485	3.548	1.825

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	74	193	66	0	1850
normalized size	1	1.	1.	2.64	6.89	2.36	0.	66.07
time (sec)	N/A	0.045	0.166	0.418	2.171	1.312	0.	46.142

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	27	48	155	135	80	63
normalized size	1	1.	1.04	1.85	5.96	5.19	3.08	2.42
time (sec)	N/A	0.033	0.103	0.06	1.069	1.34	3.662	1.327

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	45	48	124	95	0	0
normalized size	1	1.	1.61	1.71	4.43	3.39	0.	0.
time (sec)	N/A	0.042	0.103	0.658	1.908	1.379	0.	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	33	43	0	92	0	47
normalized size	1	1.	1.27	1.65	0.	3.54	0.	1.81
time (sec)	N/A	0.016	0.043	0.904	0.	1.345	0.	2.447

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	48	0	169	0	80
normalized size	1	1.	1.	1.71	0.	6.04	0.	2.86
time (sec)	N/A	0.042	0.067	0.828	0.	1.345	0.	3.49

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	164	0	0	0	0	0
normalized size	1	1.	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.592	0.246	0.	0.	0.	0.

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	213	0	0	0	0	0
normalized size	1	1.	2.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	3.169	0.276	0.	0.	0.	0.

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	133	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.258	0.328	0.	0.	0.	0.

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	197	0	0	0	0	0
normalized size	1	1.	1.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	1.304	0.219	0.	0.	0.	0.

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	64	117	0	437	0	146
normalized size	1	1.	1.02	1.86	0.	6.94	0.	2.32
time (sec)	N/A	0.093	0.116	0.11	0.	1.524	0.	1.192

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	51	0	53	0	68
normalized size	1	1.	1.	2.32	0.	2.41	0.	3.09
time (sec)	N/A	0.031	0.092	0.052	0.	1.363	0.	1.26

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	31	39	70	119	56	97
normalized size	1	1.	0.66	0.83	1.49	2.53	1.19	2.06
time (sec)	N/A	0.067	0.054	0.093	1.576	1.391	7.038	1.282

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	171	0	0	0	0
normalized size	1	1.	1.	2.95	0.	0.	0.	0.
time (sec)	N/A	0.045	0.068	2.642	0.	0.	0.	0.

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	259	0	0	0	0	0
normalized size	1	1.	1.13	0.	0.	0.	0.	0.
time (sec)	N/A	0.223	1.87	0.221	0.	0.	0.	0.

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	253	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	1.742	0.207	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	189	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.413	0.404	0.	0.	0.	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	226	226	188	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.185	0.418	0.321	0.	0.	0.	0.

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	100	299	0	0	0	0
normalized size	1	1.	0.6	1.78	0.	0.	0.	0.
time (sec)	N/A	0.142	0.517	3.447	0.	0.	0.	0.

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	91	271	0	0	0	0
normalized size	1	1.	0.65	1.95	0.	0.	0.	0.
time (sec)	N/A	0.117	0.282	3.091	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	75	238	0	0	0	0
normalized size	1	1.	0.69	2.2	0.	0.	0.	0.
time (sec)	N/A	0.084	0.103	2.964	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	55	161	0	0	0	0
normalized size	1	1.	0.69	2.01	0.	0.	0.	0.
time (sec)	N/A	0.087	0.076	2.906	0.	0.	0.	0.

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	73	213	0	0	0	0
normalized size	1	1.	0.7	2.03	0.	0.	0.	0.
time (sec)	N/A	0.117	0.172	3.575	0.	0.	0.	0.

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	85	453	0	0	0	0
normalized size	1	1.	0.62	3.33	0.	0.	0.	0.
time (sec)	N/A	0.131	0.24	4.171	0.	0.	0.	0.

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	107	575	0	0	0	0
normalized size	1	1.	0.63	3.4	0.	0.	0.	0.
time (sec)	N/A	0.163	0.448	7.365	0.	0.	0.	0.

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	103	301	0	0	0	0
normalized size	1	1.	0.61	1.78	0.	0.	0.	0.
time (sec)	N/A	0.139	0.291	3.297	0.	0.	0.	0.

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	88	273	0	0	0	0
normalized size	1	1.	0.63	1.95	0.	0.	0.	0.
time (sec)	N/A	0.104	0.128	2.857	0.	0.	0.	0.

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	76	240	0	0	0	0
normalized size	1	1.	0.68	2.14	0.	0.	0.	0.
time (sec)	N/A	0.104	0.04	3.18	0.	0.	0.	0.

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	57	163	0	0	0	0
normalized size	1	1.	0.69	1.96	0.	0.	0.	0.
time (sec)	N/A	0.101	0.064	2.713	0.	0.	0.	0.

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	73	215	0	0	0	0
normalized size	1	1.	0.66	1.95	0.	0.	0.	0.
time (sec)	N/A	0.12	0.158	3.645	0.	0.	0.	0.

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	87	455	0	0	0	0
normalized size	1	1.	0.62	3.23	0.	0.	0.	0.
time (sec)	N/A	0.141	0.142	4.066	0.	0.	0.	0.

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	107	576	0	0	0	0
normalized size	1	1.	0.61	3.31	0.	0.	0.	0.
time (sec)	N/A	0.157	0.265	7.467	0.	0.	0.	0.

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	100	301	0	0	0	0
normalized size	1	1.	0.58	1.76	0.	0.	0.	0.
time (sec)	N/A	0.12	0.059	3.313	0.	0.	0.	0.

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	89	273	0	0	0	0
normalized size	1	1.	0.61	1.88	0.	0.	0.	0.
time (sec)	N/A	0.123	0.06	3.227	0.	0.	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	78	240	0	0	0	0
normalized size	1	1.	0.67	2.07	0.	0.	0.	0.
time (sec)	N/A	0.114	0.041	2.875	0.	0.	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	54	163	0	0	0	0
normalized size	1	1.	0.64	1.92	0.	0.	0.	0.
time (sec)	N/A	0.099	0.097	2.623	0.	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	73	215	0	0	0	0
normalized size	1	1.	0.65	1.92	0.	0.	0.	0.
time (sec)	N/A	0.12	0.191	3.265	0.	0.	0.	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	87	455	0	0	0	0
normalized size	1	1.	0.61	3.18	0.	0.	0.	0.
time (sec)	N/A	0.138	0.182	3.306	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	102	578	0	0	0	0
normalized size	1	1.	0.58	3.28	0.	0.	0.	0.
time (sec)	N/A	0.162	0.236	7.057	0.	0.	0.	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	101	298	0	0	0	0
normalized size	1	1.	0.58	1.72	0.	0.	0.	0.
time (sec)	N/A	0.135	0.34	3.193	0.	0.	0.	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	88	270	0	0	0	0
normalized size	1	1.	0.61	1.88	0.	0.	0.	0.
time (sec)	N/A	0.113	0.145	3.006	0.	0.	0.	0.

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	78	237	0	0	0	0
normalized size	1	1.	0.69	2.1	0.	0.	0.	0.
time (sec)	N/A	0.093	0.049	2.878	0.	0.	0.	0.

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	160	0	0	0	0
normalized size	1	1.	0.66	1.95	0.	0.	0.	0.
time (sec)	N/A	0.068	0.047	2.728	0.	0.	0.	0.

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	73	212	0	0	0	0
normalized size	1	1.	0.69	2.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.092	3.559	0.	0.	0.	0.

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	84	405	0	0	0	0
normalized size	1	1.	0.62	3.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.147	6.625	0.	0.	0.	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	101	578	0	0	0	0
normalized size	1	1.	0.6	3.44	0.	0.	0.	0.
time (sec)	N/A	0.154	0.225	7.589	0.	0.	0.	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	104	301	0	0	0	0
normalized size	1	1.	0.59	1.71	0.	0.	0.	0.
time (sec)	N/A	0.135	0.143	2.817	0.	0.	0.	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	88	273	0	0	0	0
normalized size	1	1.	0.6	1.86	0.	0.	0.	0.
time (sec)	N/A	0.113	0.136	2.552	0.	0.	0.	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	75	240	0	0	0	0
normalized size	1	1.	0.65	2.07	0.	0.	0.	0.
time (sec)	N/A	0.094	0.092	2.651	0.	0.	0.	0.

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	163	0	0	0	0
normalized size	1	1.	0.67	1.92	0.	0.	0.	0.
time (sec)	N/A	0.077	0.047	2.652	0.	0.	0.	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	76	215	0	0	0	0
normalized size	1	1.	0.68	1.92	0.	0.	0.	0.
time (sec)	N/A	0.089	0.066	3.102	0.	0.	0.	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	87	455	0	0	0	0
normalized size	1	1.	0.62	3.25	0.	0.	0.	0.
time (sec)	N/A	0.127	0.065	3.389	0.	0.	0.	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	104	578	0	0	0	0
normalized size	1	1.	0.61	3.38	0.	0.	0.	0.
time (sec)	N/A	0.16	0.105	7.915	0.	0.	0.	0.

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	104	301	0	0	0	0
normalized size	1	1.	0.59	1.71	0.	0.	0.	0.
time (sec)	N/A	0.134	0.088	3.22	0.	0.	0.	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	91	273	0	0	0	0
normalized size	1	1.	0.62	1.86	0.	0.	0.	0.
time (sec)	N/A	0.114	0.077	3.027	0.	0.	0.	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	78	240	0	0	0	0
normalized size	1	1.	0.67	2.07	0.	0.	0.	0.
time (sec)	N/A	0.091	0.077	3.504	0.	0.	0.	0.

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	57	163	0	0	0	0
normalized size	1	1.	0.67	1.92	0.	0.	0.	0.
time (sec)	N/A	0.075	0.046	2.763	0.	0.	0.	0.

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	76	215	0	0	0	0
normalized size	1	1.	0.68	1.92	0.	0.	0.	0.
time (sec)	N/A	0.097	0.061	3.316	0.	0.	0.	0.

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	87	455	0	0	0	0
normalized size	1	1.	0.61	3.18	0.	0.	0.	0.
time (sec)	N/A	0.102	0.057	4.189	0.	0.	0.	0.

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	104	578	0	0	0	0
normalized size	1	1.	0.6	3.34	0.	0.	0.	0.
time (sec)	N/A	0.144	0.068	8.306	0.	0.	0.	0.

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	104	578	0	0	0	0
normalized size	1	1.	0.59	3.28	0.	0.	0.	0.
time (sec)	N/A	0.119	0.065	7.52	0.	0.	0.	0.

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	81	91	126	699	0	0
normalized size	1	1.	0.47	0.53	0.73	4.06	0.	0.
time (sec)	N/A	0.069	0.159	0.548	2.179	1.809	0.	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	69	74	92	639	0	0
normalized size	1	1.	0.51	0.54	0.68	4.7	0.	0.
time (sec)	N/A	0.055	0.113	0.457	2.131	1.737	0.	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	55	54	570	99	0
normalized size	1	1.	0.58	0.56	0.55	5.82	1.01	0.
time (sec)	N/A	0.022	0.112	0.439	2.15	1.666	66.148	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	39	54	508	46	0
normalized size	1	1.	0.71	0.66	0.92	8.61	0.78	0.
time (sec)	N/A	0.012	0.047	0.455	1.991	1.618	18.254	0.

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	40	54	124	589	0	0
normalized size	1	1.	0.67	0.9	2.07	9.82	0.	0.
time (sec)	N/A	0.026	0.032	0.384	2.048	1.987	0.	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	59	162	564	0	0
normalized size	1	1.	0.74	0.87	2.38	8.29	0.	0.
time (sec)	N/A	0.041	0.05	0.4	2.264	1.555	0.	0.

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	120	967	626	0	0
normalized size	1	1.	0.61	1.12	9.04	5.85	0.	0.
time (sec)	N/A	0.054	0.104	0.412	2.096	1.813	0.	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	76	139	1292	695	0	0
normalized size	1	1.	0.52	0.96	8.91	4.79	0.	0.
time (sec)	N/A	0.061	0.283	0.446	2.191	1.654	0.	0.

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	81	91	135	724	0	0
normalized size	1	1.	0.46	0.51	0.76	4.09	0.	0.
time (sec)	N/A	0.069	0.152	0.367	2.076	1.682	0.	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	69	74	100	657	0	0
normalized size	1	1.	0.49	0.53	0.71	4.69	0.	0.
time (sec)	N/A	0.056	0.112	0.394	2.052	1.804	0.	0.

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	58	55	58	583	0	0
normalized size	1	1.	0.57	0.54	0.57	5.77	0.	0.
time (sec)	N/A	0.023	0.04	0.444	2.084	1.64	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	39	54	516	0	0
normalized size	1	1.	0.69	0.64	0.89	8.46	0.	0.
time (sec)	N/A	0.013	0.063	0.237	1.914	1.707	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	40	54	128	594	0	0
normalized size	1	1.	0.65	0.87	2.06	9.58	0.	0.
time (sec)	N/A	0.026	0.042	0.237	1.897	1.961	0.	0.

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	50	59	166	572	0	0
normalized size	1	1.	0.71	0.84	2.37	8.17	0.	0.
time (sec)	N/A	0.041	0.052	0.207	2.056	1.627	0.	0.

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	65	121	1008	640	0	0
normalized size	1	1.	0.59	1.1	9.16	5.82	0.	0.
time (sec)	N/A	0.055	0.089	0.217	2.095	1.632	0.	0.

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	77	139	1339	714	0	0
normalized size	1	1.	0.52	0.93	8.99	4.79	0.	0.
time (sec)	N/A	0.063	0.037	0.28	2.191	1.711	0.	0.

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	81	91	149	748	0	0
normalized size	1	1.	0.43	0.49	0.8	4.	0.	0.
time (sec)	N/A	0.072	0.178	0.427	2.506	1.807	0.	0.

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	69	74	111	676	0	0
normalized size	1	1.	0.47	0.5	0.75	4.57	0.	0.
time (sec)	N/A	0.056	0.142	0.419	2.059	1.77	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	57	55	63	597	0	0
normalized size	1	1.	0.53	0.51	0.59	5.58	0.	0.
time (sec)	N/A	0.024	0.124	0.246	2.033	1.765	0.	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	39	54	524	0	0
normalized size	1	1.	0.65	0.6	0.83	8.06	0.	0.
time (sec)	N/A	0.014	0.077	0.223	1.904	1.701	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	40	54	134	599	0	0
normalized size	1	1.	0.61	0.82	2.03	9.08	0.	0.
time (sec)	N/A	0.026	0.06	0.185	1.821	2.081	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	59	171	581	0	0
normalized size	1	1.	0.68	0.8	2.31	7.85	0.	0.
time (sec)	N/A	0.043	0.078	0.224	1.986	1.621	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	65	121	1084	653	0	0
normalized size	1	1.	0.56	1.04	9.34	5.63	0.	0.
time (sec)	N/A	0.057	0.146	0.246	2.028	1.718	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	76	139	1431	733	0	0
normalized size	1	1.	0.48	0.89	9.11	4.67	0.	0.
time (sec)	N/A	0.064	0.18	0.257	2.078	1.963	0.	0.

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	69	74	92	645	0	0
normalized size	1	1.	0.51	0.54	0.68	4.74	0.	0.
time (sec)	N/A	0.056	0.098	0.325	2.041	1.683	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	57	55	54	576	0	0
normalized size	1	1.	0.58	0.56	0.55	5.88	0.	0.
time (sec)	N/A	0.024	0.079	0.33	2.039	1.73	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	39	54	514	46	0
normalized size	1	1.	0.71	0.66	0.92	8.71	0.78	0.
time (sec)	N/A	0.013	0.042	0.366	1.89	1.622	19.371	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	40	54	124	599	0	0
normalized size	1	1.	0.67	0.9	2.07	9.98	0.	0.
time (sec)	N/A	0.026	0.034	0.326	1.803	2.03	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	59	169	570	0	0
normalized size	1	1.	0.74	0.87	2.49	8.38	0.	0.
time (sec)	N/A	0.042	0.042	0.315	2.089	1.575	0.	0.

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	65	120	975	632	0	0
normalized size	1	1.	0.61	1.12	9.11	5.91	0.	0.
time (sec)	N/A	0.055	0.064	0.338	2.02	1.615	0.	0.

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	76	139	1292	701	0	0
normalized size	1	1.	0.52	0.96	8.91	4.83	0.	0.
time (sec)	N/A	0.065	0.081	0.378	2.065	1.687	0.	0.

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	69	74	92	651	0	0
normalized size	1	1.	0.47	0.5	0.62	4.4	0.	0.
time (sec)	N/A	0.056	0.074	0.246	2.096	1.641	0.	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	57	55	54	582	0	0
normalized size	1	1.	0.53	0.51	0.5	5.44	0.	0.
time (sec)	N/A	0.023	0.086	0.262	1.965	1.637	0.	0.

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	42	39	54	520	0	0
normalized size	1	1.	0.65	0.6	0.83	8.	0.	0.
time (sec)	N/A	0.014	0.045	0.207	1.872	1.638	0.	0.

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	40	54	124	605	0	0
normalized size	1	1.	0.61	0.82	1.88	9.17	0.	0.
time (sec)	N/A	0.027	0.038	0.309	1.873	1.895	0.	0.

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	59	180	575	0	0
normalized size	1	1.	0.68	0.8	2.43	7.77	0.	0.
time (sec)	N/A	0.042	0.053	0.315	2.044	1.719	0.	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	65	120	998	637	0	0
normalized size	1	1.	0.56	1.03	8.6	5.49	0.	0.
time (sec)	N/A	0.056	0.062	0.274	2.047	1.716	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	76	139	1327	706	0	0
normalized size	1	1.	0.48	0.89	8.45	4.5	0.	0.
time (sec)	N/A	0.064	0.08	0.262	2.063	1.685	0.	0.

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	72	74	92	651	0	0
normalized size	1	1.	0.49	0.5	0.62	4.4	0.	0.
time (sec)	N/A	0.057	0.066	0.24	2.02	1.902	0.	0.

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	60	55	54	582	0	0
normalized size	1	1.	0.56	0.51	0.5	5.44	0.	0.
time (sec)	N/A	0.024	0.061	0.22	1.979	1.977	0.	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	39	54	520	0	0
normalized size	1	1.	0.69	0.6	0.83	8.	0.	0.
time (sec)	N/A	0.014	0.043	0.23	1.863	1.928	0.	0.

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	43	54	124	605	0	0
normalized size	1	1.	0.65	0.82	1.88	9.17	0.	0.
time (sec)	N/A	0.027	0.034	0.181	1.849	2.51	0.	0.

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	50	59	180	575	0	0
normalized size	1	1.	0.68	0.8	2.43	7.77	0.	0.
time (sec)	N/A	0.041	0.05	0.331	2.029	1.914	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	65	121	1022	637	0	0
normalized size	1	1.	0.56	1.04	8.81	5.49	0.	0.
time (sec)	N/A	0.056	0.075	0.342	2.129	1.97	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	76	139	1395	706	0	0
normalized size	1	1.	0.48	0.89	8.89	4.5	0.	0.
time (sec)	N/A	0.064	0.098	0.286	2.211	1.974	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.172	0.526	0.	0.	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	89	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.133	0.252	0.	0.	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	86	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.075	0.184	0.	0.	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	86	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.094	0.462	0.	0.	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	86	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.116	0.292	0.	0.	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	94	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.089	0.344	0.	0.	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.201	0.368	0.	0.	0.	0.

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	89	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.179	0.311	0.	0.	0.	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	86	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.047	0.204	0.	0.	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	87	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.012	0.257	0.	0.	0.	0.

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.022	0.355	0.	0.	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.103	0.335	0.	0.	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.175	0.341	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	89	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.02	0.29	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.014	0.241	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	85	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.065	0.25	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	89	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.145	0.29	0.	0.	0.	0.

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	89	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.134	0.297	0.	0.	0.	0.

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.166	0.327	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	89	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.082	0.421	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	85	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.116	0.213	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	86	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.14	0.244	0.	0.	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	89	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.18	0.25	0.	0.	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	89	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.179	0.305	0.	0.	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	130	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	0.242	1.993	0.	0.	0.	0.

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.282	1.819	0.	0.	0.	0.

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.225	1.497	0.	0.	0.	0.

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	0.158	1.506	0.	0.	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	109	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.18	1.197	0.	0.	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	109	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.175	1.366	0.	0.	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	118	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.162	1.596	0.	0.	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.16	1.182	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.403	0.356	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.321	0.394	0.	0.	0.	0.

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.237	0.421	0.	0.	0.	0.

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.214	0.424	0.	0.	0.	0.

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	133	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.246	0.39	0.	0.	0.	0.

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.208	0.379	0.	0.	0.	0.

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.214	0.415	0.	0.	0.	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.213	0.391	0.	0.	0.	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	140	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.476	0.224	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.289	0.231	0.	0.	0.	0.

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	0.285	0.217	0.	0.	0.	0.

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.287	0.215	0.	0.	0.	0.

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.286	0.207	0.	0.	0.	0.

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	140	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	0.316	0.285	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [503] had the largest ratio of [0.5]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	4	1.	19	0.21
2	A	6	4	1.	19	0.21
3	A	6	4	1.	19	0.21
4	A	5	4	1.	19	0.21
5	A	1	1	1.	17	0.059
6	A	2	1	1.	10	0.1
7	A	2	2	1.	17	0.118
8	A	4	4	1.	19	0.21
9	A	5	5	1.	19	0.263
10	A	5	4	1.	19	0.21
11	A	6	4	1.	19	0.21
12	A	6	4	1.	19	0.21
13	A	11	4	1.	21	0.19
14	A	9	4	1.	21	0.19
15	A	9	4	1.	21	0.19
16	A	2	2	1.21	19	0.105
17	A	1	1	1.	12	0.083
18	A	3	3	1.	19	0.158
19	A	5	4	1.	21	0.19
20	A	7	5	1.	21	0.238
21	A	8	5	1.	21	0.238
22	A	9	4	1.	21	0.19
23	A	13	4	1.	21	0.19
24	A	11	4	1.	21	0.19
25	A	8	6	1.04	19	0.316
26	A	7	5	1.	12	0.417
27	A	6	5	1.	19	0.263
28	A	6	5	1.	21	0.238
29	A	7	5	1.	21	0.238
30	A	9	5	1.	21	0.238
31	A	11	5	1.	21	0.238
32	A	11	4	1.	21	0.19
33	A	15	4	1.	21	0.19
34	A	11	6	1.12	19	0.316
35	A	10	5	1.	12	0.417
36	A	8	6	1.	19	0.316
37	A	8	6	1.	21	0.286
38	A	8	6	1.	21	0.286
39	A	9	5	1.	21	0.238
40	A	12	5	1.	21	0.238
41	A	13	5	1.	21	0.238
42	A	15	4	1.	21	0.19
43	A	7	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
44	A	6	5	1.	21	0.238
45	A	2	2	1.	21	0.095
46	A	4	4	1.	21	0.19
47	A	2	2	1.	19	0.105
48	A	1	1	1.	12	0.083
49	A	3	3	1.	19	0.158
50	A	5	5	1.	21	0.238
51	A	6	6	1.	21	0.286
52	A	6	5	1.	21	0.238
53	A	7	6	1.	21	0.286
54	A	3	3	1.	21	0.143
55	A	6	6	1.	21	0.286
56	A	3	3	1.	21	0.143
57	A	2	2	1.	19	0.105
58	A	2	2	1.	12	0.167
59	A	4	4	1.	19	0.21
60	A	6	6	1.	21	0.286
61	A	7	7	1.	21	0.333
62	A	7	6	1.	21	0.286
63	A	4	3	1.	21	0.143
64	A	7	7	1.	21	0.333
65	A	5	5	1.	21	0.238
66	A	3	3	1.	21	0.143
67	A	3	3	1.	19	0.158
68	A	3	2	1.	12	0.167
69	A	5	4	1.	19	0.21
70	A	7	6	1.	21	0.286
71	A	8	7	1.	21	0.333
72	A	5	3	1.	21	0.143
73	A	8	7	1.	21	0.333
74	A	6	6	1.	21	0.286
75	A	5	5	1.	21	0.238
76	A	4	4	1.	21	0.19
77	A	4	3	1.	19	0.158
78	A	4	2	1.	12	0.167
79	A	6	4	1.	19	0.21
80	A	8	6	1.	21	0.286
81	A	9	7	1.	21	0.333
82	A	6	3	1.	21	0.143
83	A	9	7	1.	21	0.333
84	A	7	6	1.	21	0.286
85	A	6	6	1.	21	0.286
86	A	6	6	1.	21	0.286
87	A	5	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	3	1.	19	0.158
89	A	5	2	1.	12	0.167
90	A	7	4	1.	19	0.21
91	A	9	6	1.	21	0.286
92	A	10	7	1.	21	0.333
93	A	7	6	1.	21	0.286
94	A	7	7	1.	21	0.333
95	A	5	4	1.	23	0.174
96	A	4	4	1.	23	0.174
97	A	3	3	1.	23	0.13
98	A	2	2	1.	21	0.095
99	A	1	1	1.	14	0.071
100	A	2	2	1.	21	0.095
101	A	3	3	1.	23	0.13
102	A	4	3	1.	23	0.13
103	A	5	3	1.	23	0.13
104	A	6	6	1.	23	0.261
105	A	4	4	1.	23	0.174
106	A	3	3	1.	21	0.143
107	A	2	2	1.	14	0.143
108	A	4	4	1.	21	0.19
109	A	4	4	1.	23	0.174
110	A	5	5	1.	23	0.217
111	A	6	5	1.	23	0.217
112	A	6	6	1.	23	0.261
113	A	5	4	1.	23	0.174
114	A	4	3	1.	21	0.143
115	A	3	2	1.	14	0.143
116	A	4	4	1.	21	0.19
117	A	4	4	1.	23	0.174
118	A	4	4	1.	23	0.174
119	A	5	5	1.	23	0.217
120	A	6	5	1.	23	0.217
121	A	4	2	1.	14	0.143
122	A	7	7	1.	23	0.304
123	A	6	6	1.	23	0.261
124	A	4	4	1.	23	0.174
125	A	3	3	1.	21	0.143
126	A	2	2	1.	14	0.143
127	A	5	4	1.	21	0.19
128	A	6	5	1.	23	0.217
129	A	7	6	1.	23	0.261
130	A	8	6	1.	23	0.261
131	A	7	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
132	A	6	6	1.	23	0.261
133	A	4	4	1.	23	0.174
134	A	3	3	1.	21	0.143
135	A	3	3	1.	14	0.214
136	A	6	5	1.	21	0.238
137	A	7	6	1.	23	0.261
138	A	8	6	1.	23	0.261
139	A	7	7	1.	23	0.304
140	A	6	6	1.	23	0.261
141	A	4	4	1.	23	0.174
142	A	4	4	1.	21	0.19
143	A	4	3	1.	14	0.214
144	A	7	6	1.	21	0.286
145	A	8	7	1.	23	0.304
146	A	6	4	1.	21	0.19
147	A	5	4	1.	21	0.19
148	A	4	4	1.	21	0.19
149	A	3	3	1.	21	0.143
150	A	4	4	1.	21	0.19
151	A	5	4	1.	21	0.19
152	A	6	4	1.	21	0.19
153	A	10	4	1.	23	0.174
154	A	9	4	1.	23	0.174
155	A	7	4	1.	23	0.174
156	A	6	4	1.	23	0.174
157	A	6	4	1.	23	0.174
158	A	7	4	1.	23	0.174
159	A	9	4	1.	23	0.174
160	A	12	4	1.	23	0.174
161	A	10	4	1.	23	0.174
162	A	8	4	1.	23	0.174
163	A	8	5	1.	23	0.217
164	A	8	4	1.	23	0.174
165	A	10	4	1.	23	0.174
166	A	12	4	1.	23	0.174
167	A	16	4	1.	23	0.174
168	A	13	4	1.	23	0.174
169	A	11	4	1.	23	0.174
170	A	10	5	1.	23	0.217
171	A	10	5	1.	23	0.217
172	A	11	4	1.	23	0.174
173	A	13	4	1.	23	0.174
174	A	6	5	1.	23	0.217
175	A	5	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	4	4	1.	23	0.174
177	A	4	4	1.	23	0.174
178	A	4	4	1.	23	0.174
179	A	5	5	1.	23	0.217
180	A	6	5	1.	23	0.217
181	A	7	6	1.	23	0.261
182	A	6	6	1.	23	0.261
183	A	5	5	1.	23	0.217
184	A	5	5	1.	23	0.217
185	A	3	3	1.	23	0.13
186	A	5	5	1.	23	0.217
187	A	6	6	1.	23	0.261
188	A	7	6	1.	23	0.261
189	A	8	6	1.	23	0.261
190	A	7	6	1.	23	0.261
191	A	6	5	1.	23	0.217
192	A	6	6	1.	23	0.261
193	A	6	5	1.	23	0.217
194	A	6	5	1.	23	0.217
195	A	6	5	1.	23	0.217
196	A	7	6	1.	23	0.261
197	A	8	6	1.	23	0.261
198	A	5	3	1.	25	0.12
199	A	4	3	1.	25	0.12
200	A	3	3	1.	25	0.12
201	A	2	2	1.	25	0.08
202	A	1	1	1.	25	0.04
203	A	2	2	1.	25	0.08
204	A	3	2	1.	25	0.08
205	A	4	2	1.	25	0.08
206	A	6	5	1.	25	0.2
207	A	5	5	1.	25	0.2
208	A	4	4	1.	25	0.16
209	A	4	4	1.	25	0.16
210	A	3	3	1.	25	0.12
211	A	4	4	1.	25	0.16
212	A	5	4	1.	25	0.16
213	A	6	5	1.	25	0.2
214	A	5	5	1.	25	0.2
215	A	4	4	1.	25	0.16
216	A	4	4	1.	25	0.16
217	A	4	4	1.	25	0.16
218	A	3	3	1.	25	0.12
219	A	4	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
220	A	5	4	1.	25	0.16
221	A	2	2	1.	25	0.08
222	A	2	2	1.	25	0.08
223	A	2	2	1.	28	0.071
224	A	7	7	1.	25	0.28
225	A	6	6	1.	25	0.24
226	A	5	5	1.	25	0.2
227	A	2	2	1.	25	0.08
228	A	4	4	1.	25	0.16
229	A	5	5	1.	25	0.2
230	A	6	5	1.	25	0.2
231	A	7	6	1.	23	0.261
232	A	6	5	1.	23	0.217
233	A	5	4	1.	23	0.174
234	A	2	2	1.	23	0.087
235	A	3	3	1.	23	0.13
236	A	5	5	1.	23	0.217
237	A	6	5	1.	23	0.217
238	A	7	7	1.	25	0.28
239	A	6	6	1.	25	0.24
240	A	4	4	1.	25	0.16
241	A	4	4	1.	25	0.16
242	A	5	5	1.	25	0.2
243	A	6	5	1.	25	0.2
244	A	8	8	1.	25	0.32
245	A	7	7	1.	25	0.28
246	A	5	5	1.	25	0.2
247	A	5	5	1.	25	0.2
248	A	5	5	1.	25	0.2
249	A	6	6	1.	25	0.24
250	A	7	6	1.	25	0.24
251	A	9	8	1.	25	0.32
252	A	8	7	1.	25	0.28
253	A	6	6	1.	25	0.24
254	A	6	5	1.	25	0.2
255	A	6	5	1.	25	0.2
256	A	6	5	1.	25	0.2
257	A	7	6	1.	25	0.24
258	A	8	6	1.	25	0.24
259	A	7	6	1.	25	0.24
260	A	7	6	1.	25	0.24
261	A	2	2	1.	15	0.133
262	A	2	2	1.	17	0.118
263	A	4	3	1.	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
264	A	3	3	1.	26	0.115
265	A	2	2	1.	26	0.077
266	A	1	1	1.	26	0.038
267	A	2	2	1.	26	0.077
268	A	3	2	1.	26	0.077
269	A	4	3	1.	25	0.12
270	A	3	3	1.	25	0.12
271	A	2	2	1.	25	0.08
272	A	1	1	1.	25	0.04
273	A	2	2	1.	25	0.08
274	A	3	2	1.	25	0.08
275	A	7	7	1.	26	0.269
276	A	6	6	1.	26	0.231
277	A	5	5	1.	26	0.192
278	A	2	2	1.	26	0.077
279	A	4	4	1.	26	0.154
280	A	5	5	1.	26	0.192
281	A	6	5	1.	26	0.192
282	A	7	7	1.	25	0.28
283	A	6	6	1.	25	0.24
284	A	5	5	1.	25	0.2
285	A	2	2	1.	25	0.08
286	A	3	3	1.	25	0.12
287	A	5	5	1.	25	0.2
288	A	3	3	1.	25	0.12
289	A	3	3	1.	25	0.12
290	A	3	3	1.	25	0.12
291	A	9	6	1.	21	0.286
292	A	8	6	1.	21	0.286
293	A	7	6	1.	21	0.286
294	A	6	5	1.	21	0.238
295	A	7	6	1.	21	0.286
296	A	8	6	1.	21	0.286
297	A	9	6	1.	21	0.286
298	A	9	7	1.	23	0.304
299	A	8	7	1.	23	0.304
300	A	5	5	1.	23	0.217
301	A	7	6	1.	23	0.261
302	A	8	7	1.	23	0.304
303	A	9	7	1.	23	0.304
304	A	17	6	1.	23	0.261
305	A	15	6	1.	23	0.261
306	A	13	6	1.	23	0.261
307	A	13	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
308	A	13	6	1.	23	0.261
309	A	15	6	1.	23	0.261
310	A	17	6	1.	23	0.261
311	A	19	6	1.	23	0.261
312	A	17	6	1.	23	0.261
313	A	16	7	1.	23	0.304
314	A	16	7	1.	23	0.304
315	A	17	6	1.	23	0.261
316	A	19	6	1.	23	0.261
317	A	9	7	1.	23	0.304
318	A	8	7	1.	23	0.304
319	A	7	6	1.	23	0.261
320	A	7	6	1.	23	0.261
321	A	7	6	1.	23	0.261
322	A	8	7	1.	23	0.304
323	A	9	7	1.	23	0.304
324	A	10	8	1.	23	0.348
325	A	9	8	1.	23	0.348
326	A	8	7	1.	23	0.304
327	A	5	5	1.	23	0.217
328	A	8	7	1.	23	0.304
329	A	8	7	1.	23	0.304
330	A	9	8	1.	23	0.348
331	A	10	8	1.	23	0.348
332	A	10	8	1.	23	0.348
333	A	9	7	1.	23	0.304
334	A	9	8	1.	23	0.348
335	A	9	8	1.	23	0.348
336	A	9	8	1.	23	0.348
337	A	9	7	1.	23	0.304
338	A	10	8	1.	23	0.348
339	A	5	3	1.	25	0.12
340	A	4	3	1.	25	0.12
341	A	3	3	1.	25	0.12
342	A	2	2	1.	25	0.08
343	A	3	3	1.	25	0.12
344	A	4	4	1.	25	0.16
345	A	5	4	1.	25	0.16
346	A	6	5	1.	25	0.2
347	A	5	5	1.	25	0.2
348	A	4	4	1.	25	0.16
349	A	5	5	1.	25	0.2
350	A	5	5	1.	25	0.2
351	A	6	6	1.	25	0.24

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
352	A	7	6	1.	25	0.24
353	A	6	5	1.	25	0.2
354	A	5	5	1.	25	0.2
355	A	4	4	1.	25	0.16
356	A	5	5	1.	25	0.2
357	A	5	5	1.	25	0.2
358	A	5	5	1.	25	0.2
359	A	6	6	1.	25	0.24
360	A	7	6	1.	25	0.24
361	A	7	6	1.	23	0.261
362	A	6	6	1.	23	0.261
363	A	4	4	1.	23	0.174
364	A	3	3	1.	23	0.13
365	A	6	5	1.	23	0.217
366	A	7	6	1.	23	0.261
367	A	7	6	1.	25	0.24
368	A	6	6	1.	25	0.24
369	A	5	5	1.	25	0.2
370	A	3	3	1.36	25	0.12
371	A	6	6	1.29	25	0.24
372	A	7	7	1.	25	0.28
373	A	7	6	1.	25	0.24
374	A	6	6	1.	25	0.24
375	A	5	5	1.	25	0.2
376	A	5	5	1.	25	0.2
377	A	7	7	1.	25	0.28
378	A	8	8	1.	25	0.32
379	A	8	7	1.	25	0.28
380	A	7	7	1.	25	0.28
381	A	6	6	1.	25	0.24
382	A	6	6	1.	25	0.24
383	A	6	6	1.	25	0.24
384	A	8	8	1.	25	0.32
385	A	9	9	1.	25	0.36
386	A	9	7	1.	25	0.28
387	A	8	7	1.	25	0.28
388	A	7	6	1.	25	0.24
389	A	7	6	1.	25	0.24
390	A	7	6	1.	25	0.24
391	A	7	7	1.	25	0.28
392	A	9	8	1.	25	0.32
393	A	10	9	1.	25	0.36
394	A	8	7	1.	25	0.28
395	A	8	7	1.	25	0.28

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
396	A	3	3	1.	25	0.12
397	A	7	6	1.	21	0.286
398	A	6	5	1.	21	0.238
399	A	4	3	1.	21	0.143
400	A	3	2	1.	19	0.105
401	A	4	3	1.	21	0.143
402	A	5	4	1.	21	0.19
403	A	8	4	1.	19	0.21
404	A	7	4	1.	19	0.21
405	A	7	4	1.	19	0.21
406	A	6	4	1.	19	0.21
407	A	6	4	1.	19	0.21
408	A	5	4	1.	19	0.21
409	A	1	1	1.	17	0.059
410	A	2	1	1.	10	0.1
411	A	2	2	1.	17	0.118
412	A	4	4	1.	19	0.21
413	A	5	5	1.	19	0.263
414	A	5	4	1.	19	0.21
415	A	6	4	1.	19	0.21
416	A	6	4	1.	19	0.21
417	A	7	5	1.	21	0.238
418	A	7	5	1.	21	0.238
419	A	6	5	1.	21	0.238
420	A	2	2	1.	19	0.105
421	A	1	1	1.	12	0.083
422	A	3	3	1.	19	0.158
423	A	4	4	1.	21	0.19
424	A	5	5	1.	21	0.238
425	A	6	6	1.	21	0.286
426	A	6	5	1.	21	0.238
427	A	7	5	1.	21	0.238
428	A	8	6	1.14	21	0.286
429	A	4	3	1.	21	0.143
430	A	3	2	1.	19	0.105
431	A	2	2	1.18	12	0.167
432	A	4	4	1.	19	0.21
433	A	4	4	1.	21	0.19
434	A	4	4	1.	21	0.19
435	A	6	6	1.	21	0.286
436	A	7	7	1.	21	0.333
437	A	7	6	1.	21	0.286
438	A	9	7	1.	21	0.333
439	A	5	3	1.	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	4	2	1.	19	0.105
441	A	3	3	1.	12	0.25
442	A	5	5	1.	19	0.263
443	A	5	5	1.	21	0.238
444	A	5	5	1.	21	0.238
445	A	5	5	1.	21	0.238
446	A	7	7	1.	21	0.333
447	A	8	8	1.	21	0.381
448	A	8	7	1.	21	0.333
449	A	7	6	1.	21	0.286
450	A	6	6	1.	21	0.286
451	A	5	5	1.	21	0.238
452	A	5	5	1.	21	0.238
453	A	3	3	1.	19	0.158
454	A	2	2	1.	12	0.167
455	A	4	4	1.	19	0.21
456	A	6	6	1.	21	0.286
457	A	6	6	1.	21	0.286
458	A	7	6	1.	21	0.286
459	A	7	6	1.	21	0.286
460	A	6	6	1.28	21	0.286
461	A	5	5	1.	21	0.238
462	A	4	4	1.	21	0.19
463	A	4	4	1.	19	0.21
464	A	4	4	1.	12	0.333
465	A	5	5	1.	19	0.263
466	A	6	6	1.	21	0.286
467	A	7	6	1.	21	0.286
468	A	8	6	1.	21	0.286
469	A	7	7	1.	21	0.333
470	A	6	6	1.	21	0.286
471	A	5	5	1.	21	0.238
472	A	5	5	1.	21	0.238
473	A	5	4	1.	19	0.21
474	A	5	5	1.	12	0.417
475	A	6	6	1.	19	0.316
476	A	7	6	1.	21	0.286
477	A	8	6	1.	21	0.286
478	A	7	7	1.	21	0.333
479	A	6	6	1.	21	0.286
480	A	6	6	1.	21	0.286
481	A	6	5	1.	21	0.238
482	A	6	4	1.	19	0.21
483	A	6	5	1.	12	0.417

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
484	A	7	6	1.	19	0.316
485	A	8	6	1.	21	0.286
486	A	8	8	1.	23	0.348
487	A	7	7	1.	23	0.304
488	A	6	6	1.	21	0.286
489	A	2	2	1.	14	0.143
490	A	5	5	1.	21	0.238
491	A	9	9	1.	23	0.391
492	A	10	10	1.	23	0.435
493	A	9	8	1.	23	0.348
494	A	8	7	1.	23	0.304
495	A	7	6	1.	21	0.286
496	A	6	6	1.	14	0.429
497	A	8	8	1.	21	0.381
498	A	9	9	1.	23	0.391
499	A	10	10	1.	23	0.435
500	A	10	8	1.	23	0.348
501	A	9	7	1.	23	0.304
502	A	8	6	1.	21	0.286
503	A	7	7	1.	14	0.5
504	A	9	9	1.	21	0.429
505	A	9	9	1.	23	0.391
506	A	10	10	1.	23	0.435
507	A	11	10	1.	23	0.435
508	A	8	7	1.	14	0.5
509	A	6	6	1.	23	0.261
510	A	5	5	1.	23	0.217
511	A	4	4	1.	21	0.19
512	A	1	1	1.	14	0.071
513	A	3	3	1.	21	0.143
514	A	6	6	1.	23	0.261
515	A	7	7	1.	23	0.304
516	A	6	6	1.	23	0.261
517	A	5	5	1.	23	0.217
518	A	4	4	1.	21	0.19
519	A	1	1	1.	14	0.071
520	A	3	3	1.	21	0.143
521	A	6	6	1.	23	0.261
522	A	7	7	1.	23	0.304
523	A	7	7	1.	23	0.304
524	A	6	6	1.	23	0.261
525	A	5	5	1.	21	0.238
526	A	2	2	1.	14	0.143
527	A	2	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
528	A	9	9	1.	23	0.391
529	A	10	10	1.	23	0.435
530	A	8	8	1.	23	0.348
531	A	7	7	1.	23	0.304
532	A	6	6	1.	23	0.261
533	A	6	6	1.	21	0.286
534	A	4	4	1.	14	0.286
535	A	7	7	1.	21	0.333
536	A	10	10	1.	23	0.435
537	A	11	10	1.	23	0.435
538	A	9	9	1.	23	0.391
539	A	8	8	1.	23	0.348
540	A	7	7	1.	23	0.304
541	A	7	7	1.	23	0.304
542	A	7	6	1.	21	0.286
543	A	7	7	1.	14	0.5
544	A	10	10	1.	21	0.476
545	A	11	11	1.	23	0.478
546	A	8	7	1.	14	0.5
547	A	5	5	1.	23	0.217
548	A	4	4	1.	23	0.174
549	A	3	3	1.	21	0.143
550	A	1	1	1.	14	0.071
551	A	1	1	1.	21	0.048
552	A	6	6	1.	23	0.261
553	A	7	7	1.	23	0.304
554	A	5	5	1.	23	0.217
555	A	4	4	1.	23	0.174
556	A	3	3	1.	21	0.143
557	A	1	1	1.	14	0.071
558	A	1	1	1.	21	0.048
559	A	6	6	1.	23	0.261
560	A	7	7	1.	23	0.304
561	A	6	4	1.	21	0.19
562	A	5	4	1.	21	0.19
563	A	4	4	1.	21	0.19
564	A	3	3	1.	21	0.143
565	A	4	4	1.	21	0.19
566	A	5	4	1.	21	0.19
567	A	6	4	1.	21	0.19
568	A	7	5	1.	23	0.217
569	A	6	5	1.	23	0.217
570	A	5	5	1.	23	0.217
571	A	4	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
572	A	4	4	1.	23	0.174
573	A	5	5	1.	23	0.217
574	A	6	5	1.	23	0.217
575	A	7	6	1.	23	0.261
576	A	6	6	1.	23	0.261
577	A	5	5	1.	23	0.217
578	A	5	5	1.	23	0.217
579	A	5	5	1.	23	0.217
580	A	6	6	1.	23	0.261
581	A	7	6	1.	23	0.261
582	A	6	6	1.	23	0.261
583	A	5	5	1.	23	0.217
584	A	3	3	1.	23	0.13
585	A	1	1	1.	23	0.043
586	A	5	5	1.	23	0.217
587	A	7	7	1.	23	0.304
588	A	7	7	1.	23	0.304
589	A	6	6	1.	23	0.261
590	A	6	6	1.	23	0.261
591	A	6	6	1.	23	0.261
592	A	6	6	1.	23	0.261
593	A	7	7	1.	23	0.304
594	A	8	7	1.	23	0.304
595	A	8	8	1.	23	0.348
596	A	7	7	1.	23	0.304
597	A	7	7	1.	23	0.304
598	A	7	7	1.	23	0.304
599	A	7	7	1.	23	0.304
600	A	7	7	1.	23	0.304
601	A	8	7	1.	23	0.304
602	A	9	7	1.	23	0.304
603	A	7	7	1.	25	0.28
604	A	7	7	1.	25	0.28
605	A	1	1	1.	25	0.04
606	A	3	3	1.	25	0.12
607	A	4	4	1.	25	0.16
608	A	5	5	1.	25	0.2
609	A	6	5	1.	25	0.2
610	A	8	8	1.	25	0.32
611	A	8	8	1.	25	0.32
612	A	6	6	1.	25	0.24
613	A	5	5	1.	25	0.2
614	A	4	4	1.	25	0.16
615	A	5	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
616	A	6	5	1.	25	0.2
617	A	7	5	1.	25	0.2
618	A	8	8	1.	25	0.32
619	A	7	7	1.	25	0.28
620	A	7	7	1.	25	0.28
621	A	6	6	1.	25	0.24
622	A	5	5	1.	25	0.2
623	A	6	5	1.	25	0.2
624	A	7	5	1.	25	0.2
625	A	8	5	1.	25	0.2
626	A	8	8	1.09	25	0.32
627	A	1	1	1.	25	0.04
628	A	1	1	1.	25	0.04
629	A	3	3	1.	25	0.12
630	A	4	4	1.	25	0.16
631	A	7	7	1.	25	0.28
632	A	6	6	1.	25	0.24
633	A	4	4	1.	25	0.16
634	A	4	4	1.	25	0.16
635	A	4	4	1.	25	0.16
636	A	5	5	1.	25	0.2
637	A	6	5	1.	25	0.2
638	A	7	7	1.	25	0.28
639	A	5	5	1.	25	0.2
640	A	5	5	1.	25	0.2
641	A	5	5	1.	25	0.2
642	A	5	5	1.	25	0.2
643	A	6	5	1.	25	0.2
644	A	1	1	1.	25	0.04
645	A	1	1	1.	25	0.04
646	A	2	2	1.	25	0.08
647	A	2	2	1.	25	0.08
648	A	1	1	1.	25	0.04
649	A	1	1	1.	25	0.04
650	A	2	2	1.	25	0.08
651	A	2	2	1.	25	0.08
652	A	2	2	1.	27	0.074
653	A	2	2	1.	27	0.074
654	A	1	1	1.	27	0.037
655	A	1	1	1.	27	0.037
656	A	2	2	1.	27	0.074
657	A	2	2	1.	27	0.074
658	A	1	1	1.	27	0.037
659	A	1	1	1.	27	0.037

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
660	A	1	1	1.	25	0.04
661	A	1	1	1.	25	0.04
662	A	2	2	1.	25	0.08
663	A	2	2	1.	25	0.08
664	A	1	1	1.	25	0.04
665	A	1	1	1.	25	0.04
666	A	2	2	1.	25	0.08
667	A	2	2	1.	25	0.08
668	A	2	2	1.	27	0.074
669	A	2	2	1.	27	0.074
670	A	1	1	1.	27	0.037
671	A	1	1	1.	27	0.037
672	A	2	2	1.	27	0.074
673	A	2	2	1.	27	0.074
674	A	1	1	1.	27	0.037
675	A	1	1	1.	27	0.037
676	A	5	3	1.	23	0.13
677	A	5	3	1.	23	0.13
678	A	5	3	1.	23	0.13
679	A	5	3	1.	23	0.13
680	A	0	0	0.	0	0.
681	A	0	0	0.	0	0.
682	A	0	0	0.	0	0.
683	A	0	0	0.	0	0.
684	A	0	0	0.	0	0.
685	A	0	0	0.	0	0.
686	A	0	0	0.	0	0.
687	A	0	0	0.	0	0.
688	A	0	0	0.	0	0.
689	A	0	0	0.	0	0.
690	A	9	6	1.	21	0.286
691	A	8	6	1.	21	0.286
692	A	7	6	1.	21	0.286
693	A	6	5	1.	21	0.238
694	A	7	6	1.	21	0.286
695	A	8	6	1.	21	0.286
696	A	9	6	1.	21	0.286
697	A	10	7	1.	23	0.304
698	A	9	7	1.	23	0.304
699	A	8	7	1.	23	0.304
700	A	7	6	1.	23	0.261
701	A	7	6	1.	23	0.261
702	A	8	7	1.	23	0.304
703	A	9	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
704	A	10	7	1.	23	0.304
705	A	10	8	1.	23	0.348
706	A	9	8	1.	23	0.348
707	A	8	7	1.	23	0.304
708	A	8	7	1.	23	0.304
709	A	8	7	1.	23	0.304
710	A	9	8	1.	23	0.348
711	A	10	8	1.	23	0.348
712	A	11	10	1.	23	0.435
713	A	7	7	1.	23	0.304
714	A	3	3	1.	23	0.13
715	A	5	5	1.	23	0.217
716	A	9	8	1.	23	0.348
717	A	10	9	1.	23	0.391
718	A	12	10	1.	23	0.435
719	A	11	10	1.	23	0.435
720	A	10	9	1.	23	0.391
721	A	10	9	1.	23	0.391
722	A	10	9	1.	23	0.391
723	A	10	9	1.	23	0.391
724	A	13	11	1.	23	0.478
725	A	12	11	1.	23	0.478
726	A	11	10	1.	23	0.435
727	A	11	10	1.	23	0.435
728	A	11	10	1.	23	0.435
729	A	11	10	1.	23	0.435
730	A	6	6	1.	25	0.24
731	A	5	5	1.	25	0.2
732	A	4	4	1.	25	0.16
733	A	2	2	1.	25	0.08
734	A	8	8	1.	25	0.32
735	A	8	8	1.	25	0.32
736	A	7	6	1.	25	0.24
737	A	6	6	1.	25	0.24
738	A	5	5	1.	25	0.2
739	A	6	6	1.	25	0.24
740	A	7	7	1.	25	0.28
741	A	9	9	1.	25	0.36
742	A	9	9	1.	25	0.36
743	A	8	6	1.	25	0.24
744	A	7	6	1.	25	0.24
745	A	6	6	1.	25	0.24
746	A	7	7	1.	25	0.28
747	A	8	8	1.	25	0.32

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
748	A	8	8	1.	25	0.32
749	A	9	9	1.	25	0.36
750	A	10	9	1.	25	0.36
751	A	5	5	1.	25	0.2
752	A	4	4	1.	25	0.16
753	A	2	2	1.	25	0.08
754	A	2	2	1.	25	0.08
755	A	9	9	1.	25	0.36
756	A	8	8	1.	25	0.32
757	A	6	6	1.	25	0.24
758	A	5	5	1.	25	0.2
759	A	5	5	1.	25	0.2
760	A	5	5	1.	25	0.2
761	A	7	7	1.	25	0.28
762	A	8	8	1.	25	0.32
763	A	7	6	1.	25	0.24
764	A	6	6	1.	25	0.24
765	A	6	6	1.	25	0.24
766	A	6	6	1.	25	0.24
767	A	6	6	1.	25	0.24
768	A	8	8	1.	25	0.32
769	A	6	5	1.	21	0.238
770	A	5	4	1.	21	0.19
771	A	4	3	1.	21	0.143
772	A	3	2	1.	19	0.105
773	A	5	3	1.	21	0.143
774	A	8	3	1.	21	0.143
775	A	8	6	1.	21	0.286
776	A	7	5	1.	21	0.238
777	A	6	4	1.	19	0.21
778	A	2	2	1.	21	0.095
779	A	3	3	1.	19	0.158
780	A	1	1	1.	25	0.04
781	A	1	1	1.	27	0.037
782	A	1	1	1.	31	0.032
783	A	1	1	1.	27	0.037
784	A	1	1	1.	29	0.034
785	A	2	2	1.	25	0.08
786	A	1	1	1.	29	0.034
787	A	3	3	1.	25	0.12
788	A	3	3	1.	25	0.12
789	A	3	3	1.	25	0.12
790	A	3	3	1.	25	0.12
791	A	3	3	1.	28	0.107

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
792	A	2	2	1.	23	0.087
793	A	2	2	1.	21	0.095
794	A	3	3	1.	28	0.107
795	A	7	4	1.	25	0.16
796	A	7	4	1.	25	0.16
797	A	7	4	1.	25	0.16
798	A	7	4	1.	25	0.16
799	A	9	7	1.	31	0.226
800	A	8	7	1.	29	0.241
801	A	6	6	1.	23	0.261
802	A	6	6	1.	29	0.207
803	A	7	7	1.	31	0.226
804	A	8	7	1.	31	0.226
805	A	9	7	1.	31	0.226
806	A	9	7	1.	29	0.241
807	A	7	6	1.	23	0.261
808	A	7	7	1.	29	0.241
809	A	6	6	1.	31	0.194
810	A	7	7	1.	31	0.226
811	A	8	7	1.	31	0.226
812	A	9	7	1.	31	0.226
813	A	8	6	1.	23	0.261
814	A	8	7	1.	29	0.241
815	A	7	7	1.	31	0.226
816	A	6	6	1.	31	0.194
817	A	7	7	1.	31	0.226
818	A	8	7	1.	31	0.226
819	A	9	7	1.	31	0.226
820	A	9	7	1.	31	0.226
821	A	8	7	1.	31	0.226
822	A	7	7	1.	29	0.241
823	A	5	5	1.	23	0.217
824	A	7	7	1.	29	0.241
825	A	8	7	1.	31	0.226
826	A	9	7	1.	31	0.226
827	A	9	7	1.	31	0.226
828	A	8	7	1.	31	0.226
829	A	7	7	1.	31	0.226
830	A	6	6	1.	29	0.207
831	A	6	6	1.	23	0.261
832	A	8	7	1.	29	0.241
833	A	9	7	1.	31	0.226
834	A	9	7	1.	31	0.226
835	A	8	7	1.	31	0.226

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
836	A	7	7	1.	31	0.226
837	A	6	6	1.	31	0.194
838	A	7	7	1.	29	0.241
839	A	7	6	1.	23	0.261
840	A	9	7	1.	29	0.241
841	A	8	6	1.	23	0.261
842	A	7	5	1.	33	0.152
843	A	6	5	1.	33	0.152
844	A	2	2	1.	33	0.061
845	A	3	2	1.	33	0.061
846	A	3	3	1.	33	0.091
847	A	5	5	1.	33	0.152
848	A	6	6	1.	33	0.182
849	A	6	5	1.	33	0.152
850	A	7	5	1.	33	0.152
851	A	6	5	1.	33	0.152
852	A	2	2	1.	33	0.061
853	A	3	2	1.	33	0.061
854	A	3	3	1.	33	0.091
855	A	5	5	1.	33	0.152
856	A	6	6	1.	33	0.182
857	A	6	5	1.	33	0.152
858	A	7	5	1.	33	0.152
859	A	6	5	1.	33	0.152
860	A	2	2	1.	33	0.061
861	A	3	2	1.	33	0.061
862	A	3	3	1.	33	0.091
863	A	5	5	1.	33	0.152
864	A	6	6	1.	33	0.182
865	A	6	5	1.	33	0.152
866	A	6	5	1.	33	0.152
867	A	2	2	1.	33	0.061
868	A	3	2	1.	33	0.061
869	A	3	3	1.	33	0.091
870	A	5	5	1.	33	0.152
871	A	6	6	1.	33	0.182
872	A	6	5	1.	33	0.152
873	A	6	5	1.	33	0.152
874	A	2	2	1.	33	0.061
875	A	3	2	1.	33	0.061
876	A	3	3	1.	33	0.091
877	A	5	5	1.	33	0.152
878	A	6	6	1.	33	0.182
879	A	6	5	1.	33	0.152

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
880	A	6	5	1.	33	0.152
881	A	2	2	1.	33	0.061
882	A	3	2	1.	33	0.061
883	A	3	3	1.	33	0.091
884	A	5	5	1.	33	0.152
885	A	6	6	1.	33	0.182
886	A	6	5	1.	33	0.152
887	A	4	3	1.	31	0.097
888	A	4	3	1.	29	0.103
889	A	3	2	1.	23	0.087
890	A	4	3	1.	29	0.103
891	A	4	3	1.	31	0.097
892	A	4	3	1.	31	0.097
893	A	4	3	1.	31	0.097
894	A	4	3	1.	29	0.103
895	A	3	2	1.	23	0.087
896	A	4	3	1.	29	0.103
897	A	4	3	1.	31	0.097
898	A	4	3	1.	31	0.097
899	A	4	3	1.	31	0.097
900	A	4	3	1.	29	0.103
901	A	3	2	1.	23	0.087
902	A	4	3	1.	29	0.103
903	A	4	3	1.	31	0.097
904	A	4	3	1.	31	0.097
905	A	4	3	1.	31	0.097
906	A	4	3	1.	29	0.103
907	A	3	2	1.	23	0.087
908	A	4	3	1.	29	0.103
909	A	4	3	1.	31	0.097
910	A	4	3	1.	31	0.097
911	A	4	3	1.	29	0.103
912	A	4	3	1.	29	0.103
913	A	4	3	1.	27	0.111
914	A	3	2	1.	21	0.095
915	A	4	3	1.	27	0.111
916	A	4	3	1.	29	0.103
917	A	4	3	1.	29	0.103
918	A	4	3	1.	29	0.103
919	A	4	3	1.	31	0.097
920	A	4	3	1.	31	0.097
921	A	4	3	1.	31	0.097
922	A	4	3	1.	31	0.097
923	A	4	3	1.	31	0.097

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
924	A	4	3	1.	31	0.097
925	A	4	3	1.	31	0.097
926	A	4	3	1.	31	0.097
927	A	4	3	1.	31	0.097
928	A	4	3	1.	31	0.097
929	A	4	3	1.	31	0.097
930	A	4	3	1.	31	0.097
931	A	4	3	1.	31	0.097
932	A	4	3	1.	31	0.097

Chapter 3

Listing of integrals

3.1 $\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=114

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(}{}$$

```
[Out] (5*a*x)/16 + (a*Sin[c + d*x])/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0695691, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2633, 2635, 8}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Cos[c + d*x]),x]
```

```
[Out] (5*a*x)/16 + (a*Sin[c + d*x])/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^5(c+dx)\,dx + a\int \cos^6(c+dx)\,dx$$
$$= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right.\right.}{d}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2a\sin}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos}{}$$
$$= \frac{5ax}{16} + \frac{a\sin(c+dx)}{d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}$$

Mathematica [A] time = 0.18795, size = 75, normalized size = 0.66

$$\frac{a\left(192\sin^5(c+dx) - 640\sin^3(c+dx) + 960\sin(c+dx) + 5(45\sin(2(c+dx)) + 9\sin(4(c+dx)) + \sin(6(c+dx)) + 60c +\right.}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(960*Sin[c + d*x] - 640*Sin[c + d*x]^3 + 192*Sin[c + d*x]^5 + 5*(60*c +
60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])))/(96
0*d)
```

Maple [A] time = 0.044, size = 80, normalized size = 0.7

$$\frac{1}{d}\left(a\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5 + \frac{5\,(\cos(dx+c))^3}{4} + \frac{15\,\cos(dx+c)}{8}\right) + \frac{5\,dx}{16} + \frac{5\,c}{16}\right) + \frac{a\sin(dx+c)}{5}\left(\frac{8}{3} + (\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d*(a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.17699, size = 113, normalized size = 0.99

$$\frac{64\left(3\,\sin(dx+c)^5 - 10\,\sin(dx+c)^3 + 15\,\sin(dx+c)\right)a - 5\left(4\,\sin(2\,dx+2\,c)^3 - 60\,dx - 60\,c - 9\,\sin(4\,dx+4\,c) - 48\,\mathrm{s}\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a - 5*(4
*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2
*c))*a)/d
```

Fricas [A] time = 1.94138, size = 204, normalized size = 1.79

$$\frac{75\,adx + \left(40\,a\cos\left(dx+c\right)^{5} + 48\,a\cos\left(dx+c\right)^{4} + 50\,a\cos\left(dx+c\right)^{3} + 64\,a\cos\left(dx+c\right)^{2} + 75\,a\cos\left(dx+c\right) + 128\,a\right)\sin}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + (40*a*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 + 50*a*cos(d*x
 + c)^3 + 64*a*cos(d*x + c)^2 + 75*a*cos(d*x + c) + 128*a)*sin(d*x + c))/d
```

Sympy [A] time = 4.76251, size = 216, normalized size = 1.89

$$\begin{cases} \frac{5ax\sin^{6}(c+dx)}{16} + \frac{15ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16} + \frac{15ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16} + \frac{5ax\cos^{6}(c+dx)}{16} + \frac{5a\sin^{5}(c+dx)\cos(c+dx)}{16d} + \frac{8a\sin^{5}(c+dx)}{15d} + \\ x\left(a\cos\left(c\right)+a\right)\cos^{5}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 8*a*sin(c + d*x)**5/(15*d) + 5
*a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)
**2/(3*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + a*sin(c + d*x)*cos(c
 + d*x)**4/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**5, True))
```

Giac [A] time = 1.28096, size = 124, normalized size = 1.09

$$\frac{5}{16}\,ax + \frac{a\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{a\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{15\,a\sin\left(2\,dx+2\,c\right)}{64\,d} + \frac{5\,a\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x + 1/192*a*sin(6*d*x + 6*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 3/64*a*
sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 15/64*a*sin(2*d*x + 2*c)/d
 + 5/8*a*sin(d*x + c)/d
```

3.2 $\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=92

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c
 + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0575704, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2635, 8, 2633}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c
 + d*x]^5)/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + a\int \cos^5(c+dx)\,dx$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1-2x^2+\right.\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{2a\,s}{}$$
$$= \frac{3ax}{8} + \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

Mathematica [A] time = 0.107791, size = 65, normalized size = 0.71

$$\frac{a\left(96\sin^5(c+dx)-320\sin^3(c+dx)+480\sin(c+dx)+15(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(480*Sin[c + d*x] - 320*Sin[c + d*x]^3 + 96*Sin[c + d*x]^5 + 15*(12*(c +
 d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])))/(480*d)
```

Maple [A] time = 0.043, size = 70, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.10983, size = 93, normalized size = 1.01

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a+15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)a}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a + 15*(
12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.95596, size = 173, normalized size = 1.88

$$\frac{45\,adx+\left(24\,a\cos(dx+c)^4+30\,a\cos(dx+c)^3+32\,a\cos(dx+c)^2+45\,a\cos(dx+c)+64\,a\right)\sin(dx+c)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(45*a*d*x + (24*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^3 + 32*a*cos(d*x
 + c)^2 + 45*a*cos(d*x + c) + 64*a)*sin(d*x + c))/d
```

Sympy [A] time = 2.38951, size = 168, normalized size = 1.83

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{8a\sin^5(c+dx)}{15d} + \frac{4a\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{a\sin(c+ \\ x(a\cos(c)+a)\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)
**3*cos(c + d*x)**2/(3*d) + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + a*sin(
c + d*x)*cos(c + d*x)**4/d + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d,
0)), (x*(a*cos(c) + a)*cos(c)**4, True))
```

Giac [A] time = 1.38839, size = 104, normalized size = 1.13

$$\frac{3}{8}\,ax + \frac{a\sin(5\,dx+5\,c)}{80\,d} + \frac{a\sin(4\,dx+4\,c)}{32\,d} + \frac{5\,a\sin(3\,dx+3\,c)}{48\,d} + \frac{a\sin(2\,dx+2\,c)}{4\,d} + \frac{5\,a\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x + 1/80*a*sin(5*d*x + 5*c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 5/48*a*si
n(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 5/8*a*sin(d*x + c)/d
```

3.3 $\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0524819, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2633, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (a*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^3(c+dx)\,dx + a\int \cos^4(c+dx)\,dx$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x\right.}{d}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a\sin^3(}{3}$$
$$= \frac{3ax}{8} + \frac{a\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} -$$

Mathematica [A] time = 0.0843635, size = 73, normalized size = 0.96

$$\frac{3a(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*
Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.043, size = 60, normalized size = 0.8

$$\frac{1}{d}\left(a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{3dx}{8} + \frac{3c}{8}\right) + \frac{a\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.18399, size = 77, normalized size = 1.01

$$-\frac{32\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a - 3\left(12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\sin(2\,dx+2\,c)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.82836, size = 136, normalized size = 1.79

$$\frac{9\,adx + \left(6\,a\cos(dx+c)^3 + 8\,a\cos(dx+c)^2 + 9\,a\cos(dx+c) + 16\,a\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + (6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*a*cos(d*x + c)
 + 16*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.35328, size = 144, normalized size = 1.89

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2a\sin^3(c+dx)}{3d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{a\sin \\ x(a\cos(c)+a)\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a*
sin(c + d*x)**3/(3*d) + 5*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a*sin(c +
d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**3, True))
```

Giac [A] time = 1.28882, size = 84, normalized size = 1.11

$$\frac{3}{8}ax + \frac{a\sin(4dx+4c)}{32d} + \frac{a\sin(3dx+3c)}{12d} + \frac{a\sin(2dx+2c)}{4d} + \frac{3a\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x + 1/32*a*sin(4*d*x + 4*c)/d + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*a*sin
(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d
```

3.4 $\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.04155, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2635, 8, 2633}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (a*Sin
[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\cos(c+dx))\,dx &= a\int \cos^2(c+dx)\,dx+a\int \cos^3(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx-\frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}\\ &= \frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0657467, size = 57, normalized size = 1.06

$$\frac{a(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (a*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.04, size = 49, normalized size = 0.9

$$\frac{1}{d}\left(\frac{a\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.08752, size = 62, normalized size = 1.15

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)a-3(2dx+2c+\sin(2dx+2c))a}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(2*d*x + 2*c + sin(2*d*x +
 2*c))*a)/d
```

Fricas [A] time = 1.9071, size = 105, normalized size = 1.94

$$\frac{3\,adx+\left(2\,a\cos(dx+c)^2+3\,a\cos(dx+c)+4\,a\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*a)*sin(d*x + c))/
d
```

Sympy [A] time = 0.611027, size = 92, normalized size = 1.7

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} + \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^2(c+dx)}{d} + \frac{a\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a\cos(c) + a)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + 2*a*sin(c + d*x)
**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d + a*sin(c + d*x)*cos(c + d*x)/
(2*d), Ne(d, 0)), (x*(a*cos(c) + a)*cos(c)**2, True))
```

Giac [A] time = 1.2755, size = 63, normalized size = 1.17

$$\frac{1}{2}ax + \frac{a\sin(3dx + 3c)}{12d} + \frac{a\sin(2dx + 2c)}{4d} + \frac{3a\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*a*sin(
d*x + c)/d
```

3.5 $\int \cos(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0144199, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2734}

$$\frac{a\sin(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))\,dx=\frac{ax}{2}+\frac{a\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0459236, size = 32, normalized size = 0.84

$$\frac{a(2(c+dx)+4\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(2*(c + d*x) + 4*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.036, size = 38, normalized size = 1.

$$\frac{1}{d}\left(a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*sin(d*x+c))
```

Maxima [A] time = 1.08884, size = 46, normalized size = 1.21

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a+4\,a\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 4*a*sin(d*x + c))/d
```

Fricas [A] time = 1.6562, size = 72, normalized size = 1.89

$$\frac{adx+(a\cos(dx+c)+2\,a)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + (a*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

Sympy [A] time = 0.282475, size = 66, normalized size = 1.74

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2}+\frac{ax\cos^2(c+dx)}{2}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{a\sin(c+dx)}{d} & \text{for } d\neq 0 \\ x(a\cos(c)+a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) + a*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)*cos(c),
True))
```

Giac [A] time = 1.32989, size = 42, normalized size = 1.11

$$\frac{1}{2}\,ax+\frac{a\sin(2\,dx+2\,c)}{4\,d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/4*a*sin(2*d*x + 2*c)/d + a*sin(d*x + c)/d
```

3.6 $\int (a + a\cos(c + dx))\,dx$

Optimal. Leaf size=15

$$\frac{a\sin(c+dx)}{d}+ax$$

```
[Out] a*x + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0072543, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2637}

$$\frac{a\sin(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[a + a*Cos[c + d*x],x]
```

```
[Out] a*x + (a*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))\,dx &= ax+a\int \cos(c+dx)\,dx\\ &= ax+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0054959, size = 26, normalized size = 1.73

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Integrate[a + a*Cos[c + d*x],x]
```

```
[Out] a*x + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.025, size = 16, normalized size = 1.1

$$ax+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+cos(d*x+c)*a,x)
```

```
[Out] a*x+a*sin(d*x+c)/d
```

Maxima [A] time = 1.12463, size = 20, normalized size = 1.33

$$ax + \frac{a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cos(d*x+c),x, algorithm="maxima")
```

```
[Out] a*x + a*sin(d*x + c)/d
```

Fricas [A] time = 1.63994, size = 38, normalized size = 2.53

$$\frac{adx + a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cos(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*d*x + a*sin(d*x + c))/d
```

Sympy [A] time = 0.132864, size = 17, normalized size = 1.13

$$ax + a\left(\begin{cases} \frac{\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x\cos(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cos(d*x+c),x)
```

```
[Out] a*x + a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))
```

Giac [A] time = 1.32368, size = 20, normalized size = 1.33

$$ax + \frac{a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+a*cos(d*x+c),x, algorithm="giac")
```

```
[Out] a*x + a*sin(d*x + c)/d
```

3.7 $\int (a+a\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=16

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0201203, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2735, 3770}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))\sec(c+dx)\,dx &= ax+a\int \sec(c+dx)\,dx\\ &= ax+\frac{a\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0072224, size = 16, normalized size = 1.

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] a*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] time = 0.055, size = 30, normalized size = 1.9

$$ax+\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c),x)
```

```
[Out] a*x+1/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*c*a
```

Maxima [A] time = 1.19116, size = 38, normalized size = 2.38

$$\frac{(dx+c)a+a\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a + a*log(sec(d*x + c) + tan(d*x + c)))/d
```

Fricas [B] time = 1.72294, size = 95, normalized size = 5.94

$$\frac{2\,adx+a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

Sympy [A] time = 4.94182, size = 49, normalized size = 3.06

$$ax+a\left(\begin{cases}\frac{x\tan\left(c\right)\sec\left(c\right)}{\tan\left(c\right)+\sec\left(c\right)}+\frac{x\sec^{2}\left(c\right)}{\tan\left(c\right)+\sec\left(c\right)} & \text{for}\: d=0\\\frac{\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)}{d} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*x + a*Piecewise((x*tan(c)*sec(c)/(tan(c) + sec(c)) + x*sec(c)**2/(tan(c)
+ sec(c)), Eq(d, 0)), (log(tan(c + d*x) + sec(c + d*x))/d, True))
```

Giac [B] time = 1.46839, size = 58, normalized size = 3.62

$$\frac{(dx+c)a+a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] ((d*x + c)*a + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)))/d
```

3.8 $\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=24

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0337479, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3767, 8, 3770}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))\sec^2(c+dx)\,dx &= a\int \sec(c+dx)\,dx+a\int \sec^2(c+dx)\,dx\\
&=\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\,\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{d}\\
&=\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0105373, size = 24, normalized size = 1.

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.061, size = 32, normalized size = 1.3

$$\frac{a\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^2,x)
```

```
[Out] 1/d*a*ln(sec(d*x+c)+tan(d*x+c))+a*tan(d*x+c)/d
```

Maxima [A] time = 1.12919, size = 51, normalized size = 2.12

$$\frac{a(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*a*tan(d*x + c))/
d
```

Fricas [B] time = 1.69809, size = 162, normalized size = 6.75

$$\frac{a\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,a\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int\sec^{2}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x)
)
```

Giac [B] time = 1.48407, size = 85, normalized size = 3.54

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.9 $\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=47

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rubi [A] time = 0.0475257, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2748, 3768, 3770, 3767, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^3(c+dx)\,dx = a\int \sec^2(c+dx)\,dx + a\int \sec^3(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{a\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+d$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0136784, size = 47, normalized size = 1.

$$\frac{a\tan(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Maple [A] time = 0.067, size = 51, normalized size = 1.1

$$\frac{a\tan(dx+c)}{d} + \frac{a\sec(dx+c)\tan(dx+c)}{2d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^3,x)
```

```
[Out] a*tan(d*x+c)/d+1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 1.15867, size = 78, normalized size = 1.66

$$-\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 4a\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*a*tan(d*x + c))/d
```

Fricas [A] time = 1.66197, size = 198, normalized size = 4.21

$$\frac{a\cos(dx+c)^2\log(\sin(dx+c)+1) - a\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2(2a\cos(dx+c)+a)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cos\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int\sec^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] a*(Integral(cos(c + d*x)*sec(c + d*x)**3, x) + Integral(sec(c + d*x)**3, x)
)
```

Giac [A] time = 1.44223, size = 108, normalized size = 2.3

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 - 3*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^2)/d
```

3.10 $\int (a + a\cos(c + dx))\sec^4(c + dx)\,dx$

Optimal. Leaf size=63

$$\frac{a\tan^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0477197, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3767, 3768, 3770}

$$\frac{a\tan^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^4(c+dx)\,dx = a\int \sec^3(c+dx)\,dx + a\int \sec^4(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)dx, x, -\text{t}\right.}{d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{a\tan^3(c+}{3d}$$

Mathematica [A] time = 0.127794, size = 60, normalized size = 0.95

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.071, size = 72, normalized size = 1.1

$$\frac{a\sec(dx+c)\tan(dx+c)}{2d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2a\tan(dx+c)}{3d}+\frac{a\tan(dx+c)(\sec(dx+c))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^4,x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+2/3*a*tan(d
*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.10251, size = 95, normalized size = 1.51

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a-3a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.649, size = 236, normalized size = 3.75

$$\frac{3a\cos(dx+c)^3\log(\sin(dx+c)+1)-3a\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(4a\cos(dx+c)^2+3a\cos(dx+c)\right)}{12d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 2*a)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4472, size = 130, normalized size = 2.06

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 4*a*tan(1/2*d*x + 1/2*c)^3 + 9*
a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.11 $\int (a + a\cos(c + dx))\sec^5(c + dx)\,dx$

Optimal. Leaf size=85

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rubi [A] time = 0.0616641, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3768, 3770, 3767}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*
x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^5(c+dx)\,dx = a\int \sec^4(c+dx)\,dx + a\int \sec^5(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)dx,\right.}{d}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a\tan}{}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c}{}$$

Mathematica [A] time = 0.131248, size = 76, normalized size = 0.89

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.079, size = 92, normalized size = 1.1

$$\frac{2\,a\tan(dx+c)}{3\,d} + \frac{a\tan(dx+c)(\sec(dx+c))^2}{3\,d} + \frac{a\tan(dx+c)(\sec(dx+c))^3}{4\,d} + \frac{3\,a\sec(dx+c)\tan(dx+c)}{8\,d} + \frac{3\,a\ln(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^5,x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a*tan(d*x+c)*sec(d
*x+c)^3+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.16037, size = 128, normalized size = 1.51

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)a - 3\,a\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*a*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.739, size = 266, normalized size = 3.13

$$\frac{9\,a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(16\,a\cos\left(dx+c\right)^3+9\,a\cos\left(dx+\right.\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*a*cos(d*x
 + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45373, size = 149, normalized size = 1.75

$$\frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-49\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) - 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 49*a*tan(1/2*d*x + 1/2*c)^5 +
31*a*tan(1/2*d*x + 1/2*c)^3 - 39*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^4)/d
```

3.12 $\int (a + a\cos(c + dx))\sec^6(c + dx)\,dx$

Optimal. Leaf size=101

$$\frac{a\tan^5(c+dx)}{5d} + \frac{2a\tan^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + 3a\tan(c+$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*a*Tan[c +
d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0657615, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3767, 3768, 3770}

$$\frac{a\tan^5(c+dx)}{5d} + \frac{2a\tan^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + 3a\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*a*Tan[c +
d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^6(c+dx)\,dx = a\int \sec^5(c+dx)\,dx + a\int \sec^6(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+2x^2\right.\right.}{}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{2a}{}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec}{}$$

Mathematica [A] time = 0.22837, size = 65, normalized size = 0.64

$$\frac{a\left(45\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\left(24\tan^4(c+dx) + 80\tan^2(c+dx) + 30\sec^3(c+dx) + 45\sec(c+dx) + 120\right)\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (a*(45*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(120 + 45*Sec[c + d*x] + 30*Sec
[c + d*x]^3 + 80*Tan[c + d*x]^2 + 24*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.079, size = 112, normalized size = 1.1

$$\frac{a\tan(dx+c)(\sec(dx+c))^3}{4d} + \frac{3a\sec(dx+c)\tan(dx+c)}{8d} + \frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{8a\tan(dx+c)}{15d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^6,x)
```

```
[Out] 1/4/d*a*tan(d*x+c)*sec(d*x+c)^3+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+8/15*a*tan(d*x+c)/d+1/5/d*a*tan(d*x+c)*sec(d*x+c)^4+4/
15/d*a*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.15805, size = 144, normalized size = 1.43

$$\frac{16\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a - 15a\left(\frac{2\left(3\sin(dx+c)^3 - 5\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1} - 3\log(\sin(dx+c)+1) + \right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 15*a
*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
+ 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.73177, size = 304, normalized size = 3.01

$$\frac{45\,a\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-45\,a\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(64\,a\cos\left(dx+c\right)^{4}+45\,a\cos\left(dx\right.\right.}{240\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(45*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 2*(64*a*cos(d*x + c)^4 + 45*a*cos(d*x + c)^3 + 32*a*co
s(d*x + c)^2 + 30*a*cos(d*x + c) + 24*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41724, size = 167, normalized size = 1.65

$$\frac{45\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-130\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+464\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(45*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(45*a*tan(1/2*d*x + 1/2*c)^9 - 130*a*tan(1/2*d*x + 1/2*c)
^7 + 464*a*tan(1/2*d*x + 1/2*c)^5 - 190*a*tan(1/2*d*x + 1/2*c)^3 + 195*a*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.13 $\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{2a^2\sin^5(c+dx)}{5d}-\frac{4a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{24d}+$$

```
[Out] (11*a^2*x)/16 + (2*a^2*Sin[c + d*x])/d + (11*a^2*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (11*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) + (2*a^2*Sin[c + d*x]^5
)/(5*d)
```

Rubi [A] time = 0.129558, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 2635, 8, 2633}

$$\frac{2a^2\sin^5(c+dx)}{5d}-\frac{4a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{24d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (11*a^2*x)/16 + (2*a^2*Sin[c + d*x])/d + (11*a^2*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (11*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^2*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) + (2*a^2*Sin[c + d*x]^5
)/(5*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)+2a^2\cos^5(c+dx)+a^2\cos^6(c+dx)\right)dx$$
$$= a^2\int \cos^4(c+dx)\,dx + a^2\int \cos^6(c+dx)\,dx + \left(2a^2\right)\int \cos^5(c+dx)\,dx$$
$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{4}\left(3a^2\right)\int \cos^2(c+$$
$$= \frac{2a^2\sin(c+dx)}{d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{11a^2\cos^3(c+dx)\sin(c+dx)}{24d} +$$
$$= \frac{3a^2x}{8} + \frac{2a^2\sin(c+dx)}{d} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{16d} + \frac{11a^2\cos^3(c+dx)\sin(}{24d}$$
$$= \frac{11a^2x}{16} + \frac{2a^2\sin(c+dx)}{d} + \frac{11a^2\cos(c+dx)\sin(c+dx)}{16d} + \frac{11a^2\cos^3(c+dx)\sin}{24d}$$

Mathematica [A] time = 0.182994, size = 73, normalized size = 0.57

$$\frac{a^2(1200\sin(c+dx)+465\sin(2(c+dx))+200\sin(3(c+dx))+75\sin(4(c+dx))+24\sin(5(c+dx))+5\sin(6(c+dx))-}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(660*d*x + 1200*Sin[c + d*x] + 465*Sin[2*(c + d*x)] + 200*Sin[3*(c + d
*x)] + 75*Sin[4*(c + d*x)] + 24*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(96
0*d)
```

Maple [A] time = 0.046, size = 121, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2a^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/d*(a^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+2/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*(
1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.14151, size = 163, normalized size = 1.26

$$\frac{128\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\right.}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/960*(128*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
+ 2*c))*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^
```

```
2)/d
```

Fricas [A] time = 1.61933, size = 228, normalized size = 1.77

$$\frac{165\,a^2dx + \left(40\,a^2\cos\left(dx+c\right)^5 + 96\,a^2\cos\left(dx+c\right)^4 + 110\,a^2\cos\left(dx+c\right)^3 + 128\,a^2\cos\left(dx+c\right)^2 + 165\,a^2\cos\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(165*a^2*d*x + (40*a^2*cos(d*x + c)^5 + 96*a^2*cos(d*x + c)^4 + 110*a
^2*cos(d*x + c)^3 + 128*a^2*cos(d*x + c)^2 + 165*a^2*cos(d*x + c) + 256*a^2
)*sin(d*x + c))/d
```

Sympy [A] time = 4.57973, size = 343, normalized size = 2.66

$$\begin{cases} \frac{5a^2x\sin^6(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^2x\sin^4(c+dx)}{8} + \frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{5a^2x\cos^6(}{16} \\ x\left(a\cos\left(c\right)+a\right)^2\cos^4\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 3*a**2*x*sin(c + d*x)**4/8 + 15*a**2*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*a**2*x*cos(c +
 d*x)**6/16 + 3*a**2*x*cos(c + d*x)**4/8 + 5*a**2*sin(c + d*x)**5*cos(c + d
*x)/(16*d) + 16*a**2*sin(c + d*x)**5/(15*d) + 5*a**2*sin(c + d*x)**3*cos(c
+ d*x)**3/(6*d) + 8*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a**2*sin
(c + d*x)**3*cos(c + d*x)/(8*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*
d) + 2*a**2*sin(c + d*x)*cos(c + d*x)**4/d + 5*a**2*sin(c + d*x)*cos(c + d*
x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**4, True))
```

Giac [A] time = 1.3619, size = 143, normalized size = 1.11

$$\frac{11}{16}\,a^2x + \frac{a^2\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{a^2\sin\left(5\,dx+5\,c\right)}{40\,d} + \frac{5\,a^2\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{5\,a^2\sin\left(3\,dx+3\,c\right)}{24\,d} + \frac{31\,a^2\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 11/16*a^2*x + 1/192*a^2*sin(6*d*x + 6*c)/d + 1/40*a^2*sin(5*d*x + 5*c)/d +
5/64*a^2*sin(4*d*x + 4*c)/d + 5/24*a^2*sin(3*d*x + 3*c)/d + 31/64*a^2*sin(2
*d*x + 2*c)/d + 5/4*a^2*sin(d*x + c)/d
```

3.14 $\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2x}{4}$$

```
[Out] (3*a^2*x)/4 + (2*a^2*Sin[c + d*x])/d + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - (a^2*Sin[c + d*x]^3)/d + (a
^2*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.104287, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 2633, 2635, 8}

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/4 + (2*a^2*Sin[c + d*x])/d + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - (a^2*Sin[c + d*x]^3)/d + (a
^2*Sin[c + d*x]^5)/(5*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^3(c+dx)+2a^2\cos^4(c+dx)+a^2\cos^5(c+dx)\right)dx$$
$$= a^2\int \cos^3(c+dx)\,dx + a^2\int \cos^5(c+dx)\,dx + \left(2a^2\right)\int \cos^4(c+dx)\,dx$$
$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(3a^2\right)\int \cos^2(c+dx)\,dx - \frac{a^2\,\mathrm{Subst}\left(\int (1-$$
$$= \frac{2a^2\sin(c+dx)}{d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}$$
$$= \frac{3a^2x}{4} + \frac{2a^2\sin(c+dx)}{d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^2\cos^3(c+dx)\sin(}{2d}$$

Mathematica [A] time = 0.118239, size = 61, normalized size = 0.59

$$\frac{a^2(110\sin(c+dx)+40\sin(2(c+dx))+15\sin(3(c+dx))+5\sin(4(c+dx))+\sin(5(c+dx))+60dx)}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(60*d*x + 110*Sin[c + d*x] + 40*Sin[2*(c + d*x)] + 15*Sin[3*(c + d*x)]
 + 5*Sin[4*(c + d*x)] + Sin[5*(c + d*x)]))/(80*d)
```

Maple [A] time = 0.043, size = 96, normalized size = 0.9

$$\frac{1}{d}\left(\frac{a^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+2a^2\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/d*(1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*(2+cos(d*x+c)^2
)*sin(d*x+c))
```

Maxima [A] time = 1.13985, size = 128, normalized size = 1.24

$$\frac{16\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^2-80\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2+15(12dx+12c+}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^2 - 80
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A] time = 1.72845, size = 186, normalized size = 1.81

$$\frac{15\,a^{2}dx + \left(4\,a^{2}\cos\left(dx+c\right)^{4} + 10\,a^{2}\cos\left(dx+c\right)^{3} + 12\,a^{2}\cos\left(dx+c\right)^{2} + 15\,a^{2}\cos\left(dx+c\right) + 24\,a^{2}\right)\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/20*(15*a^2*d*x + (4*a^2*cos(d*x + c)^4 + 10*a^2*cos(d*x + c)^3 + 12*a^2*c
os(d*x + c)^2 + 15*a^2*cos(d*x + c) + 24*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.35888, size = 221, normalized size = 2.15

$$\begin{cases} \frac{3a^{2}x\sin^{4}(c+dx)}{4} + \frac{3a^{2}x\sin^{2}(c+dx)\cos^{2}(c+dx)}{2} + \frac{3a^{2}x\cos^{4}(c+dx)}{4} + \frac{8a^{2}\sin^{5}(c+dx)}{15d} + \frac{4a^{2}\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d} + \frac{3a^{2}\sin^{3}(c+dx)\cos(c+dx)}{4d} + 2 \\ x\left(a\cos\left(c\right)+a\right)^{2}\cos^{3}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/4 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/2 + 3*a**2*x*cos(c + d*x)**4/4 + 8*a**2*sin(c + d*x)**5/(15*d) + 4*a*
*2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 3*a**2*sin(c + d*x)**3*cos(c + d
*x)/(4*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**
4/d + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a**2*sin(c + d*x)*cos(c +
 d*x)**2/d, Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c)**3, True))
```

Giac [A] time = 1.32697, size = 120, normalized size = 1.17

$$\frac{3}{4}\,a^{2}x + \frac{a^{2}\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{a^{2}\sin\left(4\,dx+4\,c\right)}{16\,d} + \frac{3\,a^{2}\sin\left(3\,dx+3\,c\right)}{16\,d} + \frac{a^{2}\sin\left(2\,dx+2\,c\right)}{2\,d} + \frac{11\,a^{2}\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*x + 1/80*a^2*sin(5*d*x + 5*c)/d + 1/16*a^2*sin(4*d*x + 4*c)/d + 3/1
6*a^2*sin(3*d*x + 3*c)/d + 1/2*a^2*sin(2*d*x + 2*c)/d + 11/8*a^2*sin(d*x +
c)/d
```

3.15 $\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=87

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

```
[Out] (7*a^2*x)/8 + (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a^2*Sin[c + d*x]^3)/(3*d
)
```

Rubi [A] time = 0.0974391, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 2635, 8, 2633}

$$-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{7a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (7*a^2*x)/8 + (2*a^2*Sin[c + d*x])/d + (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(8
*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a^2*Sin[c + d*x]^3)/(3*d
)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^2(c+dx)+2a^2\cos^3(c+dx)+a^2\cos^4(c+dx)\right)dx$$
$$= a^2\int \cos^2(c+dx)\,dx + a^2\int \cos^4(c+dx)\,dx + \left(2a^2\right)\int \cos^3(c+dx)\,dx$$
$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{2}a^2\int 1\,dx + \frac{1}{4}\left(3a^2\right.$$
$$= \frac{a^2x}{2} + \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)\sin(c+d}{4d}$$
$$= \frac{7a^2x}{8} + \frac{2a^2\sin(c+dx)}{d} + \frac{7a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2\cos^3(c+dx)\sin(c+}{4d}$$

Mathematica [A] time = 0.119793, size = 53, normalized size = 0.61

$$\frac{a^2(144\sin(c+dx)+48\sin(2(c+dx))+16\sin(3(c+dx))+3\sin(4(c+dx))+84dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(84*d*x + 144*Sin[c + d*x] + 48*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)]
 + 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.043, size = 90, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2a^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a^2\left(\frac{\cos(d}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/d*(a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
^2*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A] time = 1.10975, size = 112, normalized size = 1.29

$$-\frac{64\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2-3(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^2-24(2dx+2c+\sin(2d}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*
a^2)/d
```

Fricas [A] time = 1.68364, size = 154, normalized size = 1.77

$$\frac{21\,a^2dx+\left(6\,a^2\cos\left(dx+c\right)^3+16\,a^2\cos\left(dx+c\right)^2+21\,a^2\cos\left(dx+c\right)+32\,a^2\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*a^2*d*x + (6*a^2*cos(d*x + c)^3 + 16*a^2*cos(d*x + c)^2 + 21*a^2*c
os(d*x + c) + 32*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 1.27101, size = 211, normalized size = 2.43

$$\begin{cases}\frac{3a^2x\sin^4(c+dx)}{8}+\frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{a^2x\sin^2(c+dx)}{2}+\frac{3a^2x\cos^4(c+dx)}{8}+\frac{a^2x\cos^2(c+dx)}{2}+\frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{4a^2\sin^3}{3d}\\ x\left(a\cos\left(c\right)+a\right)^2\cos^2\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + a**2*x*sin(c + d*x)**2/2 + 3*a**2*x*cos(c + d*x)**4/8 + a**2*x*co
s(c + d*x)**2/2 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 4*a**2*sin(c
+ d*x)**3/(3*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 2*a**2*sin(c
+ d*x)*cos(c + d*x)**2/d + a**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a*cos(c) + a)**2*cos(c)**2, True))
```

Giac [A] time = 1.32465, size = 97, normalized size = 1.11

$$\frac{7}{8}\,a^2x+\frac{a^2\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a^2\sin\left(3\,dx+3\,c\right)}{6\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d}+\frac{3\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 7/8*a^2*x + 1/32*a^2*sin(4*d*x + 4*c)/d + 1/6*a^2*sin(3*d*x + 3*c)/d + 1/2*
a^2*sin(2*d*x + 2*c)/d + 3/2*a^2*sin(d*x + c)/d
```

3.16 $\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=57

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+a^2x$$

```
[Out] a^2*x + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/d - (a^2*S
in[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0414491, antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2751, 2644}

$$\frac{4a^2\sin(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{3d}+a^2x+\frac{\sin(c+dx)(a\cos(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] a^2*x + (4*a^2*Sin[c + d*x])/(3*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(3*d)
+ ((a + a*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\cos(c+dx))^2\,dx &= \frac{(a+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{2}{3}\int (a+a\cos(c+dx))^2\,dx\\ &= a^2x+\frac{4a^2\sin(c+dx)}{3d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{3d}+\frac{(a+a\cos(c+dx))^2\sin(c+}{3d}\end{aligned}$$

Mathematica [A] time = 0.079044, size = 41, normalized size = 0.72

$$\frac{a^2(21\sin(c+dx)+6\sin(2(c+dx))+\sin(3(c+dx))+12dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(12*d*x + 21*Sin[c + d*x] + 6*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(1
2*d)
```

Maple [A] time = 0.039, size = 64, normalized size = 1.1

$$\frac{1}{d}\left(\frac{a^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+2a^2\left(1/2\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2\right)+a^2\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+a^2*sin(d*x+c))
```

Maxima [A] time = 1.12368, size = 82, normalized size = 1.44

$$-\frac{2\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2-3\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)a^2-6\,a^2\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 3*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^2 - 6*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.70289, size = 113, normalized size = 1.98

$$\frac{3\,a^2dx+\left(a^2\cos(dx+c)^2+3\,a^2\cos(dx+c)+5\,a^2\right)\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*d*x + (a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + 5*a^2)*sin(d*x
+ c))/d
```

Sympy [A] time = 0.609382, size = 107, normalized size = 1.88

$$\begin{cases} a^2x\sin^2(c+dx)+a^2x\cos^2(c+dx)+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^2(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d} & \text{for } d = \\ x\left(a\cos(c)+a\right)^2\cos(c) & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2 + a**2*x*cos(c + d*x)**2 + 2*a**2*sin(c +
 d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2/d + a**2*sin(c + d*x)*co
s(c + d*x)/d + a**2*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**2*cos(c),
 True))
```

Giac [A] time = 1.34546, size = 73, normalized size = 1.28

$$a^2x + \frac{a^2 \sin(3\,dx + 3\,c)}{12\,d} + \frac{a^2 \sin(2\,dx + 2\,c)}{2\,d} + \frac{7\,a^2 \sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a^2*x + 1/12*a^2*sin(3*d*x + 3*c)/d + 1/2*a^2*sin(2*d*x + 2*c)/d + 7/4*a^2*
sin(d*x + c)/d
```

3.17 $\int (a + a\cos(c + dx))^2 \, dx$

Optimal. Leaf size=45

$$\frac{2a^2\sin(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{3a^2x}{2}$$

```
[Out] (3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rubi [A] time = 0.0140222, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{2a^2\sin(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 + (2*a^2*Sin[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2 \, dx = \frac{3a^2x}{2} + \frac{2a^2\sin(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0423232, size = 34, normalized size = 0.76

$$\frac{a^2(6(c+dx)+8\sin(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(6*(c + d*x) + 8*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.039, size = 52, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right) + 2\,a^2\sin(dx+c) + a^2(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*sin(d*x+c)+a^2*(d*
x+c))
```

Maxima [A] time = 1.1332, size = 61, normalized size = 1.36

$$a^2x + \frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))a^2}{4\,d} + \frac{2\,a^2\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 2*a^2*sin(d*x + c)/d
```

Fricas [A] time = 1.60468, size = 82, normalized size = 1.82

$$\frac{3\,a^2dx + \left(a^2\cos(dx + c) + 4\,a^2\right)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*d*x + (a^2*cos(d*x + c) + 4*a^2)*sin(d*x + c))/d
```

Sympy [A] time = 0.252579, size = 78, normalized size = 1.73

$$\begin{cases} \frac{a^2x\sin^2(c+dx)}{2} + \frac{a^2x\cos^2(c+dx)}{2} + a^2x + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2a^2\sin(c+dx)}{d} & \text{for } d \neq 0 \\ x\,(a\cos(c) + a)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x + a
**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*a**2*sin(c + d*x)/d, Ne(d, 0)), (x*
(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.36878, size = 51, normalized size = 1.13

$$\frac{3}{2}\,a^2x + \frac{a^2\sin(2\,dx + 2\,c)}{4\,d} + \frac{2\,a^2\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*x + 1/4*a^2*sin(2*d*x + 2*c)/d + 2*a^2*sin(d*x + c)/d
```

3.18 $\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx$

Optimal. Leaf size=34

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.0575523, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2746, 2735, 3770}

$$\frac{a^2\sin(c+dx)}{d}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x],x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^2\sec(c+dx)\,dx &= \frac{a^2\sin(c+dx)}{d}+\int \left(a^2+2a^2\cos(c+dx)\right)\sec(c+dx)\,dx\\ &= 2a^2x+\frac{a^2\sin(c+dx)}{d}+a^2\int \sec(c+dx)\,dx\\ &= 2a^2x+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0115677, size = 47, normalized size = 1.38

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c)\cos(dx)}{d}+\frac{a^2\cos(c)\sin(dx)}{d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x],x]
```

```
[Out] 2*a^2*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Cos[d*x]*Sin[c])/d + (a^2*Co
s[c]*Sin[d*x])/d
```

Maple [A] time = 0.059, size = 51, normalized size = 1.5

$$2a^2x + \frac{a^2\sin(dx+c)}{d} + \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 2\frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c),x)
```

```
[Out] 2*a^2*x+a^2*sin(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^2*c
```

Maxima [A] time = 1.1559, size = 58, normalized size = 1.71

$$\frac{2(dx+c)a^2 + a^2\log(\sec(dx+c)+\tan(dx+c)) + a^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(d*x + c)*a^2 + a^2*log(sec(d*x + c) + tan(d*x + c)) + a^2*sin(d*x + c))
/d
```

Fricas [A] time = 1.64752, size = 131, normalized size = 3.85

$$\frac{4a^2dx + a^2\log(\sin(dx+c)+1) - a^2\log(-\sin(dx+c)+1) + 2a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) + 2
*a^2*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\cos(c+dx)\sec(c+dx)\,dx + \int \cos^2(c+dx)\sec(c+dx)\,dx + \int \sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x), x) + Integral(cos(c + d*x)**2*s
ec(c + d*x), x) + Integral(sec(c + d*x), x))
```

Giac [B] time = 1.38499, size = 107, normalized size = 3.15

$$\frac{2(dx+c)a^2 + a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a^2 + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

3.19 $\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx$

Optimal. Leaf size=34

$$\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+a^2x$$

```
[Out] a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.0583988, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 3770, 3767, 8}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] a^2*x + (2*a^2*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^2\sec^2(c+dx)\,dx &= \int \left(a^2+2a^2\sec(c+dx)+a^2\sec^2(c+dx)\right)dx\\
&= a^2x+a^2\int \sec^2(c+dx)\,dx+\left(2a^2\right)\int \sec(c+dx)\,dx\\
&= a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\,\mathrm{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\
&= a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0122583, size = 28, normalized size = 0.82

$$a^2\left(\frac{\tan(c+dx)}{d}+\frac{2\tanh^{-1}(\sin(c+dx))}{d}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] a^2*(x + (2*ArcTanh[Sin[c + d*x]])/d + Tan[c + d*x]/d)
```

Maple [A] time = 0.065, size = 50, normalized size = 1.5

$$a^2x+2\,\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\tan(dx+c)}{d}+\frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^2,x)
```

```
[Out] a^2*x+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*tan(d*x+c)/d+1/d*a^2*c
```

Maxima [A] time = 1.07548, size = 66, normalized size = 1.94

$$\frac{(dx+c)a^2+a^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a^2 + a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + a^2*
tan(d*x + c))/d
```

Fricas [B] time = 1.65433, size = 193, normalized size = 5.68

$$\frac{a^2dx\cos(dx+c)+a^2\cos(dx+c)\log(\sin(dx+c)+1)-a^2\cos(dx+c)\log(-\sin(dx+c)+1)+a^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*x*cos(d*x + c) + a^2*cos(d*x + c)*log(sin(d*x + c) + 1) - a^2*cos(d*
x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\cos(c+dx)\sec^2(c+dx)\,dx+\int\cos^2(c+dx)\sec^2(c+dx)\,dx+\int\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(cos(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sec(c + d*x)**2, x))
```

Giac [B] time = 1.33591, size = 107, normalized size = 3.15

$$\frac{(dx+c)a^2 + 2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 2*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^2*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

3.20 $\int (a + a\cos(c + dx))^2 \sec^3(c + dx)\,dx$

Optimal. Leaf size=54

$$\frac{2a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0788182, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{2a^2\tan(c+dx)}{d}+\frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```



```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sec^3(c+dx)\,dx = \int \left(a^2\sec(c+dx)+2a^2\sec^2(c+dx)+a^2\sec^3(c+dx)\right)dx$$
$$= a^2\int \sec(c+dx)\,dx + a^2\int \sec^3(c+dx)\,dx + \left(2a^2\right)\int \sec^2(c+dx)\,dx$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a^2\int \sec(c+dx)\,dx -$$
$$= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0117205, size = 54, normalized size = 1.

$$\frac{2a^2\tan(c+dx)}{d} + \frac{3a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.076, size = 58, normalized size = 1.1

$$\frac{3\,a^2\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + 2\,\frac{a^2\tan(dx+c)}{d} + \frac{a^2\sec(dx+c)\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^3,x)
```

```
[Out] 3/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*tan(d*x+c)/d+1/2/d*a^2*sec(d*x+c)
*tan(d*x+c)
```

Maxima [A] time = 1.10201, size = 119, normalized size = 2.2

$$-\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 2\,a^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) - 8\,}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 8*a^2*tan(d*x + c))/d
```

Fricas [A] time = 1.66214, size = 215, normalized size = 3.98

$$\frac{3\,a^2\cos(dx+c)^2\log(\sin(dx+c)+1) - 3\,a^2\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\left(4\,a^2\cos(dx+c)+a^2\right)\sin(dx+c}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^2*log(
-sin(d*x + c) + 1) + 2*(4*a^2*cos(d*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x
+ c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48597, size = 122, normalized size = 2.26

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.21 $\int (a + a\cos(c + dx))^2 \sec^4(c + dx)\, dx$

Optimal. Leaf size=66

$$\frac{a^2\tan^3(c+dx)}{3d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*
Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0877679, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3767, 8, 3768, 3770}

$$\frac{a^2\tan^3(c+dx)}{3d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d + (2*a^2*Tan[c + d*x])/d + (a^2*Sec[c + d*x]*
Tan[c + d*x])/d + (a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sec^4(c+dx)\,dx = \int \left(a^2\sec^2(c+dx)+2a^2\sec^3(c+dx)+a^2\sec^4(c+dx)\right)dx$$
$$= a^2\int \sec^2(c+dx)\,dx + a^2\int \sec^4(c+dx)\,dx + \left(2a^2\right)\int \sec^3(c+dx)\,dx$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + a^2\int \sec(c+dx)\,dx - \frac{a^2\,\text{Subst}(\int 1\,dx, x, -\tan}{d}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{2a^2\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2}{}$$

Mathematica [B] time = 5.54138, size = 162, normalized size = 2.45

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-2\tan(c)\cos(c+dx)-\sec(c)(-4\sin(2c+dx)+3\sin(c+2dx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3*(12*Cos[c + d*
x]^3*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]]) - Sec[c]*(13*Sin[d*x] - 4*Sin[2*c + d*x] + 3*Sin[c + 2*d*x]
 + 3*Sin[3*c + 2*d*x] + 5*Sin[2*c + 3*d*x]) - 2*Cos[c + d*x]*Tan[c]))/(48*d
)
```

Maple [A] time = 0.077, size = 78, normalized size = 1.2

$$\frac{5\,a^2\tan(dx+c)}{3d} + \frac{a^2\sec(dx+c)\tan(dx+c)}{d} + \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^2(\sec(dx+c))^2\tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^4,x)
```

```
[Out] 5/3*a^2*tan(d*x+c)/d+1/d*a^2*sec(d*x+c)*tan(d*x+c)+1/d*a^2*ln(sec(d*x+c)+ta
n(d*x+c))+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Maxima [A] time = 1.13642, size = 115, normalized size = 1.74

$$\frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2-3\,a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6\,a^2\tan(dx}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^2*tan
(d*x + c))/d
```

Fricas [A] time = 1.67818, size = 246, normalized size = 3.73

$$\frac{3\,a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(5\,a^2\cos\left(dx+c\right)^2+3\,a^2\cos\left(dx+\right.\right.}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a^2*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(5*a^2*cos(d*x + c)^2 + 3*a^2*cos(d*x + c) + a^2)*si
n(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40979, size = 143, normalized size = 2.17

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 8*a^2*tan(1/2*d*x + 1/2*c
)^3 + 9*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.22 $\int (a + a\cos(c + dx))^2 \sec^5(c + dx)\, dx$

Optimal. Leaf size=96

$$\frac{2a^2\tan^3(c+dx)}{3d} + \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{7a^2\tan(c+dx)\sec(c}{8d}$$

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*Tan[c + d*x])/d + (7*a^2*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*
a^2*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.10797, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 3768, 3770, 3767}

$$\frac{2a^2\tan^3(c+dx)}{3d} + \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{7a^2\tan(c+dx)\sec(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

```
[Out] (7*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a^2*Tan[c + d*x])/d + (7*a^2*Sec[c
 + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*
a^2*Tan[c + d*x]^3)/(3*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sec^5(c+dx)\,dx = \int \left(a^2\sec^3(c+dx)+2a^2\sec^4(c+dx)+a^2\sec^5(c+dx)\right)dx$$
$$= a^2\int \sec^3(c+dx)\,dx + a^2\int \sec^5(c+dx)\,dx + \left(2a^2\right)\int \sec^4(c+dx)\,dx$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{2}a^2\int \sec(c+dx)\,d$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2\,\mathrm{se}}{}$$
$$= \frac{7a^2\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2a^2\tan(c+dx)}{d} + \frac{7a^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2}{}$$

Mathematica [B] time = 6.37008, size = 797, normalized size = 8.3

$$-\frac{7(\cos(c+dx)a+a)^2\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d} + \frac{7(\cos(c+dx)a+a)^2\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}\right.\right.}{32d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

```
[Out] (-7*(a + a*Cos[c + d*x])^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec
[c/2 + (d*x)/2]^4)/(32*d) + (7*(a + a*Cos[c + d*x])^2*Log[Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4)/(32*d) + ((a + a*Cos[c + d*x]
)^2*Sec[c/2 + (d*x)/2]^4)/(64*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4
) + ((a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(12*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[
c + d*x])^2*Sec[c/2 + (d*x)/2]^4*(29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a + a*Cos[
c + d*x])^2*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(3*d*(Cos[c/2] - Sin[c/2])*(
Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - ((a + a*Cos[c + d*x])^2*Sec[c/2
 + (d*x)/2]^4)/(64*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + ((a + a
*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c
/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d*x])^2
*Sec[c/2 + (d*x)/2]^4*(-29*Cos[c/2] - 13*Sin[c/2]))/(192*d*(Cos[c/2] + Sin[
c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + ((a + a*Cos[c + d*x])^
2*Sec[c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(3*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.082, size = 102, normalized size = 1.1

$$\frac{7\,a^2\sec(dx+c)\tan(dx+c)}{8\,d} + \frac{7\,a^2\ln(\sec(dx+c)+\tan(dx+c))}{8\,d} + \frac{4\,a^2\tan(dx+c)}{3\,d} + \frac{2\,a^2(\sec(dx+c))^2\tan(dx+c}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^5,x)
```

```
[Out] 7/8/d*a^2*sec(d*x+c)*tan(d*x+c)+7/8/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+4/3*a^2
*tan(d*x+c)/d+2/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d+1/4*a^2*sec(d*x+c)^3*tan(d*
x+c)/d
```

Maxima [A] time = 1.12332, size = 196, normalized size = 2.04

$$\frac{32\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^2-3a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)\cdots}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.72642, size = 288, normalized size = 3.

$$\frac{21\,a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-21\,a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(32\,a^2\cos\left(dx+c\right)^3+21\,a^2\cos\cdots\right.}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(21*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 21*a^2*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(32*a^2*cos(d*x + c)^3 + 21*a^2*cos(d*x + c)^2 +
16*a^2*cos(d*x + c) + 6*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46997, size = 165, normalized size = 1.72

$$\frac{21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-21\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-77\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+83\,a^2\tan\left(\frac{1}{2}\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(21*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 21*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(21*a^2*tan(1/2*d*x + 1/2*c)^7 - 77*a^2*tan(1/2*d*x +
1/2*c)^5 + 83*a^2*tan(1/2*d*x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c))/(ta
n(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.23 $\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=129

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{2}{}$$

```
[Out] (23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^5
)/(5*d)
```

Rubi [A] time = 0.146592, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 2633, 2635, 8}

$$\frac{3a^3\sin^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{23a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3,x]
```


```
[Out] (23*a^3*x)/16 + (4*a^3*Sin[c + d*x])/d + (23*a^3*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (23*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^3*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (7*a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^5
)/(5*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^3\,dx = \int \left(a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+3a^3\cos^5(c+dx)+a^3\cos^6(c+dx)\right)$$
$$= a^3\int \cos^3(c+dx)\,dx + a^3\int \cos^6(c+dx)\,dx + \left(3a^3\right)\int \cos^4(c+dx)\,dx + (3$$
$$= \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}\left(5a^3\right)\int \cos^4$$
$$= \frac{4a^3\sin(c+dx)}{d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{23a^3\cos^3(c+dx)\sin(c+dx}{24d}$$
$$= \frac{9a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{23a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{23a^3\cos^3(c+dx)s}{24d}$$
$$= \frac{23a^3x}{16} + \frac{4a^3\sin(c+dx)}{d} + \frac{23a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{23a^3\cos^3(c+dx)}{24d}$$

Mathematica [A] time = 0.165939, size = 73, normalized size = 0.57

$$\frac{a^3(2520\sin(c+dx)+945\sin(2(c+dx))+380\sin(3(c+dx))+135\sin(4(c+dx))+36\sin(5(c+dx))+5\sin(6(c+d}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(1380*d*x + 2520*Sin[c + d*x] + 945*Sin[2*(c + d*x)] + 380*Sin[3*(c +
d*x)] + 135*Sin[4*(c + d*x)] + 36*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(
960*d)
```

Maple [A] time = 0.044, size = 143, normalized size = 1.1

$$\frac{1}{d}\left(a^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3a^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/d*(a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^3*(2+co
s(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.10023, size = 193, normalized size = 1.5

$$\frac{192\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
```

```
+ 2*c))*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 90*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A] time = 1.67549, size = 230, normalized size = 1.78

$$\frac{345\,a^3dx + \left(40\,a^3\cos\left(dx+c\right)^5 + 144\,a^3\cos\left(dx+c\right)^4 + 230\,a^3\cos\left(dx+c\right)^3 + 272\,a^3\cos\left(dx+c\right)^2 + 345\,a^3\cos\left(dx+c\right) \cdots}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(345*a^3*d*x + (40*a^3*cos(d*x + c)^5 + 144*a^3*cos(d*x + c)^4 + 230*
a^3*cos(d*x + c)^3 + 272*a^3*cos(d*x + c)^2 + 345*a^3*cos(d*x + c) + 544*a^
3)*sin(d*x + c))/d
```

Sympy [A] time = 4.70609, size = 379, normalized size = 2.94

$$\begin{cases} \frac{5a^3x\sin^6(c+dx)}{16} + \frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9a^3x\sin^4(c+dx)}{8} + \frac{15a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{9a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{5a^3x\cos^6(c+dx)}{16} \cdots \\ x\left(a\cos\left(c\right)+a\right)^3\cos^3\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(c + d*x)**6/16 + 15*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 9*a**3*x*sin(c + d*x)**4/8 + 15*a**3*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 5*a**3*x*cos(c +
 d*x)**6/16 + 9*a**3*x*cos(c + d*x)**4/8 + 5*a**3*sin(c + d*x)**5*cos(c + d
*x)/(16*d) + 8*a**3*sin(c + d*x)**5/(5*d) + 5*a**3*sin(c + d*x)**3*cos(c +
d*x)**3/(6*d) + 4*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d + 9*a**3*sin(c + d
*x)**3*cos(c + d*x)/(8*d) + 2*a**3*sin(c + d*x)**3/(3*d) + 11*a**3*sin(c +
d*x)*cos(c + d*x)**5/(16*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*a*
*3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d
, Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c)**3, True))
```

Giac [A] time = 1.33412, size = 143, normalized size = 1.11

$$\frac{23}{16}\,a^3x + \frac{a^3\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{3\,a^3\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{9\,a^3\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{19\,a^3\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{63\,a^3\sin\left(2\,dx+2\,c\right)}{64\,d} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 23/16*a^3*x + 1/192*a^3*sin(6*d*x + 6*c)/d + 3/80*a^3*sin(5*d*x + 5*c)/d +
9/64*a^3*sin(4*d*x + 4*c)/d + 19/48*a^3*sin(3*d*x + 3*c)/d + 63/64*a^3*sin(
2*d*x + 2*c)/d + 21/8*a^3*sin(d*x + c)/d
```

3.24 $\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=105

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+$$

```
[Out] (13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3)/
(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.11724, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 2635, 8, 2633}

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3,x]
```


```
[Out] (13*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (3*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (5*a^3*Sin[c + d*x]^3)/
(3*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x, x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^3\,dx = \int \left(a^3\cos^2(c+dx)+3a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)+a^3\cos^5(c+dx)\right)dx$$
$$= a^3\int \cos^2(c+dx)\,dx + a^3\int \cos^5(c+dx)\,dx + \left(3a^3\right)\int \cos^3(c+dx)\,dx + \left(3a^3\right)$$
$$= \frac{a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{2}a^3\int 1\,dx + \frac{1}{4}\left(9a\right.$$
$$= \frac{a^3x}{2} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+dx)\sin(c+}{4d}$$
$$= \frac{13a^3x}{8} + \frac{4a^3\sin(c+dx)}{d} + \frac{13a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{3a^3\cos^3(c+dx)\sin(}{4d}$$

Mathematica [A] time = 0.12669, size = 63, normalized size = 0.6

$$\frac{a^3(1380\sin(c+dx)+480\sin(2(c+dx))+170\sin(3(c+dx))+45\sin(4(c+dx))+6\sin(5(c+dx))+780dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(780*d*x + 1380*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 170*Sin[3*(c + d
*x)] + 45*Sin[4*(c + d*x)] + 6*Sin[5*(c + d*x)]))/(480*d)
```

Maple [A] time = 0.043, size = 121, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+3a^3\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/d*(1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(2+cos(d*x+c)^2)*si
n(d*x+c)+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.12049, size = 158, normalized size = 1.5

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^3-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3+45(12dx+12c+\mathrm{si}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^3 - 48
0*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 + 45*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A] time = 1.64661, size = 194, normalized size = 1.85

$$\frac{195\,a^3dx + \left(24\,a^3\cos\left(dx+c\right)^4 + 90\,a^3\cos\left(dx+c\right)^3 + 152\,a^3\cos\left(dx+c\right)^2 + 195\,a^3\cos\left(dx+c\right) + 304\,a^3\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(195*a^3*d*x + (24*a^3*cos(d*x + c)^4 + 90*a^3*cos(d*x + c)^3 + 152*a
^3*cos(d*x + c)^2 + 195*a^3*cos(d*x + c) + 304*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 2.46558, size = 272, normalized size = 2.59

$$\begin{cases} \frac{9a^3x\sin^4(c+dx)}{8} + \frac{9a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^3x\sin^2(c+dx)}{2} + \frac{9a^3x\cos^4(c+dx)}{8} + \frac{a^3x\cos^2(c+dx)}{2} + \frac{8a^3\sin^5(c+dx)}{15d} + \frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d} \\ x\left(a\cos\left(c\right)+a\right)^3\cos^2\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + a**3*x*sin(c + d*x)**2/2 + 9*a**3*x*cos(c + d*x)**4/8 + a**3*x*co
s(c + d*x)**2/2 + 8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*co
s(c + d*x)**2/(3*d) + 9*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3*si
n(c + d*x)**3/d + a**3*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**3*sin(c + d*x
)*cos(c + d*x)**3/(8*d) + 3*a**3*sin(c + d*x)*cos(c + d*x)**2/d + a**3*sin(
c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c)**2, Tru
e))
```

Giac [A] time = 1.39879, size = 119, normalized size = 1.13

$$\frac{13}{8}\,a^3x + \frac{a^3\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{3\,a^3\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{17\,a^3\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{a^3\sin\left(2\,dx+2\,c\right)}{d} + \frac{23\,a^3\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 13/8*a^3*x + 1/80*a^3*sin(5*d*x + 5*c)/d + 3/32*a^3*sin(4*d*x + 4*c)/d + 17
/48*a^3*sin(3*d*x + 3*c)/d + a^3*sin(2*d*x + 2*c)/d + 23/8*a^3*sin(d*x + c)
/d
```

3.25 $\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=85

$$-\frac{a^3\sin^3(c+dx)}{d}+\frac{4a^3\sin(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}$$

```
[Out] (15*a^3*x)/8 + (4*a^3*Sin[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d
```

Rubi [A] time = 0.0780084, antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{a^3\sin^3(c+dx)}{4d}+\frac{3a^3\sin(c+dx)}{d}+\frac{9a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{15a^3x}{8}+\frac{\sin(c+dx)(a\cos(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (15*a^3*x)/8 + (3*a^3*Sin[c + d*x])/d + (9*a^3*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + ((a + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*d) - (a^3*Sin[c + d*x]^3)/(
4*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{3}{4}\int (a+a\cos(c+dx))^3\,dx$$
$$= \frac{(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{3}{4}\int \left(a^3+3a^3\cos(c+dx)+3a^3\cos^2(c+d$$
$$= \frac{3a^3x}{4} + \frac{(a+a\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\left(3a^3\right)\int \cos^3(c+dx)\,dx + \frac{1}{4}\left(9a$$
$$= \frac{3a^3x}{4} + \frac{9a^3\sin(c+dx)}{4d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a+a\cos(c+dx))^3 s}{4d}$$
$$= \frac{15a^3x}{8} + \frac{3a^3\sin(c+dx)}{d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{(a+a\cos(c+dx))^3}{4d}$$

Mathematica [A] time = 0.118255, size = 51, normalized size = 0.6

$$\frac{a^3(104\sin(c+dx)+32\sin(2(c+dx))+8\sin(3(c+dx))+\sin(4(c+dx))+60dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(60*d*x + 104*Sin[c + d*x] + 32*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]
+ Sin[4*(c + d*x)]))/(32*d)
```

Maple [A] time = 0.04, size = 100, normalized size = 1.2

$$\frac{1}{d}\left(a^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+a^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)+3a^3(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/d*(a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^3*(
2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+
a^3*sin(d*x+c))
```

Maxima [A] time = 1.14336, size = 127, normalized size = 1.49

$$-\frac{32\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3-(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^3-24(2dx+2c+\sin(2}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^
3 - 32*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.61081, size = 151, normalized size = 1.78

$$\frac{15\,a^3dx + \left(2\,a^3\cos\left(dx+c\right)^3 + 8\,a^3\cos\left(dx+c\right)^2 + 15\,a^3\cos\left(dx+c\right) + 24\,a^3\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(15*a^3*d*x + (2*a^3*cos(d*x + c)^3 + 8*a^3*cos(d*x + c)^2 + 15*a^3*cos
(d*x + c) + 24*a^3)*sin(d*x + c))/d
```

Sympy [A] time = 1.29056, size = 224, normalized size = 2.64

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^3x\sin^2(c+dx)}{2} + \frac{3a^3x\cos^4(c+dx)}{8} + \frac{3a^3x\cos^2(c+dx)}{2} + \frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2a^3\sin^3(c}{d} \\ x\left(a\cos\left(c\right)+a\right)^3\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*
x*cos(c + d*x)**2/2 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*a**3*si
n(c + d*x)**3/d + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*a**3*sin(c
+ d*x)*cos(c + d*x)**2/d + 3*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) + a**3*si
n(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**3*cos(c), True))
```

Giac [A] time = 1.3528, size = 96, normalized size = 1.13

$$\frac{15}{8}\,a^3x + \frac{a^3\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a^3\sin\left(3\,dx+3\,c\right)}{4\,d} + \frac{a^3\sin\left(2\,dx+2\,c\right)}{d} + \frac{13\,a^3\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 15/8*a^3*x + 1/32*a^3*sin(4*d*x + 4*c)/d + 1/4*a^3*sin(3*d*x + 3*c)/d + a^3
*sin(2*d*x + 2*c)/d + 13/4*a^3*sin(d*x + c)/d
```

3.26 $\int (a + a\cos(c+dx))^3\, dx$

Optimal. Leaf size=63

$$-\frac{a^3\sin^3(c+dx)}{3d} + \frac{4a^3\sin(c+dx)}{d} + \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{5a^3x}{2}$$

```
[Out] (5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2
*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0526376, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2637, 2635, 8, 2633}

$$-\frac{a^3\sin^3(c+dx)}{3d} + \frac{4a^3\sin(c+dx)}{d} + \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/2 + (4*a^3*Sin[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2
*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\,dx = \int \left(a^3+3a^3\cos(c+dx)+3a^3\cos^2(c+dx)+a^3\cos^3(c+dx)\right)\,dx$$
$$= a^3x+a^3\int \cos^3(c+dx)\,dx+\left(3a^3\right)\int \cos(c+dx)\,dx+\left(3a^3\right)\int \cos^2(c+dx)\,dx$$
$$= a^3x+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int 1\,dx-\frac{a^3\,\text{Subst}\left(\int \left(1-x^2\right)\right.}{}$$
$$= \frac{5a^3x}{2}+\frac{4a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0648765, size = 44, normalized size = 0.7

$$\frac{a^3(45\sin(c+dx)+9\sin(2(c+dx))+\sin(3(c+dx))+30c+30dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (a^3*(30*c + 30*d*x + 45*Sin[c + d*x] + 9*Sin[2*(c + d*x)] + Sin[3*(c + d*x
)]))/(12*d)
```

Maple [A] time = 0.039, size = 74, normalized size = 1.2

$$\frac{1}{d}\left(\frac{a^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+3a^3\left(1/2\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2\right)+3a^3\sin(dx+c)+a^3(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+3*a^3*sin(d*x+c)+a^3*(d*x+c))
```

Maxima [A] time = 1.19853, size = 95, normalized size = 1.51

$$a^3x-\frac{\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3}{3d}+\frac{3(2dx+2c+\sin(2dx+2c))a^3}{4d}+\frac{3a^3\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c + si
n(2*d*x + 2*c))*a^3/d + 3*a^3*sin(d*x + c)/d
```

Fricas [A] time = 1.57316, size = 119, normalized size = 1.89

$$\frac{15a^3dx+\left(2a^3\cos(dx+c)^2+9a^3\cos(dx+c)+22a^3\right)\sin(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a^3*d*x + (2*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 22*a^3)*sin(
d*x + c))/d
```

Sympy [A] time = 0.622063, size = 121, normalized size = 1.92

$$\begin{cases} \frac{3a^3x\sin^2(c+dx)}{2} + \frac{3a^3x\cos^2(c+dx)}{2} + a^3x + \frac{2a^3\sin^3(c+dx)}{3d} + \frac{a^3\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{3a^3\sin(c+dx)}{d} & \text{for } d \\ x(a\cos(c)+a)^3 & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*x
 + 2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2/d + 3*a
**3*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*a**3*sin(c + d*x)/d, Ne(d, 0)), (x*
(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.30949, size = 74, normalized size = 1.17

$$\frac{5}{2}a^3x + \frac{a^3\sin(3dx+3c)}{12d} + \frac{3a^3\sin(2dx+2c)}{4d} + \frac{15a^3\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*x + 1/12*a^3*sin(3*d*x + 3*c)/d + 3/4*a^3*sin(2*d*x + 2*c)/d + 15/4
*a^3*sin(d*x + c)/d
```

3.27 $\int (a+a\cos(c+dx))^3 \sec(c+dx)\,dx$

Optimal. Leaf size=59

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

```
[Out] (7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0623902, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2757, 2637, 2635, 8, 3770}

$$\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x],x]
```

```
[Out] (7*a^3*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec(c+dx)\,dx = \int \left(3a^3+3a^3\cos(c+dx)+a^3\cos^2(c+dx)+a^3\sec(c+dx)\right)dx$$
$$= 3a^3x+a^3\int \cos^2(c+dx)\,dx+a^3\int \sec(c+dx)\,dx+\left(3a^3\right)\int \cos(c+dx)\,dx$$
$$= 3a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}$$
$$= \frac{7a^3x}{2}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0678953, size = 81, normalized size = 1.37

$$\frac{a^3\left(12\sin(c+dx)+\sin(2(c+dx))-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x],x]
```

```
[Out] (a^3*(14*d*x - 4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + 12*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.063, size = 72, normalized size = 1.2

$$\frac{a^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{7a^3x}{2}+\frac{7a^3c}{2d}+3\frac{a^3\sin(dx+c)}{d}+\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c),x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+7/2*a^3*x+7/2/d*a^3*c+3*a^3*sin(d*x+c)/d+1/
d*a^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.1999, size = 90, normalized size = 1.53

$$\frac{(2dx+2c+\sin(2dx+2c))a^3+12(dx+c)a^3+4a^3\log(\sec(dx+c)+\tan(dx+c))+12a^3\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(d*x + c)*a^3 + 4*a^3*log(se
c(d*x + c) + tan(d*x + c)) + 12*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.70168, size = 159, normalized size = 2.69

$$\frac{7a^3dx+a^3\log(\sin(dx+c)+1)-a^3\log(-\sin(dx+c)+1)+\left(a^3\cos(dx+c)+6a^3\right)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(7*a^3*d*x + a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (
a^3*cos(d*x + c) + 6*a^3)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int 3\cos\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int 3\cos^{2}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int \cos^{3}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int \sec\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)*sec(c + d*x), x) + Integral(3*cos(c + d*x)**2
*sec(c + d*x), x) + Integral(cos(c + d*x)**3*sec(c + d*x), x) + Integral(se
c(c + d*x), x))
```

Giac [A] time = 1.46551, size = 135, normalized size = 2.29

$$\frac{7\,(dx+c)a^{3}+2\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+7\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(7*(d*x + c)*a^3 + 2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.28 $\int (a + a\cos(c + dx))^3 \sec^2(c + dx)\, dx$

Optimal. Leaf size=48

$$\frac{a^3\sin(c+dx)}{d} + \frac{a^3\tan(c+dx)}{d} + \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} + 3a^3x$$

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Tan
[c + d*x])/d
```

Rubi [A] time = 0.0669439, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 2637, 3770, 3767, 8}

$$\frac{a^3\sin(c+dx)}{d} + \frac{a^3\tan(c+dx)}{d} + \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} + 3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/d + (a^3*Tan
[c + d*x])/d
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^2(c+dx)\,dx = \int \left(3a^3+a^3\cos(c+dx)+3a^3\sec(c+dx)+a^3\sec^2(c+dx)\right)dx$$
$$= 3a^3x + a^3\int \cos(c+dx)\,dx + a^3\int \sec^2(c+dx)\,dx + \left(3a^3\right)\int \sec(c+dx)\,dx$$
$$= 3a^3x + \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^3\sin(c+dx)}{d} - \frac{a^3\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c$$
$$= 3a^3x + \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^3\sin(c+dx)}{d} + \frac{a^3\tan(c+dx)}{d}$$

Mathematica [B] time = 0.681315, size = 211, normalized size = 4.4

$$\frac{1}{8}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(c)\cos(dx)}{d}+\frac{\cos(c)\sin(dx)}{d}+\frac{\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(3*x - (3*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]])/d + (3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d +
(Cos[d*x]*Sin[c])/d + (Cos[c]*Sin[d*x])/d + Sin[(d*x)/2]/(d*(Cos[c/2] - Sin
[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(d*x)/2]/(d*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/8
```

Maple [A] time = 0.077, size = 65, normalized size = 1.4

$$3\,a^3x + \frac{a^3\sin(dx+c)}{d} + 3\,\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{a^3\tan(dx+c)}{d} + 3\,\frac{a^3c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^2,x)
```

```
[Out] 3*a^3*x+a^3*sin(d*x+c)/d+3/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*tan(d*x+c)/d
+3/d*a^3*c
```

Maxima [A] time = 1.07648, size = 86, normalized size = 1.79

$$\frac{6\,(dx+c)a^3 + 3\,a^3(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2\,a^3\sin(dx+c) + 2\,a^3\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a^3 + 3*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) + 2*a^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.76439, size = 238, normalized size = 4.96

$$\frac{6\,a^{3}dx\cos\left(dx+c\right)+3\,a^{3}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,a^{3}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^{3}\cos\left(dx\right.\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*d*x*cos(d*x + c) + 3*a^3*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*
a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c) + a^3)*sin(d*
x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48311, size = 108, normalized size = 2.25

$$\frac{3\,(dx+c)a^{3}+3\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a^3 + 3*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^3*log(abs
(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2
*c)^4 - 1))/d
```

3.29 $\int (a + a\cos(c + dx))^3 \sec^3(c + dx)\, dx$

Optimal. Leaf size=59

$$\frac{3a^3\tan(c+dx)}{d} + \frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^3\tan(c+dx)\sec(c+dx)}{2d} + a^3x$$

```
[Out] a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^3
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0826457, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{3a^3\tan(c+dx)}{d} + \frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^3\tan(c+dx)\sec(c+dx)}{2d} + a^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

```
[Out] a^3*x + (7*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^3
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^3(c+dx)\,dx = \int \left(a^3+3a^3\sec(c+dx)+3a^3\sec^2(c+dx)+a^3\sec^3(c+dx)\right) dx$$
$$= a^3x + a^3\int \sec^3(c+dx)\,dx + \left(3a^3\right)\int \sec(c+dx)\,dx + \left(3a^3\right)\int \sec^2(c+dx)$$
$$= a^3x + \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a^3\int \sec(c$$
$$= a^3x + \frac{7a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{3a^3\tan(c+dx)}{d} + \frac{a^3\sec(c+dx)\tan(c+d}{2d}$$

Mathematica [A] time = 0.0244912, size = 50, normalized size = 0.85

$$a^3\left(\frac{3\tan(c+dx)}{d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

```
[Out] a^3*(x + (7*ArcTanh[Sin[c + d*x]])/(2*d) + (3*Tan[c + d*x])/d + (Sec[c + d*
x]*Tan[c + d*x])/(2*d))
```

Maple [A] time = 0.079, size = 71, normalized size = 1.2

$$a^3x+\frac{a^3c}{d}+\frac{7a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+3\frac{a^3\tan(dx+c)}{d}+\frac{a^3\sec(dx+c)\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^3,x)
```

```
[Out] a^3*x+1/d*a^3*c+7/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)/d+1/2/
d*a^3*sec(d*x+c)*tan(d*x+c)
```

Maxima [A] time = 1.21225, size = 134, normalized size = 2.27

$$\frac{4(dx+c)a^3-a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6a^3(\log(\sin(dx+c)+1)-\log(\sin(d}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*a^3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1)) + 12*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.80309, size = 251, normalized size = 4.25

$$\frac{4a^3dx\cos(dx+c)^2+7a^3\cos(dx+c)^2\log(\sin(dx+c)+1)-7a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\left(6a^3\cos\right.}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 + 7*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)
- 7*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^3*cos(d*x + c) + a^3
)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42431, size = 135, normalized size = 2.29

$$\frac{2\,(dx+c)a^3 + 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 7\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 + 7*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 7*a^3*log
(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*t
an(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.30 $\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0950675, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```



```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (4*a^3*Tan[c + d*x])/d + (3*a^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^4(c+dx)\,dx = \int \left(a^3\sec(c+dx)+3a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+a^3\sec^4(c+dx)\right)dx$$
$$= a^3\int \sec(c+dx)\,dx + a^3\int \sec^4(c+dx)\,dx + \left(3a^3\right)\int \sec^2(c+dx)\,dx + \left(3a^3\right)\int$$
$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(3a^3\right)\int \sec(c+dx)\,d$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^3}{}$$

Mathematica [B] time = 5.17723, size = 154, normalized size = 2.14

$$-\frac{a^3\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(-4\tan(c)\cos(c+dx)-\sec(c)(-20\sin(2c+dx)+9\sin(c+2dx)+9\sin(3c+2d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

```
[Out] -(a^3*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(60*Cos[c + d*x]^3*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 - Sec[c]*(50*Sin[d*x] - 20*Sin[2*c + d*x] + 9*Sin[c + 2*d*x] + 9*Sin[3*c +
 2*d*x] + 22*Sin[2*c + 3*d*x]) - 4*Cos[c + d*x]*Tan[c]))/(192*d)
```

Maple [A] time = 0.078, size = 80, normalized size = 1.1

$$\frac{5a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{11a^3\tan(dx+c)}{3d} + \frac{3a^3\sec(dx+c)\tan(dx+c)}{2d} + \frac{a^3\tan(dx+c)(\sec(dx+c))^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^4,x)
```

```
[Out] 5/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+11/3*a^3*tan(d*x+c)/d+3/2/d*a^3*sec(d*x
+c)*tan(d*x+c)+1/3/d*a^3*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.26993, size = 150, normalized size = 2.08

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3-9a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6a^3(\log(\sin(d}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*a^3*tan(d*x + c))/d
```

Fricas [A] time = 1.69507, size = 254, normalized size = 3.53

$$\frac{15\,a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-15\,a^3\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(22\,a^3\cos\left(dx+c\right)^2+9\,a^3\cos\right.}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(15*a^3*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^3*l
og(-sin(d*x + c) + 1) + 2*(22*a^3*cos(d*x + c)^2 + 9*a^3*cos(d*x + c) + 2*a
^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35431, size = 143, normalized size = 1.99

$$\frac{15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,a^3\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*a^3*tan(1/2*d*x + 1
/2*c)^3 + 33*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.31 $\int (a + a\cos(c + dx))^3 \sec^5(c + dx)\, dx$

Optimal. Leaf size=93

$$\frac{a^3 \tan^3(c + dx)}{d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{15a^3 \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^3 \tan(c + dx)\sec^3(c + dx)}{4d} + \frac{15a^3 \tan(c + dx)\sec(c +}{8d}$$

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
a^3*Tan[c + d*x]^3)/d
```

Rubi [A] time = 0.116538, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3767, 8, 3768, 3770}

$$\frac{a^3 \tan^3(c + dx)}{d} + \frac{4a^3 \tan(c + dx)}{d} + \frac{15a^3 \tanh^{-1}(\sin(c + dx))}{8d} + \frac{a^3 \tan(c + dx)\sec^3(c + dx)}{4d} + \frac{15a^3 \tan(c + dx)\sec(c +}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (15*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
a^3*Tan[c + d*x]^3)/d
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^5(c+dx)\,dx = \int \left(a^3\sec^2(c+dx)+3a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+a^3\sec^5(c+dx)\right)$$
$$= a^3\int \sec^2(c+dx)\,dx + a^3\int \sec^5(c+dx)\,dx + \left(3a^3\right)\int \sec^3(c+dx)\,dx + (3a$$
$$= \frac{3a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(3a^3\right)\int \sec^3($$
$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+dx)}{8d} +$$
$$= \frac{15a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{15a^3\sec(c+dx)\tan(c+dx)}{8d}$$

Mathematica [B] time = 6.33517, size = 797, normalized size = 8.57

$$-\frac{15(\cos(c+dx)a+a)^3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d} + \frac{15(\cos(c+dx)a+a)^3\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\mathrm{s}\right.}{64d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

```
[Out] (-15*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Se
c[c/2 + (d*x)/2]^6)/(64*d) + (15*(a + a*Cos[c + d*x])^3*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6)/(64*d) + ((a + a*Cos[c + d*
x])^3*Sec[c/2 + (d*x)/2]^6)/(128*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]
)^4) + ((a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(16*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a*C
os[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Co
s[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (3*(a + a
*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(8*d*(Cos[c/2] - Sin[c/
2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - ((a + a*Cos[c + d*x])^3*Se
c[c/2 + (d*x)/2]^6)/(128*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + (
(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(16*d*(Cos[c/2] +
 Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c + d
*x])^3*Sec[c/2 + (d*x)/2]^6*(-19*Cos[c/2] - 11*Sin[c/2]))/(128*d*(Cos[c/2]
+ Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (3*(a + a*Cos[c
+ d*x])^3*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(8*d*(Cos[c/2] + Sin[c/2])*(Co
s[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.086, size = 101, normalized size = 1.1

$$3\,\frac{a^3\tan(dx+c)}{d} + \frac{15\,a^3\sec(dx+c)\tan(dx+c)}{8\,d} + \frac{15\,a^3\ln(\sec(dx+c)+\tan(dx+c))}{8\,d} + \frac{a^3\tan(dx+c)(\sec(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^5,x)
```

```
[Out] 3*a^3*tan(d*x+c)/d+15/8/d*a^3*sec(d*x+c)*tan(d*x+c)+15/8/d*a^3*ln(sec(d*x+c
)+tan(d*x+c))+1/d*a^3*tan(d*x+c)*sec(d*x+c)^2+1/4/d*a^3*tan(d*x+c)*sec(d*x+
c)^3
```

Maxima [A] time = 1.21037, size = 211, normalized size = 2.27

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{3}-a^{3}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - a^3*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*a^3*tan(d*x + c
))/d
```

Fricas [A] time = 1.69643, size = 286, normalized size = 3.08

$$\frac{15\,a^{3}\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-15\,a^{3}\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(24\,a^{3}\cos\left(dx+c\right)^{3}+15\,a^{3}\cos\left(\right.\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(15*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*a^3*cos(d*x + c)^4*l
og(-sin(d*x + c) + 1) + 2*(24*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c)^2 +
8*a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4002, size = 165, normalized size = 1.77

$$\frac{15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-55\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+73\,a^{3}\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/8*(15*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 2*(15*a^3*tan(1/2*d*x + 1/2*c)^7 - 55*a^3*tan(1/2*d*x + 1
```

```
/2*c)^5 + 73*a^3*tan(1/2*d*x + 1/2*c)^3 - 49*a^3*tan(1/2*d*x + 1/2*c))/(tan
(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.32 $\int (a + a\cos(c + dx))^3 \sec^6(c + dx)\,dx$

Optimal. Leaf size=114

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{13a}{}$$

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.127425, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 3768, 3770, 3767}

$$\frac{a^3\tan^5(c+dx)}{5d}+\frac{5a^3\tan^3(c+dx)}{3d}+\frac{4a^3\tan(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{13a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*Tan[c + d*x])/d + (13*a^3*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) +
 (5*a^3*Tan[c + d*x]^3)/(3*d) + (a^3*Tan[c + d*x]^5)/(5*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^6(c+dx)\,dx = \int \left(a^3\sec^3(c+dx)+3a^3\sec^4(c+dx)+3a^3\sec^5(c+dx)+a^3\sec^6(c+dx)\right)$$
$$= a^3\int \sec^3(c+dx)\,dx + a^3\int \sec^6(c+dx)\,dx + \left(3a^3\right)\int \sec^4(c+dx)\,dx + \left(3a\right.$$
$$= \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{2}a^3\int \sec(c+$$
$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{8d} +$$
$$= \frac{13a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3\tan(c+dx)}{d} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{8d}$$

Mathematica [B] time = 1.31052, size = 487, normalized size = 4.27

$$-\frac{a^3\sec(c)\sec^5(c+dx)\left(1440\sin(2c+dx)-1500\sin(c+2dx)-1500\sin(3c+2dx)-3040\sin(2c+3dx)-390\sin(3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

```
[Out] -(a^3*Sec[c]*Sec[c + d*x]^5*(975*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 975*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 195*
Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1950*Cos[d*x]*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + 1950*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 975*Cos[2*c + 3*d*x]*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 975*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] - 195*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] - 195*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
 4640*Sin[d*x] + 1440*Sin[2*c + d*x] - 1500*Sin[c + 2*d*x] - 1500*Sin[3*c +
 2*d*x] - 3040*Sin[2*c + 3*d*x] - 390*Sin[3*c + 4*d*x] - 390*Sin[5*c + 4*d*
x] - 608*Sin[4*c + 5*d*x]))/(3840*d)
```

Maple [A] time = 0.086, size = 124, normalized size = 1.1

$$\frac{13a^3\sec(dx+c)\tan(dx+c)}{8d} + \frac{13a^3\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{38a^3\tan(dx+c)}{15d} + \frac{19a^3\tan(dx+c)(\sec(}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^6,x)
```

```
[Out] 13/8/d*a^3*sec(d*x+c)*tan(d*x+c)+13/8/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+38/15
*a^3*tan(d*x+c)/d+19/15/d*a^3*tan(d*x+c)*sec(d*x+c)^2+3/4/d*a^3*tan(d*x+c)*
sec(d*x+c)^3+1/5/d*a^3*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 1.1471, size = 242, normalized size = 2.12

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^3+240\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3-45a^3\left(\frac{2(3\sin(d}{\sin(dx+c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 45*a^3*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
 1) + 3*log(sin(d*x + c) - 1)) - 60*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1
) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.69043, size = 329, normalized size = 2.89

$$\frac{195\,a^3\cos(dx+c)^5\log(\sin(dx+c)+1)-195\,a^3\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(304\,a^3\cos(dx+c)^4+195\,a^3\right.}{240\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(195*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 195*a^3*cos(d*x + c)^
5*log(-sin(d*x + c) + 1) + 2*(304*a^3*cos(d*x + c)^4 + 195*a^3*cos(d*x + c)
^3 + 152*a^3*cos(d*x + c)^2 + 90*a^3*cos(d*x + c) + 24*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36579, size = 186, normalized size = 1.63

$$\frac{195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-195\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-910\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,a^3\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(195*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 195*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(195*a^3*tan(1/2*d*x + 1/2*c)^9 - 910*a^3*tan(1/2*d
*x + 1/2*c)^7 + 1664*a^3*tan(1/2*d*x + 1/2*c)^5 - 1330*a^3*tan(1/2*d*x + 1/
2*c)^3 + 765*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.33 $\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=127

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}$$

```
[Out] (49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)/(5
*d)
```

Rubi [A] time = 0.156639, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 2635, 8, 2633}

$$\frac{4a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{41a^4\sin(c+dx)\cos^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (49*a^4*x)/16 + (8*a^4*Sin[c + d*x])/d + (49*a^4*Cos[c + d*x]*Sin[c + d*x])
/(16*d) + (41*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a^4*Cos[c + d*x]^5
*Sin[c + d*x])/(6*d) - (4*a^4*Sin[c + d*x]^3)/d + (4*a^4*Sin[c + d*x]^5)/(5
*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\cos(c+dx))^4\,dx &= \int \left(a^4\cos^2(c+dx)+4a^4\cos^3(c+dx)+6a^4\cos^4(c+dx)+4a^4\cos^5(c+dx)+a\right.\\ &= a^4\int \cos^2(c+dx)\,dx + a^4\int \cos^6(c+dx)\,dx + \left(4a^4\right)\int \cos^3(c+dx)\,dx + \left(4a^4\right)\\ &= \frac{a^4\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a^4\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{a^4\cos^5(c+dx)\sin(}{6d}\\ &= \frac{a^4x}{2} + \frac{8a^4\sin(c+dx)}{d} + \frac{11a^4\cos(c+dx)\sin(c+dx)}{4d} + \frac{41a^4\cos^3(c+dx)\sin(c}{24d}\\ &= \frac{11a^4x}{4} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\cos^3(c+dx)\sin}{24d}\\ &= \frac{49a^4x}{16} + \frac{8a^4\sin(c+dx)}{d} + \frac{49a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{41a^4\cos^3(c+dx)\sin}{24d}\end{aligned}$$

Mathematica [A] time = 0.174498, size = 73, normalized size = 0.57

$$\frac{a^4(5280\sin(c+dx)+1905\sin(2(c+dx))+720\sin(3(c+dx))+225\sin(4(c+dx))+48\sin(5(c+dx))+5\sin(6(c+dx}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(2940*d*x + 5280*Sin[c + d*x] + 1905*Sin[2*(c + d*x)] + 720*Sin[3*(c +
 d*x)] + 225*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)] + 5*Sin[6*(c + d*x)]))/
(960*d)
```

Maple [A] time = 0.046, size = 169, normalized size = 1.3

$$\frac{1}{d}\left(a^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4a^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/d*(a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a^4
*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a^4*(2+co
s(d*x+c)^2)*sin(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.11564, size = 223, normalized size = 1.76

$$\frac{256\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^4-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 5
*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x
```

```
+ 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 180*(12*d*x + 12
*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4 + 240*(2*d*x + 2*c + sin(2*
d*x + 2*c))*a^4)/d
```

Fricas [A] time = 1.65537, size = 231, normalized size = 1.82

$$\frac{735\,a^4dx + \left(40\,a^4\cos\left(dx+c\right)^5 + 192\,a^4\cos\left(dx+c\right)^4 + 410\,a^4\cos\left(dx+c\right)^3 + 576\,a^4\cos\left(dx+c\right)^2 + 735\,a^4\cos\left(dx+\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(735*a^4*d*x + (40*a^4*cos(d*x + c)^5 + 192*a^4*cos(d*x + c)^4 + 410*
a^4*cos(d*x + c)^3 + 576*a^4*cos(d*x + c)^2 + 735*a^4*cos(d*x + c) + 1152*a
^4)*sin(d*x + c))/d
```

Sympy [A] time = 4.89644, size = 434, normalized size = 3.42

$$\begin{cases} \frac{5a^4x\sin^6(c+dx)}{16} + \frac{15a^4x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9a^4x\sin^4(c+dx)}{4} + \frac{15a^4x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{9a^4x\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{a^4x\sin^2(c\ldots}{2} \\ x\left(a\cos\left(c\right)+a\right)^4\cos^2\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((5*a**4*x*sin(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 9*a**4*x*sin(c + d*x)**4/4 + 15*a**4*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + 9*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + a**4*x*sin(c + d
*x)**2/2 + 5*a**4*x*cos(c + d*x)**6/16 + 9*a**4*x*cos(c + d*x)**4/4 + a**4*
x*cos(c + d*x)**2/2 + 5*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 32*a**4*
sin(c + d*x)**5/(15*d) + 5*a**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 16*
a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 9*a**4*sin(c + d*x)**3*cos(c +
 d*x)/(4*d) + 8*a**4*sin(c + d*x)**3/(3*d) + 11*a**4*sin(c + d*x)*cos(c + d
*x)**5/(16*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**4/d + 15*a**4*sin(c + d*x
)*cos(c + d*x)**3/(4*d) + 4*a**4*sin(c + d*x)*cos(c + d*x)**2/d + a**4*sin(
c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a*cos(c) + a)**4*cos(c)**2, Tru
e))
```

Giac [A] time = 1.38786, size = 143, normalized size = 1.13

$$\frac{49}{16}\,a^4x + \frac{a^4\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{a^4\sin\left(5\,dx+5\,c\right)}{20\,d} + \frac{15\,a^4\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{3\,a^4\sin\left(3\,dx+3\,c\right)}{4\,d} + \frac{127\,a^4\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 49/16*a^4*x + 1/192*a^4*sin(6*d*x + 6*c)/d + 1/20*a^4*sin(5*d*x + 5*c)/d +
15/64*a^4*sin(4*d*x + 4*c)/d + 3/4*a^4*sin(3*d*x + 3*c)/d + 127/64*a^4*sin(
2*d*x + 2*c)/d + 11/2*a^4*sin(d*x + c)/d
```

3.34 $\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=102

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{8a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{7a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^4x}{2}$$

```
[Out] (7*a^4*x)/2 + (8*a^4*Sin[c + d*x])/d + (7*a^4*Cos[c + d*x]*Sin[c + d*x])/(2
*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d - (8*a^4*Sin[c + d*x]^3)/(3*d) +
(a^4*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.106915, antiderivative size = 114, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2751, 2645, 2637, 2635, 8, 2633}

$$-\frac{16a^4\sin^3(c+dx)}{15d}+\frac{32a^4\sin(c+dx)}{5d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{5d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{10d}+\frac{7a^4x}{2}+\frac{\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (7*a^4*x)/2 + (32*a^4*Sin[c + d*x])/(5*d) + (27*a^4*Cos[c + d*x]*Sin[c + d*
x])/(10*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(5*d) + ((a + a*Cos[c + d*x]
)^4*Sin[c + d*x])/(5*d) - (16*a^4*Sin[c + d*x]^3)/(15*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\cos(c+dx))^4\,dx &= \frac{(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{4}{5}\int (a+a\cos(c+dx))^4\,dx\\ &= \frac{(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{4}{5}\int \left(a^4+4a^4\cos(c+dx)+6a^4\cos^2(c+d\right.\\ &= \frac{4a^4x}{5}+\frac{(a+a\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}\left(4a^4\right)\int \cos^4(c+dx)\,dx+\frac{1}{5}\left(16a\right.\\ &= \frac{4a^4x}{5}+\frac{16a^4\sin(c+dx)}{5d}+\frac{12a^4\cos(c+dx)\sin(c+dx)}{5d}+\frac{a^4\cos^3(c+dx)\sin}{5d}\\ &= \frac{16a^4x}{5}+\frac{32a^4\sin(c+dx)}{5d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{10d}+\frac{a^4\cos^3(c+dx)\sin}{5d}\\ &= \frac{7a^4x}{2}+\frac{32a^4\sin(c+dx)}{5d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{10d}+\frac{a^4\cos^3(c+dx)\sin}{5d}\end{aligned}$$

Mathematica [A] time = 0.141688, size = 63, normalized size = 0.62

$$\frac{a^4(1470\sin(c+dx)+480\sin(2(c+dx))+145\sin(3(c+dx))+30\sin(4(c+dx))+3\sin(5(c+dx))+840dx)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(840*d*x + 1470*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 145*Sin[3*(c + d
*x)] + 30*Sin[4*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.042, size = 133, normalized size = 1.3

$$\frac{1}{d}\left(\frac{a^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+4a^4\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/d*(1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a^4*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^4*(2+cos(d*x+c)^2)*
sin(d*x+c)+4*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*sin(d*x+c))
```

Maxima [A] time = 1.14421, size = 173, normalized size = 1.7

$$\frac{8\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)a^4-240\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4+15(12dx+12c+}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 - 240
*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 + 15*(12*d*x + 12*c + sin(4*d*x + 4*
c) + 8*sin(2*d*x + 2*c))*a^4 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 1
20*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.6471, size = 190, normalized size = 1.86

$$\frac{105\,a^4dx + \left(6\,a^4\cos\left(dx+c\right)^4 + 30\,a^4\cos\left(dx+c\right)^3 + 68\,a^4\cos\left(dx+c\right)^2 + 105\,a^4\cos\left(dx+c\right) + 166\,a^4\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(105*a^4*d*x + (6*a^4*cos(d*x + c)^4 + 30*a^4*cos(d*x + c)^3 + 68*a^4*
cos(d*x + c)^2 + 105*a^4*cos(d*x + c) + 166*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 2.50715, size = 280, normalized size = 2.75

$$\begin{cases} \frac{3a^4x\sin^4\left(c+dx\right)}{2} + 3a^4x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right) + 2a^4x\sin^2\left(c+dx\right) + \frac{3a^4x\cos^4\left(c+dx\right)}{2} + 2a^4x\cos^2\left(c+dx\right) + \frac{8a^4\sin^5\left(c+dx\right)}{15d} \\ x\left(a\cos\left(c\right)+a\right)^4\cos\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/2 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2 + 2*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/2 + 2*a**4*x*co
s(c + d*x)**2 + 8*a**4*sin(c + d*x)**5/(15*d) + 4*a**4*sin(c + d*x)**3*cos(
c + d*x)**2/(3*d) + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 4*a**4*sin(
c + d*x)**3/d + a**4*sin(c + d*x)*cos(c + d*x)**4/d + 5*a**4*sin(c + d*x)*c
os(c + d*x)**3/(2*d) + 6*a**4*sin(c + d*x)*cos(c + d*x)**2/d + 2*a**4*sin(c
 + d*x)*cos(c + d*x)/d + a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)*
*4*cos(c), True))
```

Giac [A] time = 1.2418, size = 120, normalized size = 1.18

$$\frac{7}{2}\,a^4x + \frac{a^4\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{a^4\sin\left(4\,dx+4\,c\right)}{8\,d} + \frac{29\,a^4\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{2\,a^4\sin\left(2\,dx+2\,c\right)}{d} + \frac{49\,a^4\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 7/2*a^4*x + 1/80*a^4*sin(5*d*x + 5*c)/d + 1/8*a^4*sin(4*d*x + 4*c)/d + 29/4
8*a^4*sin(3*d*x + 3*c)/d + 2*a^4*sin(2*d*x + 2*c)/d + 49/8*a^4*sin(d*x + c)
/d
```

3.35 $\int (a + a\cos(c + dx))^4 \, dx$

Optimal. Leaf size=87

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

```
[Out] (35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3
*d)
```

Rubi [A] time = 0.0809796, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2637, 2635, 8, 2633}

$$-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{8a^4\sin(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (35*a^4*x)/8 + (8*a^4*Sin[c + d*x])/d + (27*a^4*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a^4*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (4*a^4*Sin[c + d*x]^3)/(3
*d)
```


Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x, x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\,dx = \int \left(a^4+4a^4\cos(c+dx)+6a^4\cos^2(c+dx)+4a^4\cos^3(c+dx)+a^4\cos^4(c+dx)\right)dx$$
$$= a^4x+a^4\int \cos^4(c+dx)\,dx+\left(4a^4\right)\int \cos(c+dx)\,dx+\left(4a^4\right)\int \cos^3(c+dx)\,dx+\left(6a^4\right)\int \mathrm{c}$$
$$= a^4x+\frac{4a^4\sin(c+dx)}{d}+\frac{3a^4\cos(c+dx)\sin(c+dx)}{d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}\left(3a^4\right)$$
$$= 4a^4x+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{4a^4\,\mathrm{si}}{}$$
$$= \frac{35a^4x}{8}+\frac{8a^4\sin(c+dx)}{d}+\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{4a^4}{}$$

Mathematica [A] time = 0.104596, size = 56, normalized size = 0.64

$$\frac{a^4(672\sin(c+dx)+168\sin(2(c+dx))+32\sin(3(c+dx))+3\sin(4(c+dx))+420c+420dx)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(420*c + 420*d*x + 672*Sin[c + d*x] + 168*Sin[2*(c + d*x)] + 32*Sin[3*
(c + d*x)] + 3*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.039, size = 111, normalized size = 1.3

$$\frac{1}{d}\left(a^4\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4a^4\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+6a^4\,(1/2\,\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a
^4*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+4*a^4*sin(d*x+c)+a^4*(d*x+c))
```

Maxima [A] time = 1.11255, size = 143, normalized size = 1.64

$$a^4x-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4}{3d}+\frac{(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^4}{32d}+\frac{3(2dx+2c+\sin(2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 4/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c +
 sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c + sin(2*d*
x + 2*c))*a^4/d + 4*a^4*sin(d*x + c)/d
```

Fricas [A] time = 1.62391, size = 157, normalized size = 1.8

$$\frac{105\,a^4dx + \left(6\,a^4\cos\left(dx+c\right)^3 + 32\,a^4\cos\left(dx+c\right)^2 + 81\,a^4\cos\left(dx+c\right) + 160\,a^4\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(105*a^4*d*x + (6*a^4*cos(d*x + c)^3 + 32*a^4*cos(d*x + c)^2 + 81*a^4*
cos(d*x + c) + 160*a^4)*sin(d*x + c))/d
```

Sympy [A] time = 1.28897, size = 224, normalized size = 2.57

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + 3a^4x\sin^2\left(c+dx\right) + \frac{3a^4x\cos^4(c+dx)}{8} + 3a^4x\cos^2\left(c+dx\right) + a^4x + \frac{3a^4\sin^3(c+dx)\cdots}{8d} \\ x\left(a\cos\left(c\right)+a\right)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*x*
cos(c + d*x)**2 + a**4*x + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 8*a*
*4*sin(c + d*x)**3/(3*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*a*
*4*sin(c + d*x)*cos(c + d*x)**2/d + 3*a**4*sin(c + d*x)*cos(c + d*x)/d + 4*
a**4*sin(c + d*x)/d, Ne(d, 0)), (x*(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.26792, size = 97, normalized size = 1.11

$$\frac{35}{8}\,a^4x + \frac{a^4\sin\left(4\,dx+4\,c\right)}{32\,d} + \frac{a^4\sin\left(3\,dx+3\,c\right)}{3\,d} + \frac{7\,a^4\sin\left(2\,dx+2\,c\right)}{4\,d} + \frac{7\,a^4\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 35/8*a^4*x + 1/32*a^4*sin(4*d*x + 4*c)/d + 1/3*a^4*sin(3*d*x + 3*c)/d + 7/4
*a^4*sin(2*d*x + 2*c)/d + 7*a^4*sin(d*x + c)/d
```

3.36 $\int (a + a\cos(c + dx))^4 \sec(c + dx)\,dx$

Optimal. Leaf size=73

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

```
[Out] 6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*C
os[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.080568, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2757, 2637, 2635, 8, 2633, 3770}

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{7a^4\sin(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sin(c+dx)\cos(c+dx)}{d}+6a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x],x]
```

```
[Out] 6*a^4*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Sin[c + d*x])/d + (2*a^4*C
os[c + d*x]*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^4 \sec(c+dx)\, dx = \int \left(4a^4 + 6a^4\cos(c+dx) + 4a^4\cos^2(c+dx) + a^4\cos^3(c+dx) + a^4\sec(c+dx\right.$$

$$= 4a^4x + a^4 \int \cos^3(c+dx)\, dx + a^4 \int \sec(c+dx)\, dx + \left(4a^4\right) \int \cos^2(c+dx)\, dx$$

$$= 4a^4x + \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{6a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\sin(c+dx}{d}$$

$$= 6a^4x + \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{7a^4\sin(c+dx)}{d} + \frac{2a^4\cos(c+dx)\sin(c+dx}{d}$$

Mathematica [A] time = 0.104197, size = 91, normalized size = 1.25

$$\frac{a^4\left(81\sin(c+dx) + 12\sin(2(c+dx)) + \sin(3(c+dx)) - 12\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 12\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x],x]
```

```
[Out] (a^4*(72*d*x - 12*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + 81*Sin[c + d*x] + 12*Sin[2*(c + d*x)] + Sin
[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.066, size = 94, normalized size = 1.3

$$\frac{(\cos(dx+c))^2\sin(dx+c)\,a^4}{3d} + \frac{20\,a^4\sin(dx+c)}{3d} + 2\,\frac{a^4\cos(dx+c)\sin(dx+c)}{d} + 6\,a^4x + 6\,\frac{a^4c}{d} + \frac{a^4\ln(\sec(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c),x)
```

```
[Out] 1/3/d*sin(d*x+c)*cos(d*x+c)^2*a^4+20/3*a^4*sin(d*x+c)/d+2*a^4*cos(d*x+c)*si
n(d*x+c)/d+6*a^4*x+6/d*a^4*c+1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.14451, size = 120, normalized size = 1.64

$$-\frac{\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a^4 - 3(2dx + 2c + \sin(2dx+2c))a^4 - 12(dx+c)a^4 - 3a^4\log(\sec(dx+c) + \tan(d}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 3*(2*d*x + 2*c + sin(2*d*x +
2*c))*a^4 - 12*(d*x + c)*a^4 - 3*a^4*log(sec(d*x + c) + tan(d*x + c)) - 18*
a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.70575, size = 201, normalized size = 2.75

$$\frac{36\,a^4dx + 3\,a^4\log\left(\sin\left(dx+c\right)+1\right) - 3\,a^4\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(a^4\cos\left(dx+c\right)^2 + 6\,a^4\cos\left(dx+c\right) + 20\,a^4\right)\sin\left(dx\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(36*a^4*d*x + 3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1
) + 2*(a^4*cos(d*x + c)^2 + 6*a^4*cos(d*x + c) + 20*a^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4796, size = 157, normalized size = 2.15

$$\frac{18\,(dx+c)a^4 + 3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 3\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 38\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*(18*(d*x + c)*a^4 + 3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 + 38*a^
4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 + 1)^3)/d
```

3.37 $\int (a+a\cos(c+dx))^4\sec^2(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

```
[Out] (13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (
a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

Rubi [A] time = 0.0828627, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2757, 2637, 2635, 8, 3770, 3767}

$$\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\tan(c+dx)}{d}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}+\frac{13a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

```
[Out] (13*a^4*x)/2 + (4*a^4*ArcTanh[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (
a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^4*Tan[c + d*x])/d
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^2(c+dx)\,dx = \int \left(6a^4+4a^4\cos(c+dx)+a^4\cos^2(c+dx)+4a^4\sec(c+dx)+a^4\sec^2(c+dx)\right)$$
$$= 6a^4x+a^4\int\cos^2(c+dx)\,dx+a^4\int\sec^2(c+dx)\,dx+\left(4a^4\right)\int\cos(c+dx)\,dx+$$
$$= 6a^4x+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx)\sin(c+dx)}{2d}$$
$$= \frac{13a^4x}{2}+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\cos(c+dx)\sin(c+dx}{2d}$$

Mathematica [B] time = 1.20999, size = 241, normalized size = 3.3

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{16\sin(c)\cos(dx)}{d}+\frac{\sin(2c)\cos(2dx)}{d}+\frac{16\cos(c)\sin(dx)}{d}+\frac{\cos(2c)\sin(2dx)}{d}+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(26*x - (16*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (16*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (16*Cos[d*x]*Sin[c])/d + (Cos[2*d*x]*Sin[2*c])/d + (16*Cos[c]*Sin[d*x])/
d + (Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c
/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/64
```

Maple [A] time = 0.077, size = 86, normalized size = 1.2

$$\frac{a^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{13a^4x}{2}+\frac{13a^4c}{2d}+4\frac{a^4\sin(dx+c)}{d}+4\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^4\tan(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^2,x)
```

```
[Out] 1/2*a^4*cos(d*x+c)*sin(d*x+c)/d+13/2*a^4*x+13/2/d*a^4*c+4*a^4*sin(d*x+c)/d+
4/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*tan(d*x+c)/d
```

Maxima [A] time = 1.21079, size = 115, normalized size = 1.58

$$\frac{(2dx+2c+\sin(2dx+2c))a^4+24(dx+c)a^4+8a^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+16a^4\sin(dx+c)+}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 24*(d*x + c)*a^4 + 8*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a^4*sin(d*x + c) + 4*a^4*tan
(d*x + c))/d
```

Fricas [A] time = 1.80935, size = 270, normalized size = 3.7

$$\frac{13\,a^4dx\cos\left(dx+c\right)+4\,a^4\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-4\,a^4\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(a^4\cos\left(dx+\right.\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(13*a^4*d*x*cos(d*x + c) + 4*a^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 4
*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^4*cos(d*x + c)^2 + 8*a^4*cos(
d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34142, size = 174, normalized size = 2.38

$$\frac{13\,(dx+c)a^4+8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(7\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(13*(d*x + c)*a^4 + 8*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^4*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
+ 1/2*c)^2 - 1) + 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.38 $\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

```
[Out] 4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4*
a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0871818, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2757, 2637, 3770, 3767, 8, 3768}

$$\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+4a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

```
[Out] 4*a^4*x + (13*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (a^4*Sin[c + d*x])/d + (4*
a^4*Tan[c + d*x])/d + (a^4*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

IntegerQ[2*n]

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^4\sec^3(c+dx)\,dx &= \int \left(4a^4+a^4\cos(c+dx)+6a^4\sec(c+dx)+4a^4\sec^2(c+dx)+a^4\sec^3(c+d\right.\\ &= 4a^4x+a^4\int\cos(c+dx)\,dx+a^4\int\sec^3(c+dx)\,dx+\left(4a^4\right)\int\sec^2(c+dx)\,d\\ &= 4a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^4\sin(c+dx)}{d}+\frac{a^4\sec(c+dx)\tan(c+d}{2d}\\ &= 4a^4x+\frac{13a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\sin(c+dx)}{d}+\frac{4a^4\tan(c+dx)}{d}+\frac{a^4\,\mathrm{se}}{}\end{aligned}$$

Mathematica [B] time = 1.07651, size = 272, normalized size = 3.73

$$\frac{1}{64}a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin(c)\cos(dx)}{d}+\frac{4\cos(c)\sin(dx)}{d}+\frac{16\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(16*x - (26*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]])/d + (26*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d
 + (4*Cos[d*x]*Sin[c])/d + (4*Cos[c]*Sin[d*x])/d + 1/(d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^2) + (16*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))))/64
```

Maple [A] time = 0.085, size = 86, normalized size = 1.2

$$\frac{a^4\sin(dx+c)}{d}+4a^4x+4\frac{a^4c}{d}+\frac{13a^4\ln(\sec(dx+c)+\tan(dx+c))}{2d}+4\frac{a^4\tan(dx+c)}{d}+\frac{a^4\sec(dx+c)\tan(dx+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^3,x)
```

```
[Out] a^4*sin(d*x+c)/d+4*a^4*x+4/d*a^4*c+13/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a
^4*tan(d*x+c)/d+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d
```

Maxima [A] time = 1.15773, size = 149, normalized size = 2.04

$$\frac{16(dx+c)a^4-a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12a^4(\log(\sin(dx+c)+1)-\log(\sin}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a^4 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(
d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^4*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 4*a^4*sin(d*x + c) + 16*a^4*tan(d*x + c))/d
```

Fricas [A] time = 2.1076, size = 286, normalized size = 3.92

$$\frac{16\,a^4dx\cos\left(dx+c\right)^2+13\,a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-13\,a^4\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,a^4\cos\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a^4*d*x*cos(d*x + c)^2 + 13*a^4*cos(d*x + c)^2*log(sin(d*x + c) + 1
) - 13*a^4*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^4*cos(d*x + c)^2
+ 8*a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47891, size = 174, normalized size = 2.38

$$\frac{8\left(dx+c\right)a^4+13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{2\left(7\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a^4 + 13*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x
 + 1/2*c)^2 + 1) - 2*(7*a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.39 $\int (a + a\cos(c + dx))^4 \sec^4(c + dx)\, dx$

Optimal. Leaf size=73

$$\frac{a^4\tan^3(c+dx)}{3d}+\frac{7a^4\tan(c+dx)}{d}+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\tan(c+dx)\sec(c+dx)}{d}+a^4x$$

```
[Out] a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Tan[c + d*x])/d + (2*a^4*S
ec[c + d*x]*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0956729, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{a^4\tan^3(c+dx)}{3d}+\frac{7a^4\tan(c+dx)}{d}+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\tan(c+dx)\sec(c+dx)}{d}+a^4x$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```


```
[Out] a^4*x + (6*a^4*ArcTanh[Sin[c + d*x]])/d + (7*a^4*Tan[c + d*x])/d + (2*a^4*S
ec[c + d*x]*Tan[c + d*x])/d + (a^4*Tan[c + d*x]^3)/(3*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^4\sec^4(c+dx)\,dx &= \int \left(a^4+4a^4\sec(c+dx)+6a^4\sec^2(c+dx)+4a^4\sec^3(c+dx)+a^4\sec^4(c+dx)\right.\\ &= a^4x+a^4\int \sec^4(c+dx)\,dx+\left(4a^4\right)\int \sec(c+dx)\,dx+\left(4a^4\right)\int \sec^3(c+dx)\,dx\\ &= a^4x+\frac{4a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{2a^4\sec(c+dx)\tan(c+dx)}{d}+\left(2a^4\right)\int \sec(c+\\ &= a^4x+\frac{6a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{7a^4\tan(c+dx)}{d}+\frac{2a^4\sec(c+dx)\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0327334, size = 61, normalized size = 0.84

$$a^4\left(\frac{\tan^3(c+dx)}{3d}+\frac{7\tan(c+dx)}{d}+\frac{6\tanh^{-1}(\sin(c+dx))}{d}+\frac{2\tan(c+dx)\sec(c+dx)}{d}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

```
[Out] a^4*(x + (6*ArcTanh[Sin[c + d*x]])/d + (7*Tan[c + d*x])/d + (2*Sec[c + d*x]
*Tan[c + d*x])/d + Tan[c + d*x]^3/(3*d))
```

Maple [A] time = 0.09, size = 93, normalized size = 1.3

$$a^4x+\frac{a^4c}{d}+6\,\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{20\,a^4\tan(dx+c)}{3\,d}+2\,\frac{a^4\sec(dx+c)\tan(dx+c)}{d}+\frac{a^4\tan(dx+c)(s}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^4,x)
```

```
[Out] a^4*x+1/d*a^4*c+6/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3*a^4*tan(d*x+c)/d+2*a
^4*sec(d*x+c)*tan(d*x+c)/d+1/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 1.13387, size = 162, normalized size = 2.22

$$\frac{\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4+3\,(dx+c)a^4-3\,a^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6\,a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 3*(d*x + c)*a^4 - 3*a^4*(2*sin
(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
 1)) + 6*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*a^4*tan(d
*x + c))/d
```

Fricas [A] time = 2.013, size = 281, normalized size = 3.85

$$\frac{3\,a^4dx\cos\left(dx+c\right)^3+9\,a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-9\,a^4\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(20\,a^4\cos\right.}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cos(d*x + c)^3 + 9*a^4*cos(d*x + c)^3*log(sin(d*x + c) + 1)
- 9*a^4*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + (20*a^4*cos(d*x + c)^2 + 6*
a^4*cos(d*x + c) + a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45785, size = 157, normalized size = 2.15

$$\frac{3\left(dx+c\right)a^4+18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-18\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-38\,a^4\tan\left(\frac{1}{2}\,dx+\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 18*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 18*a^4*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a^4*tan(1/2*d*x + 1/2*c)^5 - 38*a
^4*tan(1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2
*c)^2 - 1)^3)/d
```

3.40 $\int (a+a\cos(c+dx))^4 \sec^5(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec(c}{8d}$$

```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
4*a^4*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.125709, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3770, 3767, 8, 3768}

$$\frac{4a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{27a^4\tan(c+dx)\sec(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```


```
[Out] (35*a^4*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*Tan[c + d*x])/d + (27*a^4*Sec
[c + d*x]*Tan[c + d*x])/(8*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (
4*a^4*Tan[c + d*x]^3)/(3*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^5(c+dx)\,dx = \int \left(a^4\sec(c+dx)+4a^4\sec^2(c+dx)+6a^4\sec^3(c+dx)+4a^4\sec^4(c+dx)+\right.$$
$$= a^4\int \sec(c+dx)\,dx + a^4\int \sec^5(c+dx)\,dx + \left(4a^4\right)\int \sec^2(c+dx)\,dx + \left(4a^4\right.$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^4\sec(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^3(c+dx)\tan(}{4d}$$
$$= \frac{4a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+dx)}{8d} +$$
$$= \frac{35a^4\tanh^{-1}(\sin(c+dx))}{8d} + \frac{8a^4\tan(c+dx)}{d} + \frac{27a^4\sec(c+dx)\tan(c+dx)}{8d}$$

Mathematica [B] time = 6.30807, size = 797, normalized size = 8.3

$$-\frac{35(\cos(c+dx)a+a)^4\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)}{128d} + \frac{35(\cos(c+dx)a+a)^4\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

```
[Out] (-35*(a + a*Cos[c + d*x])^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Se
c[c/2 + (d*x)/2]^8)/(128*d) + (35*(a + a*Cos[c + d*x])^4*Log[Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8)/(128*d) + ((a + a*Cos[c +
d*x])^4*Sec[c/2 + (d*x)/2]^8)/(256*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])^4) + ((a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(24*d*(
Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + ((a + a
*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*(97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(
Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (5*(a +
 a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(12*d*(Cos[c/2] - Sin
[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - ((a + a*Cos[c + d*x])^4
*Sec[c/2 + (d*x)/2]^8)/(256*d*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4)
+ ((a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(24*d*(Cos[c/2
] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + ((a + a*Cos[c
+ d*x])^4*Sec[c/2 + (d*x)/2]^8*(-97*Cos[c/2] - 65*Sin[c/2]))/(768*d*(Cos[c/
2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (5*(a + a*Cos
[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*Sin[(d*x)/2])/(12*d*(Cos[c/2] + Sin[c/2])
*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.087, size = 102, normalized size = 1.1

$$\frac{35\,a^4\ln(\sec(dx+c)+\tan(dx+c))}{8\,d} + \frac{20\,a^4\tan(dx+c)}{3\,d} + \frac{27\,a^4\sec(dx+c)\tan(dx+c)}{8\,d} + \frac{4\,a^4\tan(dx+c)(\sec(d}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^5,x)
```

```
[Out] 35/8/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+20/3*a^4*tan(d*x+c)/d+27/8*a^4*sec(d*x
+c)*tan(d*x+c)/d+4/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2+1/4*a^4*sec(d*x+c)^3*tan
(d*x+c)/d
```

Maxima [B] time = 1.15794, size = 246, normalized size = 2.56

$$64\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4-3a^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 3*a^4*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^4*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*a^4*(log(sin(
d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.95208, size = 292, normalized size = 3.04

$$\frac{105\,a^4\cos(dx+c)^4\log(\sin(dx+c)+1)-105\,a^4\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(160\,a^4\cos(dx+c)^3+81\,a^4\,\mathrm{c}\right.}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(105*a^4*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^4
*log(-sin(d*x + c) + 1) + 2*(160*a^4*cos(d*x + c)^3 + 81*a^4*cos(d*x + c)^2
 + 32*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.52136, size = 165, normalized size = 1.72

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-385\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+511\,a^4\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^7 - 385*a^4*tan(1/2*d*
```

```
x + 1/2*c)^5 + 511*a^4*tan(1/2*d*x + 1/2*c)^3 - 279*a^4*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.41 $\int (a + a\cos(c + dx))^4 \sec^6(c + dx)\, dx$

Optimal. Leaf size=111

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+\frac{7a^4\text{ta}}{}$$

```
[Out] (7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*
Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.143689, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2757, 3767, 8, 3768, 3770}

$$\frac{a^4\tan^5(c+dx)}{5d}+\frac{8a^4\tan^3(c+dx)}{3d}+\frac{8a^4\tan(c+dx)}{d}+\frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{d}+\frac{7a^4\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

```
[Out] (7*a^4*ArcTanh[Sin[c + d*x]])/(2*d) + (8*a^4*Tan[c + d*x])/d + (7*a^4*Sec[c
 + d*x]*Tan[c + d*x])/(2*d) + (a^4*Sec[c + d*x]^3*Tan[c + d*x])/d + (8*a^4*
Tan[c + d*x]^3)/(3*d) + (a^4*Tan[c + d*x]^5)/(5*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^4 \sec^6(c+dx)\,dx = \int \left(a^4\sec^2(c+dx) + 4a^4\sec^3(c+dx) + 6a^4\sec^4(c+dx) + 4a^4\sec^5(c+dx) \right.$$

$$= a^4\int \sec^2(c+dx)\,dx + a^4\int \sec^6(c+dx)\,dx + \left(4a^4\right)\int \sec^3(c+dx)\,dx + \left(4a\right.$$

$$= \frac{2a^4\sec(c+dx)\tan(c+dx)}{d} + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{d} + \left(2a^4\right)\int \sec(c+$$

$$= \frac{2a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx)}{2d} +$$

$$= \frac{7a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{7a^4\sec(c+dx)\tan(c+dx)}{2d} +$$

Mathematica [B] time = 1.36832, size = 498, normalized size = 4.49

$$-\frac{a^4\sec(c)\sec^5(c+dx)\left(960\sin(2c+dx) - 660\sin(c+2dx) - 660\sin(3c+2dx) - 1600\sin(2c+3dx) + 60\sin(4c+3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

```
[Out] -(a^4*Sec[c]*Sec[c + d*x]^5*(525*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*
Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1050*Cos[d*x]*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + 1050*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 525*Cos[2*c + 3*d*x]*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 525*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] - 105*Cos[4*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] - 105*Cos[6*c + 5*d*x]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
 2360*Sin[d*x] + 960*Sin[2*c + d*x] - 660*Sin[c + 2*d*x] - 660*Sin[3*c + 2*
d*x] - 1600*Sin[2*c + 3*d*x] + 60*Sin[4*c + 3*d*x] - 210*Sin[3*c + 4*d*x] -
 210*Sin[5*c + 4*d*x] - 332*Sin[4*c + 5*d*x]))/(960*d)
```

Maple [A] time = 0.096, size = 123, normalized size = 1.1

$$\frac{83\,a^4\tan(dx+c)}{15\,d} + \frac{7\,a^4\sec(dx+c)\tan(dx+c)}{2\,d} + \frac{7\,a^4\ln(\sec(dx+c)+\tan(dx+c))}{2\,d} + \frac{34\,a^4\tan(dx+c)(\sec(dx}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^6,x)
```

```
[Out] 83/15*a^4*tan(d*x+c)/d+7/2*a^4*sec(d*x+c)*tan(d*x+c)/d+7/2/d*a^4*ln(sec(d*x
+c)+tan(d*x+c))+34/15/d*a^4*tan(d*x+c)*sec(d*x+c)^2+a^4*sec(d*x+c)^3*tan(d*
x+c)/d+1/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4
```

Maxima [A] time = 1.12955, size = 257, normalized size = 2.32

$$\frac{4\left(3\tan(dx+c)^5 + 10\tan(dx+c)^3 + 15\tan(dx+c)\right)a^4 + 120\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)a^4 - 15\,a^4\left(\frac{2\left(3\sin(dx\right.}{\sin(dx+c)^4}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 15*a^4*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 60*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.87621, size = 325, normalized size = 2.93

$$\frac{105\,a^4\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-105\,a^4\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(166\,a^4\cos\left(dx+c\right)^4+105\,a^4\right.}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/60*(105*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*a^4*cos(d*x + c)^5
*log(-sin(d*x + c) + 1) + 2*(166*a^4*cos(d*x + c)^4 + 105*a^4*cos(d*x + c)^
3 + 68*a^4*cos(d*x + c)^2 + 30*a^4*cos(d*x + c) + 6*a^4)*sin(d*x + c))/(d*c
os(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4898, size = 186, normalized size = 1.68

$$\frac{105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+896\,a^4\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\right.\right.}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/30*(105*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*a^4*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(105*a^4*tan(1/2*d*x + 1/2*c)^9 - 490*a^4*tan(1/2*d*
x + 1/2*c)^7 + 896*a^4*tan(1/2*d*x + 1/2*c)^5 - 790*a^4*tan(1/2*d*x + 1/2*c
)^3 + 375*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.42 $\int (a + a\cos(c + dx))^4 \sec^7(c + dx)\, dx$

Optimal. Leaf size=136

$$\frac{4a^4\tan^5(c+dx)}{5d} + \frac{4a^4\tan^3(c+dx)}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d} +$$

```
[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Se
c[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*
d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (
4*a^4*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.182264, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2757, 3768, 3770, 3767}

$$\frac{4a^4\tan^5(c+dx)}{5d} + \frac{4a^4\tan^3(c+dx)}{d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```



```
[Out] (49*a^4*ArcTanh[Sin[c + d*x]])/(16*d) + (8*a^4*Tan[c + d*x])/d + (49*a^4*Se
c[c + d*x]*Tan[c + d*x])/(16*d) + (41*a^4*Sec[c + d*x]^3*Tan[c + d*x])/(24*
d) + (a^4*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a^4*Tan[c + d*x]^3)/d + (
4*a^4*Tan[c + d*x]^5)/(5*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^7(c+dx)\,dx = \int \left(a^4\sec^3(c+dx)+4a^4\sec^4(c+dx)+6a^4\sec^5(c+dx)+4a^4\sec^6(c+dx)+a^4\right.$$
$$= a^4\int \sec^3(c+dx)\,dx + a^4\int \sec^7(c+dx)\,dx + \left(4a^4\right)\int \sec^4(c+dx)\,dx + \left(4a^4\right)$$
$$= \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^4\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^4\sec^5(c+dx)\tan}{6d}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{8a^4\tan(c+dx)}{d} + \frac{11a^4\sec(c+dx)\tan(c+dx)}{4d} + \frac{41a}{}$$
$$= \frac{11a^4\tanh^{-1}(\sin(c+dx))}{4d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+dx)}{16d} +$$
$$= \frac{49a^4\tanh^{-1}(\sin(c+dx))}{16d} + \frac{8a^4\tan(c+dx)}{d} + \frac{49a^4\sec(c+dx)\tan(c+dx)}{16d} +$$

Mathematica [A] time = 0.781411, size = 211, normalized size = 1.55

$$-\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(23520\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\mathrm{s}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

```
[Out] -(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(23520*Cos[c +
 d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) - Sec[c]*(-11520*Sin[c] + 3750*Sin[d*x] + 3750*Sin[2*c +
 d*x] + 15360*Sin[c + 2*d*x] - 1920*Sin[3*c + 2*d*x] + 3845*Sin[2*c + 3*d*x
] + 3845*Sin[4*c + 3*d*x] + 6912*Sin[3*c + 4*d*x] + 735*Sin[4*c + 5*d*x] +
735*Sin[6*c + 5*d*x] + 1152*Sin[5*c + 6*d*x])))/(122880*d)
```

Maple [A] time = 0.093, size = 146, normalized size = 1.1

$$\frac{49\,a^4\sec(dx+c)\tan(dx+c)}{16\,d} + \frac{49\,a^4\ln(\sec(dx+c)+\tan(dx+c))}{16\,d} + \frac{24\,a^4\tan(dx+c)}{5\,d} + \frac{12\,a^4\tan(dx+c)(\sec(dx}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^7,x)
```

```
[Out] 49/16*a^4*sec(d*x+c)*tan(d*x+c)/d+49/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+24/
5*a^4*tan(d*x+c)/d+12/5/d*a^4*tan(d*x+c)*sec(d*x+c)^2+41/24*a^4*sec(d*x+c)^
3*tan(d*x+c)/d+4/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4+1/6*a^4*sec(d*x+c)^5*tan(d
*x+c)/d
```

Maxima [B] time = 1.13769, size = 365, normalized size = 2.68

$$\frac{128\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^4+640\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4-5\,a^4\left(\frac{2\left(15\sin(dx+c\right.}{\sin(dx+c)^6-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 6
40*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 - 5*a^4*(2*(15*sin(d*x + c)^5 - 40
*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*s
in(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) -
 180*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*a^4
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)))/d
```

Fricas [A] time = 1.70632, size = 366, normalized size = 2.69

$$\frac{735\,a^4\cos(dx+c)^6\log(\sin(dx+c)+1) - 735\,a^4\cos(dx+c)^6\log(-\sin(dx+c)+1) + 2\left(1152\,a^4\cos(dx+c)^5 + 73\right.}{480\,d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/480*(735*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 735*a^4*cos(d*x + c)^
6*log(-sin(d*x + c) + 1) + 2*(1152*a^4*cos(d*x + c)^5 + 735*a^4*cos(d*x + c
)^4 + 576*a^4*cos(d*x + c)^3 + 410*a^4*cos(d*x + c)^2 + 192*a^4*cos(d*x + c
) + 40*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50627, size = 208, normalized size = 1.53

$$\frac{735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 735\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(735\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 4165\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 970\right.}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/240*(735*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 735*a^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 2*(735*a^4*tan(1/2*d*x + 1/2*c)^11 - 4165*a^4*tan(1/2
*d*x + 1/2*c)^9 + 9702*a^4*tan(1/2*d*x + 1/2*c)^7 - 11802*a^4*tan(1/2*d*x +
 1/2*c)^5 + 7355*a^4*tan(1/2*d*x + 1/2*c)^3 - 3105*a^4*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.43 $\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}+\frac{15}{8a}$$

```
[Out] (15*x)/(8*a) - (4*Sin[c + d*x])/(a*d) + (15*Cos[c + d*x]*Sin[c + d*x])/(8*a
*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (Cos[c + d*x]^4*Sin[c + d*x
])/(d*(a + a*Cos[c + d*x])) + (4*Sin[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.106884, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2767, 2748, 2633, 2635, 8}

$$\frac{4\sin^3(c+dx)}{3ad}-\frac{4\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^4(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{15\sin(c+dx)\cos(c+dx)}{8ad}+\frac{15}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x]),x]
```


```
[Out] (15*x)/(8*a) - (4*Sin[c + d*x])/(a*d) + (15*Cos[c + d*x]*Sin[c + d*x])/(8*a
*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d) - (Cos[c + d*x]^4*Sin[c + d*x
])/(d*(a + a*Cos[c + d*x])) + (4*Sin[c + d*x]^3)/(3*a*d)
```


Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \cos^3(c+dx)(4a-5a\cos(c+dx))\,dx}{a^2}$$
$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{4\int \cos^3(c+dx)\,dx}{a} + \frac{5\int \cos^4(c+dx)\,dx}{a}$$
$$= \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^4(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{15\int \cos^2(c+dx)\,dx}{4a} + \frac{4\,\mathrm{Subst}\,(\int$$
$$= -\frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^4(c+dx)\sin}{d(a+a\cos(c}$$
$$= \frac{15x}{8a} - \frac{4\sin(c+dx)}{ad} + \frac{15\cos(c+dx)\sin(c+dx)}{8ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\cos^4(c+d}{d(a+a}$$

Mathematica [A] time = 0.318477, size = 173, normalized size = 1.47

$$\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-168\sin\left(c+\frac{dx}{2}\right)-120\sin\left(c+\frac{3dx}{2}\right)-120\sin\left(2c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{5dx}{2}\right)+40\sin\left(3c+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(360*d*x*Cos[(d*x)/2] + 360*d*x*Cos[c + (d*x)/2]
 - 552*Sin[(d*x)/2] - 168*Sin[c + (d*x)/2] - 120*Sin[c + (3*d*x)/2] - 120*S
in[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c + (5*d*x)/2] - 5
*Sin[3*c + (7*d*x)/2] - 5*Sin[4*c + (7*d*x)/2] + 3*Sin[4*c + (9*d*x)/2] + 3
*Sin[5*c + (9*d*x)/2]))/(384*a*d)
```

Maple [A] time = 0.047, size = 171, normalized size = 1.5

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{25}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-\frac{115}{12\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*tan(1/2*d*x+1/2*c)-25/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1
/2*c)^7-115/12/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-109/12/d
/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-7/4/d/a/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)+15/4/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.7319, size = 293, normalized size = 2.48

$$-\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{109\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{115\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\sin(dx+c)}{a(\cos(dx+c)+1)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 109*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 115*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 45*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a + 12*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.65273, size = 213, normalized size = 1.81

$$\frac{45\,dx\cos\left(dx+c\right)+45\,dx+\left(6\,\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}+13\,\cos\left(dx+c\right)^{2}-19\,\cos\left(dx+c\right)-64\right)\sin\left(dx+c\right)}{24\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(45*d*x*cos(d*x + c) + 45*d*x + (6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 +
 13*cos(d*x + c)^2 - 19*cos(d*x + c) - 64)*sin(d*x + c))/(a*d*cos(d*x + c)
+ a*d)
```

Sympy [A] time = 11.5405, size = 882, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((45*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) + 180*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 9
6*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 +
d*x/2)**2 + 24*a*d) + 270*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)*
*8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(
c/2 + d*x/2)**2 + 24*a*d) + 180*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d
*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*
d*tan(c/2 + d*x/2)**2 + 24*a*d) + 45*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 24*tan(c/2 + d*x/2)**9/(24*a*d*tan(c/2 + d*x/2)**8 + 96*
a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*
x/2)**2 + 24*a*d) - 246*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 9
6*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 +
d*x/2)**2 + 24*a*d) - 374*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 +
 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2
+ d*x/2)**2 + 24*a*d) - 314*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8
 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) - 66*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 +
 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2
+ d*x/2)**2 + 24*a*d), Ne(d, 0)), (x*cos(c)**5/(a*cos(c) + a), True))
```

Giac [A] time = 1.35848, size = 136, normalized size = 1.15

$$\frac{\frac{45\,(dx+c)}{a}-\frac{24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+115\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+109\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(45*(d*x + c)/a - 24*tan(1/2*d*x + 1/2*c)/a - 2*(75*tan(1/2*d*x + 1/2*
c)^7 + 115*tan(1/2*d*x + 1/2*c)^5 + 109*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

3.44 $\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{3x}{2a}$$

```
[Out] (-3*x)/(2*a) + (4*Sin[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*
d) - (Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - (4*Sin[c + d*
x]^3)/(3*a*d)
```

Rubi [A] time = 0.0923199, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2767, 2748, 2635, 8, 2633}

$$-\frac{4\sin^3(c+dx)}{3ad}+\frac{4\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{d(a\cos(c+dx)+a)}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sin[c + d*x])/(a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*
d) - (Cos[c + d*x]^3*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - (4*Sin[c + d*
x]^3)/(3*a*d)
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \cos^2(c+dx)(3a-4a\cos(c+dx))\,dx}{a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int \cos^2(c+dx)\,dx}{a} + \frac{4\int \cos^3(c+dx)\,dx}{a}$$

$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int 1\,dx}{2a} - \frac{4\,\mathrm{Subst}\left(\int \left(1-x^2\right)dx,\right.}{ad}$$

$$= -\frac{3x}{2a} + \frac{4\sin(c+dx)}{ad} - \frac{3\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{4\sin^3(c+dx}{3ad}$$

Mathematica [A] time = 0.271445, size = 143, normalized size = 1.52

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(21\sin\left(c+\frac{dx}{2}\right)+18\sin\left(c+\frac{3dx}{2}\right)+18\sin\left(2c+\frac{3dx}{2}\right)-2\sin\left(2c+\frac{5dx}{2}\right)-2\sin\left(3c+\frac{5dx}{2}\right)+\mathrm{s}\right.}{48ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-36*d*x*Cos[(d*x)/2] - 36*d*x*Cos[c + (d*x)/2]
+ 69*Sin[(d*x)/2] + 21*Sin[c + (d*x)/2] + 18*Sin[c + (3*d*x)/2] + 18*Sin[2*
c + (3*d*x)/2] - 2*Sin[2*c + (5*d*x)/2] - 2*Sin[3*c + (5*d*x)/2] + Sin[3*c
+ (7*d*x)/2] + Sin[4*c + (7*d*x)/2]))/(48*a*d)
```

Maple [A] time = 0.046, size = 136, normalized size = 1.5

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5\frac{(\tan(1/2\,dx+c/2))^5}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+\frac{16}{3\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-3}+3\frac{\tan}{da\left(1+(\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*tan(1/2*d*x+1/2*c)+5/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c
)^5+16/3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+3/d/a/(1+tan(1
/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)-3/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.72912, size = 238, normalized size = 2.53

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 3
*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.61676, size = 180, normalized size = 1.91

$$-\frac{9\,dx\cos\left(dx+c\right)+9\,dx-\left(2\,\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}+7\,\cos\left(dx+c\right)+16\right)\sin\left(dx+c\right)}{6\,\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(9*d*x*cos(d*x + c) + 9*d*x - (2*cos(d*x + c)^3 - cos(d*x + c)^2 + 7*c
os(d*x + c) + 16)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 6.5499, size = 570, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-9*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*t
an(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*d*x*tan(c/2 +
 d*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d
*tan(c/2 + d*x/2)**2 + 6*a*d) - 27*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 +
 d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*
d) - 9*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d
*tan(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)**7/(6*a*d*tan(c/2 + d*x/
2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) +
48*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)
**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 50*tan(c/2 + d*x/2)**3/(6*a*d*t
an(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**
2 + 6*a*d) + 24*tan(c/2 + d*x/2)/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/
2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*cos(c)**
4/(a*cos(c) + a), True))
```

Giac [A] time = 1.3614, size = 119, normalized size = 1.27

$$-\frac{\frac{9\,(dx+c)}{a}-\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)/a - 6*tan(1/2*d*x + 1/2*c)/a - 2*(15*tan(1/2*d*x + 1/2*c)
^5 + 16*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/
2*c)^2 + 1)^3*a))/d
```

3.45 $\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3x}{2a}$$

```
[Out] (3*x)/(2*a) - (2*Sin[c + d*x])/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) - (Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0611472, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2767, 2734}

$$-\frac{2\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos^2(c+dx)}{d(a\cos(c+dx)+a)}+\frac{3\sin(c+dx)\cos(c+dx)}{2ad}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sin[c + d*x])/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d
) - (Cos[c + d*x]^2*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{\int \cos(c+dx)(2a-3a\cos(c+dx))\,dx}{a^2}\\ &= \frac{3x}{2a}-\frac{2\sin(c+dx)}{ad}+\frac{3\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\cos^2(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.234428, size = 117, normalized size = 1.54

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-4\sin\left(c+\frac{dx}{2}\right)-3\sin\left(c+\frac{3dx}{2}\right)-3\sin\left(2c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{5dx}{2}\right)+12dx\cos\right.}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(12*d*x*Cos[(d*x)/2] + 12*d*x*Cos[c + (d*x)/2] -
 20*Sin[(d*x)/2] - 4*Sin[c + (d*x)/2] - 3*Sin[c + (3*d*x)/2] - 3*Sin[2*c +
(3*d*x)/2] + Sin[2*c + (5*d*x)/2] + Sin[3*c + (5*d*x)/2]))/(16*a*d)
```

Maple [A] time = 0.044, size = 103, normalized size = 1.4

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{3}}{da\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{2}}-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}+3\,\frac{\arctan\left(\tan\left(\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*tan(1/2*d*x+1/2*c)-3/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*
c)^3-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+3/a/d*arctan(tan(1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.65559, size = 180, normalized size = 2.37

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a+\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)
/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.58943, size = 149, normalized size = 1.96

$$\frac{3\,dx\cos\left(dx+c\right)+3\,dx+\left(\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(3*d*x*cos(d*x + c) + 3*d*x + (cos(d*x + c)^2 - cos(d*x + c) - 4)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 3.69114, size = 325, normalized size = 4.28

$$\begin{cases} \frac{3dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} + \frac{6dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} + \frac{3dx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} - \frac{2\tan^5}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4\ldots} \\ \frac{x\cos^3(c)}{a\cos(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan
(c/2 + d*x/2)**2 + 2*a*d) + 6*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/
2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 3*d*x/(2*a*d*tan(c/2 + d*x/2)*
*4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*tan(c/2 + d*x/2)**5/(2*a*d*tan(
c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 10*tan(c/2 + d*x/2)*
*3/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*tan(
c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d
), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a), True))
```

Giac [A] time = 1.37556, size = 99, normalized size = 1.3

$$\frac{\frac{3(dx+c)}{a} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} - \frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*(d*x + c)/a - 2*tan(1/2*d*x + 1/2*c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.46 $\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=43

$$\frac{\sin(c+dx)}{ad}+\frac{\sin(c+dx)}{ad(\cos(c+dx)+1)}-\frac{x}{a}$$

```
[Out] -(x/a) + Sin[c + d*x]/(a*d) + Sin[c + d*x]/(a*d*(1 + Cos[c + d*x]))
```

Rubi [A] time = 0.0798836, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2746, 12, 2735, 2648}

$$\frac{\sin(c+dx)}{ad}+\frac{\sin(c+dx)}{ad(\cos(c+dx)+1)}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(x/a) + Sin[c + d*x]/(a*d) + Sin[c + d*x]/(a*d*(1 + Cos[c + d*x]))
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{\sin(c+dx)}{ad} - \frac{\int \frac{a\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a} \\ &= \frac{\sin(c+dx)}{ad} - \int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx \\ &= -\frac{x}{a} + \frac{\sin(c+dx)}{ad} + \int \frac{1}{a+a\cos(c+dx)}\,dx \\ &= -\frac{x}{a} + \frac{\sin(c+dx)}{ad} + \frac{\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.200899, size = 89, normalized size = 2.07

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(c+\frac{dx}{2}\right)+\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{3dx}{2}\right)-2dx\cos\left(c+\frac{dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)-2dx\cos\left(\frac{dx}{2}\right)\right)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(-2*d*x*Cos[(d*x)/2] - 2*d*x*Cos[c + (d*x)/2] +
5*Sin[(d*x)/2] + Sin[c + (d*x)/2] + Sin[c + (3*d*x)/2] + Sin[2*c + (3*d*x)/
2]))/(4*a*d)
```

Maple [A] time = 0.044, size = 68, normalized size = 1.6

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\tan(1/2\,dx+c/2)}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)}-2\,\frac{\arctan(\tan(1/2\,dx+c/2))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*tan(1/2*d*x+1/2*c)+2/d/a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-
2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.66696, size = 124, normalized size = 2.88

$$-\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos
(d*x + c) + 1)))/d
```

Fricas [A] time = 1.58301, size = 116, normalized size = 2.7

$$-\frac{dx\cos(dx+c)+dx-(\cos(dx+c)+2)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) + d*x - (cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

Sympy [A] time = 1.97009, size = 129, normalized size = 3.

$$\begin{cases} -\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 + a*d) - d*x/(
a*d*tan(c/2 + d*x/2)**2 + a*d) + tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)*
*2 + a*d) + 3*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)),
(x*cos(c)**2/(a*cos(c) + a), True))
```

Giac [A] time = 1.36168, size = 78, normalized size = 1.81

$$-\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} - \frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d
*x + 1/2*c)^2 + 1)*a))/d
```

3.47 $\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0339496, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2735, 2648}

$$\frac{x}{a}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x]),x]

[Out] x/a - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{x}{a}-\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= \frac{x}{a}-\frac{\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0734609, size = 57, normalized size = 1.97

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(dx\cos\left(\frac{1}{2}(c+dx)\right)-\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x]),x]

[Out] (2*Cos[(c + d*x)/2]*(d*x*Cos[(c + d*x)/2] - Sec[c/2]*Sin[(d*x)/2]))/(a*d*(1
 + Cos[c + d*x]))
```

Maple [A] time = 0.038, size = 37, normalized size = 1.3

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*tan(1/2*d*x+1/2*c)+2/a/d*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.73606, size = 66, normalized size = 2.28

$$\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c
) + 1)))/d
```

Fricas [A] time = 1.55292, size = 89, normalized size = 3.07

$$\frac{dx\cos\left(dx+c\right)+dx-\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] (d*x*cos(d*x + c) + d*x - sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 1.00107, size = 27, normalized size = 0.93

$$\begin{cases}\frac{x}{a}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for}\: d\neq 0\\ \frac{x\cos\left(c\right)}{a\cos\left(c\right)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((x/a - tan(c/2 + d*x/2)/(a*d), Ne(d, 0)), (x*cos(c)/(a*cos(c) + a
), True))
```

Giac [A] time = 1.39856, size = 38, normalized size = 1.31

$$\frac{\frac{dx+c}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)/a - tan(1/2*d*x + 1/2*c)/a)/d
```

3.48 $\int \frac{1}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=22

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0123108, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2648}

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-1),x]
```

```
[Out] Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\cos(c+dx)}\,dx=\frac{\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [A] time = 0.0139331, size = 17, normalized size = 0.77

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-1),x]
```

```
[Out] Tan[(c + d*x)/2]/(a*d)
```

Maple [A] time = 0.032, size = 17, normalized size = 0.8

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*tan(1/2*d*x+1/2*c)
```

Maxima [A] time = 1.20812, size = 31, normalized size = 1.41

$$\frac{\sin(dx+c)}{ad(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(a*d*(cos(d*x + c) + 1))
```

Fricas [A] time = 1.56411, size = 53, normalized size = 2.41

$$\frac{\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

Sympy [A] time = 0.597071, size = 20, normalized size = 0.91

$$\begin{cases} \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad} & \text{for } d \neq 0 \\ \frac{x}{a\cos(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c)),x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)/(a*d), Ne(d, 0)), (x/(a*cos(c) + a), True))
```

Giac [A] time = 1.38707, size = 22, normalized size = 1.

$$\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] tan(1/2*d*x + 1/2*c)/(a*d)
```

3.49 $\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0475856, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2747, 3770, 2648}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(d*(a + a*Cos[c + d*x]))
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{a+a\cos(c+dx)}\,dx &= \frac{\int \sec(c+dx)\,dx}{a}-\int \frac{1}{a+a\cos(c+dx)}\,dx\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.143291, size = 103, normalized size = 2.71

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sin[(d*x)/2]))
/(a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.05, size = 58, normalized size = 1.5

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a*ln(tan(1/2*d
*x+1/2*c)+1)
```

Maxima [A] time = 1.18943, size = 101, normalized size = 2.66

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.60259, size = 181, normalized size = 4.76

$$\frac{(\cos(dx+c)+1)\log(\sin(dx+c)+1)-(\cos(dx+c)+1)\log(-\sin(dx+c)+1)-2\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - (cos(d*x + c) + 1)*log(-sin
(d*x + c) + 1) - 2*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(cos(c + d*x) + 1), x)/a
```

Giac [A] time = 1.44793, size = 73, normalized size = 1.92

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/
a - tan(1/2*d*x + 1/2*c)/a)/d
```

3.50 $\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{2\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a*d)) + (2*Tan[c + d*x])/(a*d) - Tan[c + d*x]/(d*(
a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0758197, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2768, 2748, 3767, 8, 3770}

$$\frac{2\tan(c+dx)}{ad}-\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{\tan(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a*d)) + (2*Tan[c + d*x])/(a*d) - Tan[c + d*x]/(d*(
a + a*Cos[c + d*x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int(-2a+a\cos(c+dx))\sec^2(c+dx)\,dx}{a^2}\\ &= -\frac{\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int\sec(c+dx)\,dx}{a} + \frac{2\int\sec^2(c+dx)\,dx}{a}\\ &= -\frac{\tanh^{-1}(\sin(c+dx))}{ad} - \frac{\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{2\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{ad}\\ &= -\frac{\tanh^{-1}(\sin(c+dx))}{ad} + \frac{2\tan(c+dx)}{ad} - \frac{\tan(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 0.600276, size = 188, normalized size = 3.55

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+c\right.}\right.\right.}{ad(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + S
in[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(a*d*(1 + Cos[c +
 d*x]))
```

Maple [A] time = 0.057, size = 99, normalized size = 1.9

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{da}\ln\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*tan(1/2*d*x+1/2*c)-1/d/a/(tan(1/2*d*x+1/2*c)-1)+1/d/a*ln(tan(1/2*d*x+
1/2*c)-1)-1/a/d/(tan(1/2*d*x+1/2*c)+1)-1/d/a*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.16716, size = 161, normalized size = 3.04

$$-\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
```

```
)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.6174, size = 266, normalized size = 5.02

$$-\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\cos\left(d\right.\right.}{2\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - (cos(d*x + c)
^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*cos(d*x + c) + 1)*sin(d*x
+ c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(cos(c + d*x) + 1), x)/a
```

Giac [A] time = 1.37635, size = 113, normalized size = 2.13

$$-\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))
/a - tan(1/2*d*x + 1/2*c)/a + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)
^2 - 1)*a))/d
```

3.51 $\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=83

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*Tan[c + d*x])/(a*d) + (3*Sec[c + d*x
]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x
]))
```

Rubi [A] time = 0.0931044, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$-\frac{2\tan(c+dx)}{ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\tan(c+dx)\sec(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a*d) - (2*Tan[c + d*x])/(a*d) + (3*Sec[c + d*x
]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]*Tan[c + d*x])/(d*(a + a*Cos[c + d*x
]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx &= -\frac{\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{\int(-3a+2a\cos(c+dx))\sec^3(c+dx)\,dx}{a^2}\\ &= -\frac{\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}-\frac{2\int\sec^2(c+dx)\,dx}{a}+\frac{3\int\sec^3(c+dx)\,dx}{a}\\ &= \frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}+\frac{3\int\sec(c+dx)\,dx}{2a}+\frac{2\,\mathrm{Subst}(\int 1\,dx,x,}{ad}\\ &= \frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{2\tan(c+dx)}{ad}+\frac{3\sec(c+dx)\tan(c+dx)}{2ad}-\frac{\sec(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 1.16563, size = 244, normalized size = 2.94

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{\left(\cos\left(\frac{1}{2}(c+\right.\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(-4*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*(-6*Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^(-2) - (4*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])))))/(2*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.066, size = 143, normalized size = 1.7

$$-\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{3}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{3}{2\,da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{1}{2\,da}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/d/a*tan(1/2*d*x+1/2*c)+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)^2+3/2/d/a/(tan(1/2
*d*x+1/2*c)-1)-3/2/d/a*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/a/(tan(1/2*d*x+1/2*c)
+1)^2+3/2/a/d/(tan(1/2*d*x+1/2*c)+1)+3/2/d/a*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.15837, size = 219, normalized size = 2.64

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log(si
n(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) + 1)
))/d
```

Fricas [A] time = 1.64609, size = 301, normalized size = 3.63

$$\frac{3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(4\right.}{4\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*(cos(d*x
 + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c)^2 + co
s(d*x + c) - 1)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x) + 1), x)/a
```

Giac [A] time = 1.52233, size = 136, normalized size = 1.64

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/a - 2*tan(1/2*d*x + 1/2*c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^3 - tan(1/
2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a))/d
```

3.52 $\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=103

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(2*a*d) + (4*Tan[c + d*x])/(a*d) - (3*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c +
d*x])) + (4*Tan[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.0954953, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2768, 2748, 3767, 3768, 3770}

$$\frac{4\tan^3(c+dx)}{3ad}+\frac{4\tan(c+dx)}{ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{3\tan(c+dx)\sec(c+dx)}{2ad}-\frac{\tan(c+dx)\sec^2(c+dx)}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```


```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(2*a*d) + (4*Tan[c + d*x])/(a*d) - (3*Sec[c + d*
x]*Tan[c + d*x])/(2*a*d) - (Sec[c + d*x]^2*Tan[c + d*x])/(d*(a + a*Cos[c +
d*x])) + (4*Tan[c + d*x]^3)/(3*a*d)
```


Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int(-4a+3a\cos(c+dx))\sec^4(c+dx)\,dx}{a^2}$$

$$= -\frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int\sec^3(c+dx)\,dx}{a} + \frac{4\int\sec^4(c+dx)\,dx}{a}$$

$$= -\frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\tan(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int\sec(c+dx)\,dx}{2a} - \frac{4\,\mathrm{Subst}\left(\int(1+\right.}{}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{2ad} + \frac{4\tan(c+dx)}{ad} - \frac{3\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\sec^2(c+dx)\tan(c+d}{d(a+a\cos(c+dx))}$$

Mathematica [B] time = 4.25736, size = 368, normalized size = 3.57

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(6\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{8}\sec(c)\cos\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(-12\sin(2c+dx)-6\sin(c+2dx)-6\sin(3c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(6*Sec[c/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2]*Sec[c]*Sec[c
 + d*x]^3*(9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*
Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 27*Cos[d*x]*(Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]) + 27*Cos[2*c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 9*Cos[2*c + 3*d*x]*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] - 9*Cos[4*c + 3*d*x]*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 48*Sin[d*x] - 12*Sin[2*c + d*x] - 6*Sin[c + 2*d*x] - 6*Sin[3
*c + 2*d*x] + 20*Sin[2*c + 3*d*x]))/8))/(3*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 0.069, size = 183, normalized size = 1.8

$$\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{3da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3} - \frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2} - \frac{5}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{3}{2da}\ln\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/d/a*tan(1/2*d*x+1/2*c)-1/3/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/d/a/(tan(1/2*d*
x+1/2*c)-1)^2-5/2/d/a/(tan(1/2*d*x+1/2*c)-1)+3/2/d/a*ln(tan(1/2*d*x+1/2*c)-
1)-1/3/d/a/(tan(1/2*d*x+1/2*c)+1)^3+1/d/a/(tan(1/2*d*x+1/2*c)+1)^2-5/2/a/d/
(tan(1/2*d*x+1/2*c)+1)-3/2/d/a*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [B] time = 1.17474, size = 277, normalized size = 2.69

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a-\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
+ 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(d*x
 + c) + 1)))/d
```

Fricas [A] time = 1.63692, size = 331, normalized size = 3.21

$$-\frac{9\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-9\left(\cos(dx+c)^4+\cos(dx+c)^3\right)\log(-\sin(dx+c)+1)-2}{12\left(ad\cos(dx+c)^4+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(9*(cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 9*(cos(d
*x + c)^4 + cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(16*cos(d*x + c)^3 +
 7*cos(d*x + c)^2 - cos(d*x + c) + 2)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a
*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^4(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(cos(c + d*x) + 1), x)/a
```

Giac [A] time = 1.38833, size = 154, normalized size = 1.5

$$-\frac{\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*log(abs(tan(1/2*d*x + 1/2*
c) - 1))/a - 6*tan(1/2*d*x + 1/2*c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^5 - 16*t
an(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 -
1)^3*a))/d
```

3.53 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^4(c}{3d(a\cos(c+dx)}$$

```
[Out] (-5*x)/a^2 + (12*Sin[c + d*x])/(a^2*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(a^2
*d) - (10*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (Cos[
c + d*x]^4*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) - (4*Sin[c + d*x]^3)/
(a^2*d)
```

Rubi [A] time = 0.184665, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2748, 2635, 8, 2633}

$$-\frac{4\sin^3(c+dx)}{a^2d}+\frac{12\sin(c+dx)}{a^2d}-\frac{10\sin(c+dx)\cos^3(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{5\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{5x}{a^2}-\frac{\sin(c+dx)\cos^4(c}{3d(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (-5*x)/a^2 + (12*Sin[c + d*x])/(a^2*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(a^2
*d) - (10*Cos[c + d*x]^3*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (Cos[
c + d*x]^4*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) - (4*Sin[c + d*x]^3)/
(a^2*d)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^3(c+dx)(4a-6a\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \cos^2(c+dx)\left(30a^2-36a^2\cos(\right.}{3a^4}$$

$$= -\frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{10\int \cos^2(c+dx)\,dx}{a^2} + \frac{12\int \cos^3}{}$$

$$= -\frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{5\int 1}{a^2}$$

$$= -\frac{5x}{a^2} + \frac{12\sin(c+dx)}{a^2d} - \frac{5\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{10\cos^3(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^4(c+}{3d(a+a}$$

Mathematica [A] time = 0.440387, size = 199, normalized size = 1.6

$$\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-156\sin\left(c+\frac{dx}{2}\right)+342\sin\left(c+\frac{3dx}{2}\right)+118\sin\left(2c+\frac{3dx}{2}\right)+30\sin\left(2c+\frac{5dx}{2}\right)+30\sin\left(3c+\frac{5d}{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(-360*d*x*Cos[(d*x)/2] - 360*d*x*Cos[c + (d*x)
/2] - 120*d*x*Cos[c + (3*d*x)/2] - 120*d*x*Cos[2*c + (3*d*x)/2] + 516*Sin[(
d*x)/2] - 156*Sin[c + (d*x)/2] + 342*Sin[c + (3*d*x)/2] + 118*Sin[2*c + (3*
d*x)/2] + 30*Sin[2*c + (5*d*x)/2] + 30*Sin[3*c + (5*d*x)/2] - 3*Sin[3*c + (
7*d*x)/2] - 3*Sin[4*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2] + Sin[5*c + (9*d*
x)/2]))/(192*a^2*d)
```

Maple [A] time = 0.049, size = 156, normalized size = 1.3

$$-\frac{1}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{9}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+10\frac{(\tan(1/2\,dx+c/2))^5}{a^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^3}+\frac{40}{3a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2*tan(1/2*d*x+1/2*c)+10/d/a^2/(1+ta
n(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5+40/3/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^3*tan(1/2*d*x+1/2*c)^3+6/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1
/2*c)-10/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.81837, size = 279, normalized size = 2.25

$$\frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1) -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^2)/d
```

Fricas [A] time = 1.64244, size = 278, normalized size = 2.24

$$-\frac{15\,dx\cos(dx+c)^2+30\,dx\cos(dx+c)+15\,dx-\left(\cos(dx+c)^4-\cos(dx+c)^3+6\,\cos(dx+c)^2+33\,\cos(dx+c)\ldots\right.}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(15*d*x*cos(d*x + c)^2 + 30*d*x*cos(d*x + c) + 15*d*x - (cos(d*x + c)^
4 - cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 33*cos(d*x + c) + 24)*sin(d*x + c))
/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 26.5093, size = 700, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-30*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a
**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*
d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 +
```

```
 d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 90*d*x*tan(c/2 + d
*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18
*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 30*d*x/(6*a**2*d*tan(c/2 + d*x/2)
**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**
2*d) - tan(c/2 + d*x/2)**9/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/
2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 24*tan(c/2 + d*
x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*
a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 138*tan(c/2 + d*x/2)**5/(6*a**2*d*
tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d
*x/2)**2 + 6*a**2*d) + 160*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c/2 + d*x/2)**
6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*
d) + 63*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2
+ d*x/2)**4 + 18*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(
c)**5/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.26535, size = 146, normalized size = 1.18

$$-\frac{\frac{30\,(dx+c)}{a^2} - \frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3 a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(30*(d*x + c)/a^2 - 4*(15*tan(1/2*d*x + 1/2*c)^5 + 20*tan(1/2*d*x + 1/
2*c)^3 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2) + (a^
4*tan(1/2*d*x + 1/2*c)^3 - 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.54 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2} dx$

Optimal. Leaf size=114

$$-\frac{16\sin(c+dx)}{3a^2d}-\frac{8\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (7*x)/(2*a^2) - (16*Sin[c + d*x])/(3*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*d) - (8*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) -
 (Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.15247, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2765, 2977, 2734}

$$-\frac{16\sin(c+dx)}{3a^2d}-\frac{8\sin(c+dx)\cos^2(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{7\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{7x}{2a^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (7*x)/(2*a^2) - (16*Sin[c + d*x])/(3*a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])
/(2*a^2*d) - (8*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) -
 (Cos[c + d*x]^3*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(3a-5a\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{8\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^3(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \cos(c+dx)\left(16a^2-21a^2\cos(c+\right.}{3a^4}$$
$$= \frac{7x}{2a^2} - \frac{16\sin(c+dx)}{3a^2d} + \frac{7\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{8\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^3(c+d}{3d(a+a\,c}$$

Mathematica [A] time = 0.303205, size = 177, normalized size = 1.55

$$\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(147\sin\left(c+\frac{dx}{2}\right)-239\sin\left(c+\frac{3dx}{2}\right)-63\sin\left(2c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{5dx}{2}\right)-15\sin\left(3c+\frac{5dx}{2}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(252*d*x*Cos[(d*x)/2] + 252*d*x*Cos[c + (d*x)/
2] + 84*d*x*Cos[c + (3*d*x)/2] + 84*d*x*Cos[2*c + (3*d*x)/2] - 381*Sin[(d*x
)/2] + 147*Sin[c + (d*x)/2] - 239*Sin[c + (3*d*x)/2] - 63*Sin[2*c + (3*d*x)
/2] - 15*Sin[2*c + (5*d*x)/2] - 15*Sin[3*c + (5*d*x)/2] + 3*Sin[3*c + (7*d*
x)/2] + 3*Sin[4*c + (7*d*x)/2]))/(192*a^2*d)
```

Maple [A] time = 0.046, size = 122, normalized size = 1.1

$$\frac{1}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{7}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 5\frac{(\tan(1/2\,dx+c/2))^3}{a^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - 3\frac{\tan(1/2\,dx+c/2)}{a^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} + 7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2*tan(1/2*d*x+1/2*c)-5/d/a^2/(1+tan(
1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2
*tan(1/2*d*x+1/2*c)+7/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.69111, size = 221, normalized size = 1.94

$$-\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
```

```
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^2)/d
```

Fricas [A] time = 1.64576, size = 257, normalized size = 2.25

$$\frac{21\,dx\cos(dx+c)^2 + 42\,dx\cos(dx+c) + 21\,dx + \left(3\,\cos(dx+c)^3 - 6\,\cos(dx+c)^2 - 43\,\cos(dx+c) - 32\right)\sin(dx+}{6\left(a^2d\cos(dx+c)^2 + 2\,a^2d\cos(dx+c) + a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(21*d*x*cos(d*x + c)^2 + 42*d*x*cos(d*x + c) + 21*d*x + (3*cos(d*x + c)
^3 - 6*cos(d*x + c)^2 - 43*cos(d*x + c) - 32)*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 10.7026, size = 413, normalized size = 3.62

$$\begin{cases} \frac{21dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{42dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \frac{21dx}{6a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d} + \cdots \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((21*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a*
*2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 42*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d
*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 21*d*x/(
6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) +
tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x
/2)**2 + 6*a**2*d) - 19*tan(c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**4 +
 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 71*tan(c/2 + d*x/2)**3/(6*a**2
*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - 39*tan
(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2
 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.37358, size = 128, normalized size = 1.12

$$\frac{\frac{21\,(dx+c)}{a^2} - \frac{6\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*(d*x + c)/a^2 - 6*(5*tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 21*
```

```
a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.55 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=80

$$\frac{4\sin(c+dx)}{3a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{2x}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (-2*x)/a^2 + (4*Sin[c + d*x])/(3*a^2*d) + (2*Sin[c + d*x])/(a^2*d*(1 + Cos[
c + d*x])) - (Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.169568, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2968, 3023, 12, 2735, 2648}

$$\frac{4\sin(c+dx)}{3a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{2x}{a^2}-\frac{\sin(c+dx)\cos^2(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 + (4*Sin[c + d*x])/(3*a^2*d) + (2*Sin[c + d*x])/(a^2*d*(1 + Cos[
c + d*x])) - (Cos[c + d*x]^2*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{\int \frac{\cos(c+dx)(2a-4a\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}\\
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{\int \frac{2a\cos(c+dx)-4a\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\
&= \frac{4\sin(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{\int \frac{6a^2\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^3}\\
&= \frac{4\sin(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{2\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{a}\\
&= -\frac{2x}{a^2}+\frac{4\sin(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{2\int \frac{1}{a+a\cos(c+dx)}\,dx}{a}\\
&= -\frac{2x}{a^2}+\frac{4\sin(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{2\sin(c+dx)}{d\left(a^2+a^2\cos(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.363673, size = 114, normalized size = 1.42

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-6(\sin(c+dx)-2dx)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-16\sec\left(\frac{c}{2}\right)\sin\left(\right.\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] - 16*Cos[(c + d*x)/2]^2*Sec[c/2
]*Sin[(d*x)/2] - 6*Cos[(c + d*x)/2]^3*(-2*d*x + Sin[c + d*x]) + Cos[(c + d*
x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.046, size = 88, normalized size = 1.1

$$-\frac{1}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\tan(1/2\,dx+c/2)}{a^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)}-4\,\frac{\arctan(\tan(1/2\,dx+c/2))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2*tan(1/
2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.75163, size = 159, normalized size = 1.99

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x + c)
/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.62664, size = 230, normalized size = 2.88

$$-\frac{6\,dx\cos\left(dx+c\right)^{2}+12\,dx\cos\left(dx+c\right)+6\,dx-\left(3\,\cos\left(dx+c\right)^{2}+14\,\cos\left(dx+c\right)+10\right)\sin\left(dx+c\right)}{3\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*d*x*cos(d*x + c)^2 + 12*d*x*cos(d*x + c) + 6*d*x - (3*cos(d*x + c)^
2 + 14*cos(d*x + c) + 10)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d)
```

Sympy [A] time = 5.97581, size = 201, normalized size = 2.51

$$\begin{cases} -\frac{12dx\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}-\frac{12dx}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}-\frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}+\frac{14\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d}+\frac{27\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^{2}d} & \text{for } d \\ \frac{x\cos^{3}(c)}{(a\cos(c)+a)^{2}} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*d*x*tan(c/2 + d*x/2)**2/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a*
*2*d) - 12*d*x/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) - tan(c/2 + d*x/2)
**5/(6*a**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 14*tan(c/2 + d*x/2)**3/(6*a
**2*d*tan(c/2 + d*x/2)**2 + 6*a**2*d) + 27*tan(c/2 + d*x/2)/(6*a**2*d*tan(c
/2 + d*x/2)**2 + 6*a**2*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c) + a)**2, True
))
```

Giac [A] time = 1.37909, size = 107, normalized size = 1.34

$$-\frac{\frac{12\,(dx+c)}{a^2}-\frac{12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*(d*x + c)/a^2 - 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 +
 1)*a^2) + (a^4*tan(1/2*d*x + 1/2*c)^3 - 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/
d
```

3.56 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2} dx$

Optimal. Leaf size=57

$$-\frac{5\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{x}{a^2}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] x/a^2 - (5*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(3*d*(
a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.0845487, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2758, 2735, 2648}

$$-\frac{5\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{x}{a^2}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] x/a^2 - (5*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) + Sin[c + d*x]/(3*d*(
a + a*Cos[c + d*x])^2)
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{-2a+3a\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\ &= \frac{x}{a^2}+\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{5\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}\\ &= \frac{x}{a^2}+\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{5\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.228174, size = 105, normalized size = 1.84

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6dx\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-10\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos^2\left(\frac{1}{2}(c+dx)\right.\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(6*d*x*Cos[(c + d*x)/2]^3 + Sec[c/2]*Sin[(d*x)/2] - 10*
Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*Tan[c/2]))/(3*a
^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.04, size = 56, normalized size = 1.

$$\frac{1}{6\,a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3}{2\,a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2*tan(1/2*d*x+1/2*c)+2/d/a^2*arctan(
tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.74071, size = 97, normalized size = 1.7

$$-\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.80739, size = 198, normalized size = 3.47

$$\frac{3\,dx\cos\left(dx+c\right)^2+6\,dx\cos\left(dx+c\right)+3\,dx-\left(5\,\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)}{3\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c)^2 + 6*d*x*cos(d*x + c) + 3*d*x - (5*cos(d*x + c) +
4)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [A] time = 3.12045, size = 56, normalized size = 0.98

$$\begin{cases} \frac{x}{a^2} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d} - \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d \neq 0 \\ \frac{x\cos^2(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((x/a**2 + tan(c/2 + d*x/2)**3/(6*a**2*d) - 3*tan(c/2 + d*x/2)/(2*
a**2*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.40093, size = 68, normalized size = 1.19

$$\frac{\frac{6(dx+c)}{a^2} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 9a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/a^2 + (a^4*tan(1/2*d*x + 1/2*c)^3 - 9*a^4*tan(1/2*d*x + 1/
2*c))/a^6)/d
```

3.57 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{2\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(3*d*(a^2 + a
^2*Cos[c + d*x]))
```

Rubi [A] time = 0.0384962, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2750, 2648}

$$\frac{2\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(3*d*(a^2 + a
^2*Cos[c + d*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{2\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}\\ &= -\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{2\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.112811, size = 60, normalized size = 1.09

$$\frac{\sec\left(\frac{c}{2}\right)\left(-3\sin\left(c+\frac{dx}{2}\right)+2\sin\left(c+\frac{3dx}{2}\right)+3\sin\left(\frac{dx}{2}\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(3*Sin[(d*x)/2] - 3*Sin[c + (d*x)/2] + 2*Sin[c
 + (3*d*x)/2]))/(12*a^2*d)
```

Maple [A] time = 0.036, size = 32, normalized size = 0.6

$$\frac{1}{2\,a^2d}\left(-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/2/d/a^2*(-1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.17205, size = 63, normalized size = 1.15

$$\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

Fricas [A] time = 1.69925, size = 126, normalized size = 2.29

$$\frac{(2\cos(dx+c)+1)\sin(dx+c)}{3\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c) + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)
```

Sympy [A] time = 2.07279, size = 48, normalized size = 0.87

$$\begin{cases} -\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^2d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d} & \text{for } d\neq 0\\ \frac{x\cos(c)}{(a\cos(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**3/(6*a**2*d) + tan(c/2 + d*x/2)/(2*a**2*d), N
e(d, 0)), (x*cos(c)/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.19589, size = 42, normalized size = 0.76

$$-\frac{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} - 3\,\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{6\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.58 $\int \frac{1}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*d*(a^2 + a^2*Co
s[c + d*x]))
```

Rubi [A] time = 0.0269949, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$\frac{\sin(c+dx)}{3d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2) + Sin[c + d*x]/(3*d*(a^2 + a^2*Co
s[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^2}\,dx &= \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{1}{a+a\cos(c+dx)}\,dx}{3a}\\ &= \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\sin(c+dx)}{3d\left(a^2+a^2\cos(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.049044, size = 53, normalized size = 0.96

$$\frac{\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-2),x]
```

```
[Out] (Cos[(c + d*x)/2]*(3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*a^2*d*(1
+ Cos[c + d*x])^2)
```

Maple [A] time = 0.033, size = 32, normalized size = 0.6

$$\frac{\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/2/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.11317, size = 62, normalized size = 1.13

$$\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{6a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^2*d)
```

Fricas [A] time = 1.62386, size = 123, normalized size = 2.24

$$\frac{(\cos(dx+c)+2)\sin(dx+c)}{3\left(a^{2}d\cos(dx+c)^{2}+2a^{2}d\cos(dx+c)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c) + 2)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x
 + c) + a^2*d)
```

Sympy [A] time = 1.22905, size = 44, normalized size = 0.8

$$\begin{cases} \frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{2}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^{2}d} & \text{for } d\neq 0 \\ \frac{x}{(a\cos(c)+a)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**3/(6*a**2*d) + tan(c/2 + d*x/2)/(2*a**2*d), Ne
(d, 0)), (x/(a*cos(c) + a)**2, True))
```

Giac [A] time = 1.19099, size = 42, normalized size = 0.76

$$\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(tan(1/2*d*x + 1/2*c)^3 + 3*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

3.59 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{4\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^2*d) - (4*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x]
)) - Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.111554, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2766, 2978, 12, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{4\sin(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^2*d) - (4*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x]
)) - Sin[c + d*x]/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(3a-a\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{4\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int 3a^2\sec(c+dx)\,dx}{3a^4}$$
$$= -\frac{4\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \sec(c+dx)\,dx}{a^2}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{4\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

Mathematica [B] time = 0.288208, size = 152, normalized size = 2.3

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.\right.\right.}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sin[(d*x)/
2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*Tan[c/2]
))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.053, size = 77, normalized size = 1.2

$$-\frac{1}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{3}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + \frac{1}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3-3/2/d/a^2*tan(1/2*d*x+1/2*c)-1/d/a^2*ln(tan
(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.09848, size = 132, normalized size = 2.

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x +
```

```
c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A] time = 1.68033, size = 305, normalized size = 4.62

$$\frac{3\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\log\left(-\sin\left(dx+c\right)+1}{6\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 3*(cos
(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(4*cos(d*x + c
) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec\left(c+dx\right)}{\cos^2\left(c+dx\right)+2\cos\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.39514, size = 104, normalized size = 1.58

$$\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^2 - (a^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

3.60 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=81

$$\frac{10\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (-2*ArcTanh[Sin[c + d*x]])/(a^2*d) + (10*Tan[c + d*x])/(3*a^2*d) - (2*Tan[c
 + d*x])/(a^2*d*(1 + Cos[c + d*x])) - Tan[c + d*x]/(3*d*(a + a*Cos[c + d*x]
)^2)
```

Rubi [A] time = 0.172821, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{10\tan(c+dx)}{3a^2d}-\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{2\tan(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{\tan(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*ArcTanh[Sin[c + d*x]])/(a^2*d) + (10*Tan[c + d*x])/(3*a^2*d) - (2*Tan[c
 + d*x])/(a^2*d*(1 + Cos[c + d*x])) - Tan[c + d*x]/(3*d*(a + a*Cos[c + d*x]
)^2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(4a-2a\cos(c+dx))\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(10a^2-6a^2\cos(c+dx)\right)\sec^2(c+dx)\,d}{3a^4}$$
$$= -\frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{2\int \sec(c+dx)\,dx}{a^2} + \frac{10\int \sec^2(c+dx)\,dx}{3a^2}$$
$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{10\,\mathrm{Subst}(\int 1\,dx}{3}$$
$$= -\frac{2\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{10\tan(c+dx)}{3a^2d} - \frac{2\tan(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\tan(c+dx)}{3d(a+a\cos(c+dx))^2}$$

Mathematica [B] time = 1.01288, size = 239, normalized size = 2.95

$$2\cos\left(\frac{1}{2}(c+dx)\right)\left(\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+6\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sin(}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]
*Sin[(d*x)/2] + 6*Cos[(c + d*x)/2]^3*(2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] -
Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]))) + Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(
1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.064, size = 120, normalized size = 1.5

$$\frac{1}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{5}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\frac{\ln(\tan(1/2\,dx+c/2)-1)}{a^2d}-\frac{1}{a^2d}\left(\tan\left(\frac{d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3+5/2/d/a^2*tan(1/2*d*x+1/2*c)-1/d/a^2/(tan(1/
2*d*x+1/2*c)-1)+2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c
)+1)-2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.08842, size = 196, normalized size = 2.42

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1
)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.70317, size = 387, normalized size = 4.78

$$-\frac{3\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx\right.}{3\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c)
 + 1) - 3*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x +
 c) + 1) - (10*cos(d*x + c)^2 + 14*cos(d*x + c) + 3)*sin(d*x + c))/(a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^2(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.32408, size = 143, normalized size = 1.77

$$-\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 12*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 12*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^
2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.61 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2} \, dx$

Optimal. Leaf size=119

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{8\tan(c+dx)\sec(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{\tan(c+dx)\sec(}{3d(a\cos(c+dx}$$

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (16*Tan[c + d*x])/(3*a^2*d) + (7*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*d) - (8*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(
1 + Cos[c + d*x])) - (Sec[c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2)
```

Rubi [A] time = 0.18975, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{16\tan(c+dx)}{3a^2d}+\frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{7\tan(c+dx)\sec(c+dx)}{2a^2d}-\frac{8\tan(c+dx)\sec(c+dx)}{3a^2d(\cos(c+dx)+1)}-\frac{\tan(c+dx)\sec(}{3d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (16*Tan[c + d*x])/(3*a^2*d) + (7*Sec[
c + d*x]*Tan[c + d*x])/(2*a^2*d) - (8*Sec[c + d*x]*Tan[c + d*x])/(3*a^2*d*(
1 + Cos[c + d*x])) - (Sec[c + d*x]*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{(5a-3a\cos(c+dx))\sec^3(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}$$
$$= -\frac{8\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \left(21a^2-16a^2\cos(c+dx)\right)\sec^3(c}{3a^4}$$
$$= -\frac{8\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{16\int \sec^2(c+dx)\,dx}{3a^2} + \frac{7\int \sec^3(c+}{a^2}$$
$$= \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\sec(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{7\int \sec(}{2}$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{16\tan(c+dx)}{3a^2d} + \frac{7\sec(c+dx)\tan(c+dx)}{2a^2d} - \frac{8\sec(c+dx)\tan(c+}{3a^2d(1+\cos(c+d}$$

Mathematica [B] time = 1.72626, size = 292, normalized size = 2.45

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-2\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)-2\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+3\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-\frac{}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(\right.\right.}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*(-2*Sec[c/2]*Sin[(d*x)/2] - 40*Cos[(c + d*x)/2]^2*Sec[c/2
]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]^3*(-14*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 14*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^(-2) - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) - (8*
Sin[d*x])/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - 2*Cos[(c + d*x)
```

```
/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.072, size = 162, normalized size = 1.4

$$-\frac{1}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{5}{2a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{7}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6/d/a^2*tan(1/2*d*x+1/2*c)^3-7/2/d/a^2*tan(1/2*d*x+1/2*c)+1/2/d/a^2/(tan
(1/2*d*x+1/2*c)-1)^2+5/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)-7/2/d/a^2*ln(tan(1/2*
d*x+1/2*c)-1)-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+5/2/d/a^2/(tan(1/2*d*x+1/2
*c)+1)+7/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.52658, size = 257, normalized size = 2.16

$$-\frac{\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A] time = 1.68453, size = 427, normalized size = 3.59

$$\frac{21\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-21\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(\right.}{12\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x +
 c) + 1) - 21*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin
(d*x + c) + 1) - 2*(32*cos(d*x + c)^3 + 43*cos(d*x + c)^2 + 6*cos(d*x + c)
- 3)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*c
os(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.26132, size = 165, normalized size = 1.39

$$\frac{\frac{21\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{21\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+21a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 21*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/a^2 + 6*(5*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (a^4*tan(1/2*d*x + 1/2*c)^3 + 21*a^4*ta
n(1/2*d*x + 1/2*c))/a^6)/d
```

3.62 $\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{4\tan^3(c+dx)}{a^2d}+\frac{12\tan(c+dx)}{a^2d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{10\tan(c+dx)\sec^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-$$

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^2*d) + (12*Tan[c + d*x])/(a^2*d) - (5*Sec[c +
 d*x]*Tan[c + d*x])/(a^2*d) - (10*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1
+ Cos[c + d*x])) - (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2) + (4*Tan[c + d*x]^3)/(a^2*d)
```

Rubi [A] time = 0.199197, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 3768, 3770}

$$\frac{4\tan^3(c+dx)}{a^2d}+\frac{12\tan(c+dx)}{a^2d}-\frac{5\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{5\tan(c+dx)\sec(c+dx)}{a^2d}-\frac{10\tan(c+dx)\sec^2(c+dx)}{3a^2d(\cos(c+dx)+1)}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^2*d) + (12*Tan[c + d*x])/(a^2*d) - (5*Sec[c +
 d*x]*Tan[c + d*x])/(a^2*d) - (10*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1
+ Cos[c + d*x])) - (Sec[c + d*x]^2*Tan[c + d*x])/(3*d*(a + a*Cos[c + d*x])^
2) + (4*Tan[c + d*x]^3)/(a^2*d)
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^4(c+dx)}{(a+a\cos(c+dx))^2}\,dx &= -\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \frac{(6a-4a\cos(c+dx))\sec^4(c+dx)}{a+a\cos(c+dx)}\,dx}{3a^2}\\
&= -\frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}+\frac{\int \left(36a^2-30a^2\cos(c+dx)\right)\sec
\end{aligned}
$$

$$
= -\frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{10\int \sec^3(c+dx)\,dx}{a^2}+\frac{12\int \sec
$$

$$
= -\frac{5\sec(c+dx)\tan(c+dx)}{a^2d}-\frac{10\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{\sec^2(c+dx)\tan(c+dx)}{3d(a+a\cos(c+dx))^2}-\frac{5\int
$$

$$
= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{12\tan(c+dx)}{a^2d}-\frac{5\sec(c+dx)\tan(c+dx)}{a^2d}-\frac{10\sec^2(c+dx)\tan
$$

Mathematica [B] time = 3.95732, size = 343, normalized size = 2.58

$$
960\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\sec(c)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (960*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Sec[c
 + d*x]^3*(-3*Sin[(d*x)/2] + 155*Sin[(3*d*x)/2] - 153*Sin[c - (d*x)/2] + 21
*Sin[c + (d*x)/2] - 135*Sin[2*c + (d*x)/2] + 25*Sin[c + (3*d*x)/2] + 45*Sin
[2*c + (3*d*x)/2] - 85*Sin[3*c + (3*d*x)/2] + 99*Sin[c + (5*d*x)/2] + 21*Si
n[2*c + (5*d*x)/2] + 33*Sin[3*c + (5*d*x)/2] - 45*Sin[4*c + (5*d*x)/2] + 57
*Sin[2*c + (7*d*x)/2] + 18*Sin[3*c + (7*d*x)/2] + 24*Sin[4*c + (7*d*x)/2] -
 15*Sin[5*c + (7*d*x)/2] + 24*Sin[3*c + (9*d*x)/2] + 11*Sin[4*c + (9*d*x)/2
] + 13*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.072, size = 204, normalized size = 1.5

$$\frac{1}{6a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{9}{2a^2d}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{3}{2a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-5\frac{}{a^2d\,(\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6/d/a^2*tan(1/2*d*x+1/2*c)^3+9/2/d/a^2*tan(1/2*d*x+1/2*c)-1/3/d/a^2/(tan(
1/2*d*x+1/2*c)-1)^3-3/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-5/d/a^2/(tan(1/2*d*x
+1/2*c)-1)+5/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a^2/(tan(1/2*d*x+1/2*c)+1
)^3+3/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2-5/d/a^2/(tan(1/2*d*x+1/2*c)+1)-5/d/a
^2*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.1031, size = 316, normalized size = 2.38

$$\frac{\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^2*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1) +
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [A] time = 1.67939, size = 450, normalized size = 3.38

$$-\frac{15\left(\cos(dx+c)^5+2\cos(dx+c)^4+\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^5+2\cos(dx+c)^4+\cos\right.}{6\left(a^2d\cos(dx+c)^5+2a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(15*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 + cos(d*x + c)^3)*log(sin(d*x +
 c) + 1) - 15*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 + cos(d*x + c)^3)*log(-sin
(d*x + c) + 1) - 2*(24*cos(d*x + c)^4 + 33*cos(d*x + c)^3 + 6*cos(d*x + c)^
2 - cos(d*x + c) + 1)*sin(d*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x
 + c)^4 + a^2*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec^4(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.42684, size = 182, normalized size = 1.37

$$-\frac{\frac{30\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{30\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{4\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+27a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(30*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 30*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 4*(15*tan(1/2*d*x + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^3
 + 9*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (a^4*tan(
1/2*d*x + 1/2*c)^3 + 27*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.63 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=153

$$-\frac{152\sin(c+dx)}{15a^3d}-\frac{76\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{13x}{2a^3}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{11\,\mathrm{si}}{15a}$$

```
[Out] (13*x)/(2*a^3) - (152*Sin[c + d*x])/(15*a^3*d) + (13*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3)
 - (11*Cos[c + d*x]^3*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (76*C
os[c + d*x]^2*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.264766, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2765, 2977, 2734}

$$-\frac{152\sin(c+dx)}{15a^3d}-\frac{76\sin(c+dx)\cos^2(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{13\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{13x}{2a^3}-\frac{\sin(c+dx)\cos^4(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{11\,\mathrm{si}}{15a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (13*x)/(2*a^3) - (152*Sin[c + d*x])/(15*a^3*d) + (13*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3)
 - (11*Cos[c + d*x]^3*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (76*C
os[c + d*x]^2*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)(4a-7a\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(33a^2-43a^2\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\cos^3(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{76\cos^2(c+dx)\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)} - \int c$$

$$= \frac{13x}{2a^3} - \frac{152\sin(c+dx)}{15a^3d} + \frac{13\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{\cos^4(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\cos^3(c-}{15ad(a+}$$

Mathematica [A] time = 0.638734, size = 173, normalized size = 1.13

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(15(-12\sin(c+dx)+\sin(2(c+dx))+26dx)\cos^5\left(\frac{1}{2}(c+dx)\right)+46\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)-3\tan\left(\frac{c}{2}\right.\right.}{15a^3d(\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-3*Sec[c/2]*Sin[(d*x)/2] + 46*Cos[(c + d*x)/2]^2*Sec[c
/2]*Sin[(d*x)/2] - 508*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 15*Cos[(c
 + d*x)/2]^5*(26*d*x - 12*Sin[c + d*x] + Sin[2*(c + d*x)]) - 3*Cos[(c + d*x
)/2]*Tan[c/2] + 46*Cos[(c + d*x)/2]^3*Tan[c/2]))/(15*a^3*d*(1 + Cos[c + d*x
])^3)
```

Maple [A] time = 0.047, size = 141, normalized size = 0.9

$$-\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{2}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{31}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 7\frac{(\tan(1/2\,dx+c/2))^3}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^2} - 5\frac{}{da^3(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+2/3/d/a^3*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*
tan(1/2*d*x+1/2*c)-7/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-
5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+13/d/a^3*arctan(tan(1
/2*d*x+1/2*c))
```

Maxima [A] time = 1.71221, size = 248, normalized size = 1.62

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}} + \frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.65211, size = 363, normalized size = 2.37

$$\frac{195\,dx\cos(dx+c)^3+585\,dx\cos(dx+c)^2+585\,dx\cos(dx+c)+195\,dx+\left(15\cos(dx+c)^4-45\cos(dx+c)^3-47\right.}{30\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(195*d*x*cos(d*x + c)^3 + 585*d*x*cos(d*x + c)^2 + 585*d*x*cos(d*x + c
) + 195*d*x + (15*cos(d*x + c)^4 - 45*cos(d*x + c)^3 - 479*cos(d*x + c)^2 -
 717*cos(d*x + c) - 304)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3977, size = 153, normalized size = 1.

$$\frac{\frac{390\,(dx+c)}{a^3}-\frac{60\left(7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+465\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*(d*x + c)/a^3 - 60*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5
 - 40*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.64 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3} dx$

Optimal. Leaf size=119

$$\frac{9\sin(c+dx)}{5a^3d}+\frac{3\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{3\sin(c+dx)\cos^2(c+dx)}{5ad(a\cos(c+dx)+a)^2}$$

```
[Out] (-3*x)/a^3 + (9*Sin[c + d*x])/(5*a^3*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(5*
d*(a + a*Cos[c + d*x])^3) - (3*Cos[c + d*x]^2*Sin[c + d*x])/(5*a*d*(a + a*C
os[c + d*x])^2) + (3*Sin[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.27257, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2765, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{9\sin(c+dx)}{5a^3d}+\frac{3\sin(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3x}{a^3}-\frac{\sin(c+dx)\cos^3(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{3\sin(c+dx)\cos^2(c+dx)}{5ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 + (9*Sin[c + d*x])/(5*a^3*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(5*
d*(a + a*Cos[c + d*x])^3) - (3*Cos[c + d*x]^2*Sin[c + d*x])/(5*a*d*(a + a*C
os[c + d*x])^2) + (3*Sin[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)(3a-6a\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(18a^2-27a^2\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{18a^2\cos(c+dx)-27a^2\cos^2(c+dx)}{a+a\cos(c+dx)}\,dx}{15a^4}$$

$$= \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{45a^3\cos(c+dx)}{a+a\cos(c+dx)}\,d\ldots}{15a^5}$$

$$= \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{3\int \frac{\cos(c+dx)}{a+a\cos(c+dx)}\ldots}{a^2}$$

$$= -\frac{3x}{a^3} + \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{3\int \frac{\ldots}{a+a\cos\ldots}}{a\ldots}$$

$$= -\frac{3x}{a^3} + \frac{9\sin(c+dx)}{5a^3d} - \frac{\cos^3(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{3\cos^2(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} + \frac{3\sin\ldots}{d\left(a^3+a\ldots\right.}$$

Mathematica [A] time = 0.545771, size = 161, normalized size = 1.35

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(20(\sin(c+dx)-3dx)\cos^5\left(\frac{1}{2}(c+dx)\right)-12\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\ldots\right.}{5a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] - 12*Cos[(c + d*x)/2]^2*Sec[c/2]
*Sin[(d*x)/2] + 96*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 20*Cos[(c + d
*x)/2]^5*(-3*d*x + Sin[c + d*x]) + Cos[(c + d*x)/2]*Tan[c/2] - 12*Cos[(c +
d*x)/2]^3*Tan[c/2]))/(5*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.048, size = 107, normalized size = 0.9

$$\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\tan(1/2\,dx+c/2)}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)}-6\,\frac{\arctan(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-1/2/d/a^3*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*t
an(1/2*d*x+1/2*c)+2/d/a^3*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/a
^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.66513, size = 185, normalized size = 1.55

$$\frac{\frac{40\,\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.62797, size = 325, normalized size = 2.73

$$-\frac{15\,dx\cos(dx+c)^3+45\,dx\cos(dx+c)^2+45\,dx\cos(dx+c)+15\,dx-\left(5\,\cos(dx+c)^3+39\,\cos(dx+c)^2+57\,\cos(dx}{5\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/5*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (5*cos(d*x + c)^3 + 39*cos(d*x + c)^2 + 57*cos(d*x + c) + 24)*sin
(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)
```

Sympy [A] time = 18.245, size = 240, normalized size = 2.02

$$\begin{cases} -\frac{60dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d} - \frac{60dx}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d} + \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d} - \frac{9\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3d} + \frac{75\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+20a^3} \\ \frac{x\cos^4(c)}{(a\cos(c)+a)^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-60*d*x*tan(c/2 + d*x/2)**2/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*
a**3*d) - 60*d*x/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + tan(c/2 + d*
x/2)**7/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) - 9*tan(c/2 + d*x/2)**5
/(20*a**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + 75*tan(c/2 + d*x/2)**3/(20*a
**3*d*tan(c/2 + d*x/2)**2 + 20*a**3*d) + 125*tan(c/2 + d*x/2)/(20*a**3*d*ta
n(c/2 + d*x/2)**2 + 20*a**3*d), Ne(d, 0)), (x*cos(c)**4/(a*cos(c) + a)**3,
True))
```

Giac [A] time = 1.23387, size = 130, normalized size = 1.09

$$-\frac{\frac{60\,(dx+c)}{a^3} - \frac{40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} - \frac{a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 10\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 85\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*(d*x + c)/a^3 - 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
+ 1)*a^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 - 10*a^12*tan(1/2*d*x + 1/2*c)^3 +
 85*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.65 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3} \, dx$

Optimal. Leaf size=96

$$-\frac{29\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x}{a^3}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

```
[Out] x/a^3 - (Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (7*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (29*Sin[c + d*x])/(15*d*(a^3 +
 a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.183956, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2765, 2968, 3019, 2735, 2648}

$$-\frac{29\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{x}{a^3}-\frac{\sin(c+dx)\cos^2(c+dx)}{5d(a\cos(c+dx)+a)^3}+\frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] x/a^3 - (Cos[c + d*x]^2*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (7*Sin
[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (29*Sin[c + d*x])/(15*d*(a^3 +
 a^3*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= -\frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)(2a-5a\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx}{5a^2} \\
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{2a\cos(c+dx)-5a\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2} \\
&= -\frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{-14a^2+15a^2\cos(c+dx)}{a+a\cos(c+dx)}\,dx}{15a^4} \\
&= \frac{x}{a^3} - \frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{29\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2} \\
&= \frac{x}{a^3} - \frac{\cos^2(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{29\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.232953, size = 154, normalized size = 1.6

$$
\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(60dx\cos^5\left(\frac{1}{2}(c+dx)\right)+26\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)-3\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)-3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right.}{15a^3d(\cos(c+dx)+1)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(60*d*x*Cos[(c + d*x)/2]^5 - 3*Sec[c/2]*Sin[(d*x)/2] +
26*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - 128*Cos[(c + d*x)/2]^4*Sec[c/
2]*Sin[(d*x)/2] - 3*Cos[(c + d*x)/2]*Tan[c/2] + 26*Cos[(c + d*x)/2]^3*Tan[c
/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.043, size = 75, normalized size = 0.8

$$
-\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{1}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{7}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{da^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*t
an(1/2*d*x+1/2*c)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.65458, size = 124, normalized size = 1.29

$$-\frac{\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1}-\frac{20 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.59288, size = 300, normalized size = 3.12

$$\frac{15\,dx\cos\left(dx+c\right)^3+45\,dx\cos\left(dx+c\right)^2+45\,dx\cos\left(dx+c\right)+15\,dx-\left(32\cos\left(dx+c\right)^2+51\cos\left(dx+c\right)+22\right)\sin\left(dx\right.}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cos(d*x + c)^3 + 45*d*x*cos(d*x + c)^2 + 45*d*x*cos(d*x + c) +
 15*d*x - (32*cos(d*x + c)^2 + 51*cos(d*x + c) + 22)*sin(d*x + c))/(a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 8.80605, size = 75, normalized size = 0.78

$$\begin{cases} \frac{x}{a^3}-\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d}+\frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for}\ d\neq 0\\ \frac{x\cos^3\left(c\right)}{\left(a\cos\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((x/a**3 - tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)**3/(
3*a**3*d) - 7*tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)**3/(a*cos(c
) + a)**3, True))
```

Giac [A] time = 1.41218, size = 92, normalized size = 0.96

$$\frac{\frac{60\,(dx+c)}{a^3}-\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 - 20*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.66 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3} \, dx$

Optimal. Leaf size=83

$$\frac{7\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{8\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d*(a + a
*Cos[c + d*x])^2) + (7*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.0937042, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2758, 2750, 2648}

$$\frac{7\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{8\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d*(a + a
*Cos[c + d*x])^2) + (7*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{-3a+5a\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$
$$= \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{7\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2}$$
$$= \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{7\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}$$

Mathematica [A] time = 0.185515, size = 86, normalized size = 1.04

$$\frac{\sec\left(\frac{c}{2}\right)\left(-30\sin\left(c+\frac{dx}{2}\right)+20\sin\left(c+\frac{3dx}{2}\right)-15\sin\left(2c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+40\sin\left(\frac{dx}{2}\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{240a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(40*Sin[(d*x)/2] - 30*Sin[c + (d*x)/2] + 20*Si
n[c + (3*d*x)/2] - 15*Sin[2*c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2]))/(240*
a^3*d)
```

Maple [A] time = 0.037, size = 45, normalized size = 0.5

$$\frac{1}{4\,da^3}\left(\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

Maxima [A] time = 1.22137, size = 90, normalized size = 1.08

$$\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

Fricas [A] time = 1.50235, size = 186, normalized size = 2.24

$$\frac{\left(7 \cos\left(dx+c\right)^{2}+6 \cos\left(dx+c\right)+2\right) \sin\left(dx+c\right)}{15 \left(a^{3} d \cos\left(dx+c\right)^{3}+3 a^{3} d \cos\left(dx+c\right)^{2}+3 a^{3} d \cos\left(dx+c\right)+a^{3} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(7*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 5.31673, size = 68, normalized size = 0.82

$$\begin{cases} \frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^{3}d} - \frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{3}d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^{3}d} & \text{for } d \neq 0 \\ \frac{x\cos^{2}(c)}{(a\cos(c)+a)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**5/(20*a**3*d) - tan(c/2 + d*x/2)**3/(6*a**3*d)
 + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**3,
True))
```

Giac [A] time = 1.21637, size = 62, normalized size = 0.75

$$\frac{3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}-10 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{60 a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 - 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

3.67 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{\sin(c+dx)}{5d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\sin(c+dx)}{5ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] -Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + Sin[c + d*x]/(5*a*d*(a + a*Cos
[c + d*x])^2) + Sin[c + d*x]/(5*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.0581915, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2750, 2650, 2648}

$$\frac{\sin(c+dx)}{5d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\sin(c+dx)}{5ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + Sin[c + d*x]/(5*a*d*(a + a*Cos
[c + d*x])^2) + Sin[c + d*x]/(5*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{3\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{5a}\\
&= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}+\frac{\int \frac{1}{a+a\cos(c+dx)}\,dx}{5a^2}\\
&= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}+\frac{\sin(c+dx)}{5d\left(a^3+a^3\cos(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.136478, size = 71, normalized size = 0.86

$$\frac{\sec\left(\frac{c}{2}\right)\left(-5\sin\left(c+\frac{dx}{2}\right)+5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)+5\sin\left(\frac{dx}{2}\right)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)}{80a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(5*Sin[(d*x)/2] - 5*Sin[c + (d*x)/2] + 5*Sin[c
 + (3*d*x)/2] + Sin[2*c + (5*d*x)/2]))/(80*a^3*d)
```

Maple [A] time = 0.036, size = 32, normalized size = 0.4

$$\frac{1}{4\,da^3}\left(-\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.14184, size = 63, normalized size = 0.76

$$\frac{\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{20\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(5*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)
^5)/(a^3*d)
```

Fricas [A] time = 1.51877, size = 182, normalized size = 2.19

$$\frac{\left(\cos\left(dx+c\right)^2+3\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{5\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sin(d*x + c)/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 3.41239, size = 48, normalized size = 0.58

$$\begin{cases} -\frac{\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^3d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d} & \text{for } d \neq 0 \\ \frac{x\cos(c)}{(a\cos(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)/(4*a**3*d),
Ne(d, 0)), (x*cos(c)/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.14257, size = 42, normalized size = 0.51

$$-\frac{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 - 5\,\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{20\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c))/(a^3*d)
```

3.68 $\int \frac{1}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$\frac{2\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{2\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + (2*Sin[c + d*x])/(15*a*d*(a + a
*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.0463566, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$\frac{2\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}+\frac{2\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-3),x]
```

```
[Out] Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) + (2*Sin[c + d*x])/(15*a*d*(a + a
*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))^3}\,dx &= \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{2\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{5a}\\ &= \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{2\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{2\int \frac{1}{a+a\cos(c+dx)}\,dx}{15a^2}\\ &= \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{2\sin(c+dx)}{15ad(a+a\cos(c+dx))^2}+\frac{2\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)}\end{aligned}$$

Mathematica [A] time = 0.0771736, size = 65, normalized size = 0.78

$$\frac{\left(10\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{15a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-3),x]
```

```
[Out] (Cos[(c + d*x)/2]*(10*Sin[(c + d*x)/2] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*(c
 + d*x))/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.034, size = 45, normalized size = 0.5

$$\frac{1}{4\,da^{3}}\left(\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}+\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5+2/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/
2*c))
```

Maxima [A] time = 1.16851, size = 90, normalized size = 1.08

$$\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3*d)
```

Fricas [A] time = 1.51177, size = 186, normalized size = 2.24

$$\frac{\left(2\,\cos\left(dx+c\right)^{2}+6\,\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)}{15\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(d*x + c)^2 + 6*cos(d*x + c) + 7)*sin(d*x + c)/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 2.38436, size = 63, normalized size = 0.76

$$\begin{cases}\frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{20a^{3}d}+\frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^{3}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^{3}d} & \text{for}\ d\neq 0\\ \frac{x}{(a\cos(c)+a)^{3}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**5/(20*a**3*d) + tan(c/2 + d*x/2)**3/(6*a**3*d)
 + tan(c/2 + d*x/2)/(4*a**3*d), Ne(d, 0)), (x/(a*cos(c) + a)**3, True))
```

Giac [A] time = 1.14434, size = 62, normalized size = 0.75

$$\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(3*tan(1/2*d*x + 1/2*c)^5 + 10*tan(1/2*d*x + 1/2*c)^3 + 15*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

3.69 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{22\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^3*d) - Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) -
 (7*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (22*Sin[c + d*x])/(15*d
*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.201109, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2766, 2978, 12, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{22\sin(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7\sin(c+dx)}{15ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^3*d) - Sin[c + d*x]/(5*d*(a + a*Cos[c + d*x])^3) -
 (7*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (22*Sin[c + d*x])/(15*d
*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(5a-2a\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{(15a^2-7a^2\cos(c+dx))\sec(c+dx)}{a+a\cos(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{22\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)} + \frac{\int 15a^3 \mathrm{se}}{1}$$

$$= -\frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{22\sin(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)} + \frac{\int \sec(c-}{a^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{7\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{22\sin(c+}{15d\left(a^3+a^3\mathrm{co}\right.}$$

Mathematica [B] time = 0.466445, size = 201, normalized size = 2.07

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(14\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+3\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+3\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+60\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\mathrm{lo}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(60*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Sec[c/2]*Sin[(d*
x)/2] + 14*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + 88*Cos[(c + d*x)/2]^4
*Sec[c/2]*Sin[(d*x)/2] + 3*Cos[(c + d*x)/2]*Tan[c/2] + 14*Cos[(c + d*x)/2]^
3*Tan[c/2]))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.059, size = 96, normalized size = 1.

$$-\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{1}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{7}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{da^3}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + \frac{1}{da^3}\ln\left(\tan\left(\frac{c}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-1/3/d/a^3*tan(1/2*d*x+1/2*c)^3-7/4/d/a^3*t
an(1/2*d*x+1/2*c)-1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tan(1/2*d*x+1
/2*c)+1)
```

Maxima [A] time = 1.19147, size = 161, normalized size = 1.66

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3} - \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3} + \frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/a^3)/d
```

Fricas [A] time = 1.66999, size = 424, normalized size = 4.37

$$\frac{15\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+\cdots\right.}{30\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*log(sin(d
*x + c) + 1) - 15*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
log(-sin(d*x + c) + 1) - 2*(22*cos(d*x + c)^2 + 51*cos(d*x + c) + 32)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sec\left(c+dx\right)}{\cos^{3}\left(c+dx\right)+3\cos^{2}\left(c+dx\right)+3\cos\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 1.40935, size = 127, normalized size = 1.31

$$\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{3}}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^{3}}-\frac{3\,a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+20\,a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+105\,a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^3 - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 20*a^12*tan(1/2*d*x + 1
/2*c)^3 + 105*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.70 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=112

$$\frac{24\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3\tan(c+dx)}{5ad(a\cos(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + (24*Tan[c + d*x])/(5*a^3*d) - Tan[c +
d*x]/(5*d*(a + a*Cos[c + d*x])^3) - (3*Tan[c + d*x])/(5*a*d*(a + a*Cos[c +
d*x])^2) - (3*Tan[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.278902, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{24\tan(c+dx)}{5a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{3\tan(c+dx)}{5ad(a\cos(c+dx)+a)^2}-\frac{\tan(c+dx)}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + (24*Tan[c + d*x])/(5*a^3*d) - Tan[c +
d*x]/(5*d*(a + a*Cos[c + d*x])^3) - (3*Tan[c + d*x])/(5*a*d*(a + a*Cos[c +
d*x])^2) - (3*Tan[c + d*x])/(d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{(6a-3a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2}+\frac{\int \frac{\left(27a^2-18a^2\cos(c+dx)\right)\sec^2(c+dx)}{a+a\cos(c+dx)}\,dx}{15a^4}$$

$$= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2}-\frac{3\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx)\right)}+\frac{\int \left(72a^3 \right.}{}$$

$$= -\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2}-\frac{3\tan(c+dx)}{d\left(a^3+a^3\cos(c+dx)\right)}-\frac{3\int \sec(c}{a}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{3\tan(c+dx)}{5ad(a+a\cos(c+dx))^2}-\frac{3\tan}{d\left(a^3+a^3\right.}$$

$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{24\tan(c+dx)}{5a^3d}-\frac{\tan(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{3\tan(c+dx)}{5ad(a+a\cos(c+}$$

Mathematica [B] time = 1.11605, size = 286, normalized size = 2.55

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(8\tan\left(\frac{c}{2}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\tan\left(\frac{c}{2}\right)\cos\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+20\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\frac{}{(\cos(}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(Sec[c/2]*Sin[(d*x)/2] + 8*Cos[(c + d*x)/2]^2*Sec[c/2]*
Sin[(d*x)/2] + 76*Cos[(c + d*x)/2]^4*Sec[c/2]*Sin[(d*x)/2] + 20*Cos[(c + d*
x)/2]^5*(3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]))) + Cos[(c + d*x)/2]*Tan[c/2] + 8*Cos[(c + d*x)/2]^3*Tan[c/2]))/(5*a^3*
d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.067, size = 139, normalized size = 1.2

$$\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{1}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{17}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+3\,\frac{\ln(\tan(1/2\,dx}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/20/d/a^3*tan(1/2*d*x+1/2*c)^5+1/2/d/a^3*tan(1/2*d*x+1/2*c)^3+17/4/d/a^3*t
an(1/2*d*x+1/2*c)-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d/a^3*ln(tan(1/2*d*x+1/2
*c)-1)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)-3/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.16286, size = 223, normalized size = 1.99

$$\frac{\frac{40\sin(dx+c)}{\left(a^3-\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin
(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) +
 1) - 1)/a^3)/d
```

Fricas [A] time = 1.69788, size = 506, normalized size = 4.52

$$-\frac{15\left(\cos(dx+c)^4+3\,\cos(dx+c)^3+3\,\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-15\left(\cos(dx+c)^4+3\,\cos(d\right.}{10\left(a^3d\cos(dx+c)^4+3\,a^3d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/10*(15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + cos(d*x +
 c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(
d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(24*cos(d*x + c)^3 +
57*cos(d*x + c)^2 + 39*cos(d*x + c) + 5)*sin(d*x + c))/(a^3*d*cos(d*x + c)^
4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^2(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.22283, size = 165, normalized size = 1.47

$$-\frac{\frac{60 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^3}-\frac{60 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)}{a^3}+\frac{40 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-1\right)a^3}-\frac{a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+10 a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+85 a^{12} \tan\left(\frac{1}{2} dx\right.}{a^{15}}}{20 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(60*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^3 + 40*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a
^3) - (a^12*tan(1/2*d*x + 1/2*c)^5 + 10*a^12*tan(1/2*d*x + 1/2*c)^3 + 85*a^
12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.71 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=156

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{76\tan(c+dx)\sec(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{11\tan(c+dx)\,\text{:}}{15ad(a\cos(c+}$$

```
[Out] (13*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (152*Tan[c + d*x])/(15*a^3*d) + (13*
Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - (Sec[c + d*x]*Tan[c + d*x])/(5*d*(a
+ a*Cos[c + d*x])^3) - (11*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c
+ d*x])^2) - (76*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.304536, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{152\tan(c+dx)}{15a^3d}+\frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{13\tan(c+dx)\sec(c+dx)}{2a^3d}-\frac{76\tan(c+dx)\sec(c+dx)}{15d\left(a^3\cos(c+dx)+a^3\right)}-\frac{11\tan(c+dx)\,\text{:}}{15ad(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] (13*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (152*Tan[c + d*x])/(15*a^3*d) + (13*
Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) - (Sec[c + d*x]*Tan[c + d*x])/(5*d*(a
+ a*Cos[c + d*x])^3) - (11*Sec[c + d*x]*Tan[c + d*x])/(15*a*d*(a + a*Cos[c
+ d*x])^2) - (76*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{(7a-4a\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{\left(43a^2-33a^2\cos(c+dx)\right)\sec^3(c+dx)}{a+a\cos(c+dx)}\,d}{15a^4}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{76\sec(c+dx)\tan(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)} + \int$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{76\sec(c+dx)\tan(c+dx)}{15d\left(a^3+a^3\cos(c+dx)\right)} - 15$$

$$= \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec(c+dx)\tan(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{11\sec(c+dx)\tan(c+dx)}{15ad(a+a\cos(c+dx))^2} - 76$$

$$= \frac{13\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{152\tan(c+dx)}{15a^3d} + \frac{13\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{\sec(c+dx)\tan}{5d(a+a\cos(}$$

Mathematica [B] time = 3.83344, size = 343, normalized size = 2.2

$$-\frac{24960\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(24960*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Se
c[c + d*x]^2*(-1235*Sin[(d*x)/2] + 3805*Sin[(3*d*x)/2] - 4329*Sin[c - (d*x)
```

```
/2] + 1989*Sin[c + (d*x)/2] - 3575*Sin[2*c + (d*x)/2] - 475*Sin[c + (3*d*x)
/2] + 2005*Sin[2*c + (3*d*x)/2] - 2275*Sin[3*c + (3*d*x)/2] + 2673*Sin[c +
(5*d*x)/2] + 105*Sin[2*c + (5*d*x)/2] + 1593*Sin[3*c + (5*d*x)/2] - 975*Sin
[4*c + (5*d*x)/2] + 1325*Sin[2*c + (7*d*x)/2] + 255*Sin[3*c + (7*d*x)/2] +
875*Sin[4*c + (7*d*x)/2] - 195*Sin[5*c + (7*d*x)/2] + 304*Sin[3*c + (9*d*x)
/2] + 90*Sin[4*c + (9*d*x)/2] + 214*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 +
Cos[c + d*x])^3)
```

Maple [A] time = 0.077, size = 181, normalized size = 1.2

$$-\frac{1}{20\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31}{4\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{7}{2\,da^3}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20/d/a^3*tan(1/2*d*x+1/2*c)^5-2/3/d/a^3*tan(1/2*d*x+1/2*c)^3-31/4/d/a^3*
tan(1/2*d*x+1/2*c)+1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2+7/2/d/a^3/(tan(1/2*d*
x+1/2*c)-1)-13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^3/(tan(1/2*d*x+1/2*
c)+1)^2+7/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)+13/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1
)
```

Maxima [A] time = 1.3512, size = 285, normalized size = 1.83

$$-\frac{\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3-\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}+\frac{390\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a
^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x + c
)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [A] time = 1.72531, size = 548, normalized size = 3.51

$$\frac{195\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-195\left(\cos(dx+c)^5+3\cos\right.}{60\left(a^3d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + cos(d*x +
 c)^2)*log(sin(d*x + c) + 1) - 195*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*c
```

```
os(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(304*cos(d*x + c
)^4 + 717*cos(d*x + c)^3 + 479*cos(d*x + c)^2 + 45*cos(d*x + c) - 15)*sin(d
*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x +
 c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sec^3(c+dx)}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.3276, size = 188, normalized size = 1.21

$$\frac{\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{390\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{60\left(7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}-\frac{3a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+40a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 390*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3 + 60*(7*tan(1/2*d*x + 1/2*c)^3 - 5*tan(1/2*d*x + 1/2*c))
/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*a^12*tan(1/2*d*x + 1/2*c)^5 + 40
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.72 $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4} dx$

Optimal. Leaf size=184

$$-\frac{576\sin(c+dx)}{35a^4d}-\frac{43\sin(c+dx)\cos^3(c+dx)}{35a^4d(\cos(c+dx)+1)^2}-\frac{288\sin(c+dx)\cos^2(c+dx)}{35a^4d(\cos(c+dx)+1)}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}+\frac{21x}{2a^4}-\frac{\sin}{7d}$$

```
[Out] (21*x)/(2*a^4) - (576*Sin[c + d*x])/(35*a^4*d) + (21*Cos[c + d*x]*Sin[c + d
*x])/(2*a^4*d) - (43*Cos[c + d*x]^3*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*
x])^2) - (288*Cos[c + d*x]^2*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])) -
(Cos[c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*Cos[c + d*x
]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.381852, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2765, 2977, 2734}

$$-\frac{576\sin(c+dx)}{35a^4d}-\frac{43\sin(c+dx)\cos^3(c+dx)}{35a^4d(\cos(c+dx)+1)^2}-\frac{288\sin(c+dx)\cos^2(c+dx)}{35a^4d(\cos(c+dx)+1)}+\frac{21\sin(c+dx)\cos(c+dx)}{2a^4d}+\frac{21x}{2a^4}-\frac{\sin}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (21*x)/(2*a^4) - (576*Sin[c + d*x])/(35*a^4*d) + (21*Cos[c + d*x]*Sin[c + d
*x])/(2*a^4*d) - (43*Cos[c + d*x]^3*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*
x])^2) - (288*Cos[c + d*x]^2*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])) -
(Cos[c + d*x]^5*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (2*Cos[c + d*x
]^4*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
```

```
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^4(c+dx)(5a-9a\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^3(c+dx)\left(56a^2-73a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,d}{35a^4}$$

$$= -\frac{43\cos^3(c+dx)\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3} -$$

$$= -\frac{43\cos^3(c+dx)\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^5(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^4(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^3} - 2$$

$$= \frac{21x}{2a^4} - \frac{576\sin(c+dx)}{35a^4d} + \frac{21\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{43\cos^3(c+dx)\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^5}{7d(}$$

Mathematica [A] time = 0.539259, size = 289, normalized size = 1.57

$$\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(128730\sin\left(c+\frac{dx}{2}\right)-140826\sin\left(c+\frac{3dx}{2}\right)+44310\sin\left(2c+\frac{3dx}{2}\right)-60487\sin\left(2c+\frac{5dx}{2}\right)+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(102900*d*x*Cos[(d*x)/2] + 102900*d*x*Cos[c +
(d*x)/2] + 61740*d*x*Cos[c + (3*d*x)/2] + 61740*d*x*Cos[2*c + (3*d*x)/2] +
20580*d*x*Cos[2*c + (5*d*x)/2] + 20580*d*x*Cos[3*c + (5*d*x)/2] + 2940*d*x*
Cos[3*c + (7*d*x)/2] + 2940*d*x*Cos[4*c + (7*d*x)/2] - 179830*Sin[(d*x)/2]
+ 128730*Sin[c + (d*x)/2] - 140826*Sin[c + (3*d*x)/2] + 44310*Sin[2*c + (3*
d*x)/2] - 60487*Sin[2*c + (5*d*x)/2] + 1225*Sin[3*c + (5*d*x)/2] - 12001*Si
n[3*c + (7*d*x)/2] - 3185*Sin[4*c + (7*d*x)/2] - 315*Sin[4*c + (9*d*x)/2] -
 315*Sin[5*c + (9*d*x)/2] + 35*Sin[5*c + (11*d*x)/2] + 35*Sin[6*c + (11*d*x
)/2]))/(35840*a^4*d)
```

Maple [A] time = 0.046, size = 160, normalized size = 0.9

$$\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{9}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{13}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{111}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 9\,\frac{(\tan(1/}{da^4\left(1+(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5+13/8/d/a^4*
tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*tan(1/2*d*x+1/2*c)-9/d/a^4/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-7/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2
*d*x+1/2*c)+21/d/a^4*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.75057, size = 275, normalized size = 1.49

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) - 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a^4)/d
```

Fricas [A] time = 1.76336, size = 470, normalized size = 2.55

$$\frac{735\,dx\cos(dx+c)^4+2940\,dx\cos(dx+c)^3+4410\,dx\cos(dx+c)^2+2940\,dx\cos(dx+c)+735\,dx+\left(35\cos(dx+c)^5\right.}{70\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/70*(735*d*x*cos(d*x + c)^4 + 2940*d*x*cos(d*x + c)^3 + 4410*d*x*cos(d*x +
 c)^2 + 2940*d*x*cos(d*x + c) + 735*d*x + (35*cos(d*x + c)^5 - 140*cos(d*x
+ c)^4 - 2012*cos(d*x + c)^3 - 4548*cos(d*x + c)^2 - 3873*cos(d*x + c) - 11
52)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*
cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27903, size = 173, normalized size = 0.94

$$\frac{\frac{2940\,(dx+c)}{a^4}-\frac{280\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4}+\frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-63\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+455\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3885\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^{28}}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*(d*x + c)/a^4 - 280*(9*tan(1/2*d*x + 1/2*c)^3 + 7*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (5*a^24*tan(1/2*d*x + 1/2*c
)^7 - 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 - 38
85*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.73 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=150

$$\frac{244\sin(c+dx)}{105a^4d} - \frac{88\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2} + \frac{4\sin(c+dx)}{a^4d(\cos(c+dx)+1)} - \frac{4x}{a^4} - \frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4} - \frac{12\sin(c+dx)\cos}{35ad(a\cos(c+}$$

```
[Out] (-4*x)/a^4 + (244*Sin[c + d*x])/(105*a^4*d) - (88*Cos[c + d*x]^2*Sin[c + d*
x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*Sin[c + d*x])/(a^4*d*(1 + Cos[c +
 d*x])) - (Cos[c + d*x]^4*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (12*
Cos[c + d*x]^3*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.372574, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2765, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{244\sin(c+dx)}{105a^4d} - \frac{88\sin(c+dx)\cos^2(c+dx)}{105a^4d(\cos(c+dx)+1)^2} + \frac{4\sin(c+dx)}{a^4d(\cos(c+dx)+1)} - \frac{4x}{a^4} - \frac{\sin(c+dx)\cos^4(c+dx)}{7d(a\cos(c+dx)+a)^4} - \frac{12\sin(c+dx)\cos}{35ad(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-4*x)/a^4 + (244*Sin[c + d*x])/(105*a^4*d) - (88*Cos[c + d*x]^2*Sin[c + d*
x])/(105*a^4*d*(1 + Cos[c + d*x])^2) + (4*Sin[c + d*x])/(a^4*d*(1 + Cos[c +
 d*x])) - (Cos[c + d*x]^4*Sin[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4) - (12*
Cos[c + d*x]^3*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^3(c+dx)(4a-8a\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(36a^2-52a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,d}{35a^4}$$

$$= -\frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} -$$

$$= -\frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+dx)\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} -$$

$$= \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+d}{35ad(a+a}$$

$$= \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cos^3(c+d}{35ad(a+a}$$

$$= -\frac{4x}{a^4} + \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\,\mathrm{co}}{35a}$$

$$= -\frac{4x}{a^4} + \frac{244\sin(c+dx)}{105a^4d} - \frac{88\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\cos^4(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\,\mathrm{co}}{35a}$$

Mathematica [A] time = 0.399254, size = 263, normalized size = 1.75

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(46130\sin\left(c+\frac{dx}{2}\right)-46116\sin\left(c+\frac{3dx}{2}\right)+18060\sin\left(2c+\frac{3dx}{2}\right)-19292\sin\left(2c+\frac{5dx}{2}\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^7*(29400*d*x*Cos[(d*x)/2] + 29400*d*x*Cos[c + (
d*x)/2] + 17640*d*x*Cos[c + (3*d*x)/2] + 17640*d*x*Cos[2*c + (3*d*x)/2] + 5
880*d*x*Cos[2*c + (5*d*x)/2] + 5880*d*x*Cos[3*c + (5*d*x)/2] + 840*d*x*Cos[
3*c + (7*d*x)/2] + 840*d*x*Cos[4*c + (7*d*x)/2] - 60830*Sin[(d*x)/2] + 4613
0*Sin[c + (d*x)/2] - 46116*Sin[c + (3*d*x)/2] + 18060*Sin[2*c + (3*d*x)/2]
- 19292*Sin[2*c + (5*d*x)/2] + 2100*Sin[3*c + (5*d*x)/2] - 3791*Sin[3*c + (
7*d*x)/2] - 735*Sin[4*c + (7*d*x)/2] - 105*Sin[4*c + (9*d*x)/2] - 105*Sin[5
*c + (9*d*x)/2]))/(26880*a^4*d)
```

Maple [A] time = 0.046, size = 126, normalized size = 0.8

$$-\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{23}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{49}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\tan(1/}{da^4\left(1+(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5-23/24/d/a^
4*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*tan(1/2*d*x+1/2*c)+2/d/a^4*tan(1/2*d*x+1/
2*c)/(1+tan(1/2*d*x+1/2*c)^2)-8/d/a^4*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.75742, size = 213, normalized size = 1.42

$$\frac{\frac{1680\,\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4)/d
```

Fricas [A] time = 1.71374, size = 444, normalized size = 2.96

$$-\frac{420\,dx\cos(dx+c)^4+1680\,dx\cos(dx+c)^3+2520\,dx\cos(dx+c)^2+1680\,dx\cos(dx+c)+420\,dx-\left(105\,\cos(dx+c\right.}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(420*d*x*cos(d*x + c)^4 + 1680*d*x*cos(d*x + c)^3 + 2520*d*x*cos(d*x
 + c)^2 + 1680*d*x*cos(d*x + c) + 420*d*x - (105*cos(d*x + c)^4 + 1184*cos(
d*x + c)^3 + 2636*cos(d*x + c)^2 + 2236*cos(d*x + c) + 664)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*
a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43507, size = 151, normalized size = 1.01

$$-\frac{\frac{3360\,(dx+c)}{a^4}-\frac{1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^4}+\frac{15\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-147\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5145\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*(d*x + c)/a^4 - 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a^4) + (15*a^24*tan(1/2*d*x + 1/2*c)^7 - 147*a^24*tan(1/2*d*x + 1
/2*c)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 - 5145*a^24*tan(1/2*d*x + 1/2*c))
/a^28)/d
```

3.74 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=127

$$-\frac{43\sin(c+dx)}{21a^4d(\cos(c+dx)+1)}+\frac{11\sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2}+\frac{x}{a^4}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{2\sin(c+dx)\cos^2(c+dx)}{7ad(a\cos(c+dx)+a)^3}$$

```
[Out] x/a^4 + (11*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*x])^2) - (43*Sin[c + d*x
])/(21*a^4*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^3*Sin[c + d*x])/(7*d*(a +
a*Cos[c + d*x])^4) - (2*Cos[c + d*x]^2*Sin[c + d*x])/(7*a*d*(a + a*Cos[c +
d*x])^3)
```

Rubi [A] time = 0.282547, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2968, 3019, 2735, 2648}

$$-\frac{43\sin(c+dx)}{21a^4d(\cos(c+dx)+1)}+\frac{11\sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2}+\frac{x}{a^4}-\frac{\sin(c+dx)\cos^3(c+dx)}{7d(a\cos(c+dx)+a)^4}-\frac{2\sin(c+dx)\cos^2(c+dx)}{7ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] x/a^4 + (11*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*x])^2) - (43*Sin[c + d*x
])/(21*a^4*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^3*Sin[c + d*x])/(7*d*(a +
a*Cos[c + d*x])^4) - (2*Cos[c + d*x]^2*Sin[c + d*x])/(7*a*d*(a + a*Cos[c +
d*x])^3)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
```

```
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)(3a-7a\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(20a^2-35a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} - \frac{\int \frac{20a^2\cos(c+dx)-35a^2\cos^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{35a^4}$$

$$= \frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} + \int -11$$

$$= \frac{x}{a^4} + \frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} -$$

$$= \frac{x}{a^4} + \frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\cos^3(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\cos^2(c+dx)\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} -$$

Mathematica [A] time = 0.333164, size = 224, normalized size = 1.76

$$\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(1652\sin\left(c+\frac{dx}{2}\right) - 1428\sin\left(c+\frac{3dx}{2}\right) + 756\sin\left(2c+\frac{3dx}{2}\right) - 560\sin\left(2c+\frac{5dx}{2}\right) + 168\sin\left(3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(735*d*x*Cos[(d*x)/2] + 735*d*x*Cos[c + (d*x)/
2] + 441*d*x*Cos[c + (3*d*x)/2] + 441*d*x*Cos[2*c + (3*d*x)/2] + 147*d*x*Co
s[2*c + (5*d*x)/2] + 147*d*x*Cos[3*c + (5*d*x)/2] + 21*d*x*Cos[3*c + (7*d*x
)/2] + 21*d*x*Cos[4*c + (7*d*x)/2] - 1988*Sin[(d*x)/2] + 1652*Sin[c + (d*x)
```

```
/2] - 1428*Sin[c + (3*d*x)/2] + 756*Sin[2*c + (3*d*x)/2] - 560*Sin[2*c + (5
*d*x)/2] + 168*Sin[3*c + (5*d*x)/2] - 104*Sin[3*c + (7*d*x)/2]))/(2688*a^4*
d)
```

Maple [A] time = 0.046, size = 94, normalized size = 0.7

$$\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{1}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{11}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{15}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{\arctan(\tan(1/2}{da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5+11/24/d/a^4*
tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*tan(1/2*d*x+1/2*c)+2/d/a^4*arctan(tan(1/2*d
*x+1/2*c))
```

Maxima [A] time = 1.71711, size = 151, normalized size = 1.19

$$-\frac{\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d
```

Fricas [A] time = 1.58009, size = 397, normalized size = 3.13

$$\frac{21\,dx\cos(dx+c)^4+84\,dx\cos(dx+c)^3+126\,dx\cos(dx+c)^2+84\,dx\cos(dx+c)+21\,dx-\left(52\,\cos(dx+c)^3+124\,\cos\right.}{21\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/21*(21*d*x*cos(d*x + c)^4 + 84*d*x*cos(d*x + c)^3 + 126*d*x*cos(d*x + c)^
2 + 84*d*x*cos(d*x + c) + 21*d*x - (52*cos(d*x + c)^3 + 124*cos(d*x + c)^2
+ 107*cos(d*x + c) + 32)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(
d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46203, size = 112, normalized size = 0.88

$$\frac{\frac{168\,(dx+c)}{a^4}+\frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-315\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*(d*x + c)/a^4 + (3*a^24*tan(1/2*d*x + 1/2*c)^7 - 21*a^24*tan(1/2
*d*x + 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 - 315*a^24*tan(1/2*d*x + 1
/2*c))/a^28)/d
```

3.75 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4} dx$

Optimal. Leaf size=114

$$\frac{12\sin(c+dx)}{35a^4d(\cos(c+dx)+1)} - \frac{18\sin(c+dx)}{35a^4d(\cos(c+dx)+1)^2} - \frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4} + \frac{8\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

```
[Out] (-18*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])^2) + (12*Sin[c + d*x])/(35*
a^4*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^2*Sin[c + d*x])/(7*d*(a + a*Cos[c
 + d*x])^4) + (8*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.19936, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2765, 2968, 3019, 2750, 2648}

$$\frac{12\sin(c+dx)}{35a^4d(\cos(c+dx)+1)} - \frac{18\sin(c+dx)}{35a^4d(\cos(c+dx)+1)^2} - \frac{\sin(c+dx)\cos^2(c+dx)}{7d(a\cos(c+dx)+a)^4} + \frac{8\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-18*Sin[c + d*x])/(35*a^4*d*(1 + Cos[c + d*x])^2) + (12*Sin[c + d*x])/(35*
a^4*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^2*Sin[c + d*x])/(7*d*(a + a*Cos[c
 + d*x])^4) + (8*Sin[c + d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
```

```
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos(c+dx)(2a-6a\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{\int \frac{2a\cos(c+dx)-6a\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{-24a^2+30a^2\cos(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{18\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{12\int}{}$$

$$= -\frac{18\sin(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\cos^2(c+dx)\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{8\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{}{35d}$$

Mathematica [A] time = 0.257397, size = 112, normalized size = 0.98

$$\frac{\sec\left(\frac{c}{2}\right)\left(-210\sin\left(c+\frac{dx}{2}\right)+147\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+49\sin\left(2c+\frac{5dx}{2}\right)-35\sin\left(3c+\frac{5dx}{2}\right)+12\sin\left(3\right.\right.}{2240a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(210*Sin[(d*x)/2] - 210*Sin[c + (d*x)/2] + 147
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 49*Sin[2*c + (5*d*x)/2] -
35*Sin[3*c + (5*d*x)/2] + 12*Sin[3*c + (7*d*x)/2]))/(2240*a^4*d)
```

Maple [A] time = 0.04, size = 58, normalized size = 0.5

$$\frac{1}{8da^4}\left(-\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1
/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.42387, size = 117, normalized size = 1.03

$$\frac{\frac{35 \sin(dx+c)}{\cos(dx+c)+1} - \frac{35 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{21 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

Fricas [A] time = 1.54253, size = 248, normalized size = 2.18

$$\frac{\left(12 \cos(dx+c)^3 + 13 \cos(dx+c)^2 + 8 \cos(dx+c) + 2\right) \sin(dx+c)}{35 \left(a^4 d \cos(dx+c)^4 + 4 a^4 d \cos(dx+c)^3 + 6 a^4 d \cos(dx+c)^2 + 4 a^4 d \cos(dx+c) + a^4 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(12*cos(d*x + c)^3 + 13*cos(d*x + c)^2 + 8*cos(d*x + c) + 2)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 22.5238, size = 88, normalized size = 0.77

$$\begin{cases} -\frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} - \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x\cos^3(c)}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*tan(c/2 + d*x/2)**5/(40*a**
4*d) - tan(c/2 + d*x/2)**3/(8*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d,
0)), (x*cos(c)**3/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.4228, size = 80, normalized size = 0.7

$$-\frac{5 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 21 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{280 a^4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/280*(5*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d
*x + 1/2*c)^3 - 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.76 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{13\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{13\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}-\frac{11\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) - (11*Sin[c + d*x])/(35*a*d*(a +
a*Cos[c + d*x])^3) + (13*Sin[c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) +
 (13*Sin[c + d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))
```

Rubi [A] time = 0.11202, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2758, 2750, 2650, 2648}

$$\frac{13\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{13\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}-\frac{11\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) - (11*Sin[c + d*x])/(35*a*d*(a +
a*Cos[c + d*x])^3) + (13*Sin[c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) +
 (13*Sin[c + d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx &= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{\int \frac{-4a+7a\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}\\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{11\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{13\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{35a^2}\\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{11\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{13\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{13\int \frac{}{a+}}{1}\\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}-\frac{11\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{13\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{1}{105d\left(a\right.}
\end{aligned}
$$

Mathematica [A] time = 0.248377, size = 99, normalized size = 0.88

$$
\frac{\sec\left(\frac{c}{2}\right)\left(-175\sin\left(c+\frac{dx}{2}\right)+168\sin\left(c+\frac{3dx}{2}\right)-105\sin\left(2c+\frac{3dx}{2}\right)+91\sin\left(2c+\frac{5dx}{2}\right)+13\sin\left(3c+\frac{7dx}{2}\right)+280\sin\left(\frac{dx}{2}\right)\right.}{6720a^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(280*Sin[(d*x)/2] - 175*Sin[c + (d*x)/2] + 168
*Sin[c + (3*d*x)/2] - 105*Sin[2*c + (3*d*x)/2] + 91*Sin[2*c + (5*d*x)/2] +
13*Sin[3*c + (7*d*x)/2]))/(6720*a^4*d)
```

Maple [A] time = 0.037, size = 58, normalized size = 0.5

$$
\frac{1}{8\,da^4}\left(\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*
x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.15492, size = 117, normalized size = 1.04

$$
\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(
```

```
d*x + c) + 1)^7)/(a^4*d)
```

Fricas [A] time = 1.54048, size = 251, normalized size = 2.24

$$\frac{\left(13\cos\left(dx+c\right)^{3}+52\cos\left(dx+c\right)^{2}+32\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(13*cos(d*x + c)^3 + 52*cos(d*x + c)^2 + 32*cos(d*x + c) + 8)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 10.9928, size = 87, normalized size = 0.78

$$\begin{cases} \frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}-\frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}-\frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d} & \text{for}\ d\neq 0 \\ \frac{x\cos^{2}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(40*a**4*d
) - tan(c/2 + d*x/2)**3/(24*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0)
), (x*cos(c)**2/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.3442, size = 80, normalized size = 0.71

$$\frac{15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(15*tan(1/2*d*x + 1/2*c)^7 - 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*d
*x + 1/2*c)^3 + 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.77 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4} \, dx$

Optimal. Leaf size=112

$$\frac{8\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{8\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{4\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] -Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) + (4*Sin[c + d*x])/(35*a*d*(a +
a*Cos[c + d*x])^3) + (8*Sin[c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) +
(8*Sin[c + d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))
```

Rubi [A] time = 0.0783698, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2750, 2650, 2648}

$$\frac{8\sin(c+dx)}{105d\left(a^4\cos(c+dx)+a^4\right)}+\frac{8\sin(c+dx)}{105d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{4\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] -Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) + (4*Sin[c + d*x])/(35*a*d*(a +
a*Cos[c + d*x])^3) + (8*Sin[c + d*x])/(105*d*(a^2 + a^2*Cos[c + d*x])^2) +
(8*Sin[c + d*x])/(105*d*(a^4 + a^4*Cos[c + d*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{7a}$$
$$= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{8\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{35a^2}$$
$$= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{8\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} + \frac{8\int}{}$$
$$= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{8\sin(c+dx)}{105d\left(a^2+a^2\cos(c+dx)\right)^2} + \frac{}{105}$$

Mathematica [A] time = 0.220062, size = 87, normalized size = 0.78

$$\frac{\sec\left(\frac{c}{2}\right)\left(-35\sin\left(c+\frac{dx}{2}\right)+2\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)+35\sin\left(\frac{dx}{2}\right)\right)\sec^7\left(\frac{1}{2}(c+dx)\right)}{1680a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(35*Sin[(d*x)/2] - 35*Sin[c + (d*x)/2] + 2*(21
*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*c + (7*d*x)/2])))/(168
0*a^4*d)
```

Maple [A] time = 0.037, size = 58, normalized size = 0.5

$$\frac{1}{8\,da^4}\left(-\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7-1/5*tan(1/2*d*x+1/2*c)^5+1/3*tan(1/2*d
*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.16435, size = 117, normalized size = 1.04

$$\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^4*d)
```

Fricas [A] time = 1.53257, size = 251, normalized size = 2.24

$$\frac{\left(8\cos\left(dx+c\right)^{3}+32\cos\left(dx+c\right)^{2}+52\cos\left(dx+c\right)+13\right)\sin\left(dx+c\right)}{105\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(8*cos(d*x + c)^3 + 32*cos(d*x + c)^2 + 52*cos(d*x + c) + 13)*sin(d*x
 + c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 7.78498, size = 85, normalized size = 0.76

$$\begin{cases} -\frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{4}d}-\frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{4}d}+\frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{4}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^{4}d} & \text{for}\: d\neq 0\\ \frac{x\cos\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**7/(56*a**4*d) - tan(c/2 + d*x/2)**5/(40*a**4*
d) + tan(c/2 + d*x/2)**3/(24*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0
)), (x*cos(c)/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.35611, size = 80, normalized size = 0.71

$$-\frac{15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(15*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 - 35*tan(1/2*
d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.78 $\int \frac{1}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=112

$$\frac{2\sin(c+dx)}{35d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2\sin(c+dx)}{35d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{3\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

```
[Out] Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) + (3*Sin[c + d*x])/(35*a*d*(a + a
*Cos[c + d*x])^3) + (2*Sin[c + d*x])/(35*d*(a^2 + a^2*Cos[c + d*x])^2) + (2
*Sin[c + d*x])/(35*d*(a^4 + a^4*Cos[c + d*x]))
```

Rubi [A] time = 0.0697271, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$\frac{2\sin(c+dx)}{35d\left(a^4\cos(c+dx)+a^4\right)}+\frac{2\sin(c+dx)}{35d\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{3\sin(c+dx)}{35ad(a\cos(c+dx)+a)^3}+\frac{\sin(c+dx)}{7d(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-4),x]
```

```
[Out] Sin[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) + (3*Sin[c + d*x])/(35*a*d*(a + a
*Cos[c + d*x])^3) + (2*Sin[c + d*x])/(35*d*(a^2 + a^2*Cos[c + d*x])^2) + (2
*Sin[c + d*x])/(35*d*(a^4 + a^4*Cos[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\cos(c+dx))^4}\,dx &= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{3\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{7a}\\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{3\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{6\int \frac{1}{(a+a\cos(c+dx))^2}\,dx}{35a^2}\\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{3\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{2\sin(c+dx)}{35d\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{2\int \frac{}{a}}{}\\
&= \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4}+\frac{3\sin(c+dx)}{35ad(a+a\cos(c+dx))^3}+\frac{2\sin(c+dx)}{35d\left(a^2+a^2\cos(c+dx)\right)^2}+\frac{}{35d\left(\right.}
\end{aligned}$$

Mathematica [A] time = 0.160391, size = 77, normalized size = 0.69

$$\frac{\left(35\sin\left(\frac{1}{2}(c+dx)\right)+21\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{70a^4d(\cos(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-4),x]
```

```
[Out] (Cos[(c + d*x)/2]*(35*Sin[(c + d*x)/2] + 21*Sin[(3*(c + d*x))/2] + 7*Sin[(5
*(c + d*x))/2] + Sin[(7*(c + d*x))/2]))/(70*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.035, size = 56, normalized size = 0.5

$$\frac{1}{8\,da^4}\left(\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7+3/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/
2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.14996, size = 117, normalized size = 1.04

$$\frac{\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*
x + c) + 1)^7)/(a^4*d)
```

Fricas [A] time = 1.54941, size = 248, normalized size = 2.21

$$\frac{\left(2\cos(dx+c)^3+8\cos(dx+c)^2+13\cos(dx+c)+12\right)\sin(dx+c)}{35\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 13*cos(d*x + c) + 12)*sin(d*x +
 c)/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [A] time = 5.56325, size = 83, normalized size = 0.74

$$\begin{cases} \frac{\tan^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^4d} + \frac{3\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^4d} + \frac{\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^4d} & \text{for } d \neq 0 \\ \frac{x}{(a\cos(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**7/(56*a**4*d) + 3*tan(c/2 + d*x/2)**5/(40*a**4
*d) + tan(c/2 + d*x/2)**3/(8*a**4*d) + tan(c/2 + d*x/2)/(8*a**4*d), Ne(d, 0
)), (x/(a*cos(c) + a)**4, True))
```

Giac [A] time = 1.30866, size = 80, normalized size = 0.71

$$\frac{5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{280\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(5*tan(1/2*d*x + 1/2*c)^7 + 21*tan(1/2*d*x + 1/2*c)^5 + 35*tan(1/2*d*
x + 1/2*c)^3 + 35*tan(1/2*d*x + 1/2*c))/(a^4*d)
```

3.79 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{32\sin(c+dx)}{21a^4d(\cos(c+dx)+1)}-\frac{11\sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2}-\frac{2\sin(c+dx)}{7ad(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) - (11*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*
x])^2) - (32*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*x])) - Sin[c + d*x]/(7*
d*(a + a*Cos[c + d*x])^4) - (2*Sin[c + d*x])/(7*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.286968, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2766, 2978, 12, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{32\sin(c+dx)}{21a^4d(\cos(c+dx)+1)}-\frac{11\sin(c+dx)}{21a^4d(\cos(c+dx)+1)^2}-\frac{2\sin(c+dx)}{7ad(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)}{7d(a\cos(c+dx)+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a^4*d) - (11*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*
x])^2) - (32*Sin[c + d*x])/(21*a^4*d*(1 + Cos[c + d*x])) - Sin[c + d*x]/(7*
d*(a + a*Cos[c + d*x])^4) - (2*Sin[c + d*x])/(7*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(7a-3a\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} + \frac{\int \frac{(35a^2-20a^2\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} + \int \frac{(105a^3 \cdots}{\cdots}$$

$$= -\frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} - \frac{32 \cdots}{21d\left(a^4 \cdots\right.}$$

$$= -\frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c+dx)}{7ad(a+a\cos(c+dx))^3} - \frac{32 \cdots}{21d\left(a^4 \cdots\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{11\sin(c+dx)}{21a^4d(1+\cos(c+dx))^2} - \frac{\sin(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sin(c \cdots}{7ad(a+a\,\mathrm{co} \cdots}$$

Mathematica [A] time = 0.828732, size = 185, normalized size = 1.54

$$\frac{\sec\left(\frac{c}{2}\right)\left(434\sin\left(c+\frac{dx}{2}\right) - 525\sin\left(c+\frac{3dx}{2}\right) + 147\sin\left(2c+\frac{3dx}{2}\right) - 203\sin\left(2c+\frac{5dx}{2}\right) + 21\sin\left(3c+\frac{5dx}{2}\right) - 32\sin\left(3 \cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-1344*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(-686*Sin[
(d*x)/2] + 434*Sin[c + (d*x)/2] - 525*Sin[c + (3*d*x)/2] + 147*Sin[2*c + (3
*d*x)/2] - 203*Sin[2*c + (5*d*x)/2] + 21*Sin[3*c + (5*d*x)/2] - 32*Sin[3*c
+ (7*d*x)/2]))/(84*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.059, size = 115, normalized size = 1.

$$-\frac{1}{56da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{1}{8da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{11}{24da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{15}{8da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{da^4}\ln\left(\tan\left(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-1/8/d/a^4*tan(1/2*d*x+1/2*c)^5-11/24/d/a^4
*tan(1/2*d*x+1/2*c)^3-15/8/d/a^4*tan(1/2*d*x+1/2*c)-1/d/a^4*ln(tan(1/2*d*x+
1/2*c)-1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.14041, size = 188, normalized size = 1.57

$$-\frac{\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/168*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 +
168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Fricas [A] time = 1.67843, size = 539, normalized size = 4.49

$$\frac{21\left(\cos\left(dx+c\right)^4+4\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)^2+4\,\cos\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-21\left(\cos\left(dx+c\right)^4+4\,\cos\right.}{42\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/42*(21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x
+ c) + 1)*log(sin(d*x + c) + 1) - 21*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6
*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*log(-sin(d*x + c) + 1) - 2*(32*cos(d*
x + c)^3 + 107*cos(d*x + c)^2 + 124*cos(d*x + c) + 52)*sin(d*x + c))/(a^4*d
*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d
*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39101, size = 149, normalized size = 1.24

$$\frac{\frac{168\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{168\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{3\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+315\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{a^{28}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/168*(168*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 168*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^4 - (3*a^24*tan(1/2*d*x + 1/2*c)^7 + 21*a^24*tan(1/2*d*x
+ 1/2*c)^5 + 77*a^24*tan(1/2*d*x + 1/2*c)^3 + 315*a^24*tan(1/2*d*x + 1/2*c)
)/a^28)/d
```

3.80 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=135

$$\frac{664\tan(c+dx)}{105a^4d}-\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{88\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{12\tan(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

```
[Out] (-4*ArcTanh[Sin[c + d*x]])/(a^4*d) + (664*Tan[c + d*x])/(105*a^4*d) - (88*T
an[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - (4*Tan[c + d*x])/(a^4*d*(1
+ Cos[c + d*x])) - Tan[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) - (12*Tan[c +
d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.391206, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{664\tan(c+dx)}{105a^4d}-\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{4\tan(c+dx)}{a^4d(\cos(c+dx)+1)}-\frac{88\tan(c+dx)}{105a^4d(\cos(c+dx)+1)^2}-\frac{12\tan(c+dx)}{35ad(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-4*ArcTanh[Sin[c + d*x]])/(a^4*d) + (664*Tan[c + d*x])/(105*a^4*d) - (88*T
an[c + d*x])/(105*a^4*d*(1 + Cos[c + d*x])^2) - (4*Tan[c + d*x])/(a^4*d*(1
+ Cos[c + d*x])) - Tan[c + d*x]/(7*d*(a + a*Cos[c + d*x])^4) - (12*Tan[c +
d*x])/(35*a*d*(a + a*Cos[c + d*x])^3)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(8a-4a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\left(52a^2-36a^2\cos(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} + \frac{\int (24\cdots}{\cdots}$$

$$= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{4\cdots}{d\left(a^4\cdots\right.}$$

$$= -\frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\tan(c+dx)}{35ad(a+a\cos(c+dx))^3} - \frac{4\cdots}{d\left(a^4\cdots\right.}$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{12\cdots}{35ad(a\cdots}$$

$$= -\frac{4\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{664\tan(c+dx)}{105a^4d} - \frac{88\tan(c+dx)}{105a^4d(1+\cos(c+dx))^2} - \frac{\tan(c+\cdots}{7d(a+a\cos(c\cdots}$$

Mathematica [B] time = 4.08718, size = 341, normalized size = 2.53

$$\frac{107520\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec\left(\frac{c}{2}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (107520*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*Sec[c]*Se
c[c + d*x]*(-10780*Sin[(d*x)/2] + 18788*Sin[(3*d*x)/2] - 20524*Sin[c - (d*x
)/2] + 14644*Sin[c + (d*x)/2] - 16660*Sin[2*c + (d*x)/2] - 4690*Sin[c + (3*
d*x)/2] + 14378*Sin[2*c + (3*d*x)/2] - 9100*Sin[3*c + (3*d*x)/2] + 11668*Si
n[c + (5*d*x)/2] - 630*Sin[2*c + (5*d*x)/2] + 9358*Sin[3*c + (5*d*x)/2] - 2
940*Sin[4*c + (5*d*x)/2] + 4228*Sin[2*c + (7*d*x)/2] + 315*Sin[3*c + (7*d*x
```

```
)/2] + 3493*Sin[4*c + (7*d*x)/2] - 420*Sin[5*c + (7*d*x)/2] + 664*Sin[3*c +
 (9*d*x)/2] + 105*Sin[4*c + (9*d*x)/2] + 559*Sin[5*c + (9*d*x)/2]))/(1680*a
^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.072, size = 158, normalized size = 1.2

$$\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{7}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{23}{24\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{49}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{da^4}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] 1/56/d/a^4*tan(1/2*d*x+1/2*c)^7+7/40/d/a^4*tan(1/2*d*x+1/2*c)^5+23/24/d/a^4
*tan(1/2*d*x+1/2*c)^3+49/8/d/a^4*tan(1/2*d*x+1/2*c)-1/d/a^4/(tan(1/2*d*x+1/
2*c)-1)+4/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^4/(tan(1/2*d*x+1/2*c)+1)-4/d
/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.17412, size = 251, normalized size = 1.86

$$\frac{\frac{1680\,\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/840*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(
cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x +
c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x + c
) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4)/d
```

Fricas [A] time = 1.72812, size = 632, normalized size = 4.68

$$-\frac{210\left(\cos(dx+c)^5+4\,\cos(dx+c)^4+6\,\cos(dx+c)^3+4\,\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx+c)+1)-210\left(\cos\right.}{105\left(a^4d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(210*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d
*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 210*(cos(d*x + c)^5 + 4*c
os(d*x + c)^4 + 6*cos(d*x + c)^3 + 4*cos(d*x + c)^2 + cos(d*x + c))*log(-si
n(d*x + c) + 1) - (664*cos(d*x + c)^4 + 2236*cos(d*x + c)^3 + 2636*cos(d*x
+ c)^2 + 1184*cos(d*x + c) + 105)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4*a
^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a^4
*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.53379, size = 188, normalized size = 1.39

$$-\frac{\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{3360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^4}+\frac{1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^4}-\frac{15a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+147a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+805a^{24}}{a^{28}}}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/840*(3360*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3360*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^4 + 1680*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 - 1)*a^4) - (15*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*a^24*tan(1/2*d*x + 1/2*c
)^5 + 805*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*a^24*tan(1/2*d*x + 1/2*c))/a^2
8)/d
```

3.81 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx$

Optimal. Leaf size=185

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{21\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{288\tan(c+dx)\sec(c+dx)}{35a^4d(\cos(c+dx)+1)}-\frac{43\tan(c+dx}{35a^4d(\cos(c}$$

```
[Out] (21*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (576*Tan[c + d*x])/(35*a^4*d) + (21*
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - (43*Sec[c + d*x]*Tan[c + d*x])/(35*a
^4*d*(1 + Cos[c + d*x])^2) - (288*Sec[c + d*x]*Tan[c + d*x])/(35*a^4*d*(1 +
 Cos[c + d*x])) - (Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4)
- (2*Sec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.431702, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{576\tan(c+dx)}{35a^4d}+\frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{21\tan(c+dx)\sec(c+dx)}{2a^4d}-\frac{288\tan(c+dx)\sec(c+dx)}{35a^4d(\cos(c+dx)+1)}-\frac{43\tan(c+dx}{35a^4d(\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (21*ArcTanh[Sin[c + d*x]])/(2*a^4*d) - (576*Tan[c + d*x])/(35*a^4*d) + (21*
Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) - (43*Sec[c + d*x]*Tan[c + d*x])/(35*a
^4*d*(1 + Cos[c + d*x])^2) - (288*Sec[c + d*x]*Tan[c + d*x])/(35*a^4*d*(1 +
 Cos[c + d*x])) - (Sec[c + d*x]*Tan[c + d*x])/(7*d*(a + a*Cos[c + d*x])^4)
- (2*Sec[c + d*x]*Tan[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^3)
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} + \frac{\int \frac{(9a-5a\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3} + \frac{\int \frac{\left(73a^2-56a^2\cos(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^2}\,dx}{35a^4}$$

$$= -\frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3} + \int$$

$$= -\frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{28}{3}$$

$$= -\frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\sec(c+dx)\tan(c+dx)}{5ad(a+a\cos(c+dx))^3} - \frac{28}{3}$$

$$= \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec(c+dx)\tan(c+dx)}{35a^4d(1+\cos(c+dx))^2} - \frac{\sec(c+dx)\tan(c+dx)}{7d(a+a\cos(c+dx))^4} - \frac{2\,\mathrm{s}}{5a}$$

$$= \frac{21\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{576\tan(c+dx)}{35a^4d} + \frac{21\sec(c+dx)\tan(c+dx)}{2a^4d} - \frac{43\sec(c+dx}{35a^4d(1+\mathrm{co}}$$

Mathematica [B] time = 6.24635, size = 455, normalized size = 2.46

$$-\frac{168\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^4} + \frac{168\cos^8\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^4} + \frac{\sec\left(\frac{c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (-168*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]])/(d
*(a + a*Cos[c + d*x])^4) + (168*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Se
c[c/2]*Sec[c]*Sec[c + d*x]^2*(24402*Sin[(d*x)/2] - 55556*Sin[(3*d*x)/2] + 6
1054*Sin[c - (d*x)/2] - 33614*Sin[c + (d*x)/2] + 51842*Sin[2*c + (d*x)/2] +
 12460*Sin[c + (3*d*x)/2] - 33716*Sin[2*c + (3*d*x)/2] + 34300*Sin[3*c + (3
*d*x)/2] - 39788*Sin[c + (5*d*x)/2] + 2940*Sin[2*c + (5*d*x)/2] - 26068*Sin
[3*c + (5*d*x)/2] + 16660*Sin[4*c + (5*d*x)/2] - 21351*Sin[2*c + (7*d*x)/2]
 - 1295*Sin[3*c + (7*d*x)/2] - 14911*Sin[4*c + (7*d*x)/2] + 5145*Sin[5*c +
(7*d*x)/2] - 7329*Sin[3*c + (9*d*x)/2] - 1225*Sin[4*c + (9*d*x)/2] - 5369*S
in[5*c + (9*d*x)/2] + 735*Sin[6*c + (9*d*x)/2] - 1152*Sin[4*c + (11*d*x)/2]
 - 280*Sin[5*c + (11*d*x)/2] - 872*Sin[6*c + (11*d*x)/2]))/(2240*d*(a + a*C
os[c + d*x])^4)
```

Maple [A] time = 0.083, size = 200, normalized size = 1.1

$$-\frac{1}{56\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{9}{40\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{13}{8\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{111}{8\,da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\,da^4}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^4,x)
```

```
[Out] -1/56/d/a^4*tan(1/2*d*x+1/2*c)^7-9/40/d/a^4*tan(1/2*d*x+1/2*c)^5-13/8/d/a^4
*tan(1/2*d*x+1/2*c)^3-111/8/d/a^4*tan(1/2*d*x+1/2*c)+1/2/d/a^4/(tan(1/2*d*x
+1/2*c)-1)^2+9/2/d/a^4/(tan(1/2*d*x+1/2*c)-1)-21/2/d/a^4*ln(tan(1/2*d*x+1/2
*c)-1)-1/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)^2+9/2/d/a^4/(tan(1/2*d*x+1/2*c)+1)+
21/2/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.18271, size = 312, normalized size = 1.69

$$-\frac{\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right.}{a^4}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/280*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1) + 455
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c)/(co
s(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^
4)/d
```

Fricas [A] time = 1.7083, size = 670, normalized size = 3.62

$$735\left(\cos(dx+c)^6+4\cos(dx+c)^5+6\cos(dx+c)^4+4\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-735\left(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/140*(735*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*
x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 735*(cos(d*x + c)^6 + 4*
cos(d*x + c)^5 + 6*cos(d*x + c)^4 + 4*cos(d*x + c)^3 + cos(d*x + c)^2)*log(
-sin(d*x + c) + 1) - 2*(1152*cos(d*x + c)^5 + 3873*cos(d*x + c)^4 + 4548*co
s(d*x + c)^3 + 2012*cos(d*x + c)^2 + 140*cos(d*x + c) - 35)*sin(d*x + c))/(
a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40896, size = 209, normalized size = 1.13

$$\frac{\frac{2940 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4} - \frac{2940 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4} + \frac{280\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2 a^4} - \frac{5\,a^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+63\,a^{24}\tan\left(\frac{1}{2}\,dx\right.}{}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(2940*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 2940*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/a^4 + 280*(9*tan(1/2*d*x + 1/2*c)^3 - 7*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (5*a^24*tan(1/2*d*x + 1/2*c)^7
+ 63*a^24*tan(1/2*d*x + 1/2*c)^5 + 455*a^24*tan(1/2*d*x + 1/2*c)^3 + 3885*a
^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

3.82 $\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=225

$$-\frac{7664\sin(c+dx)}{315a^5d}-\frac{28\sin(c+dx)\cos^4(c+dx)}{45a^2d(a\cos(c+dx)+a)^3}-\frac{577\sin(c+dx)\cos^3(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}-\frac{3832\sin(c+dx)\cos^2(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{31\sin}{}$$

```
[Out] (31*x)/(2*a^5) - (7664*Sin[c + d*x])/(315*a^5*d) + (31*Cos[c + d*x]*Sin[c +
 d*x])/(2*a^5*d) - (Cos[c + d*x]^6*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^
5) - (17*Cos[c + d*x]^5*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (28
*Cos[c + d*x]^4*Sin[c + d*x])/(45*a^2*d*(a + a*Cos[c + d*x])^3) - (577*Cos[
c + d*x]^3*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) - (3832*Cos[c +
 d*x]^2*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.515535, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2765, 2977, 2734}

$$-\frac{7664\sin(c+dx)}{315a^5d}-\frac{28\sin(c+dx)\cos^4(c+dx)}{45a^2d(a\cos(c+dx)+a)^3}-\frac{577\sin(c+dx)\cos^3(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}-\frac{3832\sin(c+dx)\cos^2(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{31\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (31*x)/(2*a^5) - (7664*Sin[c + d*x])/(315*a^5*d) + (31*Cos[c + d*x]*Sin[c +
 d*x])/(2*a^5*d) - (Cos[c + d*x]^6*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^
5) - (17*Cos[c + d*x]^5*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (28
*Cos[c + d*x]^4*Sin[c + d*x])/(45*a^2*d*(a + a*Cos[c + d*x])^3) - (577*Cos[
c + d*x]^3*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) - (3832*Cos[c +
 d*x]^2*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^5(c+dx)(6a-11a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^4(c+dx)\left(85a^2-111a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^3}}{63a^4}$$

$$= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\cos^4(c+dx)\sin(c+dx)}{45a^2d(a+a\cos(c+dx))^3} -$$

$$= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\cos^4(c+dx)\sin(c+dx)}{45a^2d(a+a\cos(c+dx))^3} -$$

$$= -\frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos^5(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{28\cos^4(c+dx)\sin(c+dx)}{45a^2d(a+a\cos(c+dx))^3} -$$

$$= \frac{31x}{2a^5} - \frac{7664\sin(c+dx)}{315a^5d} + \frac{31\cos(c+dx)\sin(c+dx)}{2a^5d} - \frac{\cos^6(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{17\cos}{63ad}$$

Mathematica [A] time = 0.724873, size = 345, normalized size = 1.53

$$\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(7194600\sin\left(c+\frac{dx}{2}\right) - 7472241\sin\left(c+\frac{3dx}{2}\right) + 3432975\sin\left(2c+\frac{3dx}{2}\right) - 3871989\sin\left(2c+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(4921560*d*x*Cos[(d*x)/2] + 4921560*d*x*Cos[c
+ (d*x)/2] + 3281040*d*x*Cos[c + (3*d*x)/2] + 3281040*d*x*Cos[2*c + (3*d*x)
/2] + 1406160*d*x*Cos[2*c + (5*d*x)/2] + 1406160*d*x*Cos[3*c + (5*d*x)/2] +
 351540*d*x*Cos[3*c + (7*d*x)/2] + 351540*d*x*Cos[4*c + (7*d*x)/2] + 39060*
d*x*Cos[4*c + (9*d*x)/2] + 39060*d*x*Cos[5*c + (9*d*x)/2] - 9163224*Sin[(d*
x)/2] + 7194600*Sin[c + (d*x)/2] - 7472241*Sin[c + (3*d*x)/2] + 3432975*Sin
[2*c + (3*d*x)/2] - 3871989*Sin[2*c + (5*d*x)/2] + 801675*Sin[3*c + (5*d*x)
/2] - 1186056*Sin[3*c + (7*d*x)/2] - 17640*Sin[4*c + (7*d*x)/2] - 175184*Si
n[4*c + (9*d*x)/2] - 45360*Sin[5*c + (9*d*x)/2] - 3465*Sin[5*c + (11*d*x)/2
] - 3465*Sin[6*c + (11*d*x)/2] + 315*Sin[6*c + (13*d*x)/2] + 315*Sin[7*c +
(13*d*x)/2]))/(1290240*a^5*d)
```

Maple [A] time = 0.05, size = 179, normalized size = 0.8

$$-\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9 + \frac{5}{56\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{3}{5\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 + \frac{25}{8\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{351}{16\,da^5}\,\mathrm{ta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+5/56/d/a^5*tan(1/2*d*x+1/2*c)^7-3/5/d/a^5
*tan(1/2*d*x+1/2*c)^5+25/8/d/a^5*tan(1/2*d*x+1/2*c)^3-351/16/d/a^5*tan(1/2*
d*x+1/2*c)-11/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-9/d/a^5
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+31/d/a^5*arctan(tan(1/2*d*x+
1/2*c))
```

Maxima [A] time = 1.72517, size = 302, normalized size = 1.34

$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^5+\frac{2a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}-\frac{15750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3024\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{156240\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/(a^5 + 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) + 1)
- 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9)/a^5 - 156240*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
^5)/d
```

Fricas [A] time = 1.73784, size = 591, normalized size = 2.63

$$\frac{9765\,dx\cos(dx+c)^5+48825\,dx\cos(dx+c)^4+97650\,dx\cos(dx+c)^3+97650\,dx\cos(dx+c)^2+48825\,dx\cos(dx+c}{630\left(a^5d\cos(dx+c)^5+5\,a^5d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(9765*d*x*cos(d*x + c)^5 + 48825*d*x*cos(d*x + c)^4 + 97650*d*x*cos(d
*x + c)^3 + 97650*d*x*cos(d*x + c)^2 + 48825*d*x*cos(d*x + c) + 9765*d*x +
(315*cos(d*x + c)^6 - 1575*cos(d*x + c)^5 - 28828*cos(d*x + c)^4 - 87440*co
s(d*x + c)^3 - 112119*cos(d*x + c)^2 - 66875*cos(d*x + c) - 15328)*sin(d*x
+ c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c
)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42491, size = 196, normalized size = 0.87

$$\frac{\frac{78120\,(dx+c)}{a^5}-\frac{5040\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^5}-\frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-450\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3024\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15750\,a^{40}\,\mathrm{ta}}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(78120*(d*x + c)/a^5 - 5040*(11*tan(1/2*d*x + 1/2*c)^3 + 9*tan(1/2*d
*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5) - (35*a^40*tan(1/2*d*x +
1/2*c)^9 - 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2*c)
^5 - 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*c))/
a^45)/d
```

3.83 $\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=191

$$\frac{181\sin(c+dx)}{63a^5d}-\frac{29\sin(c+dx)\cos^3(c+dx)}{63a^2d(a\cos(c+dx)+a)^3}-\frac{67\sin(c+dx)\cos^2(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}+\frac{5\sin(c+dx)}{d\left(a^5\cos(c+dx)+a^5\right)}-\frac{5x}{a^5}-\frac{\sin(c+dx)}{9d(a\cos(}$$

```
[Out] (-5*x)/a^5 + (181*Sin[c + d*x])/(63*a^5*d) - (Cos[c + d*x]^5*Sin[c + d*x])/
(9*d*(a + a*Cos[c + d*x])^5) - (5*Cos[c + d*x]^4*Sin[c + d*x])/(21*a*d*(a +
 a*Cos[c + d*x])^4) - (29*Cos[c + d*x]^3*Sin[c + d*x])/(63*a^2*d*(a + a*Cos
[c + d*x])^3) - (67*Cos[c + d*x]^2*Sin[c + d*x])/(63*a^3*d*(a + a*Cos[c + d
*x])^2) + (5*Sin[c + d*x])/(d*(a^5 + a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.490209, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2765, 2977, 2968, 3023, 12, 2735, 2648}

$$\frac{181\sin(c+dx)}{63a^5d}-\frac{29\sin(c+dx)\cos^3(c+dx)}{63a^2d(a\cos(c+dx)+a)^3}-\frac{67\sin(c+dx)\cos^2(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}+\frac{5\sin(c+dx)}{d\left(a^5\cos(c+dx)+a^5\right)}-\frac{5x}{a^5}-\frac{\sin(c+dx)}{9d(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (-5*x)/a^5 + (181*Sin[c + d*x])/(63*a^5*d) - (Cos[c + d*x]^5*Sin[c + d*x])/
(9*d*(a + a*Cos[c + d*x])^5) - (5*Cos[c + d*x]^4*Sin[c + d*x])/(21*a*d*(a +
 a*Cos[c + d*x])^4) - (29*Cos[c + d*x]^3*Sin[c + d*x])/(63*a^2*d*(a + a*Cos
[c + d*x])^3) - (67*Cos[c + d*x]^2*Sin[c + d*x])/(63*a^3*d*(a + a*Cos[c + d
*x])^2) + (5*Sin[c + d*x])/(d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^4(c+dx)(5a-10a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^3(c+dx)\left(60a^2-85a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c+dx)\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3} - \ldots$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c+dx)\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3} - \frac{67\ldots}{63\ldots}$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c+dx)\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3} - \frac{67\ldots}{63\ldots}$$

$$= \frac{181\sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c+dx)\sin\ldots}{63a^2d(a+a\cos(\ldots}$$

$$= \frac{181\sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c+dx)\sin\ldots}{63a^2d(a+a\cos(\ldots}$$

$$= -\frac{5x}{a^5} + \frac{181\sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c\ldots}{63a^2d(a+\ldots}$$

$$= -\frac{5x}{a^5} + \frac{181\sin(c+dx)}{63a^5d} - \frac{\cos^5(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{5\cos^4(c+dx)\sin(c+dx)}{21ad(a+a\cos(c+dx))^4} - \frac{29\cos^3(c\ldots}{63a^2d(a+\ldots}$$

Mathematica [A] time = 0.771424, size = 319, normalized size = 1.67

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(143010\sin\left(c+\frac{dx}{2}\right)-138726\sin\left(c+\frac{3dx}{2}\right)+73290\sin\left(2c+\frac{3dx}{2}\right)-70389\sin\left(2c+\frac{5dx}{2}\right)+2\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^9*(79380*d*x*Cos[(d*x)/2] + 79380*d*x*Cos[c + (
d*x)/2] + 52920*d*x*Cos[c + (3*d*x)/2] + 52920*d*x*Cos[2*c + (3*d*x)/2] + 2
2680*d*x*Cos[2*c + (5*d*x)/2] + 22680*d*x*Cos[3*c + (5*d*x)/2] + 5670*d*x*C
os[3*c + (7*d*x)/2] + 5670*d*x*Cos[4*c + (7*d*x)/2] + 630*d*x*Cos[4*c + (9*
d*x)/2] + 630*d*x*Cos[5*c + (9*d*x)/2] - 175014*Sin[(d*x)/2] + 143010*Sin[c
 + (d*x)/2] - 138726*Sin[c + (3*d*x)/2] + 73290*Sin[2*c + (3*d*x)/2] - 7038
9*Sin[2*c + (5*d*x)/2] + 20475*Sin[3*c + (5*d*x)/2] - 21141*Sin[3*c + (7*d*
x)/2] + 1575*Sin[4*c + (7*d*x)/2] - 3091*Sin[4*c + (9*d*x)/2] - 567*Sin[5*c
 + (9*d*x)/2] - 63*Sin[5*c + (11*d*x)/2] - 63*Sin[6*c + (11*d*x)/2]))/(6451
2*a^5*d)
```

Maple [A] time = 0.046, size = 145, normalized size = 0.8

$$\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9 - \frac{1}{14\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 + \frac{3}{8\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{3}{2\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{129}{16\,da^5}\tan\left(\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] 1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-1/14/d/a^5*tan(1/2*d*x+1/2*c)^7+3/8/d/a^5*
tan(1/2*d*x+1/2*c)^5-3/2/d/a^5*tan(1/2*d*x+1/2*c)^3+129/16/d/a^5*tan(1/2*d*
x+1/2*c)+2/d/a^5*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-10/d/a^5*arcta
n(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.65445, size = 240, normalized size = 1.26

$$\frac{\frac{2016 \sin(dx+c)}{\left(a^5+\frac{a^5 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127 \sin(dx+c)}{\cos(dx+c)+1}-\frac{1512 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{72 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{1008\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 + a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) - 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 10080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

Fricas [A] time = 1.71494, size = 541, normalized size = 2.83

$$-\frac{315\, dx \cos(dx+c)^5 + 1575\, dx \cos(dx+c)^4 + 3150\, dx \cos(dx+c)^3 + 3150\, dx \cos(dx+c)^2 + 1575\, dx \cos(dx+c) +}{63\left(a^5 d \cos(dx+c)^5 + 5\, a^5 d \cos(dx+c)^4 + 10\, a^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/63*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (63*co
s(d*x + c)^5 + 946*cos(d*x + c)^4 + 2840*cos(d*x + c)^3 + 3633*cos(d*x + c)
^2 + 2165*cos(d*x + c) + 496)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d
*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5
*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38633, size = 174, normalized size = 0.91

$$-\frac{\frac{5040\,(dx+c)}{a^5}-\frac{2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^5}-\frac{7\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-72\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+378\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1512\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8127\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*(d*x + c)/a^5 - 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2
*c)^2 + 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 - 72*a^40*tan(1/2*d*x + 1/
2*c)^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 - 1512*a^40*tan(1/2*d*x + 1/2*c)^3
 + 8127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.84 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=168

$$-\frac{34\sin(c+dx)\cos^2(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}-\frac{661\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{173\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}+\frac{x}{a^5}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

```
[Out] x/a^5 - (Cos[c + d*x]^4*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (13*Co
s[c + d*x]^3*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (34*Cos[c + d*
x]^2*Sin[c + d*x])/(105*a^2*d*(a + a*Cos[c + d*x])^3) + (173*Sin[c + d*x])/
(315*a^3*d*(a + a*Cos[c + d*x])^2) - (661*Sin[c + d*x])/(315*d*(a^5 + a^5*C
os[c + d*x]))
```

Rubi [A] time = 0.392132, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2968, 3019, 2735, 2648}

$$-\frac{34\sin(c+dx)\cos^2(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}-\frac{661\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{173\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}+\frac{x}{a^5}-\frac{\sin(c+dx)\cos^4(c+dx)}{9d(a\cos(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^5,x]
```


```
[Out] x/a^5 - (Cos[c + d*x]^4*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (13*Co
s[c + d*x]^3*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (34*Cos[c + d*
x]^2*Sin[c + d*x])/(105*a^2*d*(a + a*Cos[c + d*x])^3) + (173*Sin[c + d*x])/
(315*a^3*d*(a + a*Cos[c + d*x])^2) - (661*Sin[c + d*x])/(315*d*(a^5 + a^5*C
os[c + d*x]))
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^3(c+dx)(4a-9a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos^2(c+dx)\left(39a^2-63a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} - \int$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} - \int$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} + \frac{}{315}$$

$$= \frac{x}{a^5} - \frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} +$$

$$= \frac{x}{a^5} - \frac{\cos^4(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\cos^3(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\cos^2(c+dx)\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} +$$

Mathematica [A] time = 0.487604, size = 280, normalized size = 1.67

$$\sec\left(\frac{c}{2}\right)\sec^9\left(\frac{1}{2}(c+dx)\right)\left(100800\sin\left(c+\frac{dx}{2}\right) - 88284\sin\left(c+\frac{3dx}{2}\right) + 56700\sin\left(2c+\frac{3dx}{2}\right) - 43236\sin\left(2c+\frac{5dx}{2}\right) + 189\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(39690*d*x*Cos[(d*x)/2] + 39690*d*x*Cos[c + (d
*x)/2] + 26460*d*x*Cos[c + (3*d*x)/2] + 26460*d*x*Cos[2*c + (3*d*x)/2] + 11
340*d*x*Cos[2*c + (5*d*x)/2] + 11340*d*x*Cos[3*c + (5*d*x)/2] + 2835*d*x*Co
s[3*c + (7*d*x)/2] + 2835*d*x*Cos[4*c + (7*d*x)/2] + 315*d*x*Cos[4*c + (9*d
*x)/2] + 315*d*x*Cos[5*c + (9*d*x)/2] - 116676*Sin[(d*x)/2] + 100800*Sin[c
+ (d*x)/2] - 88284*Sin[c + (3*d*x)/2] + 56700*Sin[2*c + (3*d*x)/2] - 43236*
Sin[2*c + (5*d*x)/2] + 18900*Sin[3*c + (5*d*x)/2] - 12384*Sin[3*c + (7*d*x)
/2] + 3150*Sin[4*c + (7*d*x)/2] - 1726*Sin[4*c + (9*d*x)/2]))/(161280*a^5*d
)
```

Maple [A] time = 0.042, size = 113, normalized size = 0.7

$$-\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{3}{56\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{1}{5\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{13}{24\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{31}{16\,da^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+3/56/d/a^5*tan(1/2*d*x+1/2*c)^7-1/5/d/a^5
*tan(1/2*d*x+1/2*c)^5+13/24/d/a^5*tan(1/2*d*x+1/2*c)^3-31/16/d/a^5*tan(1/2*
d*x+1/2*c)+2/d/a^5*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.70098, size = 178, normalized size = 1.06

$$-\frac{\frac{\frac{9765\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{2730\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{270\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{10080\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) - 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 10
080*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^5)/d
```

Fricas [A] time = 1.65424, size = 514, normalized size = 3.06

$$\frac{315\,dx\cos(dx+c)^5+1575\,dx\cos(dx+c)^4+3150\,dx\cos(dx+c)^3+3150\,dx\cos(dx+c)^2+1575\,dx\cos(dx+c)+3}{315\left(a^5d\cos(dx+c)^5+5\,a^5d\cos(dx+c)^4+10\,a^5d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(315*d*x*cos(d*x + c)^5 + 1575*d*x*cos(d*x + c)^4 + 3150*d*x*cos(d*x
+ c)^3 + 3150*d*x*cos(d*x + c)^2 + 1575*d*x*cos(d*x + c) + 315*d*x - (863*c
os(d*x + c)^4 + 2740*cos(d*x + c)^3 + 3549*cos(d*x + c)^2 + 2125*cos(d*x +
c) + 488)*sin(d*x + c))/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
```

```
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**5,x)

[Out] Timed out
```

Giac [A] time = 1.33333, size = 135, normalized size = 0.8

$$\frac{\frac{5040\,(dx+c)}{a^5} - \frac{35\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 270\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 1008\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 2730\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9765\,a^{40}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{45}}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^5,x, algorithm="giac")

[Out] 1/5040*(5040*(d*x + c)/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 - 270*a^40*tan
(1/2*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 - 2730*a^40*tan(1/2*
d*x + 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.85 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5} dx$

Optimal. Leaf size=155

$$\frac{83\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{142\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}+\frac{67\sin(c+dx)}{315a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}-1$$

```
[Out] -(Cos[c + d*x]^3*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (11*Cos[c + d
*x]^2*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) + (67*Sin[c + d*x])/(31
5*a^2*d*(a + a*Cos[c + d*x])^3) - (142*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[
c + d*x])^2) + (83*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.300067, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2968, 3019, 2750, 2648}

$$\frac{83\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{142\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}+\frac{67\sin(c+dx)}{315a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^3(c+dx)}{9d(a\cos(c+dx)+a)^5}-1$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^5,x]
```


```
[Out] -(Cos[c + d*x]^3*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) - (11*Cos[c + d
*x]^2*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) + (67*Sin[c + d*x])/(31
5*a^2*d*(a + a*Cos[c + d*x])^3) - (142*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[
c + d*x])^2) + (83*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^5}\,dx &= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)(3a-8a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{\cos(c+dx)\left(22a^2-45a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^3}\,dx}{63a^4} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{\int \frac{22a^2\cos(c+dx)-45a^2\cos^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{63a^4} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx))^3} + \frac{\int \cdots}{\cdots} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx))^3} - \frac{\cdots}{315\cdots} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{11\cos^2(c+dx)\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{67\sin(c+dx)}{315a^2d(a+a\cos(c+dx))^3} - \frac{\cdots}{315\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.263809, size = 138, normalized size = 0.89

$$
\frac{\sec\left(\frac{c}{2}\right)\left(-5040\sin\left(c+\frac{dx}{2}\right)+3612\sin\left(c+\frac{3dx}{2}\right)-3360\sin\left(2c+\frac{3dx}{2}\right)+1728\sin\left(2c+\frac{5dx}{2}\right)-1260\sin\left(3c+\frac{5dx}{2}\right)+432\cdots}{80640a^5d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(5418*Sin[(d*x)/2] - 5040*Sin[c + (d*x)/2] + 3
612*Sin[c + (3*d*x)/2] - 3360*Sin[2*c + (3*d*x)/2] + 1728*Sin[2*c + (5*d*x)
```

```
/2] - 1260*Sin[3*c + (5*d*x)/2] + 432*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c +
(7*d*x)/2] + 83*Sin[4*c + (9*d*x)/2]))/(80640*a^5*d)
```

Maple [A] time = 0.039, size = 71, normalized size = 0.5

$$\frac{1}{16\,d a^5}\left(\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{4}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{6}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{4}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5-4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.13768, size = 144, normalized size = 0.93

$$\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{420\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{180\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) - 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 180*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.57359, size = 316, normalized size = 2.04

$$\frac{\left(83\,\cos\left(dx+c\right)^4+100\,\cos\left(dx+c\right)^3+84\,\cos\left(dx+c\right)^2+40\,\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{315\left(a^5 d\cos\left(dx+c\right)^5+5\,a^5 d\cos\left(dx+c\right)^4+10\,a^5 d\cos\left(dx+c\right)^3+10\,a^5 d\cos\left(dx+c\right)^2+5\,a^5 d\cos\left(dx+c\right)+a^5 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(83*cos(d*x + c)^4 + 100*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 40*cos(
d*x + c) + 8)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4085, size = 97, normalized size = 0.63

$$\frac{35 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 180 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 378 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 420 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 315 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{5040 a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 - 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 - 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

3.86 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=147

$$\frac{5\sin(c+dx)}{63d\left(a^5\cos(c+dx)+a^5\right)}+\frac{5\sin(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}-\frac{17\sin(c+dx)}{63a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}+\frac{}{7ad}$$

```
[Out] -(Cos[c + d*x]^2*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) + Sin[c + d*x]/
(7*a*d*(a + a*Cos[c + d*x])^4) - (17*Sin[c + d*x])/(63*a^2*d*(a + a*Cos[c +
 d*x])^3) + (5*Sin[c + d*x])/(63*a^3*d*(a + a*Cos[c + d*x])^2) + (5*Sin[c +
 d*x])/(63*d*(a^5 + a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.228439, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2968, 3019, 2750, 2650, 2648}

$$\frac{5\sin(c+dx)}{63d\left(a^5\cos(c+dx)+a^5\right)}+\frac{5\sin(c+dx)}{63a^3d(a\cos(c+dx)+a)^2}-\frac{17\sin(c+dx)}{63a^2d(a\cos(c+dx)+a)^3}-\frac{\sin(c+dx)\cos^2(c+dx)}{9d(a\cos(c+dx)+a)^5}+\frac{}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -(Cos[c + d*x]^2*Sin[c + d*x])/(9*d*(a + a*Cos[c + d*x])^5) + Sin[c + d*x]/
(7*a*d*(a + a*Cos[c + d*x])^4) - (17*Sin[c + d*x])/(63*a^2*d*(a + a*Cos[c +
 d*x])^3) + (5*Sin[c + d*x])/(63*a^3*d*(a + a*Cos[c + d*x])^2) + (5*Sin[c +
 d*x])/(63*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos(c+dx)(2a-7a\cos(c+dx))}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{\int \frac{2a\cos(c+dx)-7a\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4} + \frac{\int \frac{-36a^2+49a^2\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4} - \frac{17\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3} + 5\int \frac{\ }{(a+a}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4} - \frac{17\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3} + \frac{5}{63a^3d(}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\sin(c+dx)}{7ad(a+a\cos(c+dx))^4} - \frac{17\sin(c+dx)}{63a^2d(a+a\cos(c+dx))^3} + \frac{5}{63a^3d(}$$

Mathematica [A] time = 0.229102, size = 125, normalized size = 0.85

$$\frac{\sec\left(\frac{c}{2}\right)\left(-315\sin\left(c+\frac{dx}{2}\right)+273\sin\left(c+\frac{3dx}{2}\right)-147\sin\left(2c+\frac{3dx}{2}\right)+117\sin\left(2c+\frac{5dx}{2}\right)-63\sin\left(3c+\frac{5dx}{2}\right)+45\sin\left(3c\right.\right.}{16128a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(315*Sin[(d*x)/2] - 315*Sin[c + (d*x)/2] + 273
*Sin[c + (3*d*x)/2] - 147*Sin[2*c + (3*d*x)/2] + 117*Sin[2*c + (5*d*x)/2] -
 63*Sin[3*c + (5*d*x)/2] + 45*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)/2]
))/(16128*a^5*d)
```

Maple [A] time = 0.042, size = 58, normalized size = 0.4

$$\frac{1}{16\,d a^5}\left(-\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{2}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/2*d*x+1/2*c)^7-2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.13758, size = 117, normalized size = 0.8

$$\frac{\frac{63\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{42\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{18\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) - 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.52687, size = 312, normalized size = 2.12

$$\frac{\left(5\,\cos\left(dx+c\right)^4+25\,\cos\left(dx+c\right)^3+21\,\cos\left(dx+c\right)^2+10\,\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{63\left(a^5 d\cos\left(dx+c\right)^5+5\,a^5 d\cos\left(dx+c\right)^4+10\,a^5 d\cos\left(dx+c\right)^3+10\,a^5 d\cos\left(dx+c\right)^2+5\,a^5 d\cos\left(dx+c\right)+a^5 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(5*cos(d*x + c)^4 + 25*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32374, size = 80, normalized size = 0.54

$$-\frac{7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 - 18 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 42 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 63 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{1008 a^5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^7 + 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

3.87 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=139

$$\frac{2\sin(c+dx)}{45d\left(a^5\cos(c+dx)+a^5\right)}+\frac{2\sin(c+dx)}{45a^3d(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{15a^2d(a\cos(c+dx)+a)^3}-\frac{2\sin(c+dx)}{9ad(a\cos(c+dx)+a)^4}+\frac{}{9d(a}$$

```
[Out] Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) - (2*Sin[c + d*x])/(9*a*d*(a + a*
Cos[c + d*x])^4) + Sin[c + d*x]/(15*a^2*d*(a + a*Cos[c + d*x])^3) + (2*Sin[
c + d*x])/(45*a^3*d*(a + a*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(45*d*(a^5 +
 a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.144441, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2758, 2750, 2650, 2648}

$$\frac{2\sin(c+dx)}{45d\left(a^5\cos(c+dx)+a^5\right)}+\frac{2\sin(c+dx)}{45a^3d(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)}{15a^2d(a\cos(c+dx)+a)^3}-\frac{2\sin(c+dx)}{9ad(a\cos(c+dx)+a)^4}+\frac{}{9d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) - (2*Sin[c + d*x])/(9*a*d*(a + a*
Cos[c + d*x])^4) + Sin[c + d*x]/(15*a^2*d*(a + a*Cos[c + d*x])^3) + (2*Sin[
c + d*x])/(45*a^3*d*(a + a*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(45*d*(a^5 +
 a^5*Cos[c + d*x]))
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx = \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\int \frac{-5a+9a\cos(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{3a^2}$$

$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3} + \frac{2\int \frac{1}{(a+a\cos}}{15}$$

$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3} + \frac{2\sin}{45a^3d(a+}$$

$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{2\sin(c+dx)}{9ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{15a^2d(a+a\cos(c+dx))^3} + \frac{2\sin}{45a^3d(a+}$$

Mathematica [A] time = 0.223414, size = 110, normalized size = 0.79

$$\frac{\sec\left(\frac{c}{2}\right)\left(-45\sin\left(c+\frac{dx}{2}\right)+54\sin\left(c+\frac{3dx}{2}\right)-30\sin\left(2c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)+}{5760a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(81*Sin[(d*x)/2] - 45*Sin[c + (d*x)/2] + 54*Si
n[c + (3*d*x)/2] - 30*Sin[2*c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Si
n[3*c + (7*d*x)/2] + Sin[4*c + (9*d*x)/2]))/(5760*a^5*d)
```

Maple [A] time = 0.039, size = 45, normalized size = 0.3

$$\frac{1}{16\,da^5}\left(\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{2}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9-2/5*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1
/2*c))
```

Maxima [A] time = 1.17754, size = 90, normalized size = 0.65

$$\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{18\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{720\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/720*(45*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.53705, size = 312, normalized size = 2.24

$$\frac{\left(2 \cos\left(dx+c\right)^{4}+10 \cos\left(dx+c\right)^{3}+21 \cos\left(dx+c\right)^{2}+10 \cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{45\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+c\right)+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 10*cos(d*x
 + c) + 2)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [A] time = 23.3924, size = 68, normalized size = 0.49

$$\begin{cases} \frac{\tan^{9}\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^{5}d}-\frac{\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{5}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^{5}d} & \text{for}\; d\neq 0 \\ \frac{x\cos^{2}\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**5/(40*a**5*
d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x*cos(c)**2/(a*cos(c) + a)**
5, True))
```

Giac [A] time = 1.30468, size = 62, normalized size = 0.45

$$\frac{5 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-18 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+45 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{720\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/720*(5*tan(1/2*d*x + 1/2*c)^9 - 18*tan(1/2*d*x + 1/2*c)^5 + 45*tan(1/2*d*
x + 1/2*c))/(a^5*d)
```

3.88 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$\frac{2\sin(c+dx)}{63d\left(a^5\cos(c+dx)+a^5\right)}+\frac{2\sin(c+dx)}{63ad\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{\sin(c+dx)}{21a^2d(a\cos(c+dx)+a)^3}+\frac{5\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4}-\frac{}{9d(a}$$

```
[Out] -Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) + (5*Sin[c + d*x])/(63*a*d*(a +
a*Cos[c + d*x])^4) + Sin[c + d*x]/(21*a^2*d*(a + a*Cos[c + d*x])^3) + (2*Si
n[c + d*x])/(63*a*d*(a^2 + a^2*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(63*d*(a
^5 + a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.105471, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2750, 2650, 2648}

$$\frac{2\sin(c+dx)}{63d\left(a^5\cos(c+dx)+a^5\right)}+\frac{2\sin(c+dx)}{63ad\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{\sin(c+dx)}{21a^2d(a\cos(c+dx)+a)^3}+\frac{5\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4}-\frac{}{9d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) + (5*Sin[c + d*x])/(63*a*d*(a +
a*Cos[c + d*x])^4) + Sin[c + d*x]/(21*a^2*d*(a + a*Cos[c + d*x])^3) + (2*Si
n[c + d*x])/(63*a*d*(a^2 + a^2*Cos[c + d*x])^2) + (2*Sin[c + d*x])/(63*d*(a
^5 + a^5*Cos[c + d*x]))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{5\int \frac{1}{(a+a\cos(c+dx))^4}\,dx}{9a}$$
$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{5\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{21a^2}$$
$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{21a^2d(a+a\cos(c+dx))^3} + \frac{2\int \frac{}{(}}{}$$
$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{21a^2d(a+a\cos(c+dx))^3} + \frac{}{63a^3}$$
$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{5\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{\sin(c+dx)}{21a^2d(a+a\cos(c+dx))^3} + \frac{}{63a^3}$$

Mathematica [A] time = 0.17237, size = 97, normalized size = 0.68

$$\frac{\sec\left(\frac{c}{2}\right)\left(-63\sin\left(c+\frac{dx}{2}\right)+84\sin\left(c+\frac{3dx}{2}\right)+36\sin\left(2c+\frac{5dx}{2}\right)+9\sin\left(3c+\frac{7dx}{2}\right)+\sin\left(4c+\frac{9dx}{2}\right)+63\sin\left(\frac{dx}{2}\right)\right)\sec}{8064a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^9*(63*Sin[(d*x)/2] - 63*Sin[c + (d*x)/2] + 84*Si
n[c + (3*d*x)/2] + 36*Sin[2*c + (5*d*x)/2] + 9*Sin[3*c + (7*d*x)/2] + Sin[4
*c + (9*d*x)/2]))/(8064*a^5*d)
```

Maple [A] time = 0.037, size = 58, normalized size = 0.4

$$\frac{1}{16\,da^5}\left(-\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{2}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] 1/16/d/a^5*(-1/9*tan(1/2*d*x+1/2*c)^9-2/7*tan(1/2*d*x+1/2*c)^7+2/3*tan(1/2*
d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.16703, size = 117, normalized size = 0.82

$$\frac{\frac{63\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{18\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{1008\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(63*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 18*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*sin(d*x + c)^9/(cos(d
```

```
*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.56325, size = 312, normalized size = 2.18

$$\frac{\left(2 \cos\left(dx+c\right)^{4}+10 \cos\left(dx+c\right)^{3}+21 \cos\left(dx+c\right)^{2}+25 \cos\left(dx+c\right)+5\right) \sin\left(dx+c\right)}{63 \left(a^{5} d \cos\left(dx+c\right)^{5}+5 a^{5} d \cos\left(dx+c\right)^{4}+10 a^{5} d \cos\left(dx+c\right)^{3}+10 a^{5} d \cos\left(dx+c\right)^{2}+5 a^{5} d \cos\left(dx+c\right)+a^{5} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/63*(2*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 21*cos(d*x + c)^2 + 25*cos(d*x
 + c) + 5)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10
*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^
5*d)
```

Sympy [A] time = 19.2459, size = 85, normalized size = 0.59

$$\begin{cases} -\frac{\tan^{9}\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^{5}d}-\frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{56a^{5}d}+\frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{24a^{5}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^{5}d} & \text{for } d \neq 0 \\ \frac{x\cos\left(c\right)}{\left(a\cos\left(c\right)+a\right)^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((-tan(c/2 + d*x/2)**9/(144*a**5*d) - tan(c/2 + d*x/2)**7/(56*a**5
*d) + tan(c/2 + d*x/2)**3/(24*a**5*d) + tan(c/2 + d*x/2)/(16*a**5*d), Ne(d,
 0)), (x*cos(c)/(a*cos(c) + a)**5, True))
```

Giac [A] time = 1.37065, size = 80, normalized size = 0.56

$$-\frac{7 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9}+18 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7}-42 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}-63 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{1008 a^{5} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(7*tan(1/2*d*x + 1/2*c)^9 + 18*tan(1/2*d*x + 1/2*c)^7 - 42*tan(1/2*
d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c))/(a^5*d)
```

3.89 $\int \frac{1}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=143

$$\frac{8\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{8\sin(c+dx)}{315ad\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{4\sin(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}+\frac{4\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4}$$

```
[Out] Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) + (4*Sin[c + d*x])/(63*a*d*(a + a
*Cos[c + d*x])^4) + (4*Sin[c + d*x])/(105*a^2*d*(a + a*Cos[c + d*x])^3) + (
8*Sin[c + d*x])/(315*a*d*(a^2 + a^2*Cos[c + d*x])^2) + (8*Sin[c + d*x])/(31
5*d*(a^5 + a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.0913571, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2650, 2648}

$$\frac{8\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}+\frac{8\sin(c+dx)}{315ad\left(a^2\cos(c+dx)+a^2\right)^2}+\frac{4\sin(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}+\frac{4\sin(c+dx)}{63ad(a\cos(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-5),x]
```

```
[Out] Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) + (4*Sin[c + d*x])/(63*a*d*(a + a
*Cos[c + d*x])^4) + (4*Sin[c + d*x])/(105*a^2*d*(a + a*Cos[c + d*x])^3) + (
8*Sin[c + d*x])/(315*a*d*(a^2 + a^2*Cos[c + d*x])^2) + (8*Sin[c + d*x])/(31
5*d*(a^5 + a^5*Cos[c + d*x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^5}\,dx = \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\int \frac{1}{(a+a\cos(c+dx))^4}\,dx}{9a}$$
$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{4\int \frac{1}{(a+a\cos(c+dx))^3}\,dx}{21a^2}$$
$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} + \frac{8\int \frac{1}{(a+a}{1}$$
$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} + \frac{8}{315a^3d(}$$
$$= \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{4\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{4\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} + \frac{8}{315a^3d(}$$

Mathematica [A] time = 0.15382, size = 89, normalized size = 0.62

$$\frac{\left(126\sin\left(\frac{1}{2}(c+dx)\right)+84\sin\left(\frac{3}{2}(c+dx)\right)+36\sin\left(\frac{5}{2}(c+dx)\right)+9\sin\left(\frac{7}{2}(c+dx)\right)+\sin\left(\frac{9}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{315a^5d(\cos(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-5),x]
```

```
[Out] (Cos[(c + d*x)/2]*(126*Sin[(c + d*x)/2] + 84*Sin[(3*(c + d*x))/2] + 36*Sin[
(5*(c + d*x))/2] + 9*Sin[(7*(c + d*x))/2] + Sin[(9*(c + d*x))/2]))/(315*a^5
*d*(1 + Cos[c + d*x])^5)
```

Maple [A] time = 0.034, size = 71, normalized size = 0.5

$$\frac{1}{16\,da^5}\left(\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{4}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{6}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{4}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] 1/16/d/a^5*(1/9*tan(1/2*d*x+1/2*c)^9+4/7*tan(1/2*d*x+1/2*c)^7+6/5*tan(1/2*d
*x+1/2*c)^5+4/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.13397, size = 144, normalized size = 1.01

$$\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{420\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{180\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{5040\,a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/5040*(315*sin(d*x + c)/(cos(d*x + c) + 1) + 420*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 180*sin(d*x + c)^7/(
```

```
cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^5*d)
```

Fricas [A] time = 1.54747, size = 316, normalized size = 2.21

$$\frac{\left(8\cos\left(dx+c\right)^{4}+40\cos\left(dx+c\right)^{3}+84\cos\left(dx+c\right)^{2}+100\cos\left(dx+c\right)+83\right)\sin\left(dx+c\right)}{315\left(a^{5}d\cos\left(dx+c\right)^{5}+5\,a^{5}d\cos\left(dx+c\right)^{4}+10\,a^{5}d\cos\left(dx+c\right)^{3}+10\,a^{5}d\cos\left(dx+c\right)^{2}+5\,a^{5}d\cos\left(dx+c\right)+a^{5}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 84*cos(d*x + c)^2 + 100*cos(d
*x + c) + 83)*sin(d*x + c)/(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 +
 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) +
 a^5*d)
```

Sympy [A] time = 15.2683, size = 102, normalized size = 0.71

$$\begin{cases} \frac{\tan^{9}\left(\frac{c}{2}+\frac{dx}{2}\right)}{144a^{5}d}+\frac{\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{28a^{5}d}+\frac{3\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{40a^{5}d}+\frac{\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{12a^{5}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16a^{5}d} & \text{for } d\neq 0 \\ \frac{x}{\left(a\cos\left(c\right)+a\right)^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Piecewise((tan(c/2 + d*x/2)**9/(144*a**5*d) + tan(c/2 + d*x/2)**7/(28*a**5*
d) + 3*tan(c/2 + d*x/2)**5/(40*a**5*d) + tan(c/2 + d*x/2)**3/(12*a**5*d) +
tan(c/2 + d*x/2)/(16*a**5*d), Ne(d, 0)), (x/(a*cos(c) + a)**5, True))
```

Giac [A] time = 1.43179, size = 97, normalized size = 0.68

$$\frac{35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+180\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+378\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+420\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{5040\,a^{5}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(35*tan(1/2*d*x + 1/2*c)^9 + 180*tan(1/2*d*x + 1/2*c)^7 + 378*tan(1/
2*d*x + 1/2*c)^5 + 420*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c))/(
a^5*d)
```

3.90 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=153

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{488\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{173\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}-\frac{34\sin(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}-\frac{13}{63ad(a\cos}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a^5*d) - Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) -
 (13*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (34*Sin[c + d*x])/(105
*a^2*d*(a + a*Cos[c + d*x])^3) - (173*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c
 + d*x])^2) - (488*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.377481, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2766, 2978, 12, 3770}

$$\frac{\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{488\sin(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{173\sin(c+dx)}{315a^3d(a\cos(c+dx)+a)^2}-\frac{34\sin(c+dx)}{105a^2d(a\cos(c+dx)+a)^3}-\frac{13}{63ad(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(a^5*d) - Sin[c + d*x]/(9*d*(a + a*Cos[c + d*x])^5) -
 (13*Sin[c + d*x])/(63*a*d*(a + a*Cos[c + d*x])^4) - (34*Sin[c + d*x])/(105
*a^2*d*(a + a*Cos[c + d*x])^3) - (173*Sin[c + d*x])/(315*a^3*d*(a + a*Cos[c
 + d*x])^2) - (488*Sin[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} + \frac{\int \frac{(9a-4a\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} + \frac{\int \frac{(63a^2-39a^2\cos(c+dx))\sec(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} + \int$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} - 315$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} - 315$$

$$= -\frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin(c+dx)}{105a^2d(a+a\cos(c+dx))^3} - 315$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{\sin(c+dx)}{9d(a+a\cos(c+dx))^5} - \frac{13\sin(c+dx)}{63ad(a+a\cos(c+dx))^4} - \frac{34\sin}{105a^2d(a+}$$

Mathematica [A] time = 1.82841, size = 211, normalized size = 1.38

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(\sec\left(\frac{c}{2}\right)\left(-25515\sin\left(c+\frac{dx}{2}\right)+29757\sin\left(c+\frac{3dx}{2}\right)-11235\sin\left(2c+\frac{3dx}{2}\right)+14733\sin\left(2c+\frac{5dx}{2}\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] -(Cos[(c + d*x)/2]*(80640*Cos[(c + d*x)/2]^9*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*(35973*S
in[(d*x)/2] - 25515*Sin[c + (d*x)/2] + 29757*Sin[c + (3*d*x)/2] - 11235*Sin
[2*c + (3*d*x)/2] + 14733*Sin[2*c + (5*d*x)/2] - 2835*Sin[3*c + (5*d*x)/2]
+ 4077*Sin[3*c + (7*d*x)/2] - 315*Sin[4*c + (7*d*x)/2] + 488*Sin[4*c + (9*d
*x)/2])))/(2520*a^5*d*(1 + Cos[c + d*x])^5)
```

Maple [A] time = 0.068, size = 134, normalized size = 0.9

$$-\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9 - \frac{3}{56\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 - \frac{1}{5\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{13}{24\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{31}{16\,da^5}\,\mathrm{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-3/56/d/a^5*tan(1/2*d*x+1/2*c)^7-1/5/d/a^5
*tan(1/2*d*x+1/2*c)^5-13/24/d/a^5*tan(1/2*d*x+1/2*c)^3-31/16/d/a^5*tan(1/2*
d*x+1/2*c)-1/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^5*ln(tan(1/2*d*x+1/2*c)+1
)
```

Maxima [A] time = 1.15144, size = 215, normalized size = 1.41

$$-\frac{\frac{\frac{9765\sin(dx+c)}{\cos(dx+c)+1}+\frac{2730\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^5}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*((9765*sin(d*x + c)/(cos(d*x + c) + 1) + 2730*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 270*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^5 - 50
40*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*x + c)/(co
s(d*x + c) + 1) - 1)/a^5)/d
```

Fricas [A] time = 1.69057, size = 671, normalized size = 4.39

$$\frac{315\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\log(\sin(dx+c)+1)-31}{630\,(a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/630*(315*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(
d*x + c)^2 + 5*cos(d*x + c) + 1)*log(sin(d*x + c) + 1) - 315*(cos(d*x + c)^
5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 + 10*cos(d*x + c)^2 + 5*cos(d*x +
c) + 1)*log(-sin(d*x + c) + 1) - 2*(488*cos(d*x + c)^4 + 2125*cos(d*x + c)^
3 + 3549*cos(d*x + c)^2 + 2740*cos(d*x + c) + 863)*sin(d*x + c))/(a^5*d*cos
(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*c
os(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.51151, size = 170, normalized size = 1.11

$$\frac{\frac{5040 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^5}-\frac{5040 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^5}-\frac{35 a^{40} \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+270 a^{40} \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+1008 a^{40} \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+2730 a^{40} \mathrm{t}}{a^{45}}}{5040 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2*
d*x + 1/2*c) - 1))/a^5 - (35*a^40*tan(1/2*d*x + 1/2*c)^9 + 270*a^40*tan(1/2
*d*x + 1/2*c)^7 + 1008*a^40*tan(1/2*d*x + 1/2*c)^5 + 2730*a^40*tan(1/2*d*x
+ 1/2*c)^3 + 9765*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.91 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5} dx$

Optimal. Leaf size=168

$$\frac{496\tan(c+dx)}{63a^5d} - \frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{5\tan(c+dx)}{d\left(a^5\cos(c+dx)+a^5\right)} - \frac{67\tan(c+dx)}{63a^3d(a\cos(c+dx)+a)^2} - \frac{29\tan(c+dx)}{63a^2d(a\cos(c+dx)+$$

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^5*d) + (496*Tan[c + d*x])/(63*a^5*d) - Tan[c
+ d*x]/(9*d*(a + a*Cos[c + d*x])^5) - (5*Tan[c + d*x])/(21*a*d*(a + a*Cos[c
 + d*x])^4) - (29*Tan[c + d*x])/(63*a^2*d*(a + a*Cos[c + d*x])^3) - (67*Tan
[c + d*x])/(63*a^3*d*(a + a*Cos[c + d*x])^2) - (5*Tan[c + d*x])/(d*(a^5 + a
^5*Cos[c + d*x]))
```

Rubi [A] time = 0.532824, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2748, 3767, 8, 3770}

$$\frac{496\tan(c+dx)}{63a^5d} - \frac{5\tanh^{-1}(\sin(c+dx))}{a^5d} - \frac{5\tan(c+dx)}{d\left(a^5\cos(c+dx)+a^5\right)} - \frac{67\tan(c+dx)}{63a^3d(a\cos(c+dx)+a)^2} - \frac{29\tan(c+dx)}{63a^2d(a\cos(c+dx)+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```


```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(a^5*d) + (496*Tan[c + d*x])/(63*a^5*d) - Tan[c
+ d*x]/(9*d*(a + a*Cos[c + d*x])^5) - (5*Tan[c + d*x])/(21*a*d*(a + a*Cos[c
 + d*x])^4) - (29*Tan[c + d*x])/(63*a^2*d*(a + a*Cos[c + d*x])^3) - (67*Tan
[c + d*x])/(63*a^3*d*(a + a*Cos[c + d*x])^2) - (5*Tan[c + d*x])/(d*(a^5 + a
^5*Cos[c + d*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{\int \frac{(10a-5a\cos(c+dx))\sec^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4}+\frac{\int \frac{\left(85a^2-60a^2\cos(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^3}\,dx}{63a^4}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4}-\frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3}+\frac{\int \frac{(57}{}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4}-\frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3}-\frac{}{63a^3}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4}-\frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3}-\frac{}{63a^3}$$

$$= -\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4}-\frac{29\tan(c+dx)}{63a^2d(a+a\cos(c+dx))^3}-\frac{}{63a^3}$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}-\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{5\tan(c+dx)}{21ad(a+a\cos(c+dx))^4}-\frac{29}{63a^2d(a}$$

$$= -\frac{5\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{496\tan(c+dx)}{63a^5d}-\frac{\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{5\tan(c+d}{21ad(a+a\cos(c}$$

Mathematica [B] time = 6.33967, size = 453, normalized size = 2.7

$$\frac{160\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^5}-\frac{160\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^5}+\frac{\sec\left(\frac{c}{2}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (160*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]])/(d
*(a + a*Cos[c + d*x])^5) - (160*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/2
```

```
] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^5) + (Cos[c/2 + (d*x)/2]*S
ec[c/2]*Sec[c]*Sec[c + d*x]*(-33978*Sin[(d*x)/2] + 52002*Sin[(3*d*x)/2] - 5
6952*Sin[c - (d*x)/2] + 43722*Sin[c + (d*x)/2] - 47208*Sin[2*c + (d*x)/2] -
 18144*Sin[c + (3*d*x)/2] + 41796*Sin[2*c + (3*d*x)/2] - 28350*Sin[3*c + (3
*d*x)/2] + 34578*Sin[c + (5*d*x)/2] - 5691*Sin[2*c + (5*d*x)/2] + 28719*Sin
[3*c + (5*d*x)/2] - 11550*Sin[4*c + (5*d*x)/2] + 15517*Sin[2*c + (7*d*x)/2]
 - 504*Sin[3*c + (7*d*x)/2] + 13186*Sin[4*c + (7*d*x)/2] - 2835*Sin[5*c + (
7*d*x)/2] + 4149*Sin[3*c + (9*d*x)/2] + 252*Sin[4*c + (9*d*x)/2] + 3582*Sin
[5*c + (9*d*x)/2] - 315*Sin[6*c + (9*d*x)/2] + 496*Sin[4*c + (11*d*x)/2] +
63*Sin[5*c + (11*d*x)/2] + 433*Sin[6*c + (11*d*x)/2]))/(2016*d*(a + a*Cos[c
 + d*x])^5)
```

Maple [A] time = 0.068, size = 177, normalized size = 1.1

$$\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{1}{14\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{3}{8\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{3}{2\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{129}{16\,da^5}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] 1/144/d/a^5*tan(1/2*d*x+1/2*c)^9+1/14/d/a^5*tan(1/2*d*x+1/2*c)^7+3/8/d/a^5*
tan(1/2*d*x+1/2*c)^5+3/2/d/a^5*tan(1/2*d*x+1/2*c)^3+129/16/d/a^5*tan(1/2*d*
x+1/2*c)-1/d/a^5/(tan(1/2*d*x+1/2*c)-1)+5/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/
d/a^5/(tan(1/2*d*x+1/2*c)+1)-5/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.17826, size = 278, normalized size = 1.65

$$\frac{\frac{2016\,\sin(dx+c)}{\left(a^5-\frac{a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{8127\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1512\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{72\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{7\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{5040\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\frac{5040\,\log}{}}{1008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/1008*(2016*sin(d*x + c)/((a^5 - a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*
(cos(d*x + c) + 1)) + (8127*sin(d*x + c)/(cos(d*x + c) + 1) + 1512*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 72*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
)/a^5 - 5040*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^5 + 5040*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

Fricas [A] time = 1.76818, size = 755, normalized size = 4.49

$$-\frac{315\left(\cos(dx+c)^6+5\,\cos(dx+c)^5+10\,\cos(dx+c)^4+10\,\cos(dx+c)^3+5\,\cos(dx+c)^2+\cos(dx+c)\right)\log(\sin(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/126*(315*(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos
(d*x + c)^3 + 5*cos(d*x + c)^2 + cos(d*x + c))*log(sin(d*x + c) + 1) - 315*
(cos(d*x + c)^6 + 5*cos(d*x + c)^5 + 10*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 5*cos(d*x + c)^2 + cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(496*cos(d*x
+ c)^5 + 2165*cos(d*x + c)^4 + 3633*cos(d*x + c)^3 + 2840*cos(d*x + c)^2 +
946*cos(d*x + c) + 63)*sin(d*x + c))/(a^5*d*cos(d*x + c)^6 + 5*a^5*d*cos(d*
x + c)^5 + 10*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 5*a^5*d*cos(
d*x + c)^2 + a^5*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33544, size = 209, normalized size = 1.24

$$-\frac{\frac{5040 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^5}-\frac{5040 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^5}+\frac{2016 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^5}-\frac{7a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+72a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+378a^{40}\text{ta}}{}}{1008d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/1008*(5040*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 5040*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^5 + 2016*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^
2 - 1)*a^5) - (7*a^40*tan(1/2*d*x + 1/2*c)^9 + 72*a^40*tan(1/2*d*x + 1/2*c)
^7 + 378*a^40*tan(1/2*d*x + 1/2*c)^5 + 1512*a^40*tan(1/2*d*x + 1/2*c)^3 + 8
127*a^40*tan(1/2*d*x + 1/2*c))/a^45)/d
```

3.92 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx$

Optimal. Leaf size=224

$$-\frac{7664\tan(c+dx)}{315a^5d}+\frac{31\tanh^{-1}(\sin(c+dx))}{2a^5d}+\frac{31\tan(c+dx)\sec(c+dx)}{2a^5d}-\frac{3832\tan(c+dx)\sec(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{577\tan(c-}{315a^3d(a\,c}$$

```
[Out] (31*ArcTanh[Sin[c + d*x]])/(2*a^5*d) - (7664*Tan[c + d*x])/(315*a^5*d) + (3
1*Sec[c + d*x]*Tan[c + d*x])/(2*a^5*d) - (Sec[c + d*x]*Tan[c + d*x])/(9*d*(
a + a*Cos[c + d*x])^5) - (17*Sec[c + d*x]*Tan[c + d*x])/(63*a*d*(a + a*Cos[
c + d*x])^4) - (28*Sec[c + d*x]*Tan[c + d*x])/(45*a^2*d*(a + a*Cos[c + d*x]
)^3) - (577*Sec[c + d*x]*Tan[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) -
 (3832*Sec[c + d*x]*Tan[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```

Rubi [A] time = 0.539683, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2766, 2978, 2748, 3768, 3770, 3767, 8}

$$-\frac{7664\tan(c+dx)}{315a^5d}+\frac{31\tanh^{-1}(\sin(c+dx))}{2a^5d}+\frac{31\tan(c+dx)\sec(c+dx)}{2a^5d}-\frac{3832\tan(c+dx)\sec(c+dx)}{315d\left(a^5\cos(c+dx)+a^5\right)}-\frac{577\tan(c-}{315a^3d(a\,c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```


```
[Out] (31*ArcTanh[Sin[c + d*x]])/(2*a^5*d) - (7664*Tan[c + d*x])/(315*a^5*d) + (3
1*Sec[c + d*x]*Tan[c + d*x])/(2*a^5*d) - (Sec[c + d*x]*Tan[c + d*x])/(9*d*(
a + a*Cos[c + d*x])^5) - (17*Sec[c + d*x]*Tan[c + d*x])/(63*a*d*(a + a*Cos[
c + d*x])^4) - (28*Sec[c + d*x]*Tan[c + d*x])/(45*a^2*d*(a + a*Cos[c + d*x]
)^3) - (577*Sec[c + d*x]*Tan[c + d*x])/(315*a^3*d*(a + a*Cos[c + d*x])^2) -
 (3832*Sec[c + d*x]*Tan[c + d*x])/(315*d*(a^5 + a^5*Cos[c + d*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^5}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5}+\frac{\int \frac{(11a-6a\cos(c+dx))\sec^3(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{9a^2}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4}+\frac{\int \frac{\left(111a^2-85a^2\cos(c+dx)\right)\sec^3(c+dx)}{(a+a\cos(c+dx))^3}\,d}{63a^4}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4}-\frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3}+\frac{\int}{}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4}-\frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3}-\frac{5}{3}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4}-\frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3}-\frac{5}{3}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4}-\frac{28\sec(c+dx)\tan(c+dx)}{45a^2d(a+a\cos(c+dx))^3}-\frac{5}{3}$$

$$= \frac{31\sec(c+dx)\tan(c+dx)}{2a^5d}-\frac{\sec(c+dx)\tan(c+dx)}{9d(a+a\cos(c+dx))^5}-\frac{17\sec(c+dx)\tan(c+dx)}{63ad(a+a\cos(c+dx))^4}-\frac{28}{45}$$

$$= \frac{31\tanh^{-1}(\sin(c+dx))}{2a^5d}-\frac{7664\tan(c+dx)}{315a^5d}+\frac{31\sec(c+dx)\tan(c+dx)}{2a^5d}-\frac{\sec(c+dx)\,t}{9d(a+a\cos}$$

Mathematica [B] time = 6.32031, size = 507, normalized size = 2.26

$$-\frac{496\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^5}+\frac{496\cos^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d(a\cos(c+dx)+a)^5}+\frac{\sec}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^5,x]
```

```
[Out] (-496*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]])/(
d*(a + a*Cos[c + d*x])^5) + (496*Cos[c/2 + (d*x)/2]^10*Log[Cos[c/2 + (d*x)/
2] + Sin[c/2 + (d*x)/2]])/(d*(a + a*Cos[c + d*x])^5) + (Cos[c/2 + (d*x)/2]*
Sec[c/2]*Sec[c]*Sec[c + d*x]^2*(1472562*Sin[(d*x)/2] - 2822886*Sin[(3*d*x)/
2] + 3057654*Sin[c - (d*x)/2] - 1885854*Sin[c + (d*x)/2] + 2644362*Sin[2*c
+ (d*x)/2] + 867048*Sin[c + (3*d*x)/2] - 1868436*Sin[2*c + (3*d*x)/2] + 182
1498*Sin[3*c + (3*d*x)/2] - 2083537*Sin[c + (5*d*x)/2] + 339885*Sin[2*c + (
5*d*x)/2] - 1456687*Sin[3*c + (5*d*x)/2] + 966735*Sin[4*c + (5*d*x)/2] - 11
95641*Sin[2*c + (7*d*x)/2] + 46515*Sin[3*c + (7*d*x)/2] - 874341*Sin[4*c +
(7*d*x)/2] + 367815*Sin[5*c + (7*d*x)/2] - 494579*Sin[3*c + (9*d*x)/2] - 31
815*Sin[4*c + (9*d*x)/2] - 374879*Sin[5*c + (9*d*x)/2] + 87885*Sin[6*c + (9
*d*x)/2] - 128187*Sin[4*c + (11*d*x)/2] - 18585*Sin[5*c + (11*d*x)/2] - 998
37*Sin[6*c + (11*d*x)/2] + 9765*Sin[7*c + (11*d*x)/2] - 15328*Sin[5*c + (13
*d*x)/2] - 3150*Sin[6*c + (13*d*x)/2] - 12178*Sin[7*c + (13*d*x)/2]))/(4032
0*d*(a + a*Cos[c + d*x])^5)
```

Maple [A] time = 0.076, size = 219, normalized size = 1.

$$-\frac{1}{144\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{5}{56\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{3}{5\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{25}{8\,da^5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{351}{16\,da^5}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^5,x)
```

```
[Out] -1/144/d/a^5*tan(1/2*d*x+1/2*c)^9-5/56/d/a^5*tan(1/2*d*x+1/2*c)^7-3/5/d/a^5
*tan(1/2*d*x+1/2*c)^5-25/8/d/a^5*tan(1/2*d*x+1/2*c)^3-351/16/d/a^5*tan(1/2*
d*x+1/2*c)+1/2/d/a^5/(tan(1/2*d*x+1/2*c)-1)^2+11/2/d/a^5/(tan(1/2*d*x+1/2*c
)-1)-31/2/d/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^5/(tan(1/2*d*x+1/2*c)+1)^2
+11/2/d/a^5/(tan(1/2*d*x+1/2*c)+1)+31/2/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.13478, size = 339, normalized size = 1.51

$$-\frac{\frac{5040\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^5-\frac{2a^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{110565\sin(dx+c)}{\cos(dx+c)+1}+\frac{15750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3024\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^5}-\frac{78120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^5}+\cdots}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/5040*(5040*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 11*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3)/(a^5 - 2*a^5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^5*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4) + (110565*sin(d*x + c)/(cos(d*x + c) + 1)
+ 15750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3024*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9)/a^5 - 78120*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a
^5 + 78120*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^5)/d
```

Fricas [A] time = 1.74552, size = 807, normalized size = 3.6

$$\frac{9765\left(\cos(dx+c)^7+5\cos(dx+c)^6+10\cos(dx+c)^5+10\cos(dx+c)^4+5\cos(dx+c)^3+\cos(dx+c)^2\right)\log(\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/1260*(9765*(cos(d*x + c)^7 + 5*cos(d*x + c)^6 + 10*cos(d*x + c)^5 + 10*co
s(d*x + c)^4 + 5*cos(d*x + c)^3 + cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 9
765*(cos(d*x + c)^7 + 5*cos(d*x + c)^6 + 10*cos(d*x + c)^5 + 10*cos(d*x + c
)^4 + 5*cos(d*x + c)^3 + cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(15328*
cos(d*x + c)^6 + 66875*cos(d*x + c)^5 + 112119*cos(d*x + c)^4 + 87440*cos(d
*x + c)^3 + 28828*cos(d*x + c)^2 + 1575*cos(d*x + c) - 315)*sin(d*x + c))/(
a^5*d*cos(d*x + c)^7 + 5*a^5*d*cos(d*x + c)^6 + 10*a^5*d*cos(d*x + c)^5 + 1
0*a^5*d*cos(d*x + c)^4 + 5*a^5*d*cos(d*x + c)^3 + a^5*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43188, size = 231, normalized size = 1.03

$$\frac{\frac{78120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^5}-\frac{78120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^5}+\frac{5040\left(11\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^5}-\frac{35a^{40}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+450a^{40}\mathrm{ta}}{}}{5040d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/5040*(78120*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 78120*log(abs(tan(1/
2*d*x + 1/2*c) - 1))/a^5 + 5040*(11*tan(1/2*d*x + 1/2*c)^3 - 9*tan(1/2*d*x
+ 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^5) - (35*a^40*tan(1/2*d*x + 1/2
*c)^9 + 450*a^40*tan(1/2*d*x + 1/2*c)^7 + 3024*a^40*tan(1/2*d*x + 1/2*c)^5
+ 15750*a^40*tan(1/2*d*x + 1/2*c)^3 + 110565*a^40*tan(1/2*d*x + 1/2*c))/a^4
5)/d
```

3.93 $\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6} dx$

Optimal. Leaf size=184

$$-\frac{118\sin(c+dx)\cos^2(c+dx)}{693a^2d(a\cos(c+dx)+a)^4}+\frac{146\sin(c+dx)}{693a^6d(\cos(c+dx)+1)}-\frac{268\sin(c+dx)}{693a^6d(\cos(c+dx)+1)^2}+\frac{130\sin(c+dx)}{693a^6d(\cos(c+dx)+1)^3}-\frac{\sin(}{11d}$$

```
[Out] (130*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c + d*x])^3) - (268*Sin[c + d*x])/(6
93*a^6*d*(1 + Cos[c + d*x])^2) + (146*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c +
 d*x])) - (Cos[c + d*x]^4*Sin[c + d*x])/(11*d*(a + a*Cos[c + d*x])^6) - (14
*Cos[c + d*x]^3*Sin[c + d*x])/(99*a*d*(a + a*Cos[c + d*x])^5) - (118*Cos[c
+ d*x]^2*Sin[c + d*x])/(693*a^2*d*(a + a*Cos[c + d*x])^4)
```

Rubi [A] time = 0.409626, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2765, 2977, 2968, 3019, 2750, 2648}

$$-\frac{118\sin(c+dx)\cos^2(c+dx)}{693a^2d(a\cos(c+dx)+a)^4}+\frac{146\sin(c+dx)}{693a^6d(\cos(c+dx)+1)}-\frac{268\sin(c+dx)}{693a^6d(\cos(c+dx)+1)^2}+\frac{130\sin(c+dx)}{693a^6d(\cos(c+dx)+1)^3}-\frac{\sin(}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^6,x]
```

```
[Out] (130*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c + d*x])^3) - (268*Sin[c + d*x])/(6
93*a^6*d*(1 + Cos[c + d*x])^2) + (146*Sin[c + d*x])/(693*a^6*d*(1 + Cos[c +
 d*x])) - (Cos[c + d*x]^4*Sin[c + d*x])/(11*d*(a + a*Cos[c + d*x])^6) - (14
*Cos[c + d*x]^3*Sin[c + d*x])/(99*a*d*(a + a*Cos[c + d*x])^5) - (118*Cos[c
+ d*x]^2*Sin[c + d*x])/(693*a^2*d*(a + a*Cos[c + d*x])^4)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+a\cos(c+dx))^6}\,dx = -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{\int \frac{\cos^3(c+dx)(4a-10a\cos(c+dx))}{(a+a\cos(c+dx))^5}\,dx}{11a^2}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos^2(c+dx)\left(42a^2-76a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^4}\,d}{99a^4}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{118\cos^2(c+dx)\sin(c+dx)}{693a^2d(a+a\cos(c+dx))^4} -$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{118\cos^2(c+dx)\sin(c+dx)}{693a^2d(a+a\cos(c+dx))^4} -$$

$$= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{118}{69}$$

$$= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{118}{69}$$

$$= \frac{130\sin(c+dx)}{693a^6d(1+\cos(c+dx))^3} - \frac{\cos^4(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{14\cos^3(c+dx)\sin(c+dx)}{99ad(a+a\cos(c+dx))^5} - \frac{118}{69}$$

Mathematica [A] time = 0.343251, size = 164, normalized size = 0.89

$$\sec\left(\frac{c}{2}\right)\left(-33726\sin\left(c+\frac{dx}{2}\right)+25080\sin\left(c+\frac{3dx}{2}\right)-23100\sin\left(2c+\frac{3dx}{2}\right)+12540\sin\left(2c+\frac{5dx}{2}\right)-11550\sin\left(3c+\frac{5}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + a*Cos[c + d*x])^6,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^11*(33726*Sin[(d*x)/2] - 33726*Sin[c + (d*x)/2]
+ 25080*Sin[c + (3*d*x)/2] - 23100*Sin[2*c + (3*d*x)/2] + 12540*Sin[2*c + (
5*d*x)/2] - 11550*Sin[3*c + (5*d*x)/2] + 4565*Sin[3*c + (7*d*x)/2] - 3465*S
in[4*c + (7*d*x)/2] + 913*Sin[4*c + (9*d*x)/2] - 693*Sin[5*c + (9*d*x)/2] +
 146*Sin[5*c + (11*d*x)/2]))/(709632*a^6*d)
```

Maple [A] time = 0.041, size = 84, normalized size = 0.5

$$\frac{1}{32\,da^6}\left(-\frac{1}{11}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}+\frac{5}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}-\frac{10}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+2\,(\tan(1/2\,dx+c/2))^5-\frac{5}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+cos(d*x+c)*a)^6,x)
```

```
[Out] 1/32/d/a^6*(-1/11*tan(1/2*d*x+1/2*c)^11+5/9*tan(1/2*d*x+1/2*c)^9-10/7*tan(1
/2*d*x+1/2*c)^7+2*tan(1/2*d*x+1/2*c)^5-5/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x
+1/2*c))
```

Maxima [A] time = 1.13969, size = 171, normalized size = 0.93

$$\frac{\frac{693\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{1155\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1386\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{990\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{385\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{63\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{22176\,a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] 1/22176*(693*sin(d*x + c)/(cos(d*x + c) + 1) - 1155*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 1386*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 990*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 385*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 63*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11)/(a^6*d)
```

Fricas [A] time = 1.53696, size = 382, normalized size = 2.08

$$\frac{\left(146\,\cos(dx+c)^5+183\,\cos(dx+c)^4+184\,\cos(dx+c)^3+124\,\cos(dx+c)^2+48\,\cos(dx+c)+8\right)\sin(dx}{693\left(a^6d\cos(dx+c)^6+6\,a^6d\cos(dx+c)^5+15\,a^6d\cos(dx+c)^4+20\,a^6d\cos(dx+c)^3+15\,a^6d\cos(dx+c)^2+6\,a^6d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] 1/693*(146*cos(d*x + c)^5 + 183*cos(d*x + c)^4 + 184*cos(d*x + c)^3 + 124*c
os(d*x + c)^2 + 48*cos(d*x + c) + 8)*sin(d*x + c)/(a^6*d*cos(d*x + c)^6 + 6
*a^6*d*cos(d*x + c)^5 + 15*a^6*d*cos(d*x + c)^4 + 20*a^6*d*cos(d*x + c)^3 +
 15*a^6*d*cos(d*x + c)^2 + 6*a^6*d*cos(d*x + c) + a^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+a*cos(d*x+c))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35491, size = 115, normalized size = 0.62

$$-\frac{63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-385\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+990\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-1386\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+1155\tan\left(\frac{1}{2}dx+\right.}{22176\,a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+a*cos(d*x+c))^6,x, algorithm="giac")
```

```
[Out] -1/22176*(63*tan(1/2*d*x + 1/2*c)^11 - 385*tan(1/2*d*x + 1/2*c)^9 + 990*tan
(1/2*d*x + 1/2*c)^7 - 1386*tan(1/2*d*x + 1/2*c)^5 + 1155*tan(1/2*d*x + 1/2*
c)^3 - 693*tan(1/2*d*x + 1/2*c))/(a^6*d)
```

3.94 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6} dx$

Optimal. Leaf size=176

$$\frac{61\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)}+\frac{61\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^2}-\frac{241\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^3}+\frac{9\sin(c+dx)}{77a^2d(a\cos(c+dx)+a)^4}-\frac{\sin(c}{11d(}$$

```
[Out] (-241*Sin[c + d*x])/(1155*a^6*d*(1 + Cos[c + d*x])^3) + (61*Sin[c + d*x])/(
1155*a^6*d*(1 + Cos[c + d*x])^2) + (61*Sin[c + d*x])/(1155*a^6*d*(1 + Cos[c
 + d*x])) - (Cos[c + d*x]^3*Sin[c + d*x])/(11*d*(a + a*Cos[c + d*x])^6) - (
4*Cos[c + d*x]^2*Sin[c + d*x])/(33*a*d*(a + a*Cos[c + d*x])^5) + (9*Sin[c +
 d*x])/(77*a^2*d*(a + a*Cos[c + d*x])^4)
```

Rubi [A] time = 0.318357, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2765, 2977, 2968, 3019, 2750, 2650, 2648}

$$\frac{61\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)}+\frac{61\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^2}-\frac{241\sin(c+dx)}{1155a^6d(\cos(c+dx)+1)^3}+\frac{9\sin(c+dx)}{77a^2d(a\cos(c+dx)+a)^4}-\frac{\sin(c}{11d(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^6,x]
```



```
[Out] (-241*Sin[c + d*x])/(1155*a^6*d*(1 + Cos[c + d*x])^3) + (61*Sin[c + d*x])/(
1155*a^6*d*(1 + Cos[c + d*x])^2) + (61*Sin[c + d*x])/(1155*a^6*d*(1 + Cos[c
 + d*x])) - (Cos[c + d*x]^3*Sin[c + d*x])/(11*d*(a + a*Cos[c + d*x])^6) - (
4*Cos[c + d*x]^2*Sin[c + d*x])/(33*a*d*(a + a*Cos[c + d*x])^5) + (9*Sin[c +
 d*x])/(77*a^2*d*(a + a*Cos[c + d*x])^4)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^6}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{\int \frac{\cos^2(c+dx)(3a-9a\cos(c+dx))}{(a+a\cos(c+dx))^5}\,dx}{11a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} - \frac{\int \frac{\cos(c+dx)\left(24a^2-57a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^4}\,dx}{99a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} - \frac{\int \frac{24a^2\cos(c+dx)-57a^2\cos^2(c+dx)}{(a+a\cos(c+dx))^4}\,dx}{99a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} + \frac{9\sin(c+dx)}{77a^2d(a+a\cos(c+dx))^4} + \int$$

$$= -\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} + \frac{}{7}$$

$$= -\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} + \frac{}{7}$$

$$= -\frac{241\sin(c+dx)}{1155a^6d(1+\cos(c+dx))^3} - \frac{\cos^3(c+dx)\sin(c+dx)}{11d(a+a\cos(c+dx))^6} - \frac{4\cos^2(c+dx)\sin(c+dx)}{33ad(a+a\cos(c+dx))^5} + \frac{}{7}$$

Mathematica [A] time = 0.297256, size = 151, normalized size = 0.86

$$\frac{\sec\left(\frac{c}{2}\right)\left(-12936\sin\left(c+\frac{dx}{2}\right)+10890\sin\left(c+\frac{3dx}{2}\right)-9240\sin\left(2c+\frac{3dx}{2}\right)+6600\sin\left(2c+\frac{5dx}{2}\right)-3465\sin\left(3c+\frac{5dx}{2}\right)+2\cdots\right.}{118272\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^6,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^11*(15246*Sin[(d*x)/2] - 12936*Sin[c + (d*x)/2]
+ 10890*Sin[c + (3*d*x)/2] - 9240*Sin[2*c + (3*d*x)/2] + 6600*Sin[2*c + (5*
d*x)/2] - 3465*Sin[3*c + (5*d*x)/2] + 2200*Sin[3*c + (7*d*x)/2] - 1155*Sin[
4*c + (7*d*x)/2] + 671*Sin[4*c + (9*d*x)/2] + 61*Sin[5*c + (11*d*x)/2]))/(1
182720*a^6*d)
```

Maple [A] time = 0.04, size = 84, normalized size = 0.5

$$\frac{1}{32\,da^6}\left(\frac{1}{11}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}+\frac{2}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+\frac{2}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\mathrm{ta}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^6,x)
```

```
[Out] 1/32/d/a^6*(1/11*tan(1/2*d*x+1/2*c)^11-1/3*tan(1/2*d*x+1/2*c)^9+2/7*tan(1/2
*d*x+1/2*c)^7+2/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2
*c))
```

Maxima [A] time = 1.14884, size = 171, normalized size = 0.97

$$\frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1}-\frac{1155\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{462\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{330\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{385\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{105\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{36960\,a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="maxima")
```

```
[Out] 1/36960*(1155*sin(d*x + c)/(cos(d*x + c) + 1) - 1155*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 462*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 330*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 385*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 105*sin(
d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^6*d)
```

Fricas [A] time = 1.59349, size = 383, normalized size = 2.18

$$\frac{\left(61\cos(dx+c)^5+366\cos(dx+c)^4+368\cos(dx+c)^3+248\cos(dx+c)^2+96\cos(dx+c)+16\right)\sin(d\cdots}{1155\left(a^6d\cos(dx+c)^6+6\,a^6d\cos(dx+c)^5+15\,a^6d\cos(dx+c)^4+20\,a^6d\cos(dx+c)^3+15\,a^6d\cos(dx+c)^2+6\,a^6d\,\mathrm{c}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="fricas")
```

```
[Out] 1/1155*(61*cos(d*x + c)^5 + 366*cos(d*x + c)^4 + 368*cos(d*x + c)^3 + 248*c
os(d*x + c)^2 + 96*cos(d*x + c) + 16)*sin(d*x + c)/(a^6*d*cos(d*x + c)^6 +
6*a^6*d*cos(d*x + c)^5 + 15*a^6*d*cos(d*x + c)^4 + 20*a^6*d*cos(d*x + c)^3
+ 15*a^6*d*cos(d*x + c)^2 + 6*a^6*d*cos(d*x + c) + a^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43857, size = 115, normalized size = 0.65

$$\frac{105 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{11} - 385 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{9} + 330 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{7} + 462 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{5} - 1155 \tan\left(\frac{1}{2} dx + \right.}{36960 a^6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^6,x, algorithm="giac")
```

```
[Out] 1/36960*(105*tan(1/2*d*x + 1/2*c)^11 - 385*tan(1/2*d*x + 1/2*c)^9 + 330*tan
(1/2*d*x + 1/2*c)^7 + 462*tan(1/2*d*x + 1/2*c)^5 - 1155*tan(1/2*d*x + 1/2*c
)^3 + 1155*tan(1/2*d*x + 1/2*c))/(a^6*d)
```

3.95 $\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{32\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{64\sin(c+dx)\sqrt{a\cos}}{315d}$$

```
[Out] (32*a*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a*Cos[c + d*x]^3*
Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Cos[c + d*x]^4*Sin[c +
 d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (64*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(315*d) + (32*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.241181, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2770, 2759, 2751, 2646}

$$\frac{2a\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{32\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{105ad}-\frac{64\sin(c+dx)\sqrt{a\cos}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]],x]
```



```
[Out] (32*a*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a*Cos[c + d*x]^3*
Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Cos[c + d*x]^4*Sin[c +
 d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (64*Sqrt[a + a*Cos[c + d*x]]*Sin[c
+ d*x])/(315*d) + (32*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(105*a*d)
```


Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{8}{9}\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$= \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{16}{21}\int \cos^2(c+$$
$$= \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{32(a+a\cos(c+}{1}$$
$$= \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} - \frac{64\sqrt{a+a\cos(c}{31}$$
$$= \frac{32a\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{16a\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^4(c+dx)\sin(}{9d\sqrt{a+a\cos(c+}$$

Mathematica [A] time = 0.262265, size = 92, normalized size = 0.58

$$\frac{\left(1890\sin\left(\frac{1}{2}(c+dx)\right)+420\sin\left(\frac{3}{2}(c+dx)\right)+252\sin\left(\frac{5}{2}(c+dx)\right)+45\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c\right.}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(1890*Sin[(c + d*x)/2] + 420*S
in[(3*(c + d*x))/2] + 252*Sin[(5*(c + d*x))/2] + 45*Sin[(7*(c + d*x))/2] +
35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

Maple [A] time = 0.932, size = 97, normalized size = 0.6

$$\frac{2a\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(560\left(\cos\left(1/2\,dx+c/2\right)\right)^8-800\left(\cos\left(1/2\,dx+c/2\right)\right)^6+552\left(\cos\left(1/2\,dx+c/2\right)\right)^4-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 2/315*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(560*cos(1/2*d*x+1/2*c)^8-800
*cos(1/2*d*x+1/2*c)^6+552*cos(1/2*d*x+1/2*c)^4-104*cos(1/2*d*x+1/2*c)^2+107
)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.98302, size = 107, normalized size = 0.68

$$\frac{\left(35\sqrt{2}\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+45\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+252\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+420\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+1890\sqrt{2}\sin\left(\right.\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2520*(35*sqrt(2)*sin(9/2*d*x + 9/2*c) + 45*sqrt(2)*sin(7/2*d*x + 7/2*c) +
 252*sqrt(2)*sin(5/2*d*x + 5/2*c) + 420*sqrt(2)*sin(3/2*d*x + 3/2*c) + 1890
*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.56197, size = 203, normalized size = 1.28

$$\frac{2\left(35\,\cos\left(dx+c\right)^{4}+40\,\cos\left(dx+c\right)^{3}+48\,\cos\left(dx+c\right)^{2}+64\,\cos\left(dx+c\right)+128\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*cos(d*x + c)^4 + 40*cos(d*x + c)^3 + 48*cos(d*x + c)^2 + 64*cos(d
*x + c) + 128)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^4, x)
```

3.96 $\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{12\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{8\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{4a\sin(c+}{5d\sqrt{a\cos(c+}}$$

```
[Out] (4*a*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Cos[c + d*x]^3*Sin
[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) - (8*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(35*d) + (12*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rubi [A] time = 0.175556, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2770, 2759, 2751, 2646}

$$\frac{2a\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{12\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35ad}-\frac{8\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d}+\frac{4a\sin(c+}{5d\sqrt{a\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (4*a*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Cos[c + d*x]^3*Sin
[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) - (8*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(35*d) + (12*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(35*a*d)
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{6}{7}\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$= \frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{12(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35ad} + \frac{12\int\left(\frac{3a}{2}-a\right.}{}$$
$$= \frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{8\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d} + \frac{12(a+a\cos(c}{}$$
$$= \frac{4a\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{8\sqrt{a+a\cos(c+dx)}\sin(c}{35d}$$

Mathematica [A] time = 0.156356, size = 80, normalized size = 0.66

$$\frac{\left(105\sin\left(\frac{1}{2}(c+dx)\right)+35\sin\left(\frac{3}{2}(c+dx)\right)+7\sin\left(\frac{5}{2}(c+dx)\right)+5\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(105*Sin[(c + d*x)/2] + 35*Sin
[(3*(c + d*x))/2] + 7*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2]))/(140*
d)
```

Maple [A] time = 0.694, size = 84, normalized size = 0.7

$$\frac{2a\sqrt{2}}{35d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(40\,(\cos(1/2\,dx+c/2))^6-36\,(\cos(1/2\,dx+c/2))^4+22\,(\cos(1/2\,dx+c/2))^2+9\right)\frac{}{\sqrt{(c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 2/35*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(40*cos(1/2*d*x+1/2*c)^6-36*co
s(1/2*d*x+1/2*c)^4+22*cos(1/2*d*x+1/2*c)^2+9)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2
*a)^(1/2)/d
```

Maxima [A] time = 1.9278, size = 88, normalized size = 0.72

$$\frac{\left(5\sqrt{2}\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+7\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+35\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+105\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/140*(5*sqrt(2)*sin(7/2*d*x + 7/2*c) + 7*sqrt(2)*sin(5/2*d*x + 5/2*c) + 35
*sqrt(2)*sin(3/2*d*x + 3/2*c) + 105*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.55646, size = 169, normalized size = 1.39

$$\frac{2\left(5\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+8\cos\left(dx+c\right)+16\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 8*cos(d*x + c) + 16)*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.97 $\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=86

$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}-\frac{4\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{14a\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (14*a*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (4*Sqrt[a + a*Cos[c +
 d*x]]*Sin[c + d*x])/(15*d) + (2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
5*a*d)
```

Rubi [A] time = 0.114655, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2759, 2751, 2646}

$$\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5ad}-\frac{4\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{14a\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (14*a*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (4*Sqrt[a + a*Cos[c +
 d*x]]*Sin[c + d*x])/(15*d) + (2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(
5*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad} + \frac{2\int\left(\frac{3a}{2}-a\cos(c+dx)\right)\sqrt{a+a\cos(c+dx)}}{5a}$$
$$= -\frac{4\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5ad} + \frac{7}{15}$$
$$= \frac{14a\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{4\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2(a+a\cos(c+dx))}{5}$$

Mathematica [A] time = 0.0988188, size = 68, normalized size = 0.79

$$\frac{\left(30\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(30*Sin[(c + d*x)/2] + 5*Sin[(
3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)
```

Maple [A] time = 0.818, size = 71, normalized size = 0.8

$$\frac{2a\sqrt{2}}{15d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(12\left(\cos\left(1/2\,dx+c/2\right)\right)^4-4\left(\cos\left(1/2\,dx+c/2\right)\right)^2+7\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 2/15*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(12*cos(1/2*d*x+1/2*c)^4-4*cos
(1/2*d*x+1/2*c)^2+7)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.92158, size = 69, normalized size = 0.8

$$\frac{\left(3\sqrt{2}\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sqrt{2}\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+30\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*sin(3/2*d*x + 3/2*c) + 30*
sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.52935, size = 142, normalized size = 1.65

$$\frac{2\sqrt{a\cos(dx+c)+a}\left(3\cos(dx+c)^2+4\cos(dx+c)+8\right)\sin(dx+c)}{15\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c)^2 + 4*cos(d*x + c) + 8)*sin(d
*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.98 $\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2a\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0456321, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2751, 2646}

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{2a\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(3*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+a\cos(c+dx)}\,dx &= \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}\int \sqrt{a+a\cos(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.070667, size = 54, normalized size = 0.96

$$\frac{\left(3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sin[(c + d*x)/2] + Sin[(3*(
c + d*x))/2]))/(3*d)
```

Maple [A] time = 0.661, size = 58, normalized size = 1.

$$\frac{2a\sqrt{2}}{3d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 2/3*cos(1/2*d*x+1/2*c)*a*sin(1/2*d*x+1/2*c)*(2*cos(1/2*d*x+1/2*c)^2+1)*2^(1
/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.89027, size = 49, normalized size = 0.88

$$\frac{\left(\sqrt{2}\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+3\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)
/d
```

Fricas [A] time = 1.55326, size = 112, normalized size = 2.

$$\frac{2\,\sqrt{a\cos\left(dx+c\right)+a}\left(\cos\left(dx+c\right)+2\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) + 2)*sin(d*x + c)/(d*cos(d*x + c
) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(dx+c)+a}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*cos(d*x + c), x)
```

3.99 $\int \sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=26

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.0129286, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2646}

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.0299949, size = 29, normalized size = 1.12

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]],x]

[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d
```

Maple [A] time = 0.513, size = 43, normalized size = 1.7

$$2\,\frac{a\cos(1/2\,dx+c/2)\sin(1/2\,dx+c/2)\sqrt{2}}{\sqrt{(\cos(1/2\,dx+c/2))^2\,a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^
(1/2)/d
```

Maxima [A] time = 1.73806, size = 27, normalized size = 1.04

$$\frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

Fricas [A] time = 1.56692, size = 84, normalized size = 3.23

$$\frac{2\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos(c+dx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*cos(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a), x)
```

3.100 $\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
```

Rubi [A] time = 0.0513787, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2773, 206}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx &= -\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0472732, size = 50, normalized size = 1.35

$$\frac{\sqrt{2}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(
c + d*x)/2])/d
```

Maple [B] time = 2.346, size = 180, normalized size = 4.9

$$\frac{1}{d}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\ln\left(-4\frac{\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}-a\sqrt{2}\cos\left(1/2\,dx+c/2\right)+2\,a}{-2\,\cos\left(1/2\,dx+c/2\right)+\sqrt{2}}\right)+\ln\left(4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c),x)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(ln(-4/(-2*cos(1/
2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(
1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))/
sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [A] time = 1.66574, size = 383, normalized size = 10.35

$$\left[\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{2\,d},\ \frac{\sqrt{-a}\arctan\left(\frac{2\,\sqrt{a\cos(dx+c)+a}\sqrt{-a}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x
+ c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 +
cos(d*x + c)^2))/d, sqrt(-a)*arctan(2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sin
(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x), x)
```

Giac [B] time = 3.63405, size = 153, normalized size = 4.14

$$\frac{a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{d|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*
c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/(d*abs(a
))
```

3.101 $\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*T
an[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.103651, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2772, 2773, 206}

$$\frac{a\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```


```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*T
an[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx = \frac{a\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx$$
$$= \frac{a\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.093715, size = 79, normalized size = 1.27

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(Sqrt[2]*ArcTanh[
Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 2.484, size = 379, normalized size = 6.1

$$\frac{1}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-2\,a\left(\ln\left(-4\,\frac{\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}-a\sqrt{2}\cos\left(1/2\,dx+c/2\right)+2\,a}{-2\,\cos\left(1/2\,dx+c/2\right)+\sqrt{2}}\right)+\ln\left(4\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^2,x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*(ln(-4/(-2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))*si
n(1/2*d*x+1/2*c)^2+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+ln(4/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2
^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 1.97755, size = 1580, normalized size = 25.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((4*sqrt(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (4*sqrt(2)*sin(1/2*d*x +
 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*
sin(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + 4*sq
rt(2)*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - 2*(2*sqrt(2)*sin(3/2*d*x + 3/
2*c) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) - 4*(sqrt(2)*cos(2*d*x + 2*c
) + sqrt(2))*sin(5/2*d*x + 5/2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) + 4*sqrt
(2)*sin(1/2*d*x + 1/2*c) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*d)
```

Fricas [B] time = 1.68303, size = 377, normalized size = 6.08

$$\frac{\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,\sqrt{a\cos(dx+}}{4\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*co
s(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*
x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) +
a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\cos(c+dx)+1)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**2,x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sec(c + d*x)**2, x)
```

Giac [B] time = 3.63426, size = 336, normalized size = 5.42

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{8\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}a^{\frac{3}{2}}-a^{\frac{5}{2}}\right)}{\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}-6\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\cdots\right.\right.}}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))/abs(a) + 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2*a^(3/2) - a^(5/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.102 $\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx$

Optimal. Leaf size=102

$$\frac{3a\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (3*a*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Sec[c + d*x]*Tan[c
 + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.162247, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2772, 2773, 206}

$$\frac{3a\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (3*a*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Sec[c + d*x]*Tan[c
 + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx = \frac{a\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{4}\int \sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx$$

$$= \frac{3a\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{8}\int \sqrt{a+a\cos(c+dx)}\sec$$

$$= \frac{3a\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\right.}{4d}$$

$$= \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{3a\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.186922, size = 94, normalized size = 0.92

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(\sin\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{3}{2}(c+dx)\right)+3\sqrt{2}\cos^2(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(3*Sqrt[2]*ArcT
anh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + Sin[(c + d*x)/2] + 3*Sin[(3*
(c + d*x))/2]))/(8*d)
```

Maple [B] time = 2.656, size = 545, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*a*(ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2
)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))
)*sin(1/2*d*x+1/2*c)^4+(-12*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a
-12*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-12*a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2))*sin(1/2*d*x+1/2*c)^2+3*ln(-4/(-2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+10*a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(1/2)/(2*cos(1/2*d*x+1/2*c)
-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*d*
x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 22.5067, size = 3567, normalized size = 34.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c
) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*c
os(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 12*(log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 24*sqrt(2)*
cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 8*sqrt(2)*cos(5/2*d*x + 5/2*c)*sin(
2*d*x + 2*c) + 2*(6*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) -
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2))*cos(2*d*x + 2*c) + 6*sqrt(2)*sin(7/2*d*x + 7/2*c) + 2*sqrt(2)*si
n(5/2*d*x + 5/2*c) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) - 6*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(4*d*x + 4*c) - 4*(2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 6*sqrt(2)*
sin(1/2*d*x + 1/2*c) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 4*(3*(log(2*cos(1/2*d*x + 1/2*c)^2
```

```
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c) - 3*sqrt(2)*cos(7/2*d*
x + 7/2*c) - sqrt(2)*cos(5/2*d*x + 5/2*c) + sqrt(2)*cos(3/2*d*x + 3/2*c) +
3*sqrt(2)*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) + 12*(2*sqrt(2)*cos(2*d*x
+ 2*c) + sqrt(2))*sin(7/2*d*x + 7/2*c) + 4*(2*sqrt(2)*cos(2*d*x + 2*c) + sq
rt(2))*sin(5/2*d*x + 5/2*c) + 8*(sqrt(2)*cos(3/2*d*x + 3/2*c) + 3*sqrt(2)*c
os(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*sqrt(2)*sin(3/2*d*x + 3/2*c) - 12
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*
sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*sqrt(a)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
 2*c) + 1)*d)
```

Fricas [A] time = 1.72247, size = 414, normalized size = 4.06

$$\frac{3\left(\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\sqrt{a\cos(dx+}}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
c) + a)*(3*cos(d*x + c) + 2)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x +
c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.85229, size = 443, normalized size = 4.34

$$\frac{\sqrt{2}\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{|a|}-\frac{8\left(5\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{6}a^{\frac{3}{2}}+19\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}\right.}\right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*sqrt(2)*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))/abs(a) - 8*(5*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*a^(3/2) + 19*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(5/2) - 17*(sqrt(a)*tan(1/2*d*x + 1/2*c)
 - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(7/2) + a^(9/2))/((sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.103 $\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{5a\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{5a\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (5*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (5*a*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (5*a*Sec[c + d*x]*Tan
[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.218053, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2772, 2773, 206}

$$\frac{5a\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{5a\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```


```
[Out] (5*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (5*a*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (5*a*Sec[c + d*x]*Tan
[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Sec[c + d*x]^2*Tan[c + d*x]
)/(3*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\sec^4(c+dx)\,dx = \frac{a\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{6}\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$= \frac{5a\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{8}\int \sqrt{a+a\cos(c$$
$$= \frac{5a\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec^2(c+dx)\tan(c+}{3d\sqrt{a+a\cos(c+dx}$$
$$= \frac{5a\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\sec^2(c+dx)\tan(c+}{3d\sqrt{a+a\cos(c+dx}$$
$$= \frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{5a\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\sec(c+dx)\tan(c-}{12d\sqrt{a+a\cos(c+}$$

Mathematica [A] time = 0.334332, size = 109, normalized size = 0.79

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(42\sin\left(\frac{1}{2}(c+dx)\right)+5\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)+30\sqrt{2}\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(30*Sqrt[2]*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 42*Sin[(c + d*x)/2] + 5*(Si
n[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

Maple [B] time = 2.813, size = 709, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)*sec(d*x+c)^4,x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-120*a*(ln(-4/(-2*co
s(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a
*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
)))*sin(1/2*d*x+1/2*c)^6+60*(2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-160*a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)-90*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-
90*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+66
*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+15*ln(4/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1
/2*d*x+1/2*c)+2*a))*a+15*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a
^(1/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/si
```

```
n(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.69106, size = 444, normalized size = 3.22

$$\frac{15\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\,\sqrt{a\cos\left(dx+\right.}}{96\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(15*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*
sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
 c) + a)*(15*cos(d*x + c)^2 + 10*cos(d*x + c) + 8)*sin(d*x + c))/(d*cos(d*x
 + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.83765, size = 554, normalized size = 4.01

$$\sqrt{2}\left(\frac{15\,\sqrt{2}a^{\frac{3}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{8\left(63\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{10}a^{\frac{3}{2}}-369\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/96*sqrt(2)*(15*sqrt(2)*a^(3/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt
(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2
)*abs(a) - 6*a))/abs(a) + 8*(63*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^10*a^(3/2) - 369*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*a^(5/2) + 1638*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(7/2) - 1074*(sqrt(a)*tan(
1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(9/2) + 171*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(11/2)
- 13*a^(13/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*a + a^2)^3)/d
```

3.104 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{34a^2\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{68a^2\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{68\sin(c+dx)(a\cos(c+dx)+}{105d}$$

```
[Out] (68*a^2*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (34*a^2*Cos[c + d*x
]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Cos[c + d*x]^4*S
in[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (136*a*Sqrt[a + a*Cos[c + d*x
]]*Sin[c + d*x])/(315*d) + (68*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10
5*d)
```

Rubi [A] time = 0.247446, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2763, 21, 2770, 2759, 2751, 2646}

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)}{9d\sqrt{a\cos(c+dx)+a}}+\frac{34a^2\sin(c+dx)\cos^3(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{68a^2\sin(c+dx)}{45d\sqrt{a\cos(c+dx)+a}}+\frac{68\sin(c+dx)(a\cos(c+dx)+}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (68*a^2*Sin[c + d*x])/(45*d*Sqrt[a + a*Cos[c + d*x]]) + (34*a^2*Cos[c + d*x
]^3*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Cos[c + d*x]^4*S
in[c + d*x])/(9*d*Sqrt[a + a*Cos[c + d*x]]) - (136*a*Sqrt[a + a*Cos[c + d*x
]]*Sin[c + d*x])/(315*d) + (68*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10
5*d)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{2}{9}\int \frac{\cos^3(c+dx)\left(\frac{17a^2}{2}+\frac{17}{2}a^2\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}$$
$$= \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{9}(17a)\int \cos^3(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$
$$= \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{1}{21}(34a)\int$$
$$= \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} + \frac{68(a+a\,\mathrm{co}}{}$$
$$= \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)\sin(c+dx)}{9d\sqrt{a+a\cos(c+dx)}} - \frac{136a\sqrt{a+a}}{}$$
$$= \frac{68a^2\sin(c+dx)}{45d\sqrt{a+a\cos(c+dx)}} + \frac{34a^2\cos^3(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\cos^4(c+dx)}{9d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.243161, size = 93, normalized size = 0.57

$$\frac{a\left(3780\sin\left(\frac{1}{2}(c+dx)\right)+1050\sin\left(\frac{3}{2}(c+dx)\right)+378\sin\left(\frac{5}{2}(c+dx)\right)+135\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)\sec(}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3780*Sin[(c + d*x)/2] + 105
0*Sin[(3*(c + d*x))/2] + 378*Sin[(5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2
] + 35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

Maple [A] time = 0.759, size = 99, normalized size = 0.6

$$\frac{4a^2\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(280\left(\cos\left(1/2\,dx+c/2\right)\right)^8-220\left(\cos\left(1/2\,dx+c/2\right)\right)^6+114\left(\cos\left(1/2\,dx+c/2\right)\right)^4+47\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 4/315*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(280*cos(1/2*d*x+1/2*c)^8-2
20*cos(1/2*d*x+1/2*c)^6+114*cos(1/2*d*x+1/2*c)^4+47*cos(1/2*d*x+1/2*c)^2+94
)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 2.0319, size = 113, normalized size = 0.7

$$\frac{\left(35\sqrt{2}a\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+135\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+378\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+1050\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3780\sqrt{2}a\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2520*(35*sqrt(2)*a*sin(9/2*d*x + 9/2*c) + 135*sqrt(2)*a*sin(7/2*d*x + 7/2
*c) + 378*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 1050*sqrt(2)*a*sin(3/2*d*x + 3/2
*c) + 3780*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.58266, size = 219, normalized size = 1.35

$$\frac{2\left(35\,a\cos\left(dx+c\right)^4+85\,a\cos\left(dx+c\right)^3+102\,a\cos\left(dx+c\right)^2+136\,a\cos\left(dx+c\right)+272\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a*cos(d*x + c)^4 + 85*a*cos(d*x + c)^3 + 102*a*cos(d*x + c)^2 + 1
36*a*cos(d*x + c) + 272*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
 + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

3.105 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=116

$$\frac{152a^2\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}-\frac{4\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{38a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}$$

```
[Out] (152*a^2*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (38*a*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(105*d) - (4*(a + a*Cos[c + d*x])^(3/2)*Sin[c +
 d*x])/(35*d) + (2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rubi [A] time = 0.141282, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2647, 2646}

$$\frac{152a^2\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{5/2}}{7ad}-\frac{4\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d}+\frac{38a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (152*a^2*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) + (38*a*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(105*d) - (4*(a + a*Cos[c + d*x])^(3/2)*Sin[c +
 d*x])/(35*d) + (2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*a*d)
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} + \frac{2\int\left(\frac{5a}{2}-a\cos(c+dx)\right)(a+a\cos(c+$$
$$= -\frac{4(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7ad} +$$
$$= \frac{38a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} - \frac{4(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} +$$
$$= \frac{152a^2\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{38a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105d} - \frac{4(a+a\cos$$

Mathematica [A] time = 0.171932, size = 81, normalized size = 0.7

$$\frac{a\left(735\sin\left(\frac{1}{2}(c+dx)\right)+175\sin\left(\frac{3}{2}(c+dx)\right)+63\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx}}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(735*Sin[(c + d*x)/2] + 175*
Sin[(3*(c + d*x))/2] + 63*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c + d*x))/2]))/
(420*d)
```

Maple [A] time = 0.848, size = 86, normalized size = 0.7

$$\frac{4a^2\sqrt{2}}{105d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(60\left(\cos\left(1/2\,dx+c/2\right)\right)^6-12\left(\cos\left(1/2\,dx+c/2\right)\right)^4+19\left(\cos\left(1/2\,dx+c/2\right)\right)^2+38\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 4/105*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(60*cos(1/2*d*x+1/2*c)^6-12
*cos(1/2*d*x+1/2*c)^4+19*cos(1/2*d*x+1/2*c)^2+38)*2^(1/2)/(cos(1/2*d*x+1/2*
c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.9518, size = 93, normalized size = 0.8

$$\frac{\left(15\sqrt{2}a\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+63\sqrt{2}a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+175\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+735\sqrt{2}a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/420*(15*sqrt(2)*a*sin(7/2*d*x + 7/2*c) + 63*sqrt(2)*a*sin(5/2*d*x + 5/2*c
) + 175*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 735*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*sqrt(a)/d
```

Fricas [A] time = 1.54267, size = 186, normalized size = 1.6

$$\frac{2\left(15\,a\cos\left(dx+c\right)^{3}+39\,a\cos\left(dx+c\right)^{2}+52\,a\cos\left(dx+c\right)+104\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*a*cos(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 52*a*cos(d*x + c) + 104*
a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.106 $\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=86

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (8*a^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(5*d) + (2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(5*d)
```

Rubi [A] time = 0.0653159, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2647, 2646}

$$\frac{8a^2\sin(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(5*d) + (2*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/
(5*d)
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{3/2}\,dx=\frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{3}{5}\int (a+a\cos(c+dx))^{3/2}\,dx$$

$$=\frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d}+\frac{2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}($$

$$=\frac{8a^2\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}}+\frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d}+\frac{2(a+a\cos(c+$$

Mathematica [A] time = 0.101397, size = 67, normalized size = 0.78

$$\frac{a\left(20\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(20*Sin[(c + d*x)/2] + 5*Sin
[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(10*d)
```

Maple [A] time = 0.744, size = 71, normalized size = 0.8

$$\frac{4a^2\sqrt{2}}{5d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^4+\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 4/5*cos(1/2*d*x+1/2*c)*a^2*sin(1/2*d*x+1/2*c)*(2*cos(1/2*d*x+1/2*c)^4+cos(1
/2*d*x+1/2*c)^2+2)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.94805, size = 72, normalized size = 0.84

$$\frac{\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*(sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 2
0*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.55303, size = 146, normalized size = 1.7

$$\frac{2\left(a\cos\left(dx+c\right)^2+3\,a\cos\left(dx+c\right)+6\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 6*a)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.107 $\int (a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=59

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

```
[Out] (8*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0293274, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{8a^2\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*Sqrt[a + a*Cos[c
 + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}\,dx &= \frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}(4a)\int \sqrt{a+a\cos(c+dx)}\,dx\\ &= \frac{8a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}+\frac{2a\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0640326, size = 55, normalized size = 0.93

$$\frac{a\left(9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(9*Sin[(c + d*x)/2] + Sin[(3
*(c + d*x))/2]))/(3*d)
```

Maple [A] time = 0.83, size = 58, normalized size = 1.

$$\frac{4a^2\sqrt{2}}{3d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 4/3*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(cos(1/2*d*x+1/2*c)^2+2)*2^(1
/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.92992, size = 51, normalized size = 0.86

$$\frac{\left(\sqrt{2}a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+9\sqrt{2}a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*sqr
t(a)/d
```

Fricas [A] time = 1.54189, size = 117, normalized size = 1.98

$$\frac{2\left(a\cos\left(dx+c\right)+5a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(d*x + c) + 5*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x
 + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + a)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2), x)
```

3.108 $\int (a + a\cos(c + dx))^{3/2} \sec(c + dx)\, dx$

Optimal. Leaf size=66

$$\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}} + \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2
*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.106958, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2763, 21, 2773, 206}

$$\frac{2a^2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}} + \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2
*a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}\sec(c+dx)\,dx &= \frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+2\int \frac{\left(\frac{a^2}{2}+\frac{1}{2}a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx\\ &= \frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+a\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx\\ &= \frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.07092, size = 65, normalized size = 0.98

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcTanh[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*Sin[(c + d*x)/2]))/d
```

Maple [B] time = 2.353, size = 207, normalized size = 3.1

$$\frac{1}{d}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}+\ln\left(-4\,\frac{\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}-a\sqrt{2}\,\mathrm{c}}{-2\,\cos\left(1/2\,dx+c/2\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c),x)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*
a))*a+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1
/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/sin(1/2*d*x+1/2*c)/(co
s(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [B] time = 1.63034, size = 344, normalized size = 5.21

$$\frac{(a\cos(dx+c)+a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\sqrt{a\cos(dx+c)+a}a\sin}{2(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((a*cos(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*a*sin(d*
x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 6.06331, size = 201, normalized size = 3.05

$$\frac{\frac{2\sqrt{2}a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{a^{\frac{5}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (2*sqrt(2)*a^2*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + a^
(5/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a))/d
```

3.109 $\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=65

$$\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (3*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a
^2*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.118048, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 21, 2773, 206}

$$\frac{a^2\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (3*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a
^2*Tan[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx &= \frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - a\int \frac{\left(-\frac{3a}{2}-\frac{3}{2}a\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx \\
&= \frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}(3a)\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx \\
&= \frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= \frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^2\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.115174, size = 81, normalized size = 1.25

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+3\sqrt{2}\cos(c+dx)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]*(3*Sqrt[2]*ArcT
anh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 2.273, size = 381, normalized size = 5.9

$$\frac{1}{d}\sqrt{a}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-6\,a\left(\ln\left(-4\,\frac{\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}-a\sqrt{2}\cos\left(1/2\,dx+c/2\right)+2\,a}{-2\,\cos\left(1/2\,dx+c/2\right)+\sqrt{2}}\right)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^2,x)
```

```
[Out] a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-6*a*(ln(-4/(-2*
cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a)))*sin(1/2*d*x+1/2*c)^2+2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
+3*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*ln(4/(2*cos(1/2*d*x+1/
2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos
(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+1/2*
c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 2.13424, size = 1774, normalized size = 27.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(2)*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 6*sqrt(2)*a*cos(5
/2*d*x + 5/2*c)*sin(2*d*x + 2*c) + (2*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 6*sq
rt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 + (2*sqrt(2)*a*sin(
3/2*d*x + 3/2*c) + 6*sqrt(2)*a*sin(1/2*d*x + 1/2*c) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c
)^2 - 4*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) -
 2*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) - 5*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
))*cos(2*d*x + 2*c) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 2*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin
(7/2*d*x + 7/2*c) - 6*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(5/2*d*x
+ 5/2*c) + 2*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + sqrt(2)*a*cos(1/2*d*x + 1/
2*c))*sin(2*d*x + 2*c))*sqrt(a)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [B] time = 1.65082, size = 387, normalized size = 5.95

$$\frac{3\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\sqrt{a\cos\left(dx\right.}}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*
sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
```

```
c) + a)*a*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.20447, size = 339, normalized size = 5.22

$$\frac{\sqrt{2}a^{\frac{7}{2}}\left(\frac{3\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\sqrt{2}|a|-6\,a\right|}\right)}{a|a|}+\frac{8\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^4-6\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\right.\right.}\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*a^(7/2)*(3*sqrt(2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*
abs(a) - 6*a))/(a*abs(a)) + 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^2 - a)/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a))/d
```

3.110 $\int (a + a\cos(c + dx))^{3/2} \sec^3(c + dx)\, dx$

Optimal. Leaf size=106

$$\frac{7a^2\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (7*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (7*a^2*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sec[c + d*x]*T
an[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.175759, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 21, 2772, 2773, 206}

$$\frac{7a^2\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (7*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (7*a^2*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sec[c + d*x]*T
an[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx &= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}-\frac{1}{2}a\int\frac{\left(-\frac{7a}{2}-\frac{7}{2}a\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx\\ &= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}+\frac{1}{4}(7a)\int\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\,dx\\ &= \frac{7a^2\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}+\frac{1}{8}(7a)\int\sqrt{a+a\cos(c+dx)}\ldots\\ &= \frac{7a^2\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}}-\frac{\left(7a^2\right)\text{Subst}\left(\int\frac{1}{a-x^2}\ldots\right.}{4\ldots}\\ &= \frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d}+\frac{7a^2\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sec(c+dx)\tan(\ldots}{2d\sqrt{a+a\cos(c+\ldots}}\end{aligned}$$

Mathematica [A] time = 0.219935, size = 97, normalized size = 0.92

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+7\sin\left(\frac{3}{2}(c+dx)\right)+7\sqrt{2}\cos^2(c+dx)\tanh^{-1}\ldots\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^2*(7*Sqrt[2]*Ar
cTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - 3*Sin[(c + d*x)/2] + 7*Sin
[(3*(c + d*x))/2]))/(8*d)
```

Maple [B] time = 2.506, size = 545, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(28*a*(ln(-4/
(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
```

```
*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-28*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)-28*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-28*ln(4/(2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^2+18*a^(1/2)*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)+7*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))*a+7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/(2*cos(1/2*d*x+1/2*c)
-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*d*
x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 5.75661, size = 4342, normalized size = 40.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
+ 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*
sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a
*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/
2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 4*a*sin(5/2*d*x + 5/2*c) - 12
*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*c)^2 + 4
*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*s
in(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c)^2 + (7*sqrt(2)*a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) + 4*a*sin(5/2*d*x + 5/2*c) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*
d*x + 1/2*c))*sin(4*d*x + 4*c)^2 - 160*a*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2
*c) - 168*a*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 72*a*cos(3/2*d*x + 3/2*
c)*sin(2*d*x + 2*c) + 4*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*
sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 12*a*sin(3/
```

```
2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c)^2 + 7*sqrt(2)*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) + 4*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*
cos(13/2*d*x + 13/2*c) - 12*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos
(11/2*d*x + 11/2*c) - 48*(a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c))*cos(9/
2*d*x + 9/2*c) + 2*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
+ 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 2*(7*sqrt(2)*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/
2*c))*cos(2*d*x + 2*c) + 40*a*sin(7/2*d*x + 7/2*c) + 2*(4*a*cos(2*d*x + 2*c
) + 23*a)*sin(5/2*d*x + 5/2*c) + 6*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*
x + 1/2*c))*cos(4*d*x + 4*c) + 4*(7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c)
+ 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 6*
a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) - 4*(a
*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(13/2*d*x + 13/2*c) + 12*(
a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(11/2*d*x + 11/2*c) + 48*
(a*cos(4*d*x + 4*c) + 2*a*cos(2*d*x + 2*c) + a)*sin(9/2*d*x + 9/2*c) + 4*(4
*a*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 20*a*cos(7/2*d*x + 7/2*c) - 21*a
*cos(5/2*d*x + 5/2*c) - 9*a*cos(3/2*d*x + 3/2*c) + (7*sqrt(2)*a*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*sin(1/2*d*x + 1/2*c) + 2) + 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 7*sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - 12*a*sin(3/2*d*x + 3/2*c) - 56*a*sin(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*sin(4*d*x + 4*c) + 80*(2*a*cos(2*d*x + 2*c) + a)*sin(7/2*d*x
+ 7/2*c) + 8*(2*a*cos(2*d*x + 2*c)^2 + 2*a*sin(2*d*x + 2*c)^2 + 23*a*cos(2*
d*x + 2*c) + 11*a)*sin(5/2*d*x + 5/2*c) + 24*a*sin(3/2*d*x + 3/2*c) - 56*a*
sin(1/2*d*x + 1/2*c))*sqrt(a)/((sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(
2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*d)
```

Fricas [A] time = 1.69114, size = 425, normalized size = 4.01

$$\frac{7\left(a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,(7\,a\cos\,(d\ldots}{16\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(7*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(7*a*cos(d*x
+ c) + 2*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^3 + d*co
s(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.11094, size = 433, normalized size = 4.08

$$7\,a^{\frac{3}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\,\sqrt{2}+3\right)\right|\right)-7\,a^{\frac{3}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(7*a^(3/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 7*a^(3/2)*log(abs((sqrt(a)*tan(1/
2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
) + 4*sqrt(2)*(7*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a))^6*a^(5/2) - 95*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4*a^(7/2) + 53*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*a^(9/2) - 5*a^(11/2))/((sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/
2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

3.111 $\int (a + a\cos(c + dx))^{3/2}\sec^4(c + dx)\,dx$

Optimal. Leaf size=144

$$\frac{11a^2\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (11*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
 + (11*a^2*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (11*a^2*Sec[c + d
*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sec[c + d*x]^2*Tan
[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.236609, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 21, 2772, 2773, 206}

$$\frac{11a^2\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\tan(c+dx)\sec(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

```
[Out] (11*a^(3/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
 + (11*a^2*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (11*a^2*Sec[c + d
*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sec[c + d*x]^2*Tan
[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx = \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{1}{3}a\int \frac{\left(-\frac{11a}{2}-\frac{11}{2}a\cos(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(11a)\int \sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\,dx$$
$$= \frac{11a^2\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(11a)\int \sqrt{a+a}$$
$$= \frac{11a^2\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec^2(c+dx)\tan(c}{3d\sqrt{a+a\cos(c+d}}$$
$$= \frac{11a^2\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sec^2(c+dx)\tan(c}{3d\sqrt{a+a\cos(c+d}}$$
$$= \frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{11a^2\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\sec(c+dx)\tan(}{12d\sqrt{a+a\cos(c+}}$$

Mathematica [A] time = 0.341624, size = 110, normalized size = 0.76

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sqrt{a(\cos(c+dx)+1)}\left(54\sin\left(\frac{1}{2}(c+dx)\right)+11\left(\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)+66\sqrt{2}\right.}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(66*Sqrt[2]*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + 54*Sin[(c + d*x)/2] + 11*
(Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2])))/(96*d)
```

Maple [B] time = 2.765, size = 710, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(1/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-264*a*(ln(-
4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+132*(2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4-22*(16*a^(1/2)*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+9*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+9*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2
*c)^2+126*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+33*ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+33*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^
(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2
*a))*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/s
in(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.74485, size = 458, normalized size = 3.18

$$\frac{33\left(a\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\left(33a\right.}{96\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(33*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(33*a*cos(d*
x + c)^2 + 22*a*cos(d*x + c) + 8*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/
(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.88223, size = 541, normalized size = 3.76

$$33\,a^{\frac{3}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-a\left(2\,\sqrt{2}+3\right)\right|\right)-33\,a^{\frac{3}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/48*(33*a^(3/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 33*a^(3/2)*log(abs((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) -
3))) + 4*sqrt(2)*(33*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^10*a^(5/2) - 303*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^8*a^(7/2) + 2394*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sq
rt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(9/2) - 1806*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(11/2) + 309*(sqrt(a)*tan(1
/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(13/2) - 19*a^(15
/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4
 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*
a + a^2)^3)/d
```

3.112 $\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=203

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}+\frac{46a^3\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{710a^3\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}-5$$

```
[Out] (284*a^3*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*x]]) + (710*a^3*Cos[c + d
*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (46*a^3*Cos[c + d*x]
^4*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*x]]) - (568*a^2*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(693*d) + (2*a^2*Cos[c + d*x]^4*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(11*d) + (284*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x]
)/(231*d)
```

Rubi [A] time = 0.362981, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2763, 2981, 2770, 2759, 2751, 2646}

$$\frac{2a^2\sin(c+dx)\cos^4(c+dx)\sqrt{a\cos(c+dx)+a}}{11d}+\frac{46a^3\sin(c+dx)\cos^4(c+dx)}{99d\sqrt{a\cos(c+dx)+a}}+\frac{710a^3\sin(c+dx)\cos^3(c+dx)}{693d\sqrt{a\cos(c+dx)+a}}-5$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (284*a^3*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*x]]) + (710*a^3*Cos[c + d
*x]^3*Sin[c + d*x])/(693*d*Sqrt[a + a*Cos[c + d*x]]) + (46*a^3*Cos[c + d*x]
^4*Sin[c + d*x])/(99*d*Sqrt[a + a*Cos[c + d*x]]) - (568*a^2*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(693*d) + (2*a^2*Cos[c + d*x]^4*Sqrt[a + a*Cos[c +
d*x]]*Sin[c + d*x])/(11*d) + (284*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x]
)/(231*d)
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\cos(c+dx))^{5/2}\,dx &= \frac{2a^2\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{11d}+\frac{2}{11}\int \cos^3(c+dx)\sqrt{a+a\,\mathrm{co}}\\ &= \frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\cos^4(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx}{11d}\\ &= \frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}+\frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\cos^4(c+}{}\\ &= \frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}+\frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\cos^4(c+}{}\\ &= \frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}+\frac{46a^3\cos^4(c+dx)\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}}-\frac{568a^2\sqrt{a+a}}{}\\ &= \frac{284a^3\sin(c+dx)}{99d\sqrt{a+a\cos(c+dx)}}+\frac{710a^3\cos^3(c+dx)\sin(c+dx)}{693d\sqrt{a+a\cos(c+dx)}}+\frac{46a^3\cos^4(c+dx)\,\mathrm{s}}{99d\sqrt{a+a\cos(}}\end{aligned}$$

Mathematica [A] time = 0.480828, size = 107, normalized size = 0.53

$$\frac{a^2\left(31878\sin\left(\frac{1}{2}(c+dx)\right)+8778\sin\left(\frac{3}{2}(c+dx)\right)+3465\sin\left(\frac{5}{2}(c+dx)\right)+1287\sin\left(\frac{7}{2}(c+dx)\right)+385\sin\left(\frac{9}{2}(c+dx)\right)+6\right.}{11088d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(31878*Sin[(c + d*x)/2] +
8778*Sin[(3*(c + d*x))/2] + 3465*Sin[(5*(c + d*x))/2] + 1287*Sin[(7*(c + d*
x))/2] + 385*Sin[(9*(c + d*x))/2] + 63*Sin[(11*(c + d*x))/2]))/(11088*d)
```

Maple [A] time = 0.794, size = 112, normalized size = 0.6

$$\frac{8a^3\sqrt{2}}{693d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(504\left(\cos\left(1/2\,dx+c/2\right)\right)^{10}-364\left(\cos\left(1/2\,dx+c/2\right)\right)^{8}+178\left(\cos\left(1/2\,dx+c/2\right)\right)^{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 8/693*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(504*cos(1/2*d*x+1/2*c)^10-
364*cos(1/2*d*x+1/2*c)^8+178*cos(1/2*d*x+1/2*c)^6+75*cos(1/2*d*x+1/2*c)^4+1
00*cos(1/2*d*x+1/2*c)^2+200)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.90858, size = 150, normalized size = 0.74

$$\frac{\left(63\sqrt{2}a^2\sin\left(\frac{11}{2}dx+\frac{11}{2}c\right)+385\sqrt{2}a^2\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+1287\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+3465\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+\right.}{11088\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11088*(63*sqrt(2)*a^2*sin(11/2*d*x + 11/2*c) + 385*sqrt(2)*a^2*sin(9/2*d*
x + 9/2*c) + 1287*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 3465*sqrt(2)*a^2*sin(5
/2*d*x + 5/2*c) + 8778*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 31878*sqrt(2)*a^2
*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.56357, size = 269, normalized size = 1.33

$$\frac{2\left(63a^2\cos\left(dx+c\right)^5+224a^2\cos\left(dx+c\right)^4+355a^2\cos\left(dx+c\right)^3+426a^2\cos\left(dx+c\right)^2+568a^2\cos\left(dx+c\right)+1136\right.}{693\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*a^2*cos(d*x + c)^5 + 224*a^2*cos(d*x + c)^4 + 355*a^2*cos(d*x + c
)^3 + 426*a^2*cos(d*x + c)^2 + 568*a^2*cos(d*x + c) + 1136*a^2)*sqrt(a*cos(
d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.113 $\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=146

$$\frac{208a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{832a^3\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}-\frac{4\sin(c+dx)(\cdots}{}$$

```
[Out] (832*a^3*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (208*a^2*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (26*a*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(105*d) - (4*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) +
 (2*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*a*d)
```

Rubi [A] time = 0.160045, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2647, 2646}

$$\frac{208a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{315d}+\frac{832a^3\sin(c+dx)}{315d\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)(a\cos(c+dx)+a)^{7/2}}{9ad}-\frac{4\sin(c+dx)(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (832*a^3*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]]) + (208*a^2*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(315*d) + (26*a*(a + a*Cos[c + d*x])^(3/2)*S
in[c + d*x])/(105*d) - (4*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*d) +
 (2*(a + a*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*a*d)
```


Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{2(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad} + \frac{2\int\left(\frac{7a}{2}-a\cos(c+dx)\right)(a+a\cos(c+dx}{9a}$$
$$= -\frac{4(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + \frac{2(a+a\cos(c+dx))^{7/2}\sin(c+dx)}{9ad} + \frac{13}{21}$$
$$= \frac{26a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d} - \frac{4(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{63d} + 2$$
$$= \frac{208a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} + \frac{26a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{105d} -$$
$$= \frac{832a^3\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{208a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{315d} + 26a(a+a\,c$$

Mathematica [A] time = 0.283529, size = 95, normalized size = 0.65

$$\frac{a^2\left(8190\sin\left(\frac{1}{2}(c+dx)\right)+2100\sin\left(\frac{3}{2}(c+dx)\right)+756\sin\left(\frac{5}{2}(c+dx)\right)+225\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(\right.}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(8190*Sin[(c + d*x)/2] + 2
100*Sin[(3*(c + d*x))/2] + 756*Sin[(5*(c + d*x))/2] + 225*Sin[(7*(c + d*x))
/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*d)
```

Maple [A] time = 0.766, size = 99, normalized size = 0.7

$$\frac{8a^3\sqrt{2}}{315d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(140\,(\cos(1/2\,dx+c/2))^8-20\,(\cos(1/2\,dx+c/2))^6+39\,(\cos(1/2\,dx+c/2))^4+52\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 8/315*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(140*cos(1/2*d*x+1/2*c)^8-2
0*cos(1/2*d*x+1/2*c)^6+39*cos(1/2*d*x+1/2*c)^4+52*cos(1/2*d*x+1/2*c)^2+104)
*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 2.00699, size = 127, normalized size = 0.87

$$\frac{\left(35\sqrt{2}a^2\sin\left(\frac{9}{2}dx+\frac{9}{2}c\right)+225\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+756\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+2100\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+8190\right.}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2520*(35*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 225*sqrt(2)*a^2*sin(7/2*d*x +
 7/2*c) + 756*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 2100*sqrt(2)*a^2*sin(3/2*d
*x + 3/2*c) + 8190*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.51386, size = 234, normalized size = 1.6

$$\frac{2\left(35\,a^2\cos\left(dx+c\right)^4+130\,a^2\cos\left(dx+c\right)^3+219\,a^2\cos\left(dx+c\right)^2+292\,a^2\cos\left(dx+c\right)+584\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\,s}{315\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*a^2*cos(d*x + c)^4 + 130*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x + c
)^2 + 292*a^2*cos(d*x + c) + 584*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.114 $\int \cos(c + dx)(a + a\cos(c + dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{64a^3 \sin(c + dx)}{21d\sqrt{a\cos(c + dx) + a}} + \frac{16a^2 \sin(c + dx)\sqrt{a\cos(c + dx) + a}}{21d} + \frac{2a\sin(c + dx)(a\cos(c + dx) + a)^{3/2}}{7d} + \frac{2\sin(c + dx)(a\cos}{7d}$$

```
[Out] (64*a^3*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(7*d) + (2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.0869676, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2647, 2646}

$$\frac{64a^3 \sin(c + dx)}{21d\sqrt{a\cos(c + dx) + a}} + \frac{16a^2 \sin(c + dx)\sqrt{a\cos(c + dx) + a}}{21d} + \frac{2a\sin(c + dx)(a\cos(c + dx) + a)^{3/2}}{7d} + \frac{2\sin(c + dx)(a\cos}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (64*a^3*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(7*d) + (2*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{5}{7}\int (a+a\cos(c+dx))^{5/2}\,dx$$
$$= \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d} + \frac{2(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + $$
$$= \frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{7d} + $$
$$= \frac{64a^3\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{21d} + 2a(a+a\cos$$

Mathematica [A] time = 0.225013, size = 84, normalized size = 0.72

$$\frac{a^2\left(315\sin\left(\frac{1}{2}(c+dx)\right)+77\sin\left(\frac{3}{2}(c+dx)\right)+3\left(7\sin\left(\frac{5}{2}(c+dx)\right)+\sin\left(\frac{7}{2}(c+dx)\right)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)}}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(315*Sin[(c + d*x)/2] + 77
*Sin[(3*(c + d*x))/2] + 3*(7*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2])))
/(84*d)
```

Maple [A] time = 0.694, size = 86, normalized size = 0.7

$$\frac{8a^3\sqrt{2}}{21d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(6\left(\cos(1/2\,dx+c/2)\right)^6+3\left(\cos(1/2\,dx+c/2)\right)^4+4\left(\cos(1/2\,dx+c/2)\right)^2+8\right)\frac{}{\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 8/21*cos(1/2*d*x+1/2*c)*a^3*sin(1/2*d*x+1/2*c)*(6*cos(1/2*d*x+1/2*c)^6+3*co
s(1/2*d*x+1/2*c)^4+4*cos(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*
a)^(1/2)/d
```

Maxima [A] time = 1.94901, size = 104, normalized size = 0.9

$$\frac{\left(3\sqrt{2}a^2\sin\left(\frac{7}{2}dx+\frac{7}{2}c\right)+21\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+77\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+315\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{84d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/84*(3*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 21*sqrt(2)*a^2*sin(5/2*d*x + 5/2
*c) + 77*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 315*sqrt(2)*a^2*sin(1/2*d*x + 1
/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.56016, size = 193, normalized size = 1.66

$$\frac{2\left(3 a^{2} \cos (dx+c)^{3}+12 a^{2} \cos (dx+c)^{2}+23 a^{2} \cos (dx+c)+46 a^{2}\right) \sqrt{a \cos (dx+c)+a} \sin (dx+c)}{21 (d \cos (dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*a^2*cos(d*x + c)^3 + 12*a^2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) +
46*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.115 $\int (a + a\cos(c + dx))^{5/2}\, dx$

Optimal. Leaf size=89

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (64*a^3*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(5*d)
```

Rubi [A] time = 0.0500422, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{64a^3\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{16a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15d}+\frac{2a\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (64*a^3*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (16*a^2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*(a + a*Cos[c + d*x])^(3/2)*Sin[c
+ d*x])/(5*d)
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{5/2}\,dx &= \frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{5}(8a)\int (a+a\cos(c+dx))^{3/2}\,dx\\ &= \frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2a(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}+\frac{1}{15}\left(32a^2\right)\\ &= \frac{64a^3\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}}+\frac{16a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15d}+\frac{2a(a+a\cos(c+dx))^{3/2}}{5d}\end{aligned}$$

Mathematica [A] time = 0.107517, size = 71, normalized size = 0.8

$$\frac{a^2\left(150\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(150*Sin[(c + d*x)/2] + 25
*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))/(30*d)
```

Maple [A] time = 0.764, size = 73, normalized size = 0.8

$$\frac{8a^3\sqrt{2}}{15d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\,(\cos(1/2\,dx+c/2))^4+4\,(\cos(1/2\,dx+c/2))^2+8\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 8/15*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(3*cos(1/2*d*x+1/2*c)^4+4*co
s(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [A] time = 1.89147, size = 81, normalized size = 0.91

$$\frac{\left(3\,\sqrt{2}a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\,\sqrt{2}a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\,\sqrt{2}a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/30*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.52733, size = 161, normalized size = 1.81

$$\frac{2\left(3\,a^2\cos(dx+c)^2+14\,a^2\cos(dx+c)+43\,a^2\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{15\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 43*a^2)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.116 $\int (a + a\cos(c + dx))^{5/2} \sec(c + dx)\, dx$

Optimal. Leaf size=98

$$\frac{14a^3\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d} + \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (1
4*a^3*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.202207, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2763, 2981, 2773, 206}

$$\frac{14a^3\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d} + \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (1
4*a^3*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[
c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \sqrt{a+a\cos(c+dx)}\left(\frac{3a^2}{2}+\frac{7}{2}a^2\,\text{co}\right.$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + a^2\int \sqrt{a+a\,\text{c}}$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} - \frac{(2a^3)\,\text{Subst}\left(\right.}{}$$

$$= \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{14a^3\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+a}}{3d}$$

Mathematica [A] time = 0.433532, size = 89, normalized size = 0.91

$$\frac{2a^2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{1-\cos(c+dx)}(\cos(c+dx)+8)+3\tanh^{-1}\left(\sqrt{1-\cos(c+dx)}\right)\right)}{3d\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (2*a^2*Sqrt[a*(1 + Cos[c + d*x])]*(3*ArcTanh[Sqrt[1 - Cos[c + d*x]]] + Sqrt
[1 - Cos[c + d*x]]*(8 + Cos[c + d*x]))*Tan[(c + d*x)/2])/(3*d*Sqrt[1 - Cos[
c + d*x]])
```

Maple [B] time = 2.497, size = 244, normalized size = 2.5

$$\frac{1}{3d}a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\sqrt{2}\sqrt{a(\sin(1/2\,dx+c/2))^2}\sqrt{a}(\sin(1/2\,dx+c/2))^2+18\sqrt{a}\sqrt{2}\sqrt{a(\sin(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c),x)
```

```
[Out] 1/3*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*2^(1/2)*(
a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+18*a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+3*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)/sin(1/2*d*x+1/2*c
)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [A] time = 1.70054, size = 386, normalized size = 3.94

$$\frac{3\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(a^2\cos(dx+c)+8\,a^2\right)}{6\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cos(d*x + c) + a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
 + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c
) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(a^2*cos(d*x + c) + 8*a^2)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 7.02483, size = 234, normalized size = 2.39

$$\frac{\frac{3\,a^{\frac{7}{2}}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-4\,\sqrt{2}|a|-6\,a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+4\,\sqrt{2}|a|-6\,a\right|}\right)}{|a|}+\frac{2\left(7\,\sqrt{2}a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,\sqrt{2}a^4\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^(7/2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x +
```

```
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/a
bs(a) + 2*(7*sqrt(2)*a^4*tan(1/2*d*x + 1/2*c)^2 + 9*sqrt(2)*a^4)*tan(1/2*d*
x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2))/d
```

3.117 $\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=92

$$\frac{a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}+\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (5*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a
^3*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c + d*x
]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.19817, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 2981, 2773, 206}

$$\frac{a^3\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sqrt{a\cos(c+dx)+a}}{d}+\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] (5*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a
^3*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c + d*x
]]*Tan[c + d*x])/d
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \frac{a^2\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{d} - a\int\left(-\frac{5a}{2}-\frac{1}{2}a\cos(c+dx)\right)\sqrt{a+a\,\mathrm{co}$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{2}\left(5a^2\right)\int\sqrt{a\,-}$$

$$= \frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\tan(c+dx)}{d} - \frac{\left(5a^3\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx}}{d}$$

Mathematica [C] time = 34.2941, size = 1547, normalized size = 16.82

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] ((-5/32 + (5*I)/32)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16
*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*
x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)
*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*S
qrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) +
(8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2
)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*
x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt
[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*(a*(1
+ Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(((-1 - I) + Sqrt[2]*E^((I/2)*
c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x
)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)) + I*E^((2*I)*(c + d*x)))^2) - (((5*I
)/8)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d
*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)
/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*d) - (((5
*I)/8)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 +
(d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x
)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*d) - (5*
(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*S
in[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(16*Sqrt[2]*d) - (5*(a*(1 + Cos[c
+ d*x]))^(5/2)*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)
/2]]*Sec[c/2 + (d*x)/2]^5)/(16*Sqrt[2]*d) + (Cos[(d*x)/2]*(a*(1 + Cos[c + d
*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[c/2])/(2*d) - (((5*I)/4)*ArcTan[((2*I)
*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2
+ 4*Sin[c/2]^2]]*(a*(1 + Cos[c + d*x]))^(5/2)*Cot[c/2]*Sec[c/2 + (d*x)/2]^5
)/(d*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (5*(a*(1 + Cos[c + d*x]))^(5
/2)*Csc[c/2]*Sec[c/2 + (d*x)/2]^5*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[
(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcT
an[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Co
```

```
s[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2])
)/(4*Sqrt[2]*d*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + (Cos[c/2]*(a*(1 + Cos[c + d
*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(2*d) + ((a*(1 + Cos[c + d*x
]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(8*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])) - ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(8*d*(Cos[c/2 +
(d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 2.658, size = 408, normalized size = 4.4

$$\frac{1}{d}a^{\frac{3}{2}}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\left(-8\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}-10\ln\left(-4\,\frac{\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}-a}{-2\cos\left(1/2\,dx+\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^2,x)
```

```
[Out] a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-8*a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-10*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a))*a-10*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/
2*c)^2+6*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+5*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+5*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)
)*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*
d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.72404, size = 427, normalized size = 4.64

$$\frac{5\left(a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\left(2\,a^{2}\cos\right.}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) -
```

```
 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*a^2*cos(d
*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2 + d
*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 4.60415, size = 371, normalized size = 4.03

$$\frac{4\sqrt{2}a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+5a^{\frac{5}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)-5a^{\frac{5}{2}}\log\left(\left|\left(\sqrt{a}\tan\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*sqrt(2)*a^3*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
+ 5*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 5*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d
*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) +
 4*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
 + a))^2*a^(7/2) - a^(9/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2
*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

3.118 $\int (a + a\cos(c + dx))^{5/2} \sec^3(c + dx)\, dx$

Optimal. Leaf size=106

$$\frac{9a^3 \tan(c + dx)}{4d\sqrt{a\cos(c + dx) + a}} + \frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} + \frac{a^2 \tan(c + dx)\sec(c + dx)\sqrt{a\cos(c + dx) + a}}{2d}$$

```
[Out] (19*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
 + (9*a^3*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Co
s[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.22241, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2762, 2980, 2773, 206}

$$\frac{9a^3 \tan(c + dx)}{4d\sqrt{a\cos(c + dx) + a}} + \frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d} + \frac{a^2 \tan(c + dx)\sec(c + dx)\sqrt{a\cos(c + dx) + a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] (19*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
 + (9*a^3*Tan[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Co
s[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \frac{a^2\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}a\int\left(-\frac{9a}{2}-\frac{5}{2}a\cos(c+d\right.$$

$$= \frac{9a^3\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{8}$$

$$= \frac{9a^3\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} - \frac{(1}{}$$

$$= \frac{19a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{9a^3\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+}}{}$$

Mathematica [C] time = 33.8539, size = 1693, normalized size = 15.97

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] ((-19/128 + (19*I)/128)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16
- 16*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2
)*d*x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((
3*I)*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*
I)*Sqrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x
) + (8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*
I)/2)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c
+ d*x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) +
Sqrt[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*(a
*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(((-1 - I) + Sqrt[2]*E^((I
/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c +
 d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)) + I*E^((2*I)*(c + d*x)))^2) - ((
(19*I)/32)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/
4 + (d*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 +
 (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*d)
- (((19*I)/32)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Si
n[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/
4 + (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*
d) - (19*(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] -
Sqrt[2]*Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(64*Sqrt[2]*d) - (19*(a*(
1 + Cos[c + d*x]))^(5/2)*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c
/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(64*Sqrt[2]*d) - (((19*I)/16)*ArcTan[(
(2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/
2]^2 + 4*Sin[c/2]^2]]*(a*(1 + Cos[c + d*x]))^(5/2)*Cot[c/2]*Sec[c/2 + (d*x)
/2]^5)/(d*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (19*(a*(1 + Cos[c + d*x
]))^(5/2)*Csc[c/2]*Sec[c/2 + (d*x)/2]^5*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] +
2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2
]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2
+ 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/
```

```
2]^2]))/(16*Sqrt[2]*d*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + ((a*(1 + Cos[c + d*x
]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(16*d*(Cos[c/2] - Sin[c/2])*(C
os[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a*(1 + Cos[c + d*x]))^(5/2)*
Sec[c/2 + (d*x)/2]^5*(11*Cos[c/2] - 9*Sin[c/2]))/(32*d*(Cos[c/2] - Sin[c/2]
)*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + ((a*(1 + Cos[c + d*x]))^(5/2
)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(16*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 +
 (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2
+ (d*x)/2]^5*(-11*Cos[c/2] - 9*Sin[c/2]))/(32*d*(Cos[c/2] + Sin[c/2])*(Cos[
c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 2.701, size = 545, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(76*a*(ln(-4/
(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-76*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(
a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)
+2*a))*a-76*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-44*a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))*sin(1/2*d*x+1/2*c)^2+19*ln(-4/(-2*cos(1/2
*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1
/2)*cos(1/2*d*x+1/2*c)+2*a))*a+19*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1
/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a
))*a+26*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/(2*cos(1/2*d*x+1/2*
c)+2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*
d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [B] time = 24.9808, size = 4950, normalized size = 46.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(150*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 154*sqrt(2)*
a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - (3*sqrt(2)*a^2*sin(7/2*d*x + 7
/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*
x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt
(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x +
```

4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d
*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - (3*
sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) + 5*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) - 17*
sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2))*sin(4*d*x + 4*c)^2 + 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c)
+ 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*s
in(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
- 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 1
9*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*c
os(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)
^2 - 3*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(
15/2*d*x + 15/2*c) - 5*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*
d*x + 2*c))*cos(13/2*d*x + 13/2*c) + 11*(sqrt(2)*a^2*sin(4*d*x + 4*c) + 2*s
qrt(2)*a^2*sin(2*d*x + 2*c))*cos(11/2*d*x + 11/2*c) + 45*(sqrt(2)*a^2*sin(4
*d*x + 4*c) + 2*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/2*d*x + 9/2*c) - (11*sq
rt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 38*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - 4*(17*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 55*sqrt(2)*a^2*sin(
1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(4*sqrt(2)*a^2*co
s(2*d*x + 2*c) + 27*sqrt(2)*a^2)*sin(7/2*d*x + 7/2*c) + (20*sqrt(2)*a^2*cos
(2*d*x + 2*c) + 87*sqrt(2)*a^2)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c) - 2*
(11*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 99*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c)

```
+ 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*a^2*log(2*cos(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*s
qrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 38*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2
*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*
sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(15/2*d*x + 15/2*c) + 5*(sqr
t(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*s
in(13/2*d*x + 13/2*c) - 11*(sqrt(2)*a^2*cos(4*d*x + 4*c) + 2*sqrt(2)*a^2*co
s(2*d*x + 2*c) + sqrt(2)*a^2)*sin(11/2*d*x + 11/2*c) - 45*(sqrt(2)*a^2*cos(
4*d*x + 4*c) + 2*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(9/2*d*x +
9/2*c) - (12*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) + 20*sqrt(2)
*a^2*sin(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 75*sqrt(2)*a^2*cos(7/2*d*x + 7
/2*c) - 77*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c) - 45*sqrt(2)*a^2*cos(3/2*d*x +
3/2*c) - 11*sqrt(2)*a^2*cos(1/2*d*x + 1/2*c) - 4*(17*sqrt(2)*a^2*sin(3/2*d*
x + 3/2*c) + 55*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*
d*x + 2*c))*sin(4*d*x + 4*c) - 6*(2*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 2*sqrt
(2)*a^2*sin(2*d*x + 2*c)^2 + 27*sqrt(2)*a^2*cos(2*d*x + 2*c) + 13*sqrt(2)*a
^2)*sin(7/2*d*x + 7/2*c) - 2*(10*sqrt(2)*a^2*cos(2*d*x + 2*c)^2 + 10*sqrt(2
)*a^2*sin(2*d*x + 2*c)^2 + 87*sqrt(2)*a^2*cos(2*d*x + 2*c) + 41*sqrt(2)*a^2
)*sin(5/2*d*x + 5/2*c) + 2*(45*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) + 11*sqrt(2
)*a^2*cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c))*sqrt(a)/((2*(2*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(
4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2
 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.66186, size = 439, normalized size = 4.14

$$\frac{19\left(a^2\cos(dx+c)^3+a^2\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(11a^2c\right.}{16\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(19*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x +
c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(11*a^2*
cos(d*x + c) + 2*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 5.20708, size = 447, normalized size = 4.22

$$\frac{\sqrt{2}a^{\frac{11}{2}}\left(\frac{19\sqrt{2}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)}{a^{2}|a|}+\frac{8\left(19\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{6}-171\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}\right.}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*a^(11/2)*(19*sqrt(2)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/(a^2*abs(a)) + 8*(19*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^6 - 171*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqr
t(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a + 89*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^2 - 9*a^3)/(((sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*a^2))/d
```

3.119 $\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{25a^3\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{13a^3\tan(c+}{12d\sqrt{a\cos(}}$$

```
[Out] (25*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
 + (25*a^3*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (13*a^3*Sec[c + d
*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c +
 d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.282293, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 2980, 2772, 2773, 206}

$$\frac{25a^3\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}+\frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{13a^3\tan(c+}{12d\sqrt{a\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] (25*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
 + (25*a^3*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (13*a^3*Sec[c + d
*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c +
 d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
```

```
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \frac{a^2\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d} - \frac{1}{3}a\int\left(-\frac{13a}{2}-\frac{9}{2}a\cos(c+\right.$$

$$= \frac{13a^3\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sec^2(c+dx)\tan(c+}{3d}$$

$$= \frac{25a^3\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+}}{}$$

$$= \frac{25a^3\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+}}{}$$

$$= \frac{25a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{25a^3\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\sec(c+dx)\,t}{12d\sqrt{a+a\cos}}$$

Mathematica [C] time = 33.5679, size = 1825, normalized size = 12.67

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] ((-25/256 + (25*I)/256)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16
- 16*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2
)*d*x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((
3*I)*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*
I)*Sqrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x
) + (8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*
I)/2)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c
+ d*x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) +
Sqrt[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*(a
*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(((-1 - I) + Sqrt[2]*E^((I
/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c +
 d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)) + I*E^((2*I)*(c + d*x)))^2) - ((
(25*I)/64)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/
4 + (d*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 +
```

```
(d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*d)
- (((25*I)/64)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Si
n[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/
4 + (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[2]*
d) - (25*(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] -
Sqrt[2]*Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(128*Sqrt[2]*d) - (25*(a*
(1 + Cos[c + d*x]))^(5/2)*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[
c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(128*Sqrt[2]*d) - (((25*I)/32)*ArcTan
[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[
c/2]^2 + 4*Sin[c/2]^2]]*(a*(1 + Cos[c + d*x]))^(5/2)*Cot[c/2]*Sec[c/2 + (d*
x)/2]^5)/(d*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (25*(a*(1 + Cos[c + d
*x]))^(5/2)*Csc[c/2]*Sec[c/2 + (d*x)/2]^5*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2]
+ 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt
[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-
2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[
c/2]^2]))/(32*Sqrt[2]*d*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + ((a*(1 + Cos[c + d
*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(48*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*
x)/2])^3) + (5*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/
2])/(32*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2
) + (5*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(5*Cos[c/2] - 3*Si
n[c/2]))/(64*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/
2])) - ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(48*d*(Cos[c/2 +
 (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (5*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/
2 + (d*x)/2]^5*Sin[(d*x)/2])/(32*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2])^2) - (5*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x
)/2]^5*(5*Cos[c/2] + 3*Sin[c/2]))/(64*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 2.53, size = 709, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^4,x)
```

```
[Out] 1/6*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-600*a*(ln(-
4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+300*(2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+3*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+3*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-450*ln(4/(2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-450*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a))*a-736*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))*sin(1/2*d*x+
1/2*c)^2+75*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+75*ln(-4/(-2*cos(
1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2
^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+234*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)
^2)^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^
3/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.73438, size = 471, normalized size = 3.27

$$\frac{75\left(a^2\cos(dx+c)^4+a^2\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\left(75\right.}{96\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(75*(a^2*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x +
c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(75*a^2*
cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 8*a^2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.45762, size = 541, normalized size = 3.76

$$\frac{75\,a^{\frac{5}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\,\sqrt{2}+3\right)\right|\right)-75\,a^{\frac{5}{2}}\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/48*(75*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 75*a^(5/2)*log(abs((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) -
```

```
3))) + 4*sqrt(2)*(75*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1
/2*c)^2 + a))^10*a^(7/2) - 1125*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^8*a^(9/2) + 6174*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(11/2) - 4314*(sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(13/2) + 807*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(15/2) - 49*a^(
17/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))
^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*a + a^2)^3)/d
```

3.120 $\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx$

Optimal. Leaf size=182

$$\frac{163a^3\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{17a^3\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sec^3(c+d}{4d}$$

```
[Out] (163*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(64*
d) + (163*a^3*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + (163*a^3*Sec[
c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (17*a^3*Sec[c + d*
x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c
+ d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.344594, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2762, 2980, 2772, 2773, 206}

$$\frac{163a^3\tan(c+dx)}{64d\sqrt{a\cos(c+dx)+a}}+\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{64d}+\frac{17a^3\tan(c+dx)\sec^2(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\tan(c+dx)\sec^3(c+d}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

```
[Out] (163*a^(5/2)*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(64*
d) + (163*a^3*Tan[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]]) + (163*a^3*Sec[
c + d*x]*Tan[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x]]) + (17*a^3*Sec[c + d*
x]^2*Tan[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c
+ d*x]]*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx &= \frac{a^2\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+dx)}{4d}-\frac{1}{4}a\int\left(-\frac{17a}{2}-\frac{13}{2}a\cos(c+d\right.\\
&=\frac{17a^3\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sqrt{a+a\cos(c+dx)}\sec^3(c+dx)\tan(c+dx)}{4d}\\
&=\frac{163a^3\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}}+\frac{17a^3\sec^2(c+dx)\tan(c+dx)}{24d\sqrt{a+a\cos(c+dx)}}+\frac{a^2\sqrt{a+a\cos}}{}\\
&=\frac{163a^3\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{163a^3\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}}+\frac{17a^3\sec^2(c+dx)\tan}{24d\sqrt{a+a\cos(}}\\
&=\frac{163a^3\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{163a^3\sec(c+dx)\tan(c+dx)}{96d\sqrt{a+a\cos(c+dx)}}+\frac{17a^3\sec^2(c+dx)\tan}{24d\sqrt{a+a\cos(}}\\
&=\frac{163a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d}+\frac{163a^3\tan(c+dx)}{64d\sqrt{a+a\cos(c+dx)}}+\frac{163a^3\sec(c+dx)\tan}{96d\sqrt{a+a\cos(}}\end{aligned}$$

Mathematica [C] time = 34.9342, size = 2069, normalized size = 11.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

```
[Out] ((-163/2048 + (163*I)/2048)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) +
(16 - 16*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*
I)/2)*d*x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*
E^((3*I)*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4
+ 4*I)*Sqrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)
*d*x) + (8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(
((3*I)/2)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)
*(c + d*x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)
) + Sqrt[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*
x*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(((-1 - I) + Sqrt[2]*E
^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*
```

```
(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)) + I*E^((2*I)*(c + d*x)))^2)
- (((163*I)/512)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*
Sin[c/4 + (d*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin
[c/4 + (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(Sqrt[
2]*d) - (((163*I)/512)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sq
rt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4]
- Sin[c/4 + (d*x)/4])]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5)/(
Sqrt[2]*d) - (163*(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 - Sqrt[2]*Cos[c/2 + (d
*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(1024*Sqrt[2]*d)
 - (163*(a*(1 + Cos[c + d*x]))^(5/2)*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - S
qrt[2]*Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^5)/(1024*Sqrt[2]*d) - (((163*
I)/256)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sq
rt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*(a*(1 + Cos[c + d*x]))^(5/2)*Cot[c/2]
*Sec[c/2 + (d*x)/2]^5)/(d*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (163*(a
*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2]*Sec[c/2 + (d*x)/2]^5*(-(d*x*Cos[c/2]) +
 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2
] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(
d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[
c/2]^2 + 4*Sin[c/2]^2]))/(256*Sqrt[2]*d*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + ((
a*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(64*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^4) + ((a*(1 + Cos
[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(23*Cos[c/2] - 17*Sin[c/2]))/(384*d*
(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (43*(a
*(1 + Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(256*d*(Cos[c
/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + ((a*(1 + Cos
[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(163*Cos[c/2] - 77*Sin[c/2]))/(512*d
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + ((a*(1
+ Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(64*d*(Cos[c/2] +
 Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^4) + ((a*(1 + Cos[c +
d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(-23*Cos[c/2] - 17*Sin[c/2]))/(384*d*(Cos
[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (43*(a*(1
+ Cos[c + d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*Sin[(d*x)/2])/(256*d*(Cos[c/2]
+ Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + ((a*(1 + Cos[c +
 d*x]))^(5/2)*Sec[c/2 + (d*x)/2]^5*(-163*Cos[c/2] - 77*Sin[c/2]))/(512*d*(C
os[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 3.157, size = 872, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^5,x)
```

```
[Out] 1/24*a^(3/2)*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7824*a*(ln(
-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2
)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a)))*sin(1/2*d*x+1/2*c)^8-7824*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)+2*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2*ln(-4/(-2*cos
(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*
2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^6+1304*(11*a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+9*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))
*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*
c)+2*a))*a+9*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1
```

```
/2*c)^4+(-3912*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-3912*ln(-4/(-2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-9212*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2))*sin(1/2*d*x+1/2*c)^2+489*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2
))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/
2*c)+2*a))*a+489*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*
sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a+2094*a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^4
/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^4/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*
a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.75102, size = 512, normalized size = 2.81

$$\frac{489\left(a^2\cos(dx+c)^5+a^2\cos(dx+c)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+4\,(489\,a \cdots}{768\,(d\cos(dx+c)^5+d\cos(d \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/768*(489*(a^2*cos(d*x + c)^5 + a^2*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x
 + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x
+ c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(489*a
^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 184*a^2*cos(d*x + c) + 48*a^2)
*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^
4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 3.32993, size = 649, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/384*(489*a^(5/2)*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d
*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 489*a^(5/2)*log(abs((sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2)
 - 3))) + 4*sqrt(2)*(489*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^14*a^(7/2) - 10269*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^12*a^(9/2) + 69885*(sqrt(a)*tan(1/2*d*x + 1/2
*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*a^(11/2) - 259233*(sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*a^(13/2) + 209979
*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(1
5/2) - 55511*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2
+ a))^4*a^(17/2) + 6687*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2*a^(19/2) - 299*a^(21/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c)
- sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

3.121 $\int (a + a\cos(c+dx))^{7/2}\,dx$

Optimal. Leaf size=119

$$\frac{256a^4\sin(c+dx)}{35d\sqrt{a\cos(c+dx)+a}} + \frac{64a^3\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d} + \frac{24a^2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d} + \frac{2a\sin(c+dx)(a}{}$$

```
[Out] (256*a^4*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (64*a^3*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(35*d) + (24*a^2*(a + a*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(35*d) + (2*a*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.0682629, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{256a^4\sin(c+dx)}{35d\sqrt{a\cos(c+dx)+a}} + \frac{64a^3\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{35d} + \frac{24a^2\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{35d} + \frac{2a\sin(c+dx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(7/2),x]
```


```
[Out] (256*a^4*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (64*a^3*Sqrt[a + a
*Cos[c + d*x]]*Sin[c + d*x])/(35*d) + (24*a^2*(a + a*Cos[c + d*x])^(3/2)*Si
n[c + d*x])/(35*d) + (2*a*(a + a*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```


Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^{7/2}\,dx &= \frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}(12a)\int (a+a\cos(c+dx))^{5/2}\,dx\\
&= \frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2a(a+a\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{35}\left(96a^2\right)\int\\
&= \frac{64a^3\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d} + \frac{24a^2(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2a(a+a\cos}{}\\
&= \frac{256a^4\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{64a^3\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{35d} + \frac{24a^2(a+a\cos(c+dx))^{3/2}}{35d}
\end{aligned}$$

Mathematica [A] time = 0.252157, size = 83, normalized size = 0.7

$$\frac{a^3\left(1225\sin\left(\frac{1}{2}(c+dx)\right) + 245\sin\left(\frac{3}{2}(c+dx)\right) + 49\sin\left(\frac{5}{2}(c+dx)\right) + 5\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+}}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (a^3*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(1225*Sin[(c + d*x)/2] + 2
45*Sin[(3*(c + d*x))/2] + 49*Sin[(5*(c + d*x))/2] + 5*Sin[(7*(c + d*x))/2])
)/(140*d)
```

Maple [A] time = 0.746, size = 86, normalized size = 0.7

$$\frac{16\,a^4\sqrt{2}}{35\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(5\,(\cos(1/2\,dx+c/2))^6+6\,(\cos(1/2\,dx+c/2))^4+8\,(\cos(1/2\,dx+c/2))^2+16\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 16/35*a^4*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*(5*cos(1/2*d*x+1/2*c)^6+6*c
os(1/2*d*x+1/2*c)^4+8*cos(1/2*d*x+1/2*c)^2+16)*2^(1/2)/(cos(1/2*d*x+1/2*c)^
2*a)^(1/2)/d
```

Maxima [A] time = 1.9217, size = 104, normalized size = 0.87

$$\frac{\left(5\,\sqrt{2}a^3\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+49\,\sqrt{2}a^3\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+245\,\sqrt{2}a^3\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+1225\,\sqrt{2}a^3\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/140*(5*sqrt(2)*a^3*sin(7/2*d*x + 7/2*c) + 49*sqrt(2)*a^3*sin(5/2*d*x + 5/
2*c) + 245*sqrt(2)*a^3*sin(3/2*d*x + 3/2*c) + 1225*sqrt(2)*a^3*sin(1/2*d*x
+ 1/2*c))*sqrt(a)/d
```

Fricas [A] time = 1.58563, size = 194, normalized size = 1.63

$$\frac{2\left(5\,a^3\cos(dx+c)^3+27\,a^3\cos(dx+c)^2+71\,a^3\cos(dx+c)+177\,a^3\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{35\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*a^3*cos(d*x + c)^3 + 27*a^3*cos(d*x + c)^2 + 71*a^3*cos(d*x + c) +
177*a^3)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.122 $\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=174

$$\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{62\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}-\frac{148\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d) - (148*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*Co
s[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Cos[c + d*x
]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (62*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(105*a*d)
```

Rubi [A] time = 0.371551, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2778, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2\sin(c+dx)\cos^3(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\cos^2(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{62\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{105ad}-\frac{148\sin(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d) - (148*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]) - (2*Co
s[c + d*x]^2*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Cos[c + d*x
]^3*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (62*Sqrt[a + a*Cos[c + d
*x]]*Sin[c + d*x])/(105*a*d)
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
```

```
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)(-6a+a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx}{7a}$$

$$= -\frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)\left(2a^2-\frac{31}{2}a^2\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{35a^2}$$

$$= -\frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{2\int \frac{2a^2\cos(c+dx)-\frac{31}{2}a^2\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{35a^2}$$

$$= -\frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{62\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{105ad}$$

$$= -\frac{148\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + 62\sqrt{a}$$

$$= -\frac{148\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^3(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + 62\sqrt{a}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{148\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} - \frac{2\cos^2(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos}{7d}$$

Mathematica [A] time = 0.189554, size = 130, normalized size = 0.75

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(-525\sin\left(\frac{1}{2}(c+dx)\right)+175\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)-420\log\left(\cos\left(\right.\right.\right.}{210d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(-420*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 420*Log[
Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] - 525*Sin[(c + d*x)/2] + 175*Sin[(3*(c
 + d*x))/2] - 21*Sin[(5*(c + d*x))/2] + 15*Sin[(7*(c + d*x))/2]))/(210*d*Sq
rt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 1.724, size = 194, normalized size = 1.1

$$-\frac{\sqrt{2}}{105\,d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^{2}}\,(\sin(1/2\,dx+c/2))^{6}-336\,\sqrt{a}\sqrt{a\,(\sin(1/2\,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/105*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*a^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6-336*a^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+280*a^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-105*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/
2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.68245, size = 431, normalized size = 2.48

$$\frac{4\left(15\cos(dx+c)^{3}-3\cos(dx+c)^{2}+31\cos(dx+c)-43\right)\sqrt{a\cos(dx+c)+a}\,\sin(dx+c)+\frac{105\sqrt{2}(a\cos(dx+c)+a)\log\left(-\right.}{}}{210\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/210*(4*(15*cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 31*cos(d*x + c) - 43)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c) + 105*sqrt(2)*(a*cos(d*x + c) + a)*log(-(
cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) -
2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*co
s(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.6378, size = 159, normalized size = 0.91

$$-\frac{\sqrt{2}\left(\frac{105\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{8\left(35a^{3}+\left(23a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+28a^{3}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{7}{2}}}\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/105*sqrt(2)*(105*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*
d*x + 1/2*c)^2 + a)))/sqrt(a) + 8*(35*a^3 + (23*a^3*tan(1/2*d*x + 1/2*c)^2
+ 28*a^3)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^3/(a*tan(1/2*d*x + 1
/2*c)^2 + a)^(7/2))/d
```

3.123 $\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}+\frac{28\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d)) + (28*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C
os[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) - (2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```

Rubi [A] time = 0.239252, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2778, 2968, 3023, 2751, 2649, 206}

$$\frac{2\sin(c+dx)\cos^2(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{15ad}+\frac{28\sin(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d)) + (28*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*C
os[c + d*x]^2*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) - (2*Sqrt[a + a*
Cos[c + d*x]]*Sin[c + d*x])/(15*a*d)
```


Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\cos(c+dx)(-4a+a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-4a\cos(c+dx)+a\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15ad} - \frac{2\int \frac{\frac{a^2}{2}-7a^2\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{15a^2}$$

$$= \frac{28\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15ad} - \int$$

$$= \frac{28\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{15ad} + \frac{2\,\mathrm{S}}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{28\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\cos^2(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{2\sqrt{a}}{}$$

Mathematica [A] time = 0.164344, size = 118, normalized size = 0.84

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(60\sin\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+dx)\right)+3\sin\left(\frac{5}{2}(c+dx)\right)+30\log\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)-}{15d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*(30*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 30*Log[Cos
[(c + d*x)/4] + Sin[(c + d*x)/4]] + 60*Sin[(c + d*x)/2] - 5*Sin[(3*(c + d*x
))/2] + 3*Sin[(5*(c + d*x))/2]))/(15*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 1.327, size = 183, normalized size = 1.3

$$\frac{\sqrt{2}}{15d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}(\sin(1/2\,dx+c/2))^4-20\sqrt{a}\sqrt{a(\sin(1/2\,dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/15*cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+30*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)
^(1/2)-15*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a
))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.63261, size = 397, normalized size = 2.84

$$\frac{4\sqrt{a\cos(dx+c)+a}\left(3\cos(dx+c)^2-\cos(dx+c)+13\right)\sin(dx+c)+\frac{15\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\,\mathrm{si}}{\sqrt{a}}}{\cos(dx+c)^2+2\cos(}\right.}{\sqrt{a}}}{30\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(4*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c)^2 - cos(d*x + c) + 13)*sin
(d*x + c) + 15*sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.62716, size = 157, normalized size = 1.12

$$\frac{\sqrt{2}\left(\frac{15\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{\sqrt{a}}+\frac{2\left(\left(17a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+20a^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+15a^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{5}{2}}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*(15*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/sqrt(a) + 2*((17*a^2*tan(1/2*d*x + 1/2*c)^2 + 20*a^2)*ta
n(1/2*d*x + 1/2*c)^2 + 15*a^2)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)
^2 + a)^(5/2))/d
```

3.124 $\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}-\frac{4\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d) - (4*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.125209, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2759, 2751, 2649, 206}

$$\frac{2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3ad}-\frac{4\sin(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d) - (4*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sqrt[a
 + a*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
```


Rule 2759

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin[e
 + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ
[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{a}{2}-a\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$
$$= -\frac{4\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} + \int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= -\frac{4\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(}{\sqrt{a+a\,\mathrm{co}}}\right.}{d}$$
$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{4\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3ad}$$

Mathematica [A] time = 0.112476, size = 104, normalized size = 1.

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(-3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)-3\log\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)+3\log\left(\sin\left(\frac{1}{4}(c+dx\right.\right.\right.}{3d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(-3*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 3*Log[Co
s[(c + d*x)/4] + Sin[(c + d*x)/4]] - 3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))
/2]))/(3*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 1.491, size = 135, normalized size = 1.3

$$\frac{1}{3d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\sqrt{2}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}\sqrt{a}\,(\sin(1/2\,dx+c/2))^2+3\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\,(\sin(1/}}{\cos(1/2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/3*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*2^(1/2)*(a*sin(1/
2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*sin(1/2*d*x+1/2*c)^2+3*2^(1/2)*ln(4/cos(1/2*d
*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*
x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.64812, size = 367, normalized size = 3.53

$$\frac{4\sqrt{a\cos(dx+c)+a}(\cos(dx+c)-1)\sin(dx+c)+\frac{3\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{6(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sin(d*x + c) + 3*sqrt(2)
*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c)
+ a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.77496, size = 107, normalized size = 1.03

$$-\frac{\sqrt{2}\left(\frac{4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{\frac{3}{2}}}+\frac{3\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{a}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(2)*(4*a*tan(1/2*d*x + 1/2*c)^3/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/
2) + 3*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^
2 + a)))/sqrt(a))/d
```

3.125 $\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.0510623, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2751, 2649, 206}

$$\frac{2\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.0433876, size = 53, normalized size = 0.73

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(ArcTanh[Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2]))/(d*S
qrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 1.441, size = 120, normalized size = 1.6

$$\frac{\sqrt{2}}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}-\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}+a}{\cos\left(1/2\,dx+c/2\right)}\right)a\right)a^{-\frac{3}{2}}\Bigg(\mathrm{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)-ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a))*a)/a^(3/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.62093, size = 339, normalized size = 4.64

$$\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}+4\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(a*cos(d*x + c) + a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2
 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/
(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [A] time = 2.08252, size = 100, normalized size = 1.37

$$\frac{\sqrt{2}\left(\frac{\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{a}}+\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c
)^2 + a)))/sqrt(a) + 2*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))/d
```

3.126 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rubi [A] time = 0.0220375, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2649, 206}

$$\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx &= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] time = 0.0116876, size = 40, normalized size = 0.87

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])
])
```

Maple [C] time = 0.16, size = 54, normalized size = 1.2

$$\frac{\sqrt{2}}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\mathrm{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},1\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a}}\left(\mathrm{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/csgn(cos(1/2*d*x+1/2*c))*cos(1/2
*d*x+1/2*c)*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

Maxima [B] time = 1.8937, size = 122, normalized size = 2.65

$$\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\right.}{2\,\sqrt{a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

Fricas [A] time = 1.58576, size = 347, normalized size = 7.54

$$\left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^{2}-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)}{2\,\sqrt{a}d},-\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}}{\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)
*sqrt(-1/a)/sin(d*x + c))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*cos(c + d*x) + a), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*cos(d*x + c) + a), x)
```

3.127 $\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rubi [A] time = 0.114229, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2780, 2649, 206, 2773}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) -
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]
)/(Sqrt[a]*d)
```

Rule 2780

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{a} - \int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] time = 0.0522652, size = 65, normalized size = 0.76

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \sqrt{2}\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*(ArcTanh[Sin[(c + d*x)/2]] - Sqrt[2]*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]])
*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 2.721, size = 224, normalized size = 2.6

$$-\frac{1}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\sqrt{2}\ln\left(4\frac{\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}+a}{\cos\left(1/2\,dx+c/2\right)}\right)-\ln\left(-4\frac{\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right.\right.}}{-2\cos\left(1/\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(4/cos(1/2*d*
x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-ln(-4/(-2*cos(1/2*d*x+
1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*c
os(1/2*d*x+1/2*c)+2*a))-ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))/a^(1/2
)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(a*cos(d*x + c) + a), x)
```

Fricas [B] time = 1.69845, size = 450, normalized size = 5.29

$$\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x
+ c) + 1)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*
cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [B] time = 4.89447, size = 219, normalized size = 2.58

$$\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{a}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{a}}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt
(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*
tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))/abs(a) + log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*
x + 1/2*c)^2 + a))^2)/sqrt(a))/d
```

3.128 $\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=108

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]/(Sqrt[a]*d)) + (
Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])
/(Sqrt[a]*d) + Tan[c + d*x]/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.212789, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2779, 2985, 2649, 206, 2773}

$$\frac{\tan(c+dx)}{d\sqrt{a\cos(c+dx)+a}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]/(Sqrt[a]*d)) + (
Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])
/(Sqrt[a]*d) + Tan[c + d*x]/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx &= \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(a-a\cos(c+dx))\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a} \\
&= \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{2a} + \int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx \\
&= \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin}{\sqrt{a+a}}\right.}{d} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\tan(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 26.0774, size = 1540, normalized size = 14.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((1/4 - I/4)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*I)*E^(
((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x) - (3
4 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*c + ((
5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sqrt[2]*
E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (8*I)*E
^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)*(c +
d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x)) - 1
6*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[2]*E^(
(4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2 + (d*
x)/2])/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((
I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))
 + I*E^((2*I)*(c + d*x)))^2*Sqrt[a*(1 + Cos[c + d*x])]) + (I*ArcTan[(Cos[c/
4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(-Cos[c/4 +
 (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*
x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + (I*ArcTan[(Cos[c/4 + (d*x)/
4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] +
 Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/2])/(Sqr
t[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) - (2*Cos[c/2 + (d*x)/2]*Log[Cos[c/4 + (d
*x)/4] - Sin[c/4 + (d*x)/4]])/(d*Sqrt[a*(1 + Cos[c + d*x])]) + (2*Cos[c/2 +
 (d*x)/2]*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]])/(d*Sqrt[a*(1 + Cos[
c + d*x])]) + (Cos[c/2 + (d*x)/2]*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt
[2]*Sin[c/2 + (d*x)/2]])/(2*Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + (Cos[c/
2 + (d*x)/2]*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2
]])/(2*Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + ((2*I)*ArcTan[((2*I)*Cos[c/2
] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[
```

```
c/2]^2]]*Cos[c/2 + (d*x)/2]*Cot[c/2])/(d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-2
 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) - (Sqrt[2]*Cos[c/2 + (d*x)/2]*Csc[c/2]*(-(
d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d
*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*
Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/S
qrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]))/(d*Sqrt[a*(1 + Cos[c + d*x])]*(4*Co
s[c/2]^2 + 4*Sin[c/2]^2)) + Cos[c/2 + (d*x)/2]/(d*Sqrt[a*(1 + Cos[c + d*x])
]*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - Cos[c/2 + (d*x)/2]/(d*Sqrt[a
*(1 + Cos[c + d*x])]*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 3.109, size = 462, normalized size = 4.3

$$\frac{1}{d}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,a\left(-2\sqrt{2}\ln\left(4\,\frac{\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}+a}{\cos\left(1/2\,dx+c/2\right)}\right)+\ln\left(-4\,\frac{\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(1/2\,dx\right.\right.}}{-2\,\mathrm{c}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(-2*2^(1/2)*ln(4/cos
(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))+ln(-4/(-2*cos(1
/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^
(1/2)*cos(1/2*d*x+1/2*c)+2*a))+ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a)))
*sin(1/2*d*x+1/2*c)^2+2*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a))*a+2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)
-ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*
c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-ln(4/(2*cos(1/2*d*x+1/2*c)
+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2
*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/(2*cos(1/2*d*x+
1/2*c)-2^(1/2))/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.74919, size = 647, normalized size = 5.99

$$\frac{\left(\cos\left(dx+c\right)^2+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\,\sqrt{a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\sqrt{2}\left(a\cos(dx+c)^2\right.}{}}{4\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/4*((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) + 4*sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(1/2),x)

[Out] Integral(sec(c + d*x)**2/sqrt(a*(cos(c + d*x) + 1)), x)

Giac [B] time = 5.07722, size = 392, normalized size = 3.63

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt{a}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{2\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{a}}-\frac{8\left(3\cdots\right.}{\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\cdots}\right.}\cdots\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] -1/4*sqrt(2)*(sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))/abs(a) + 2*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - a^(3/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d

3.129 $\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=147

$$-\frac{\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d) - Tan[c + d*x]/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (Sec[c + d*
x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.34026, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2779, 2984, 2985, 2649, 206, 2773}

$$-\frac{\tan(c+dx)}{4d\sqrt{a\cos(c+dx)+a}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\tan(c+dx)\sec(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]]
)])/(Sqrt[a]*d) - Tan[c + d*x]/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (Sec[c + d*
x]*Tan[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx &= \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(a-3a\cos(c+dx))\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a} \\
&= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\left(-\frac{7a^2}{2}+\frac{1}{2}a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a^2} \\
&= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{7\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{8a} - \\
&= -\frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \\
&= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\tan(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\sec(c+}{2d\sqrt{a+}}
\end{aligned}$$

Mathematica [C] time = 29.7051, size = 1791, normalized size = 12.18

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-7/16 + (7*I)/16)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16
*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*
x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)
*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*S
qrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) +
(8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2
```

```
)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*
x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt
[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/
2 + (d*x)/2])/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[
2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c
+ d*x)) + I*E^((2*I)*(c + d*x)))^2*Sqrt[a*(1 + Cos[c + d*x])]) - (((7*I)/4)
*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/
4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])
]*Cos[c/2 + (d*x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) - (((7*I)/4)*A
rcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4]
)/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*C
os[c/2 + (d*x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + (2*Cos[c/2 + (d
*x)/2]*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(d*Sqrt[a*(1 + Cos[c +
 d*x])]) - (2*Cos[c/2 + (d*x)/2]*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4
]])/(d*Sqrt[a*(1 + Cos[c + d*x])]) - (7*Cos[c/2 + (d*x)/2]*Log[2 - Sqrt[2]*
Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(8*Sqrt[2]*d*Sqrt[a*(1 +
Cos[c + d*x])]) - (7*Cos[c/2 + (d*x)/2]*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2]
- Sqrt[2]*Sin[c/2 + (d*x)/2]])/(8*Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) - (
((7*I)/2)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/
Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]*Cot[c/2])/(d*Sqr
t[a*(1 + Cos[c + d*x])]*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (7*Cos[c/
2 + (d*x)/2]*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Sin
[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*Co
s[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4
*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]))/(2*Sqrt[2]
*d*Sqrt[a*(1 + Cos[c + d*x])]*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + (Cos[c/2 + (
d*x)/2]*Sin[(d*x)/2])/(2*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2] - Sin[c/2])
*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 + (d*x)/2]*(-Cos[c
/2] + 3*Sin[c/2]))/(4*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2] - Sin[c/2])*(C
os[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c/2 + (d*x)/2]*Sin[(d*x)/2]
)/(2*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2]
 + Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 + (d*x)/2]*(Cos[c/2] + 3*Sin[c/2]))/(4
*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2]))
```

Maple [B] time = 3.047, size = 671, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/2*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*(8*2^(1/2)*ln(4
/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-7*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-7*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*
(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c
)+2*a)))*sin(1/2*d*x+1/2*c)^4+(-32*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+28*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/
2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1
/2*c)+2*a))*a+28*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-4*a^(1/2)*2^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))*sin(1/2*d*x+1/2*c)^2+8*2^(1/2)*ln(4/c
os(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-7*ln(-4/(-2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-7*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
```

```
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a))*a-2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(3/2)/(2*co
s(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2*c)+2^(1/2))^2/sin(1/2*d*x+1/
2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.71313, size = 684, normalized size = 4.65

$$\frac{7\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)-4\,\sqrt{a\cos\left(dx+\right.}}{16\left(ad\cos\left(dx+c\right)^{3}+ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(7*(cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*s
in(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x +
c) + a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*sqrt(2)*(a*cos(d*x + c)^3 + a*c
os(d*x + c)^2)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{3}(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [B] time = 4.71562, size = 501, normalized size = 3.41

$$\frac{\sqrt{2}\left(\frac{7\sqrt{2}\sqrt{a}\log\left(\frac{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+4\sqrt{2}|a|-6a\right|}\right)}{|a|}+\frac{8\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{a}}-\frac{8\left(17\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\right.\right.\right.}{}\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(7*sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) - s
qrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)
*abs(a) - 6*a))/abs(a) + 8*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(17*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 57*(sqrt(a)*tan(1/2*d*x + 1
/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 19*(sqrt(a)*tan(1/2
*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 3*a^(7/2))/
((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*
(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a
^2)^2)/d
```

3.130 $\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{7\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\tan(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (-9*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*Sqrt[a]*d)
 + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]
])])/(Sqrt[a]*d) + (7*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) - (Sec[c
 + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (Sec[c + d*x]^2*Tan
[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.488682, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2779, 2984, 2985, 2649, 206, 2773}

$$\frac{7\tan(c+dx)}{8d\sqrt{a\cos(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8\sqrt{a}d}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\tan(c+dx)\sec^2(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{\tan(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (-9*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*Sqrt[a]*d)
 + (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]
])])/(Sqrt[a]*d) + (7*Tan[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) - (Sec[c
 + d*x]*Tan[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (Sec[c + d*x]^2*Tan
[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{(a-5a\cos(c+dx))\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{6a}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\left(-\frac{21a^2}{2}+\frac{3}{2}a^2\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{12a^2}$$

$$= \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\left(\frac{27a^3}{4}-\frac{21}{4}\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{9\int\sqrt{a+\cdots}}{\cdots}$$

$$= \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{\sec^2(c+dx)\tan(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{9\,\mathrm{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8\sqrt{a}d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{7\tan(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} - \frac{\sec\cdots}{12\cdots}$$

Mathematica [C] time = 28.422, size = 1921, normalized size = 10.61

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((9/32 - (9*I)/32)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*
I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x
) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*
c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sq
rt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (
8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)
*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x
)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[
2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2
 + (d*x)/2])/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2
]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c +
 d*x)) + I*E^((2*I)*(c + d*x)))^2*Sqrt[a*(1 + Cos[c + d*x])]) + (((9*I)/8)*
ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4
])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]
*Cos[c/2 + (d*x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) + (((9*I)/8)*Ar
cTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])
/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Co
s[c/2 + (d*x)/2])/(Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) - (2*Cos[c/2 + (d*
x)/2]*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(d*Sqrt[a*(1 + Cos[c +
d*x])]) + (2*Cos[c/2 + (d*x)/2]*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]
])/(d*Sqrt[a*(1 + Cos[c + d*x])]) + (9*Cos[c/2 + (d*x)/2]*Log[2 - Sqrt[2]*C
os[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(16*Sqrt[2]*d*Sqrt[a*(1 +
Cos[c + d*x])]) + (9*Cos[c/2 + (d*x)/2]*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2]
- Sqrt[2]*Sin[c/2 + (d*x)/2]])/(16*Sqrt[2]*d*Sqrt[a*(1 + Cos[c + d*x])]) +
(((9*I)/4)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])
/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]*Cot[c/2])/(d*Sq
rt[a*(1 + Cos[c + d*x])]*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) - (9*Cos[c
/2 + (d*x)/2]*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Si
n[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*C
os[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 +
4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]))/(4*Sqrt[2
]*d*Sqrt[a*(1 + Cos[c + d*x])]*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) + Cos[c/2 + (
d*x)/2]/(6*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*
x)/2])^3) - (Cos[c/2 + (d*x)/2]*Sin[(d*x)/2])/(4*d*Sqrt[a*(1 + Cos[c + d*x]
)]*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Co
s[c/2 + (d*x)/2]*(7*Cos[c/2] - 9*Sin[c/2]))/(8*d*Sqrt[a*(1 + Cos[c + d*x])]
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) - Cos[c/2
 + (d*x)/2]/(6*d*Sqrt[a*(1 + Cos[c + d*x])]*(Cos[c/2 + (d*x)/2] + Sin[c/2 +
 (d*x)/2])^3) - (Cos[c/2 + (d*x)/2]*Sin[(d*x)/2])/(4*d*Sqrt[a*(1 + Cos[c +
d*x])]*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) +
 (Cos[c/2 + (d*x)/2]*(-7*Cos[c/2] - 9*Sin[c/2]))/(8*d*Sqrt[a*(1 + Cos[c + d
*x])]*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 3.372, size = 875, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/6*cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*a*(16*2^(1/2)*ln
(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))-9*ln(-4/(
-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))-9*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2)
)*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2
*c)+2*a)))*sin(1/2*d*x+1/2*c)^6+(576*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/
```

```
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+168*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+
1/2*c)^2)^(1/2)-324*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-324*ln(
4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a)*sin(1/2*d*x+1/2*c)^4+(-288*2^(1
/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a-1
60*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+162*ln(-4/(-2*cos(1/2*d*x
+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-a*2^(1/2)*
cos(1/2*d*x+1/2*c)+2*a))*a+162*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)
*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*
a)*sin(1/2*d*x+1/2*c)^2+48*2^(1/2)*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)+a))*a+54*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)-27*ln(-4/(-2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)-a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*a-27*ln(4/(2*cos(1/2*
d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/
2)*cos(1/2*d*x+1/2*c)+2*a))*a)/a^(3/2)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^3/(2*
cos(1/2*d*x+1/2*c)+2^(1/2))^3/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(
1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.7805, size = 717, normalized size = 3.96

$$\frac{27\left(\cos\left(dx+c\right)^{4}+\cos\left(dx+c\right)^{3}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\,\sqrt{a\cos(dx+c)+a}\,\sqrt{a}(\cos(dx+c)-2)\sin(dx+c)+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)+4\,\sqrt{a\cos\left(\cdots\right.}}{96\left(ad\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(27*(cos(d*x + c)^4 + cos(d*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*
sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x +
 c) + a)*(21*cos(d*x + c)^2 - 2*cos(d*x + c) + 8)*sin(d*x + c) + 48*sqrt(2)
*(a*cos(d*x + c)^4 + a*cos(d*x + c)^3)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqr
t(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x +
 c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c
)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{4}\left(c+dx\right)}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [B] time = 4.61957, size = 609, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*sqrt(2)*(27*sqrt(2)*sqrt(a)*log(abs(2*(sqrt(a)*tan(1/2*d*x + 1/2*c) -
 sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqr
t(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(
2)*abs(a) - 6*a))/abs(a) + 48*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2)/sqrt(a) - 8*(165*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^10*sqrt(a) - 1323*(sqrt(a)*tan(1/2*
d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^8*a^(3/2) + 3906*(sqrt(a
)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^6*a^(5/2) - 21
18*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^
(7/2) + 393*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2*a^(9/2) - 31*a^(11/2))/((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

3.131 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=183

$$-\frac{13\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{10a^2d}-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{9\sin(c+dx)\cos^2(c+}{10ad\sqrt{a\cos(c+dx)}}$$

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x
])^(3/2)) + (31*Sin[c + d*x])/(5*a*d*Sqrt[a + a*Cos[c + d*x]]) + (9*Cos[c +
 d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]]) - (13*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(10*a^2*d)
```

Rubi [A] time = 0.399834, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2765, 2983, 2968, 3023, 2751, 2649, 206}

$$-\frac{13\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{10a^2d}-\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{9\sin(c+dx)\cos^2(c+}{10ad\sqrt{a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-15*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2
*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x
])^(3/2)) + (31*Sin[c + d*x])/(5*a*d*Sqrt[a + a*Cos[c + d*x]]) + (9*Cos[c +
 d*x]^2*Sin[c + d*x])/(10*a*d*Sqrt[a + a*Cos[c + d*x]]) - (13*Sqrt[a + a*Co
s[c + d*x]]*Sin[c + d*x])/(10*a^2*d)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos^2(c+dx)\left(3a-\frac{9}{2}a\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}dx}{2a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(-9a^2+\frac{39}{4}a^2\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}}{5a^3}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-9a^2\cos(c+dx)+\frac{39}{4}a^2\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}}{5a^3}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - \frac{13\sqrt{a+a\cos(c+dx)}\sin(c+}{10a^2d}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - 13$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + \frac{9\cos^2(c+dx)\sin(c+dx)}{10ad\sqrt{a+a\cos(c+dx)}} - 13$$

$$= -\frac{15\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{31\sin(c+dx)}{5ad\sqrt{a+a\cos(c+dx)}} + 9$$

Mathematica [A] time = 1.41112, size = 226, normalized size = 1.23

$$\cos^3\left(\frac{1}{2}(c+dx)\right)\left(200\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right) - 20\sin\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right) + 4\sin\left(\frac{5c}{2}\right)\cos\left(\frac{5dx}{2}\right) + 200\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right) - 20\cos\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(150*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 150*Log
[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] + 200*Cos[(d*x)/2]*Sin[c/2] - 20*Cos[
(3*d*x)/2]*Sin[(3*c)/2] + 4*Cos[(5*d*x)/2]*Sin[(5*c)/2] + 200*Cos[c/2]*Sin[
(d*x)/2] - 20*Cos[(3*c)/2]*Sin[(3*d*x)/2] + 4*Cos[(5*c)/2]*Sin[(5*d*x)/2] +
 5/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - 5/(Cos[(c + d*x)/4] + Sin[(c +
 d*x)/4])^2))/(10*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 1.523, size = 265, normalized size = 1.5

$$\frac{\sqrt{2}}{20\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-32\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}(\sin(1/2\,dx+c/2))^6 + 32\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}(\sin(1/\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/20/cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-32*a^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+32*a^(1/2)*(a*sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+75*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2
)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*sin(1/2*d*x+1/2*c)^2-80*a^(1/2)*(a*s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-75*ln(4/cos(1/2*d*x+1/2*c)*
(a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a+85*a^(1/2)*(a*sin(1/2*d*x+1/2
*c)^2)^(1/2))/a^(5/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.64834, size = 497, normalized size = 2.72

$$\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^2+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left(4\cos\left(dx+\right.\right.}{40\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(75*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(4*cos(d*x +
 c)^3 - 4*cos(d*x + c)^2 + 36*cos(d*x + c) + 49)*sqrt(a*cos(d*x + c) + a)*s
in(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.07941, size = 185, normalized size = 1.01

$$\frac{\frac{75\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\left(\left(\left(5\sqrt{2}a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+127\sqrt{2}a\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+175\sqrt{2}a\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+85\sqrt{2}a\right)\tan\left(\frac{1}{2}\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{5}{2}}}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/20*(75*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a)))/a^(3/2) + (((5*sqrt(2)*a*tan(1/2*d*x + 1/2*c)^2 + 127*sq
rt(2)*a)*tan(1/2*d*x + 1/2*c)^2 + 175*sqrt(2)*a)*tan(1/2*d*x + 1/2*c)^2 + 8
5*sqrt(2)*a)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(5/2))/d
```

3.132 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{7\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{13\sin(c+dx)}{3ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (11*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2)) - (13*Sin[c + d*x])/(3*a*d*Sqrt[a + a*Cos[c + d*x]]) + (7*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(6*a^2*d)
```

Rubi [A] time = 0.26108, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2968, 3023, 2751, 2649, 206}

$$\frac{7\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{6a^2d}+\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\cos^2(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-\frac{13\sin(c+dx)}{3ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (11*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x]
)^(3/2)) - (13*Sin[c + d*x])/(3*a*d*Sqrt[a + a*Cos[c + d*x]]) + (7*Sqrt[a +
 a*Cos[c + d*x]]*Sin[c + d*x])/(6*a^2*d)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(2a-\frac{7}{2}a\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{2a\cos(c+dx)-\frac{7}{2}a\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d} - \frac{\int \frac{-\frac{7a^2}{4}+\frac{13}{2}a^2\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{3a^3}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + \frac{7\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + \frac{7\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{6a^2d}$$

$$= \frac{11\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^2(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{13\sin(c+dx)}{3ad\sqrt{a+a\cos(c+dx)}} + 7\sqrt{}$$

Mathematica [A] time = 0.951746, size = 196, normalized size = 1.35

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-72\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)+8\sin\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)-72\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+8\cos\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)-\frac{}{\left(\cos\left(\frac{1}{4}(c+dx)\right.\right.}\right.}{6d(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(-66*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] + 66*Log[
Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] - 72*Cos[(d*x)/2]*Sin[c/2] + 8*Cos[(3*
d*x)/2]*Sin[(3*c)/2] - 72*Cos[c/2]*Sin[(d*x)/2] + 8*Cos[(3*c)/2]*Sin[(3*d*x
)/2] - 3/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 + 3/(Cos[(c + d*x)/4] + Si
```

```
n[(c + d*x)/4])^2))/(6*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 1.602, size = 234, normalized size = 1.6

$$\frac{\sqrt{2}}{12\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(16\,\sqrt{a}\sqrt{a\,(\sin\,(1/2\,dx+c/2))^{2}}\,(\sin\,(1/2\,dx+c/2))^{4}+8\,\sqrt{a}\sqrt{a\,(\sin\,(1/2\,dx+c/2))^{2}}\,(\sin\,(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/12/cos(1/2*d*x+1/2*c)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4+8*a^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2-33*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a))*a*sin(1/2*d*x+1/2*c)^2-27*a^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+33*ln(4/cos(1/2*d*x+1/2*c)*(a^(1/2)*(a*sin(1/2*d*x
+1/2*c)^2)^(1/2)+a))*a)/a^(5/2)/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)
^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.63858, size = 471, normalized size = 3.25

$$\frac{33\,\sqrt{2}\left(\cos\,(dx+c)^{2}+2\,\cos\,(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)+4\,\sqrt{a\cos\,(dx+}{24\left(a^{2}d\cos\,(dx+c)^{2}+2\,a^{2}d\cos\,(dx+c)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(33*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(
d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d
*x + c) + a)*(4*cos(d*x + c)^2 - 12*cos(d*x + c) - 19)*sin(d*x + c))/(a^2*d
*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.12237, size = 155, normalized size = 1.07

$$-\frac{\frac{\left(\left(3\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+46\sqrt{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+27\sqrt{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}}+\frac{33\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/12*(((3*sqrt(2)*tan(1/2*d*x + 1/2*c)^2 + 46*sqrt(2))*tan(1/2*d*x + 1/2*c
)^2 + 27*sqrt(2))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2)
 + 33*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a)))/a^(3/2))/d
```

3.133 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (2*Sin
[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.133844, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2751, 2649, 206}

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2\sin(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (2*Sin
[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{-\frac{3a}{2}+2a\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{7\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{7\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}$$
$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{2\sin(c+dx)}{ad\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.45932, size = 164, normalized size = 1.56

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(16\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)+16\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{1}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{1}{\left(\sin\left(\frac{1}{4}(c+dx)\right)+\cos\left(\frac{1}{4}(c+dx)\right)\right)^2}+14\right.}{2d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(14*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 14*Log[C
os[(c + d*x)/4] + Sin[(c + d*x)/4]] + 16*Cos[(d*x)/2]*Sin[c/2] + 16*Cos[c/2
]*Sin[(d*x)/2] + (Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^(-2) - (Cos[(c + d*x
)/4] + Sin[(c + d*x)/4])^(-2)))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 1.456, size = 174, normalized size = 1.7

$$-\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(7\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}+2a}{\cos(1/2\,dx+c/2)}\right)(\cos(1/2\,dx+c/2))^2a-8\sqrt{2}\sqrt{a(\sin(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*2^(1/2)*ln(2*(2*a
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+
1/2*c)^2*a-8*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2
*c)^2-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(5/2)/sin(1/2*d*x+1
/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.64861, size = 440, normalized size = 4.19

$$\frac{7\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+4\sqrt{a\cos\left(dx+\right.}}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*co
s(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*x
 + c) + a)*(4*cos(d*x + c) + 5)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2
*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.05735, size = 138, normalized size = 1.31

$$\frac{\frac{\left(\frac{\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a}+\frac{9\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}+\frac{7\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a + 9*sqrt(2)/a)*tan(1/2*d*x + 1/2*c)/
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a) + 7*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x
 + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(3/2))/d
```

3.134 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0590195, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2750, 2649, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$
$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}$$
$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.0954928, size = 54, normalized size = 0.7

$$\frac{3\cos^3\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{1}{2}\sin(c+dx)}{d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 - Sin[c + d*x]/2)/(d*(a*(1
+ Cos[c + d*x]))^(3/2))
```

Maple [B] time = 1.43, size = 140, normalized size = 1.8

$$\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(3\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}+2a}{\cos(1/2\,dx+c/2)}\right)(\cos(1/2\,dx+c/2))^2a - \sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2)*ln(2*(2*a^
(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1
/2*c)^2*a-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(5/2)/sin(1/2*d
*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.59655, size = 412, normalized size = 5.35

$$\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\sqrt{a\cos(dx}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*co
s(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*sqrt(a*cos(d*x
 + c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.80432, size = 109, normalized size = 1.42

$$-\frac{\frac{3\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a)))/a^(3/2) + sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(
1/2*d*x + 1/2*c)/a^2)/d
```

3.135 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[
2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0389913, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])]/(2*Sqrt[
2]*a^(3/2)*d) + Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx &= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}\\
&= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{2ad}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0660621, size = 63, normalized size = 0.82

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\tan\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + Tan[(c +
d*x)/2]))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 1.49, size = 138, normalized size = 1.8

$$\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\sqrt{2}\ln\left(2\,\frac{2\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a+\sqrt{a}\sqrt{2}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2^(1/2)*ln(2
*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2
*d*x+1/2*c)^2*a+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x
+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.68665, size = 409, normalized size = 5.31

$$\frac{\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\sqrt{a\cos(dx+c)}}{8\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*x
+ c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(
d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*sqrt(a*cos(d*x +
```

```
c) + a)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos{(c+dx)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*cos(c + d*x) + a)**(-3/2), x)
```

Giac [A] time = 2.10006, size = 109, normalized size = 1.42

$$-\frac{\frac{\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x +
1/2*c)^2 + a)))/a^(3/2) - sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/
2*d*x + 1/2*c)/a^2)/d
```

3.136 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) -
(5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.220885, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2766, 2985, 2649, 206, 2773}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) -
(5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*S
qrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(2a-\frac{1}{2}a\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,dx}{a^2} - \frac{5\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} + \frac{5\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,\right.}{2ad}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [C] time = 22.7986, size = 1787, normalized size = 15.68

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((-1 + I)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*I)*E^(((3
*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x) - (34 -
 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*c + ((5*I
)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sqrt[2]*E^(
(4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (8*I)*E^((
I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)*(c + d*x
)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x)) - 16*S
qrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[2]*E^((4*
I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2 + (d*x)/
2]^3)/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((I
/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))
+ I*E^((2*I)*(c + d*x)))^2*(a*(1 + Cos[c + d*x]))^(3/2)) - ((2*I)*Sqrt[2]*A
rcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4]
)/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*
Cos[c/2 + (d*x)/2]^3)/(d*(a*(1 + Cos[c + d*x]))^(3/2)) + (5*Cos[c/2 + (d*x)
/2]^3*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(d*(a*(1 + Cos[c + d*x]
))^(3/2)) - (5*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x
)/4]])/(d*(a*(1 + Cos[c + d*x]))^(3/2)) - (Sqrt[2]*Cos[c/2 + (d*x)/2]^3*Log
[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(d*(a*(1 + C
os[c + d*x]))^(3/2)) + ((1 - I)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x
)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 +
(d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/2]^3*((1 + I)*Cos[c/4] + Sq
```

```
rt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4])*((-1 - I)*Cos[c/4]
+ Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4]))/(Sqrt[2]*d*(a*
(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] + Sin[c/2])) - ((1/2 + I/2)*Cos[c/2 + (
d*x)/2]^3*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]]*
((1 + I)*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4
])*((-1 - I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin
[c/4]))/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] + Sin[c/2])) - ((
8*I)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[
-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]^3*Cot[c/2])/(d*(a*(1
+ Cos[c + d*x]))^(3/2)*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (4*Sqrt[2]
*Cos[c/2 + (d*x)/2]^3*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*Cos[(d*
x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*ArcTan[
((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c
/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]))/(
d*(a*(1 + Cos[c + d*x]))^(3/2)*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) - Cos[c/2 + (
d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 +
 (d*x)/4])^2) + Cos[c/2 + (d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos
[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2)
```

Maple [B] time = 3.078, size = 290, normalized size = 2.5

$$-\frac{1}{4d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5\sqrt{2}\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin\left(1/2\,dx+c/2\right)\right)^2}+2\,a}{\cos\left(1/2\,dx+c/2\right)}\right)\left(\cos\left(1/2\,dx+c/2\right)\right)^2a-4\,\ln\left(-4\,\frac{a\sqrt{2}\cos\left(1/\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/a^(5/2)/cos(1/2*d*x+1/2*c)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*2^(1/2)*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(
1/2*d*x+1/2*c)^2*a-4*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a
*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d
*x+1/2*c)^2*a-4*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin
(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2
*c)^2*a+a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/
(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*cos(d*x + c) + a)^(3/2), x)
```

Fricas [B] time = 1.78237, size = 687, normalized size = 6.03

$$\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+4\left(\cos\left(dx+c\right)^{2}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*log(-(a*cos(d*
x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x + c) - 2*a*co
s(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2
- 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)
/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sqrt(a*cos(d*x + c) + a)*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 3.48685, size = 255, normalized size = 2.24

$$\frac{\frac{5\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)}{a^{\frac{3}{2}}}-\frac{2\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{2}}+\frac{8\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right.\right.\right.}{a^{\frac{3}{2}}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(5*sqrt(2)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a))^2)/a^(3/2) - 2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1
/2*d*x + 1/2*c)/a^2 + 8*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(3/2) - 8*log(abs((sqrt(
a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt
(2) - 3)))/a^(3/2))/d
```

3.137 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (-3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 (9*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (3*Tan
[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.374043, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2984, 2985, 2649, 206, 2773}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{3\tan(c+dx)}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{\tan(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
 (9*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(2*
Sqrt[2]*a^(3/2)*d) - Tan[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (3*Tan
[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(3a-\frac{3}{2}a\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-3a^2+\frac{3}{2}a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^3}$$

$$= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{3\int \sqrt{a+a\cos(c+dx)}\sec(c+dx)\,d}{2a^2}$$

$$= -\frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\tan(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx}}\right.}{ad}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{}{2ad}$$

Mathematica [A] time = 0.466104, size = 103, normalized size = 0.72

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)(2\sec(c+dx)+3)+9\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sqrt{2}\sin\left(\right.\right.}{2ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (9*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] - 6*Sqrt[2]*ArcTanh[Sqrt[2]*S
in[(c + d*x)/2]]*Cos[(c + d*x)/2] + (3 + 2*Sec[c + d*x])*Tan[(c + d*x)/2])/
(2*a*d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 3.164, size = 567, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(18*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d
*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-12*ln(-4
*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/
2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^4*a-12*ln(4/(2*c
os(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+
a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^4*a-9*2^(1/2)*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*
x+1/2*c)^2*a+6*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1
/2*c)^2+6*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*
x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^2
*a+6*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/
2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^2*a-a^(
1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(5/2)/cos(1/2*d*x+1/2*c)/(2*
cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2*d*x+1/2*
c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.7959, size = 771, normalized size = 5.35

$$\frac{9\sqrt{2}\left(\cos\left(dx+c\right)^{3}+2\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)+\cdots}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(9*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*l
og(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(d*x
+ c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 6*(
cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x +
c)^3 - 7*a*cos(d*x + c)^2 + 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c
) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*co
s(d*x + c) + a)*(3*cos(d*x + c) + 2)*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 +
2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}(c+dx)}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 3.37803, size = 435, normalized size = 3.02

$$-\frac{\frac{9\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}\right)}{a^{\frac{3}{2}}}-\frac{2\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{2}}-\frac{16\sqrt{2}\left(3\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\cdots\right.\right.}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{4}-6\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*(9*sqrt(2)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2)/a^(3/2) - 2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(
1/2*d*x + 1/2*c)/a^2 - 16*sqrt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*sqrt(a) - a^(3/2))/(((sqrt(a)*tan(1/2*d*x +
 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x +
1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*a) + 12*log(abs((sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*s
qrt(2) + 3)))/a^(3/2) - 12*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(3/2))/d
```

3.138 $\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{7\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{\tan(c+dx)\sec(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}-\frac{\tan(c}{2d(a}$$

```
[Out] (19*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*a^(3/2)*d)
 - (13*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/
(2*Sqrt[2]*a^(3/2)*d) - (7*Tan[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]) -
 (Sec[c + d*x]*Tan[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (Sec[c + d*
x]*Tan[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.499431, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2984, 2985, 2649, 206, 2773}

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{13\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{7\tan(c+dx)}{4ad\sqrt{a\cos(c+dx)+a}}+\frac{\tan(c+dx)\sec(c+dx)}{ad\sqrt{a\cos(c+dx)+a}}-\frac{\tan(c}{2d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*a^(3/2)*d)
 - (13*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/
(2*Sqrt[2]*a^(3/2)*d) - (7*Tan[c + d*x])/(4*a*d*Sqrt[a + a*Cos[c + d*x]]) -
 (Sec[c + d*x]*Tan[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (Sec[c + d*
x]*Tan[c + d*x])/(a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(4a-\frac{5}{2}a\cos(c+dx)\right)\sec^3(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(-7a^2+6a^2\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a^3}$$

$$= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{\int \frac{\left(\frac{19a^3}{2}-\frac{7}{2}\right.}{}}{}$$

$$= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} + \frac{19\int \sqrt{a}}{}$$

$$= -\frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(c+dx)\tan(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\sec(c+dx)\tan(c+dx)}{ad\sqrt{a+a\cos(c+dx)}} - \frac{19\,\mathrm{Subst}}{}$$

$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4a^{3/2}d} - \frac{13\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{7\tan(c+dx)}{4ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec(}{2d(a}$$

Mathematica [C] time = 27.0107, size = 1941, normalized size = 10.49

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((-19/8 + (19*I)/8)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16
*I)*E^(((3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*
x) - (34 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)
*c + ((5*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*S
qrt[2]*E^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) +
(8*I)*E^((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2
)*(c + d*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*
x)) - 16*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt
[2]*E^((4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/
2 + (d*x)/2]^3)/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqr
t[2]*E^((I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(
c + d*x)) + I*E^((2*I)*(c + d*x)))^2*(a*(1 + Cos[c + d*x]))^(3/2)) - (((19*
I)/2)*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (
d*x)/4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x
)/4])]*Cos[c/2 + (d*x)/2]^3)/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)) - (((
19*I)/2)*ArcTan[(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4
+ (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d
*x)/4])]*Cos[c/2 + (d*x)/2]^3)/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)) + (
13*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(d*(a
*(1 + Cos[c + d*x]))^(3/2)) - (13*Cos[c/2 + (d*x)/2]^3*Log[Cos[c/4 + (d*x)/
4] + Sin[c/4 + (d*x)/4]])/(d*(a*(1 + Cos[c + d*x]))^(3/2)) - (19*Cos[c/2 +
(d*x)/2]^3*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]]
)/(4*Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)) - (19*Cos[c/2 + (d*x)/2]^3*Log
[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(4*Sqrt[2]*d
*(a*(1 + Cos[c + d*x]))^(3/2)) - ((19*I)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[
2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[
c/2 + (d*x)/2]^3*Cot[c/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*Sqrt[-2 + 4*Cos[
c/2]^2 + 4*Sin[c/2]^2]) + (19*Cos[c/2 + (d*x)/2]^3*Csc[c/2]*(-(d*x*Cos[c/2]
) + 2*Log[Sqrt[2] + 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[
c/2] + ((4*I)*Sqrt[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Ta
n[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*C
os[c/2]^2 + 4*Sin[c/2]^2]))/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(3/2)*(4*Cos[
c/2]^2 + 4*Sin[c/2]^2)) - Cos[c/2 + (d*x)/2]^3/(2*d*(a*(1 + Cos[c + d*x]))^
(3/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) + Cos[c/2 + (d*x)/2]^3/(
2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^
2) + (Cos[c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(C
os[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c/2
 + (d*x)/2]^3*(-5*Cos[c/2] + 7*Sin[c/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)
*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c/
2 + (d*x)/2]^3*Sin[(d*x)/2])/(d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] + Si
n[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c/2 + (d*x)/2]^
3*(5*Cos[c/2] + 7*Sin[c/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2)*(Cos[c/2] +
Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 3.483, size = 807, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/2*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(104*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*a-76*ln(
-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(
1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a-76*ln(4/(2
*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^6*a-104*2^(1/2)*ln(
2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/
2*d*x+1/2*c)^4*a+28*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*
d*x+1/2*c)^4+76*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(
1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/
2*c)^4*a+76*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^
4*a+26*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*
d*x+1/2*c))*cos(1/2*d*x+1/2*c)^2*a-22*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2
)*a^(1/2)*cos(1/2*d*x+1/2*c)^2-19*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/
2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2
)))*cos(1/2*d*x+1/2*c)^2*a-19*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*
2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*c
os(1/2*d*x+1/2*c)^2*a+2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(
5/2)/cos(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)-2^(1/2))^2/(2*cos(1/2*d*x+1/2
*c)+2^(1/2))^2/sin(1/2*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.80454, size = 809, normalized size = 4.37

$$\frac{26\sqrt{2}\left(\cos\left(dx+c\right)^4+2\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/16*(26*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(
a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sin(
d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) +
 19*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*log((a*cos
(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(a)*(cos(
d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*sq
rt(a*cos(d*x + c) + a)*(7*cos(d*x + c)^2 + 3*cos(d*x + c) - 2)*sin(d*x + c)
)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Giac [B] time = 3.88501, size = 543, normalized size = 2.94

$$\frac{13\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a^{\frac{3}{2}}}-\frac{2\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}+\frac{19\log\left(\left|\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}\right.\right.\right.}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(13*sqrt(2)*log((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/
2*c)^2 + a))^2)/a^(3/2) - 2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(
1/2*d*x + 1/2*c)/a^2 + 19*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(3/2) - 19*log(abs((sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*s
qrt(2) - 3)))/a^(3/2) - 4*sqrt(2)*(29*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(
a*tan(1/2*d*x + 1/2*c)^2 + a))^6*sqrt(a) - 133*(sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4*a^(3/2) + 55*(sqrt(a)*tan(1/2*d*x
 + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a^(5/2) - 7*a^(7/2))/(((s
qrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sq
rt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)
^2*a))/d
```

3.139 $\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=183

$$\frac{95\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}-\frac{197\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}+\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (163*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*
x])^(5/2)) - (17*Cos[c + d*x]^2*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^
(3/2)) - (197*Sin[c + d*x])/(24*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + (95*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(48*a^3*d)
```

Rubi [A] time = 0.41154, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2765, 2977, 2968, 3023, 2751, 2649, 206}

$$\frac{95\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{48a^3d}-\frac{197\sin(c+dx)}{24a^2d\sqrt{a\cos(c+dx)+a}}+\frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^3(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (163*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*
x])^(5/2)) - (17*Cos[c + d*x]^2*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^
(3/2)) - (197*Sin[c + d*x])/(24*a^2*d*Sqrt[a + a*Cos[c + d*x]]) + (95*Sqrt[
a + a*Cos[c + d*x]]*Sin[c + d*x])/(48*a^3*d)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos^2(c+dx)\left(3a-\frac{11}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(17a^2-\frac{95}{4}a^2\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}dx}{8a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{17a^2\cos(c+dx)-\frac{95}{4}a^2\cos^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}dx}{8a^4}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{95\sqrt{a+a\cos(c+dx)}\sin(c+d}{48a^3d}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{197\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx)}} + \frac{95}{}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{197\sin(c+dx)}{24a^2d\sqrt{a+a\cos(c+dx)}} + \frac{95}{}$$

$$= \frac{163\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^3(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\cos^2(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - $$

Mathematica [B] time = 6.35395, size = 587, normalized size = 3.21

$$-\frac{40\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(a(\cos(c+dx)+1))^{5/2}} + \frac{8\sin\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d(a(\cos(c+dx)+1))^{5/2}} - \frac{40\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{d(a(\cos(c+dx)+1))^{5/2}} + \frac{8\cos\left(\frac{3c}{2}\right)}{3d(a(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-163*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(4
*d*(a*(1 + Cos[c + d*x]))^(5/2)) + (163*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 +
(d*x)/4] + Sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) - (40*Co
s[(d*x)/2]*Cos[c/2 + (d*x)/2]^5*Sin[c/2]])/(d*(a*(1 + Cos[c + d*x]))^(5/2))
+ (8*Cos[(3*d*x)/2]*Cos[c/2 + (d*x)/2]^5*Sin[(3*c)/2]])/(3*d*(a*(1 + Cos[c +
 d*x]))^(5/2)) - (40*Cos[c/2]*Cos[c/2 + (d*x)/2]^5*Sin[(d*x)/2]])/(d*(a*(1 +
 Cos[c + d*x]))^(5/2)) + (8*Cos[(3*c)/2]*Cos[c/2 + (d*x)/2]^5*Sin[(3*d*x)/2
])/(3*d*(a*(1 + Cos[c + d*x]))^(5/2)) + Cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + C
os[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^4) - (29*Cos[
c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - S
in[c/4 + (d*x)/4])^2) - Cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5
/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^4) + (29*Cos[c/2 + (d*x)/2]^5
)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4
])^2)
```

Maple [A] time = 1.463, size = 242, normalized size = 1.3

$$\frac{1}{96d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(128\sqrt{2}\sqrt{a(\sin(1/2dx+c/2))^2}\sqrt{a}(\cos(1/2dx+c/2))^6 + 489\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+c/}}{\cos(1/2dx+c/2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/96/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(128*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^6+489*2^(1/2)*ln(2*(2*
a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x
+1/2*c)^4*a-512*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+
1/2*c)^4-87*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*
c)^2+6*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(7/2)/sin(1/2*d*x+
1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.72145, size = 566, normalized size = 3.09

$$\frac{489 \sqrt{2}\left(\cos(dx+c)^3 + 3\cos(dx+c)^2 + 3\cos(dx+c) + 1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2 - 2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c) - 2a\cos(dx+c}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{192\left(a^3d\cos(dx+c)^3 + 3a^3d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(489*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a
)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1)) + 4*(32*cos(d*x + c)^3 - 160*cos(d*x + c)^2 - 503*cos(d*x + c) - 299)*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(
d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.44584, size = 197, normalized size = 1.08

$$\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a}-\frac{23\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\frac{668\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\frac{465\sqrt{2}}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a\right)^{\frac{3}{2}}}-\frac{489\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{5}{2}}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/96*(((3*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a - 23*sqrt(2)/a)*tan(1/2*d*x +
 1/2*c)^2 - 668*sqrt(2)/a)*tan(1/2*d*x + 1/2*c)^2 - 465*sqrt(2)/a)*tan(1/2*
d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) - 489*sqrt(2)*log(abs(-sq
rt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/
d
```

3.140 $\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=145

$$\frac{9\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{13\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^2*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*
x])^(5/2)) + (13*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (9*Sin
[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.270186, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2968, 3019, 2751, 2649, 206}

$$\frac{9\sin(c+dx)}{4a^2d\sqrt{a\cos(c+dx)+a}}-\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^2(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}+\frac{13\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(1
6*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^2*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*
x])^(5/2)) + (13*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (9*Sin
[c + d*x])/(4*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos(c+dx)\left(2a-\frac{9}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{2a\cos(c+dx)-\frac{9}{2}a\cos^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{-\frac{39a^2}{4}+9a^2\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx}{8a^4}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx)}} - \frac{75\int}{}$$

$$= -\frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{9\sin(c+dx)}{4a^2d\sqrt{a+a\cos(c+dx)}} + \frac{75\,\mathrm{Su}}{}$$

$$= -\frac{75\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^2(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{}{4}$$

Mathematica [A] time = 4.38222, size = 216, normalized size = 1.49

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(128\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)+128\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\frac{21}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{21}{\left(\sin\left(\frac{1}{4}(c+dx)\right)+\cos\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{}{(\mathrm{co}}\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(150*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - 150*Log
[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] + 128*Cos[(d*x)/2]*Sin[c/2] + 128*Cos
[c/2]*Sin[(d*x)/2] - (Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^(-4) + 21/(Cos[(
c + d*x)/4] - Sin[(c + d*x)/4])^2 + (Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^(
```

```
-4) - 21/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2))/(8*d*(a*(1 + Cos[c + d*x
]))^(5/2))
```

Maple [A] time = 1.507, size = 208, normalized size = 1.4

$$-\frac{1}{32d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(75\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2dx+c/2))^2}+2a}{\cos(1/2dx+c/2)}\right)(\cos(1/2dx+c/2))^4a-64\sqrt{2}\sqrt{a(\sin(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/32*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(75*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-64*2^(
1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^4-21*2^(1/2)
*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+2*a^(1/2)*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/cos(1/2*d*x+1/2*c)^3/a^(7/2)/sin(1/2*d*
x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.68844, size = 532, normalized size = 3.67

$$\frac{75\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) + 4*sqrt(a*cos(d*x + c) + a)*(32*cos(d*x + c)^2 + 85*cos(d*x + c) + 49)*
sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.7545, size = 167, normalized size = 1.15

$$-\frac{\frac{\left(\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{2}}-\frac{17\sqrt{2}}{a^{2}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\frac{83\sqrt{2}}{a^{2}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}-\frac{75\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(((2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/a^2 - 17*sqrt(2)/a^2)*tan(1/2*d*x
 + 1/2*c)^2 - 83*sqrt(2)/a^2)*tan(1/2*d*x + 1/2*c)/sqrt(a*tan(1/2*d*x + 1/2
*c)^2 + a) - 75*sqrt(2)*log(abs(-sqrt(a)*tan(1/2*d*x + 1/2*c) + sqrt(a*tan(
1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.141 $\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (19*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (13*S
in[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.136926, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2758, 2750, 2649, 206}

$$\frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (19*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (13*S
in[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2758

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{-\frac{5a}{2}+4a\cos(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$
$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{19\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}$$
$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{19\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{16a^2d}$$
$$= \frac{19\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 1.12884, size = 103, normalized size = 0.96

$$\frac{-18\sin(c+dx) - 13\sin(2(c+dx)) - 152\cos^5\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{4}(c+dx)\right) - \sin\left(\frac{1}{4}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{4}(c+dx)\right. \right.\right.}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-152*Cos[(c + d*x)/2]^5*(Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - Log[Co
s[(c + d*x)/4] + Sin[(c + d*x)/4]]) - 18*Sin[c + d*x] - 13*Sin[2*(c + d*x)]
)/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 1.465, size = 174, normalized size = 1.6

$$\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(19\sqrt{2}\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\left(\sin(1/2\,dx+c/2)\right)^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)\left(\cos(1/2\,dx+c/2)\right)^4 a - 13\sqrt{2}\sqrt{a\left(\sin(1/2\,dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(19*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^4*a-13*2^(1
/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^2+2*a^(1/2)*2
^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/cos(1/2*d*x+1/2*c)^3/a^(7/2)/sin(1/2
*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.6551, size = 504, normalized size = 4.71

$$\frac{19\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) - 4*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c) + 9)*sin(d*x + c))/(a^3*d*
cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.40209, size = 139, normalized size = 1.3

$$\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{3}}-\frac{11\sqrt{2}}{a^{3}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{19\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{5}{2}}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/
a^3 - 11*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 19*sqrt(2)*log(abs(-sqrt(a)*ta
n(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.142 $\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (5*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.080487, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2750, 2650, 2649, 206}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{5\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (5*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx}{8a}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin}{\sqrt{a+a}}\right.}{16a^2d}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.248004, size = 65, normalized size = 0.61

$$\frac{2\sin(c+dx) + 5\sin(2(c+dx)) + 40\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (40*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 2*Sin[c + d*x] + 5*Sin[2
*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 1.579, size = 174, normalized size = 1.6

$$\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5\sqrt{2}\ln\left(2\,\frac{2\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)(\cos(1/2\,dx+c/2))^4\,a+5\sqrt{2}\sqrt{a\,(\sin(1/2\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*2^(1/2)*ln(2*(2
*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*
x+1/2*c)^4*a+5*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1
/2*c)^2-2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(7/2)/sin(1/2*d
*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.68154, size = 501, normalized size = 4.68

$$\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*s
in(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
) + 4*sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c) + 1)*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.61996, size = 139, normalized size = 1.3

$$-\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{3}}-\frac{3\sqrt{2}}{a^{3}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{5\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{5}{2}}}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2
/a^3 - 3*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) + 5*sqrt(2)*log(abs(-sqrt(a)*tan
(1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.143 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.0633022, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2650, 2649, 206}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{3\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16*
Sqrt[2]*a^(5/2)*d) + Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) + (3*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+a\cos(c+dx))^{3/2}}\,dx}{8a}$$
$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}$$
$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{16a^2d}$$
$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.156517, size = 65, normalized size = 0.61

$$\frac{14\sin(c+dx)+3\sin(2(c+dx))+24\cos^5\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (24*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^5 + 14*Sin[c + d*x] + 3*Sin[
2*(c + d*x)])/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 1.468, size = 174, normalized size = 1.6

$$\frac{1}{32d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(3\sqrt{2}\ln\left(2\frac{2\sqrt{a}\sqrt{a(\sin(1/2\,dx+c/2))^2}+2a}{\cos(1/2\,dx+c/2)}\right)(\cos(1/2\,dx+c/2))^4a+3\sqrt{2}\sqrt{a(\sin(1/2\,dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*2^(1/2)
*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*co
s(1/2*d*x+1/2*c)^4*a+3*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1
/2*d*x+1/2*c)^2+2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d
*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.62614, size = 501, normalized size = 4.68

$$\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^{2}-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right.}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*s
in(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)
) + 4*sqrt(a*cos(d*x + c) + a)*(3*cos(d*x + c) + 7)*sin(d*x + c))/(a^3*d*co
s(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.04978, size = 139, normalized size = 1.3

$$\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}{a^{3}}+\frac{5\sqrt{2}}{a^{3}}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{3\sqrt{2}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right|\right)}{a^{\frac{5}{2}}}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/32*(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^2/
a^3 + 5*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 3*sqrt(2)*log(abs(-sqrt(a)*tan(
1/2*d*x + 1/2*c) + sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)))/a^(5/2))/d
```

3.144 $\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{11\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) -
(43*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (11*S
in[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.334748, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2766, 2978, 2985, 2649, 206, 2773}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{11\sin(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) -
(43*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(16
*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (11*S
in[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(4a-\frac{3}{2}a\cos(c+dx)\right)\sec(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(8a^2-\frac{11}{4}a^2\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,d}{8a^4}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\cos(c+dx)}\sec(c+}{a^3}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\frac{a\sin}{\sqrt{a+a\,c}}\right.}{a^2d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{}{16}$$

Mathematica [C] time = 23.2702, size = 1919, normalized size = 13.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((-2 + 2*I)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*I)*E^((
(3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x) - (34
 - 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*c + ((5
*I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sqrt[2]*E
^((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (8*I)*E^
((I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)*(c + d
```

```
*x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x)) - 16
*Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[2]*E^((
4*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2 + (d*x
)/2]^5)/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^(
(I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x)
) + I*E^((2*I)*(c + d*x)))^2*(a*(1 + Cos[c + d*x]))^(5/2)) - ((4*I)*Sqrt[2]
*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/
4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])
]*Cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2)) + (43*Cos[c/2 + (d
*x)/2]^5*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c +
 d*x]))^(5/2)) - (43*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] + Sin[c/4
+ (d*x)/4]])/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) - (2*Sqrt[2]*Cos[c/2 + (d*x
)/2]^5*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/(d
*(a*(1 + Cos[c + d*x]))^(5/2)) + ((1 - I)*Sqrt[2]*ArcTan[(Cos[c/4 + (d*x)/4
] + Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] +
Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/2]^5*((1
+ I)*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4])*(
(-1 - I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4
]))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2] + Sin[c/2])) - ((1 + I)*Cos[c
/2 + (d*x)/2]^5*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x
)/2]]*((1 + I)*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*S
in[c/4])*((-1 - I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[
2]*Sin[c/4]))/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2] + Sin[c/2])
) - ((16*I)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4]
)/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]^5*Cot[c/2])/(d
*(a*(1 + Cos[c + d*x]))^(5/2)*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]) + (8*
Sqrt[2]*Cos[c/2 + (d*x)/2]^5*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log[Sqrt[2] + 2*
Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((4*I)*Sqrt[2]*
ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4])/Sqrt[-2 +
4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]
^2]))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(4*Cos[c/2]^2 + 4*Sin[c/2]^2)) - Cos[
c/2 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - Si
n[c/4 + (d*x)/4])^4) - (11*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x])
)^(5/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) + Cos[c/2 + (d*x)/2]^5
/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4]
)^4) + (11*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4
 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2)
```

Maple [B] time = 3.241, size = 325, normalized size = 2.3

$$-\frac{1}{32\,d}\sqrt{a\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(43\,\sqrt{2}\ln\left(2\,\frac{2\,\sqrt{a}\sqrt{a\,(\sin(1/2\,dx+c/2))^2}+2\,a}{\cos(1/2\,dx+c/2)}\right)(\cos(1/2\,dx+c/2))^4\,a-32\,\ln\left(-4\,\frac{a\sqrt{2}\cos(1}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/32/a^(7/2)/cos(1/2*d*x+1/2*c)^3*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(43*2^(1/
2)*ln(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*
cos(1/2*d*x+1/2*c)^4*a-32*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/
2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(
1/2*d*x+1/2*c)^4*a-32*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*
(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d
*x+1/2*c)^4*a+11*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x
+1/2*c)^2+2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*x+1/2
```

```
*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(a*cos(d*x + c) + a)^(5/2), x)
```

Fricas [B] time = 1.77469, size = 807, normalized size = 5.6

$$\frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+c)-2a\cos(dx+c)}{\cos(dx+c)^2+2\cos(dx+c)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) + 32*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*lo
g((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*sqrt(a*cos(d*x + c) + a)*sqrt(
a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) - 4*sqrt(a*cos(d*x + c) + a)*(11*cos(d*x + c) + 15)*sin(d*x + c))/(a^3*d*
cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 3.18545, size = 285, normalized size = 1.98

$$-\frac{2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3}+\frac{13\sqrt{2}}{a^3}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{43\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right.}{a^{\frac{5}{2}}}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)
^2/a^3 + 13*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 43*sqrt(2)*log((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^(5/2) - 64*log
(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2
- a*(2*sqrt(2) + 3)))/a^(5/2) + 64*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) -
sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3)))/a^(5/2))/d
```

3.145 $\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{35\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{15\tan(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-$$

```
[Out] (-5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 (115*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(
16*Sqrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (15
*Tan[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (35*Tan[c + d*x])/(16*
a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.519235, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2766, 2978, 2984, 2985, 2649, 206, 2773}

$$\frac{35\tan(c+dx)}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{15\tan(c+dx)}{16ad(a\cos(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-5*ArcTanh[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
 (115*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(
16*Sqrt[2]*a^(5/2)*d) - Tan[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (15
*Tan[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (35*Tan[c + d*x])/(16*
a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\left(5a-\frac{5}{2}a\cos(c+dx)\right)\sec^2(c+dx)}{(a+a\cos(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(\frac{35a^2}{2}-\frac{45}{4}a^2\cos(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+a\cos(c+dx)}}}{8a^4}$$

$$= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} + \int$$

$$= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} - 5$$

$$= -\frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\tan(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\tan(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} + 5$$

$$= -\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\tan(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} -$$

Mathematica [C] time = 23.2167, size = 2051, normalized size = 11.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((5 - 5*I)*(1 + E^(I*c))*(Sqrt[2] - (1 - I)*E^((I/2)*c) + (16 - 16*I)*E^(((
3*I)/2)*c + I*d*x) + (20 + 20*I)*Sqrt[2]*E^((2*I)*c + ((3*I)/2)*d*x) - (34
- 34*I)*E^(((5*I)/2)*c + (2*I)*d*x) - (20 + 20*I)*Sqrt[2]*E^((3*I)*c + ((5*
I)/2)*d*x) + (16 - 16*I)*E^(((7*I)/2)*c + (3*I)*d*x) + (4 + 4*I)*Sqrt[2]*E^
((4*I)*c + ((7*I)/2)*d*x) - (1 - I)*E^(((9*I)/2)*c + (4*I)*d*x) + (8*I)*E^(
(I/2)*(c + d*x)) - 16*Sqrt[2]*E^(I*(c + d*x)) - (40*I)*E^(((3*I)/2)*(c + d*
x)) + 34*Sqrt[2]*E^((2*I)*(c + d*x)) + (40*I)*E^(((5*I)/2)*(c + d*x)) - 16*
Sqrt[2]*E^((3*I)*(c + d*x)) - (8*I)*E^(((7*I)/2)*(c + d*x)) + Sqrt[2]*E^((4
*I)*(c + d*x)) - (4 + 4*I)*Sqrt[2]*E^((I/2)*(2*c + d*x)))*x*Cos[c/2 + (d*x)
/2]^5)/(((-1 - I) + Sqrt[2]*E^((I/2)*c))*(-1 + E^(I*c))*(I - 2*Sqrt[2]*E^((
I/2)*(c + d*x)) - (4*I)*E^(I*(c + d*x)) + 2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))
 + I*E^((2*I)*(c + d*x)))^2*(a*(1 + Cos[c + d*x]))^(5/2)) + ((10*I)*Sqrt[2]
*ArcTan[(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/
4])/(-Cos[c/4 + (d*x)/4] + Sqrt[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])
]*Cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2)) - (115*Cos[c/2 + (
d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c
+ d*x]))^(5/2)) + (115*Cos[c/2 + (d*x)/2]^5*Log[Cos[c/4 + (d*x)/4] + Sin[c/
4 + (d*x)/4]])/(4*d*(a*(1 + Cos[c + d*x]))^(5/2)) + (5*Sqrt[2]*Cos[c/2 + (d
*x)/2]^5*Log[2 - Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Sin[c/2 + (d*x)/2]])/
(d*(a*(1 + Cos[c + d*x]))^(5/2)) - ((5 - 5*I)*ArcTan[(Cos[c/4 + (d*x)/4] +
Sin[c/4 + (d*x)/4] - Sqrt[2]*Sin[c/4 + (d*x)/4])/(Cos[c/4 + (d*x)/4] + Sqrt
[2]*Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])]*Cos[c/2 + (d*x)/2]^5*((1 + I)
*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4])*((-1
- I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/4] - I*Sqrt[2]*Sin[c/4]))/
(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2] + Sin[c/2])) + ((5/2 + (5
*I)/2)*Cos[c/2 + (d*x)/2]^5*Log[2 + Sqrt[2]*Cos[c/2 + (d*x)/2] - Sqrt[2]*Si
n[c/2 + (d*x)/2]]*((1 + I)*Cos[c/4] + Sqrt[2]*Cos[c/4] - (1 - I)*Sin[c/4] -
 I*Sqrt[2]*Sin[c/4])*((-1 - I)*Cos[c/4] + Sqrt[2]*Cos[c/4] + (1 - I)*Sin[c/
```

```
4] - I*Sqrt[2]*Sin[c/4]))/(Sqrt[2]*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2]
 + Sin[c/2])) + ((40*I)*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*
Tan[(d*x)/4])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2 + (d*x)/2]^5*
Cot[c/2])/(d*(a*(1 + Cos[c + d*x]))^(5/2)*Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/
2]^2]) - (20*Sqrt[2]*Cos[c/2 + (d*x)/2]^5*Csc[c/2]*(-(d*x*Cos[c/2]) + 2*Log
[Sqrt[2] + 2*Cos[(d*x)/2]*Sin[c/2] + 2*Cos[c/2]*Sin[(d*x)/2]]*Sin[c/2] + ((
4*I)*Sqrt[2]*ArcTan[((2*I)*Cos[c/2] - I*(-Sqrt[2] + 2*Sin[c/2])*Tan[(d*x)/4
])/Sqrt[-2 + 4*Cos[c/2]^2 + 4*Sin[c/2]^2]]*Cos[c/2])/Sqrt[-2 + 4*Cos[c/2]^2
 + 4*Sin[c/2]^2]))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(4*Cos[c/2]^2 + 4*Sin[c/
2]^2)) + Cos[c/2 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 +
(d*x)/4] - Sin[c/4 + (d*x)/4])^4) + (19*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 +
Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] - Sin[c/4 + (d*x)/4])^2) - Cos[c/2
 + (d*x)/2]^5/(8*d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/4 + (d*x)/4] + Sin[c
/4 + (d*x)/4])^4) - (19*Cos[c/2 + (d*x)/2]^5)/(8*d*(a*(1 + Cos[c + d*x]))^(
5/2)*(Cos[c/4 + (d*x)/4] + Sin[c/4 + (d*x)/4])^2) + (4*Cos[c/2 + (d*x)/2]^5
)/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])
) - (4*Cos[c/2 + (d*x)/2]^5)/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [B] time = 3.185, size = 601, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/16*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*(230*2^(1/2)*ln(2*(2*a^(1/2)*(a*sin(1/2
*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(1/2*d*x+1/2*c)^6*a-160*ln
(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^
(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+1/2*c)^6*a-160*ln(4/
(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1
/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c)^6*a-115*2^(1/2)*l
n(2*(2*a^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*a)/cos(1/2*d*x+1/2*c))*cos(
1/2*d*x+1/2*c)^4*a+70*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/
2*d*x+1/2*c)^4+80*ln(-4*(a*2^(1/2)*cos(1/2*d*x+1/2*c)-a^(1/2)*2^(1/2)*(a*si
n(1/2*d*x+1/2*c)^2)^(1/2)-2*a)/(2*cos(1/2*d*x+1/2*c)-2^(1/2)))*cos(1/2*d*x+
1/2*c)^4*a+80*ln(4/(2*cos(1/2*d*x+1/2*c)+2^(1/2))*(a^(1/2)*2^(1/2)*(a*sin(1
/2*d*x+1/2*c)^2)^(1/2)+a*2^(1/2)*cos(1/2*d*x+1/2*c)+2*a))*cos(1/2*d*x+1/2*c
)^4*a-15*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^(1/2)*cos(1/2*d*x+1/2*c)^
2-2*a^(1/2)*2^(1/2)*(a*sin(1/2*d*x+1/2*c)^2)^(1/2))/a^(7/2)/cos(1/2*d*x+1/2
*c)^3/(2*cos(1/2*d*x+1/2*c)-2^(1/2))/(2*cos(1/2*d*x+1/2*c)+2^(1/2))/sin(1/2
*d*x+1/2*c)/(cos(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.81368, size = 891, normalized size = 5.12

$$115\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sin(dx+}{\cos(dx+c)^2+2\cos(dx+c)+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/64*(115*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + c
os(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c)
 + a)*sqrt(a)*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 80*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2
 + cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*sqr
t(a*cos(d*x + c) + a)*sqrt(a)*(cos(d*x + c) - 2)*sin(d*x + c) + 8*a)/(cos(d
*x + c)^3 + cos(d*x + c)^2)) + 4*sqrt(a*cos(d*x + c) + a)*(35*cos(d*x + c)^
2 + 55*cos(d*x + c) + 16)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.15441, size = 464, normalized size = 2.67

$$2\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^3}+\frac{21\sqrt{2}}{a^3}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{115\sqrt{2}\log\left(\left(\sqrt{a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(2*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*tan(1/2*d*x + 1/2*c)^
2/a^3 + 21*sqrt(2)/a^3)*tan(1/2*d*x + 1/2*c) - 115*sqrt(2)*log((sqrt(a)*tan
(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/a^(5/2) + 128*sq
rt(2)*(3*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2*sqrt(a) - a^(3/2))/(((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
 + 1/2*c)^2 + a))^4 - 6*(sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x
+ 1/2*c)^2 + a))^2*a + a^2)*a^2) - 160*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c
) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3)))/a^(5/2) + 1
60*log(abs((sqrt(a)*tan(1/2*d*x + 1/2*c) - sqrt(a*tan(1/2*d*x + 1/2*c)^2 +
a))^2 + a*(2*sqrt(2) - 3)))/a^(5/2))/d
```

3.146 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin(c+dx)}{21\ldots}$$

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.0776239, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2635, 2639, 2641}

$$\frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10a\sin(c+dx)}{21\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*a*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*a*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^{\frac{5}{2}}(c+dx)\,dx + a\int \cos^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(3a)\int \sqrt{\cos(c$$
$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5d}$$
$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10a\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2}{}$$

Mathematica [C] time = 6.16369, size = 490, normalized size = 4.41

$$a\left(-\frac{3\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos(t}}\right.}{10d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*Cot[c])/
(5*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[3
*d*x]*Sin[3*c])/(28*d) + (23*Cos[c]*Sin[d*x])/(84*d) + (Cos[2*c]*Sin[2*d*x]
)/(10*d) + (Cos[3*c]*Sin[3*d*x])/(28*d)) - (5*(1 + Cos[c + d*x])*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt
[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
 ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (3*(1 + Cos[c + d*x])*Csc[c]
*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + Ar
cTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[
c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))
```

Maple [A] time = 2.169, size = 270, normalized size = 2.4

$$-\frac{2a}{105d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8-528\left(\sin(1/2\,dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+25*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\cos(dx+c)^{3}+a\cos(dx+c)^{2}\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.147 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0666751, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2635, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]),x]
```

```
[Out] (6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\,dx = a\int \cos^{\frac{3}{2}}(c+dx)\,dx + a\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx}}$$
$$= \frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + 2a\cos$$

Mathematica [C] time = 5.35802, size = 232, normalized size = 2.67

$$a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-18\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hypergeo}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((9*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*Cos[
c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ
[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*
Sin[c] + 2*Cos[c + d*x]*(-18*Cot[c] + 10*Sin[c + d*x] + 3*Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.177, size = 219, normalized size = 2.5

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\left(\cos(1/2\,dx+c/2)\right)^7-28\left(\cos(1/2\,dx+c/2)\right)^5+5\sqrt{(\sin(1/2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.148 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0505184, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2639, 2635, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]),x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))\,dx = a\int \sqrt{\cos(c+dx)}\,dx + a\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [C] time = 4.45005, size = 222, normalized size = 3.64

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyperg}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((3*(3*Cos[c - d*x - ArcTan[Tan[c]
]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 4*Cos[c
 + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*S
in[c] - 4*Cos[c + d*x]*(3*Cot[c] - Sin[c + d*x]) - 6*Cos[c]*Csc[d*x + ArcTa
n[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d
*x]])
```

Maple [B] time = 2.171, size = 225, normalized size = 3.7

$$-\frac{2a}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+\sqrt{2\left(\sin(1/2\,dx+c/2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\cos(dx+c)+a)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.149 $\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=35

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

Rubi [A] time = 0.0388943, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2748, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx &= a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+a\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [C] time = 23.4065, size = 155, normalized size = 4.43

$$a\sqrt{\cos(c+dx)}(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\tan\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+dx\right)\right)}{\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-2*Sqrt[Cos[d*
x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Tan[d*x + A
rcTan[Tan[c]]] - (HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Tan[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(
2*d)
```

Maple [A] time = 1.904, size = 150, normalized size = 4.3

$$-2\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}a\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(\mathrm{EllipticF}\left(\mathrm{cc}\right.\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a\cos(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a\cos(dx+c)+a}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.150 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0496062, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/d + (2
*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 9.22089, size = 209, normalized size = 3.67

$$a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyperge}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(4*Cos[d*x]*Csc[c] - ((3*Cos[c - d
*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[S
ec[c]^2] - 4*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x -
 ArcTan[Cot[c]]]*Sin[c] + 2*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hypergeometric
PFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[S
in[d*x + ArcTan[Tan[c]]]^2]))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.306, size = 146, normalized size = 2.6

$$-2\,\frac{a\left(\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\,\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)+\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2}\right.}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*a*((2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.151 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 0.0594036, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2636, 2641, 2639}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/d + (2*a*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a + a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - a\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 6.14702, size = 444, normalized size = 5.35

$$a\left(\frac{\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+dx\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-1}(\tan\right.}}\right.}{2d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((Csc[c]*Sec[
c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c + d*x]*(Sin[
c] + 3*Sin[d*x]))/(3*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/
4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -
 ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]
^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])
/(3*d*Sqrt[1 + Cot[c]^2]) + ((1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(2*d))
```

Maple [B] time = 3.569, size = 369, normalized size = 4.5

$$\frac{2a}{3d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\,\text{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.152 $\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0739601, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2636, 2639, 2641}

$$\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*a*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = a\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \frac{1}{5}(3a)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(3a)\int \sqrt{\cos(}$$
$$= -\frac{6aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\sin(c+}{5d\sqrt{\cos(c+}}$$

Mathematica [C] time = 6.19831, size = 477, normalized size = 4.3

$$a\left(\frac{3\csc(c)(\cos(c+dx)+1)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+dx\right)\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-1}(\right.}}}{10d}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((3*Csc[c]*Se
c[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*Sec[c + d*x]
^2*(3*Sin[c] + 5*Sin[d*x]))/(15*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 9*Sin
[d*x]))/(15*d)) - ((1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[C
ot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c
]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqr
t[1 + Cot[c]^2]) + (3*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hype
rgeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[
c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(
10*d))
```

Maple [B] time = 3.729, size = 384, normalized size = 3.5

$$-4\,\frac{\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)(\sin(1/2\,dx+c/2))^2}\,a}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}\,d}\left(-1/12\,\frac{\cos(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2}}{\left((\cos(1/2\,dx+c/2))^2-1/2\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)/cos(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/12*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.153 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=147

$$\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\mathrm{s}}{}$$

```
[Out] (32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2,
2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.137932, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2635, 2639, 2641}

$$\frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{4a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{32a^2\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (32*a^2*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a^2*EllipticF[(c + d*x)/2,
2])/(21*d) + (20*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (32*a^2*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^{\frac{5}{2}}(c+dx)+2a^2\cos^{\frac{7}{2}}(c+dx)+a^2\cos^{\frac{9}{2}}(c+dx)\right)dx$$
$$= a^2\int \cos^{\frac{5}{2}}(c+dx)\,dx + a^2\int \cos^{\frac{9}{2}}(c+dx)\,dx + \left(2a^2\right)\int \cos^{\frac{7}{2}}(c+dx)\,dx$$
$$= \frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{4a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\cos^{\frac{7}{2}}(c+dx}{9d}$$
$$= \frac{6a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{20a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{32a^2\cos^{\frac{3}{2}}(c+dx)\sin(c}{45d}$$
$$= \frac{32a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

Mathematica [C] time = 6.13067, size = 532, normalized size = 3.62

$$-\frac{4\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-1}\right.}}\right.}{15d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((-8*Cot[c])
/(15*d) + (23*Cos[d*x]*Sin[c])/(84*d) + (37*Cos[2*d*x]*Sin[2*c])/(360*d) +
(Cos[3*d*x]*Sin[3*c])/(28*d) + (Cos[4*d*x]*Sin[4*c])/(144*d) + (23*Cos[c]*S
in[d*x])/(84*d) + (37*Cos[2*c]*Sin[2*d*x])/(360*d) + (Cos[3*c]*Sin[3*d*x])/
(28*d) + (Cos[4*c]*Sin[4*d*x])/(144*d)) - (5*(a + a*Cos[c + d*x])^2*Csc[c]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 +
 (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*S
qrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*
x - ArcTan[Cot[c]]]])/(21*d*Sqrt[1 + Cot[c]^2]) - (4*(a + a*Cos[c + d*x])^2
*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d
*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x
 + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(15*d)
```

Maple [A] time = 2.44, size = 260, normalized size = 1.8

$$-\frac{4a^2}{315d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(560\left(\cos(1/2\,dx+c/2)\right)^{11}-960\left(\cos(1/2\,dx+c/2)\right)^9+608\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(560*cos
(1/2*d*x+1/2*c)^11-960*cos(1/2*d*x+1/2*c)^9+608*cos(1/2*d*x+1/2*c)^7-96*cos
(1/2*d*x+1/2*c)^5-205*cos(1/2*d*x+1/2*c)^3+75*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+93*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2\cos(dx+c)^4+2\,a^2\cos(dx+c)^3+a^2\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*s
qrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.154 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=121

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+$$

```
[Out] (12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2]
)/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```

Rubi [A] time = 0.118619, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2635, 2641, 2639}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{4a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^2\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (12*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (8*a^2*EllipticF[(c + d*x)/2, 2]
)/(7*d) + (8*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (4*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d
)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2\,dx = \int \left(a^2\cos^{\frac{3}{2}}(c+dx)+2a^2\cos^{\frac{5}{2}}(c+dx)+a^2\cos^{\frac{7}{2}}(c+dx)\right)dx$$
$$= a^2\int \cos^{\frac{3}{2}}(c+dx)\,dx + a^2\int \cos^{\frac{7}{2}}(c+dx)\,dx + \left(2a^2\right)\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{4a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\cos^{\frac{5}{2}}(c+$$
$$= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{8a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{7d}$$
$$= \frac{12a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{8a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{7d}$$

Mathematica [C] time = 6.11489, size = 500, normalized size = 4.13

$$-\frac{3\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos(t}}\right.}{10d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((-3*Cot[c])
/(5*d) + (17*Cos[d*x]*Sin[c])/(56*d) + (Cos[2*d*x]*Sin[2*c])/(10*d) + (Cos[
3*d*x]*Sin[3*c])/(56*d) + (17*Cos[c]*Sin[d*x])/(56*d) + (Cos[2*c]*Sin[2*d*x
])/(10*d) + (Cos[3*c]*Sin[3*d*x])/(56*d)) - (2*(a + a*Cos[c + d*x])^2*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^4*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]
*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[
d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (3*(a + a*Cos[c + d*x])^
2*Csc[c]*Sec[c/2 + (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*
x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + A
rcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Ta
n[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d)
```

Maple [A] time = 2.161, size = 272, normalized size = 2.3

$$-\frac{4a^2}{35d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(40\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8-116\left(\sin(1/2\,dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2 \cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2\cos(dx+c)^3+2\,a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.155 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=95

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0932659, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2639, 2635, 2641}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2]
)/(3*d) + (4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a^2*Cos[c + d*
x]^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2\,dx=\int\left(a^2\sqrt{\cos(c+dx)}+2a^2\cos^{\frac{3}{2}}(c+dx)+a^2\cos^{\frac{5}{2}}(c+dx)\right)dx$$
$$=a^2\int\sqrt{\cos(c+dx)}\,dx+a^2\int\cos^{\frac{5}{2}}(c+dx)\,dx+\left(2a^2\right)\int\cos^{\frac{3}{2}}(c+dx)\,dx$$
$$=\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+$$
$$=\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}+$$

Mathematica [C] time = 5.64705, size = 235, normalized size = 2.47

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-24\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyperg}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((12*(3*Cos[c - d*x - ArcTan[T
an[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20
*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[
c]]]*Sin[c] + Cos[c + d*x]*(-48*Cot[c] + 20*Sin[c + d*x] + 3*Sin[2*(c + d*x
)]) - 24*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan
[c]]]^2]))/(60*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.284, size = 250, normalized size = 2.6

$$-\frac{4a^2}{15d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-12\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+32\left(\sin\left(1/2\,dx+c/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-13*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sqrt(cos(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.156 $\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0807783, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2641, 2639, 2635}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = \int \left(\frac{a^2}{\sqrt{\cos(c+dx)}} + 2a^2\sqrt{\cos(c+dx)} + a^2\cos^{\frac{3}{2}}(c+dx)\right)dx$$
$$= a^2\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + a^2\int \cos^{\frac{3}{2}}(c+dx)\,dx + \left(2a^2\right)\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}a^2\int \frac{}{\sqrt{\mathrm{co}}}$$
$$= \frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [C] time = 5.02634, size = 224, normalized size = 3.34

$$a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\mathrm{Hyper}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*((3*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 8*C
os[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]
]]*Sin[c] + 2*Cos[c + d*x]*(-6*Cot[c] + Sin[c + d*x]) - 6*Cos[c]*Csc[d*x +
ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan
[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[
c + d*x]])
```

Maple [B] time = 2.049, size = 228, normalized size = 3.4

$$-\frac{4a^2}{3d}\sqrt{\left(2(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2(\sin(1/2\,dx+c/2))^4\cos(1/2\,dx+c/2)+2\sqrt{2(\sin(1/2\,dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\cos(dx+c)^2 + 2a^2\cos(dx+c) + a^2}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/sqrt(cos(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.157 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (4*a^2*EllipticF[(c + d*x)/2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rubi [A] time = 0.0804822, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2639, 2641}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*EllipticF[(c + d*x)/2, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \left(\frac{a^2}{\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2}{\sqrt{\cos(c+dx)}} + a^2\sqrt{\cos(c+dx)}\right)dx$$
$$= a^2\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + a^2\int \sqrt{\cos(c+dx)}\,dx + \left(2a^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - a^2\int \sqrt{\cos(c+dx)}\,dx$$
$$= \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.16049, size = 39, normalized size = 0.89

$$\frac{2a^2\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right) + \frac{\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a^2*(2*EllipticF[(c + d*x)/2, 2] + Sin[c + d*x]/Sqrt[Cos[c + d*x]]))/d
```

Maple [A] time = 2.259, size = 104, normalized size = 2.4

$$-4\,\frac{a^2\left(\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-(\sin(1/2\,dx+c/2))^2\cos(1/2}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(3/2),x)
```

```
[Out] -4*a^2*((2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^(3/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.158 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.092857, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2641, 2639}

$$\frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```


```
[Out] (-4*a^2*EllipticE[(c + d*x)/2, 2])/d + (8*a^2*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a^2*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]])
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^2}{\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^2}{\cos^{\frac{3}{2}}(c+dx)} + \frac{a^2}{\sqrt{\cos(c+dx)}}\right)dx$$
$$= a^2\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a^2\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(2a^2\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}a^2\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - (2$$
$$= -\frac{4a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 6.1551, size = 454, normalized size = 4.99

$$\frac{\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-1}\right.}}\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((Csc[c]*Sec
[c])/d + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(Sin
[c] + 6*Sin[d*x]))/(6*d)) - (2*(a + a*Cos[c + d*x])^2*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Se
c[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 +
 Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot
[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + ((a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 +
 (d*x)/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c
]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]
]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[
1 + Tan[c]^2]]))/(2*d)
```

Maple [B] time = 3.694, size = 371, normalized size = 4.1

$$\frac{4a^2}{3d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\text{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*(2*sin(1/2
```

```
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.159 $\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]])
```

Rubi [A] time = 0.118515, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2639, 2641}

$$\frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{16a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-16*a^2*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Cos[c + d*x]^(3/2)) + (16*a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x
]])
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \left(\frac{a^2}{\cos^{\frac{7}{2}}(c+dx)} + \frac{2a^2}{\cos^{\frac{5}{2}}(c+dx)} + \frac{a^2}{\cos^{\frac{3}{2}}(c+dx)}\right) dx$$
$$= a^2 \int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a^2 \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \left(2a^2\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{5}\left(3a^2\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}$$
$$= -\frac{2a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16a}{5d}$$
$$= -\frac{16a^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{16}{5a}$$

Mathematica [C] time = 6.19474, size = 487, normalized size = 4.02

$$\frac{2\csc(c)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^2\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c)\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos(\tan}}\right.}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2*Sec[c/2 + (d*x)/2]^4*((4*Csc[c]*S
ec[c])/(5*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(10*d) + (Sec[c]*Sec[c + d*
x]^2*(3*Sin[c] + 10*Sin[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 12
*Sin[d*x]))/(15*d)) - ((a + a*Cos[c + d*x])^2*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(3*d*Sqrt[1 + Cot[c]^2]) + (2*(a + a*Cos[c + d*x])^2*Csc[c]*Sec[c/2 + (d*x)
/2]^4*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]
*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt
[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/S
qrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]))/(5*d)
```

Maple [B] time = 3.846, size = 386, normalized size = 3.2

$$-8\,\frac{\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)(\sin(1/2\,dx+c/2))^2a^2}}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(-1/12\,\frac{\cos(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin}}{\left((\cos(1/2\,dx+c/2))^2-1/2\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2/cos(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/12*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/cos(d*x + c)^(7/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

3.160 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=147

$$\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a^3\sin(}{}$$

```
[Out] (68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.152808, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2635, 2641, 2639}

$$\frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^3\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{6a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{68a^3\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3,x]
```


```
[Out] (68*a^3*EllipticE[(c + d*x)/2, 2])/(15*d) + (44*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (44*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (68*a^3*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (6*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^3*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3\,dx = \int \left(a^3\cos^{\frac{3}{2}}(c+dx)+3a^3\cos^{\frac{5}{2}}(c+dx)+3a^3\cos^{\frac{7}{2}}(c+dx)+a^3\cos^{\frac{9}{2}}(c+dx)\right.$$
$$= a^3\int\cos^{\frac{3}{2}}(c+dx)\,dx + a^3\int\cos^{\frac{9}{2}}(c+dx)\,dx + \left(3a^3\right)\int\cos^{\frac{5}{2}}(c+dx)\,dx + \left(\right.$$
$$= \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{6a^3\cos^{\frac{5}{2}}(c+\right.}{}$$
$$= \frac{18a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{44a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}$$
$$= \frac{68a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{44a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{44a^3\sqrt{\cos(c+dx)}\sin(c+d}{21d}$$

Mathematica [C] time = 6.13385, size = 532, normalized size = 3.62

$$-\frac{17\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos}}\right.}{60d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*((-17*Cot[c]
)/(30*d) + (97*Cos[d*x]*Sin[c])/(336*d) + (73*Cos[2*d*x]*Sin[2*c])/(720*d)
+ (3*Cos[3*d*x]*Sin[3*c])/(112*d) + (Cos[4*d*x]*Sin[4*c])/(288*d) + (97*Cos
[c]*Sin[d*x])/(336*d) + (73*Cos[2*c]*Sin[2*d*x])/(720*d) + (3*Cos[3*c]*Sin[
3*d*x])/(112*d) + (Cos[4*c]*Sin[4*d*x])/(288*d)) - (11*(a + a*Cos[c + d*x])
^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (17*(a + a*Cos[
c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d)
```

Maple [A] time = 2.115, size = 260, normalized size = 1.8

$$-\frac{4a^3}{315d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(560\left(\cos(1/2\,dx+c/2)\right)^{11}-600\left(\cos(1/2\,dx+c/2)\right)^9+212\,(c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a^3\cos(dx+c)^4+3\,a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+a^3\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2
+ a^3*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.161 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=121

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52a^3\,\mathrm{s}}{}$$

```
[Out] (28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2
])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(7*d)
```

Rubi [A] time = 0.128461, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2639, 2635, 2641}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{52a^3\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3,x]
```


```
[Out] (28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2, 2
])/(21*d) + (52*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (6*a^3*Cos[c
+ d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(7*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3\,dx = \int \left(a^3\sqrt{\cos(c+dx)} + 3a^3\cos^{\frac{3}{2}}(c+dx) + 3a^3\cos^{\frac{5}{2}}(c+dx) + a^3\cos^{\frac{7}{2}}(c+dx)\right)$$

$$= a^3\int \sqrt{\cos(c+dx)}\,dx + a^3\int \cos^{\frac{7}{2}}(c+dx)\,dx + \left(3a^3\right)\int \cos^{\frac{3}{2}}(c+dx)\,dx + \left(3a\right.$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d} + \frac{6a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5d}$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

$$= \frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{52a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}$$

Mathematica [C] time = 6.11681, size = 500, normalized size = 4.13

$$-\frac{7\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-1}\right.}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*((-7*Cot[c])
/(10*d) + (107*Cos[d*x]*Sin[c])/(336*d) + (3*Cos[2*d*x]*Sin[2*c])/(40*d) +
(Cos[3*d*x]*Sin[3*c])/(112*d) + (107*Cos[c]*Sin[d*x])/(336*d) + (3*Cos[2*c]
*Sin[2*d*x])/(40*d) + (Cos[3*c]*Sin[3*d*x])/(112*d)) - (13*(a + a*Cos[c + d
*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcT
an[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*S
qrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (7*(a + a*C
os[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}
, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sq
rt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) -
((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(20*d)
```

Maple [A] time = 2.362, size = 272, normalized size = 2.3

$$-\frac{4a^3}{105d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(120\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8-432\left(\sin(1/2\,dx+c/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+65*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.162 $\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

```
[Out] (36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2]
)/d + (2*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.109022, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2641, 2639, 2635}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2]
)/d + (2*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(5*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \int \left(\frac{a^3}{\sqrt{\cos(c+dx)}} + 3a^3\sqrt{\cos(c+dx)} + 3a^3\cos^{\frac{3}{2}}(c+dx) + a^3\cos^{\frac{5}{2}}(c+dx)\right)dx$$
$$= a^3\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + a^3\int \cos^{\frac{5}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sqrt{\cos(c+dx)}\,dx + \left(3a^3\right)\int \cos$$
$$= \frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\cos^{\frac{3}{2}}(c$$
$$= \frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\cos^{\frac{3}{2}}(c$$

Mathematica [C] time = 5.63404, size = 233, normalized size = 2.56

$$a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-18\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hype}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*((9*(3*Cos[c - d*x - ArcTan[Ta
n[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 20*
Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometri
cPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c
]]]*Sin[c] + Cos[c + d*x]*(-36*Cot[c] + 10*Sin[c + d*x] + Sin[2*(c + d*x)])
 - 18*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]
]]^2]))/(40*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.391, size = 250, normalized size = 2.8

$$-\frac{4a^3}{5d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+14\left(\sin(1/2\,dx+c/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{3}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.163 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.110365, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2757, 2636, 2639, 2641, 2635}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*a^3*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^3*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(3*d)
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx=\int\left(\frac{a^3}{\cos^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\cos(c+dx)}}+3a^3\sqrt{\cos(c+dx)}+a^3\cos^{\frac{3}{2}}(c+dx)\right)dx$$
$$=a^3\int\frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx+a^3\int\cos^{\frac{3}{2}}(c+dx)\,dx+\left(3a^3\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx+\left(3a^3\right)\int\sqrt{\cos(
$$
$$=\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+dx}{3d}$$
$$=\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^3\sqrt{\cos(c+dx)}\sin(c+d}{3d}$$

Mathematica [C] time = 4.6832, size = 240, normalized size = 2.64

$$a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-6\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyperge}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-3*Cos[d*x]*Csc[c] - 9*Cos[2*
c + d*x]*Csc[c] + 9*Cos[c - d*x - ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] + 3
*Cos[c + d*x + ArcTan[Tan[c]]]*Cot[c]*Sqrt[Sec[c]^2] - 20*Cos[c + d*x]*Sqrt
[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2},
{5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Sin[
2*(c + d*x)] - 6*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*HypergeometricPFQ[{-1/2,
-1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + Ar
cTan[Tan[c]]]^2]))/(24*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.25, size = 172, normalized size = 1.9

$$-\frac{4a^3}{3d}\left(2\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+5\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\text{EllipticF}\left(\cos(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.164 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (6*a^3*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.104893, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2641, 2639}

$$\frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^3*EllipticE[(c + d*x)/2, 2])/d + (20*a^3*EllipticF[(c + d*x)/2, 2])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (6*a^3*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\cos^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\cos^{\frac{3}{2}}(c+dx)} + \frac{3a^3}{\sqrt{\cos(c+dx)}} + a^3\sqrt{\cos(c+dx)}\right) dx$$

$$= a^3 \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a^3 \int \sqrt{\cos(c+dx)}\,dx + \left(3a^3\right) \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right) \int \frac{}{\sqrt{\text{c}}}$$

$$= \frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{6a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}a^3 \int$$

$$= -\frac{4a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{20a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^3\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 6.18745, size = 463, normalized size = 5.09

$$\frac{\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3 \left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))+\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos\left(\tan^{-}\right.}}\right.}{4d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*(-((-5 + Cos
[2*c])*Csc[c]*Sec[c])/(8*d) + (Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(12*d) + (Se
c[c]*Sec[c + d*x]*(Sin[c] + 9*Sin[d*x]))/(12*d)) - (5*(a + a*Cos[c + d*x])^
3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(6*d*Sqrt[1 + Cot[c]^2]) + ((a + a*Cos[c + d
*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(4*d)
```

Maple [B] time = 3.625, size = 371, normalized size = 4.1

$$\frac{4a^3}{3d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(10\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\,\text{EllipticF}\left(\cos(1/2\,dx+c/2),\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(10*(2*sin(1/
```

```
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2-18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

3.165 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2
])/d + (2*a^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a^3*Sin[c + d*x])
/(d*Cos[c + d*x]^(3/2)) + (36*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.127116, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2639, 2641}

$$\frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{36a^3\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-36*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^3*EllipticF[(c + d*x)/2, 2
])/d + (2*a^3*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*a^3*Sin[c + d*x])
/(d*Cos[c + d*x]^(3/2)) + (36*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\cos^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\cos^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\cos^{\frac{3}{2}}(c+dx)} + \frac{a^3}{\sqrt{\cos(c+dx)}}\right)dx$$

$$= a^3 \int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a^3 \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(3a^3\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \left(3a^3\right)\int \frac{}{\text{co}}$$

$$= \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{5}\left(3a^3\right)\int$$

$$= -\frac{6a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{36a}{5d\sqrt{}}$$

$$= -\frac{36a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{36}{5d}$$

Mathematica [C] time = 6.2025, size = 485, normalized size = 4.15

$$\frac{9\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c)\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos(\text{ta}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*((9*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(20*d) + (Sec[c]*Sec[c + d
*x]^2*(Sin[c] + 5*Sin[d*x]))/(20*d) + (Sec[c]*Sec[c + d*x]*(5*Sin[c] + 18*S
in[d*x]))/(20*d)) - ((a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4,
1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(2
*d*Sqrt[1 + Cot[c]^2]) + (9*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2
]^6*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(20*d)
```

Maple [B] time = 3.726, size = 386, normalized size = 3.3

$$-16\,\frac{\sqrt{-\left(-2\,(\cos(1/2\,dx+c/2))^2+1\right)(\sin(1/2\,dx+c/2))^2}\,a^3}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}\,d}\left(\frac{7\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\text{E}}{10\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(7/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

3.166 $\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+d}{5d\sqrt{\cos(c+d}}$$

```
[Out] (-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^
(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.14811, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2641, 2639}

$$\frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{52a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{28a^3\sin(c+d}{5d\sqrt{\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-28*a^3*EllipticE[(c + d*x)/2, 2])/(5*d) + (52*a^3*EllipticF[(c + d*x)/2,
2])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Cos[c + d*x]^(5/2)) + (52*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^
(3/2)) + (28*a^3*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^3}{\cos^{\frac{9}{2}}(c+dx)} + \frac{3a^3}{\cos^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\cos^{\frac{5}{2}}(c+dx)} + \frac{a^3}{\cos^{\frac{3}{2}}(c+dx)}\right) dx$$

$$= a^3 \int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}\,dx + a^3 \int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right) \int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + \left(3a^3\right) \int \frac{}{\text{co}}$$

$$= \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{7}\left(5a^3\right) \int \frac{}{\text{co}}$$

$$= -\frac{2a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{52a}{21d}$$

$$= -\frac{28a^3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{52a^3F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^3\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{5}{2}$$

Mathematica [C] time = 6.23853, size = 515, normalized size = 3.5

$$\frac{7\csc(c)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^3\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c)\right)\right)}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos(\text{ta}}}\right.}{20d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3*Sec[c/2 + (d*x)/2]^6*((7*Csc[c]*S
ec[c])/(10*d) + (Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(28*d) + (Sec[c]*Sec[c + d
*x]^3*(5*Sin[c] + 21*Sin[d*x]))/(140*d) + (Sec[c]*Sec[c + d*x]^2*(63*Sin[c]
 + 130*Sin[d*x]))/(420*d) + (Sec[c]*Sec[c + d*x]*(65*Sin[c] + 147*Sin[d*x])
)/(210*d)) - (13*(a + a*Cos[c + d*x])^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}
, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^6*Sec[d*x - ArcTan
[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin
[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*
Sqrt[1 + Cot[c]^2]) + (7*(a + a*Cos[c + d*x])^3*Csc[c]*Sec[c/2 + (d*x)/2]^6
*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 +
Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]]))/(20*d)
```

Maple [B] time = 3.825, size = 439, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3/cos(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-13/168*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-7/20*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-3/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-1/44
8*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

3.167 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=173

$$\frac{904a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{11d}+\frac{8a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{150a^4 s}{}$$

```
[Out] (128*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (904*a^4*EllipticF[(c + d*x)/2
, 2])/(231*d) + (904*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (128*a^
4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (150*a^4*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(77*d) + (8*a^4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*a^4*
Cos[c + d*x]^(9/2)*Sin[c + d*x])/(11*d)
```

Rubi [A] time = 0.20459, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2635, 2641, 2639}

$$\frac{904a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)}{11d}+\frac{8a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{150a^4 s}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (128*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (904*a^4*EllipticF[(c + d*x)/2
, 2])/(231*d) + (904*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(231*d) + (128*a^
4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (150*a^4*Cos[c + d*x]^(5/2)*Sin
[c + d*x])/(77*d) + (8*a^4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d) + (2*a^4*
Cos[c + d*x]^(9/2)*Sin[c + d*x])/(11*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^4\,dx = \int \left(a^4\cos^{\frac{3}{2}}(c+dx)+4a^4\cos^{\frac{5}{2}}(c+dx)+6a^4\cos^{\frac{7}{2}}(c+dx)+4a^4\cos^{\frac{9}{2}}(c+d\right.$$

$$= a^4\int \cos^{\frac{3}{2}}(c+dx)\,dx + a^4\int \cos^{\frac{11}{2}}(c+dx)\,dx + \left(4a^4\right)\int \cos^{\frac{5}{2}}(c+dx)\,dx +$$

$$= \frac{2a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{8a^4\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{12a^4\cos^{\frac{5}{2}}(c}{}$$

$$= \frac{24a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{74a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}$$

$$= \frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{74a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{904a^4\sqrt{\cos(c+dx)}\sin(c+}{231d}$$

$$= \frac{128a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{904a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d} + \frac{904a^4\sqrt{\cos(c+dx)}\sin(c}{231d}$$

Mathematica [C] time = 3.63354, size = 271, normalized size = 1.57

$$a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\frac{59136\sec(c)\left(\csc(c)\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(3\cos\left(c-\tan^{-1}(\tan(c))-dx\right)+\cos\left(c+\tan^{-1}(\tan(c))+dx\right)\right)-}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(t\right.}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(-108480*Cos[c + d*x]*Sqrt[Cos
[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + Cos[c +
d*x]*(-236544*Cot[c] + 122610*Sin[c + d*x] + 45584*Sin[2*(c + d*x)] + 14445
*Sin[3*(c + d*x)] + 3080*Sin[4*(c + d*x)] + 315*Sin[5*(c + d*x)]) + (59136*
Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]
^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c +
d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Sec
[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(443520*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.187, size = 273, normalized size = 1.6

$$-\frac{8a^4}{3465d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5040\left(\cos(1/2\,dx+c/2)\right)^{13}-5320\left(\cos(1/2\,dx+c/2)\right)^{11}+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^4,x)
```

```
[Out] -8/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(5040*c
os(1/2*d*x+1/2*c)^13-5320*cos(1/2*d*x+1/2*c)^11+1740*cos(1/2*d*x+1/2*c)^9+3
26*cos(1/2*d*x+1/2*c)^7+678*cos(1/2*d*x+1/2*c)^5-4465*cos(1/2*d*x+1/2*c)^3+
1695*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-3696*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2001*cos(1/2*
```

```
d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4\cos(dx+c)^5+4\,a^4\cos(dx+c)^4+6\,a^4\cos(dx+c)^3+4\,a^4\cos(dx+c)^2+a^4\cos(dx+c)\right)\sqrt{\cos(dx+c)}, x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^5 + 4*a^4*cos(d*x + c)^4 + 6*a^4*cos(d*x + c)^3
+ 4*a^4*cos(d*x + c)^2 + a^4*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^(3/2), x)
```

3.168 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=147

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{152a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{122a^4}{}$$

```
[Out] (152*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (32*a^4*EllipticF[(c + d*x)/2,
 2])/(7*d) + (32*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (122*a^4*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (8*a^4*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.163782, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2639, 2635, 2641}

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{152a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^4\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d}+\frac{8a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{122a^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4,x]
```


```
[Out] (152*a^4*EllipticE[(c + d*x)/2, 2])/(15*d) + (32*a^4*EllipticF[(c + d*x)/2,
 2])/(7*d) + (32*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(7*d) + (122*a^4*Cos[
c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (8*a^4*Cos[c + d*x]^(5/2)*Sin[c + d*x
])/(7*d) + (2*a^4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^4\,dx = \int \left(a^4\sqrt{\cos(c+dx)} + 4a^4\cos^{\frac{3}{2}}(c+dx) + 6a^4\cos^{\frac{5}{2}}(c+dx) + 4a^4\cos^{\frac{7}{2}}(c+dx)\right.$$

$$= a^4\int \sqrt{\cos(c+dx)}\,dx + a^4\int \cos^{\frac{9}{2}}(c+dx)\,dx + \left(4a^4\right)\int \cos^{\frac{3}{2}}(c+dx)\,dx + \left(4a\right.$$

$$= \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{12a^4\cos^{\frac{3}{2}}(c+dx)\sin(c}{5d}$$

$$= \frac{46a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{8a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{32a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{7d} +$$

$$= \frac{152a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d} + \frac{32a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{7d}$$

Mathematica [C] time = 6.14535, size = 532, normalized size = 3.62

$$-\frac{19\csc(c)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^4\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(c))\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos(\tan}}\right.}{60d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*((-19*Cot[c]
)/(30*d) + (17*Cos[d*x]*Sin[c])/(56*d) + (127*Cos[2*d*x]*Sin[2*c])/(1440*d)
 + (Cos[3*d*x]*Sin[3*c])/(56*d) + (Cos[4*d*x]*Sin[4*c])/(576*d) + (17*Cos[c
]*Sin[d*x])/(56*d) + (127*Cos[2*c]*Sin[2*d*x])/(1440*d) + (Cos[3*c]*Sin[3*d
*x])/(56*d) + (Cos[4*c]*Sin[4*d*x])/(576*d)) - (2*(a + a*Cos[c + d*x])^4*Cs
c[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2 + (d*x)/2]^8*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]
]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + S
in[d*x - ArcTan[Cot[c]]]])/(7*d*Sqrt[1 + Cot[c]^2]) - (19*(a + a*Cos[c + d*
x])^4*Csc[c]*Sec[c/2 + (d*x)/2]^8*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Co
s[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Co
s[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x
 + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTa
n[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(60*d)
```

Maple [A] time = 2.234, size = 260, normalized size = 1.8

$$-\frac{8a^4}{315d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(280\left(\cos(1/2\,dx+c/2)\right)^{11}-120\left(\cos(1/2\,dx+c/2)\right)^9+34\left(\cos(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^4,x)
```

```
[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos
(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(
1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(cos(d*x + c)), x)
```

3.169 $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=121

$$\frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{94a^4\sin}{}$$

```
[Out] (64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2,
2])/(21*d) + (94*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (8*a^4*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^4*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(7*d)
```

Rubi [A] time = 0.139796, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2641, 2639, 2635}

$$\frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{94a^4\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2,
2])/(21*d) + (94*a^4*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (8*a^4*Cos[c
 + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^4*Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(7*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\cos(c+dx)}}dx = \int \left(\frac{a^4}{\sqrt{\cos(c+dx)}} + 4a^4\sqrt{\cos(c+dx)} + 6a^4\cos^{\frac{3}{2}}(c+dx) + 4a^4\cos^{\frac{5}{2}}(c+dx) + a^4\cos^{\frac{7}{2}}(c$$

$$= a^4\int \frac{1}{\sqrt{\cos(c+dx)}}dx + a^4\int \cos^{\frac{7}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sqrt{\cos(c+dx)}\,dx + \left(4a^4\right)\int \cos$$

$$= \frac{8a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{d} + \frac{8a^4\cos^{\frac{3}{2}}(c$$

$$= \frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{6a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{94a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{8a^4\cos^{\frac{3}{2}}$$

$$= \frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{94a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{8a^4\text{co}$$

Mathematica [C] time = 6.1675, size = 500, normalized size = 4.13

$$-\frac{2\csc(c)\sec^8\left(\frac{c}{2}+\frac{dx}{2}\right)(a\cos(c+dx)+a)^4\left(\frac{\tan(c)\sin\left(\tan^{-1}(\tan(c))+dx\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},-\frac{1}{4}\right\},\left\{\frac{3}{4}\right\},\cos^2\left(\tan^{-1}(\tan(\right.\right.}{\sqrt{\tan^2(c)+1}\sqrt{1-\cos\left(\tan^{-1}(\tan(c))+dx\right)}\sqrt{\cos\left(\tan^{-1}(\tan(c))+dx\right)+1}\sqrt{\cos(c)\sqrt{\tan^2(c)+1}\cos(t}}\right.}{5d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Sqrt[Cos[c + d*x]],x]
```

```
[Out] Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^4*Sec[c/2 + (d*x)/2]^8*((-4*Cot[c])
/(5*d) + (191*Cos[d*x]*Sin[c])/(672*d) + (Cos[2*d*x]*Sin[2*c])/(20*d) + (Co
s[3*d*x]*Sin[3*c])/(224*d) + (191*Cos[c]*Sin[d*x])/(672*d) + (Cos[2*c]*Sin[
2*d*x])/(20*d) + (Cos[3*c]*Sin[3*d*x])/(224*d)) - (17*(a + a*Cos[c + d*x])^
4*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2 + (d*x)/2]^8*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Co
t[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1
 + Sin[d*x - ArcTan[Cot[c]]]])/(42*d*Sqrt[1 + Cot[c]^2]) - (2*(a + a*Cos[c
+ d*x])^4*Csc[c]*Sec[c/2 + (d*x)/2]^8*((HypergeometricPFQ[{-1/2, -1/4}, {3/
4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1
- Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + A
rcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d
*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d)
```

Maple [A] time = 1.958, size = 272, normalized size = 2.3

$$-\frac{8a^4}{105d}\sqrt{\left(2\left(\cos(1/2dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(60\cos(1/2dx+c/2)\left(\sin(1/2dx+c/2)\right)^8-258\left(\sin(1/2dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(1/2),x)
```

```
[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/
2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(cos(d*x + c)), x)
```

3.170 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^4\sin(c}{d\sqrt{\cos(c}}$$

```
[Out] (56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (8*a^4*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*d) + (2*a^4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.12211, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2757, 2636, 2639, 2641, 2635}

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^4\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^4\sin(c}{d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(3/2),x]
```


```
[Out] (56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2, 2
])/(3*d) + (2*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (8*a^4*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(3*d) + (2*a^4*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\cos(c+dx)}} + 6a^4\sqrt{\cos(c+dx)} + 4a^4\cos^{\frac{3}{2}}(c+dx) + a^4\cos^{\frac{5}{2}}(c+$$

$$= a^4\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + a^4\int \cos^{\frac{5}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(4a^4\right)\int \cos$$

$$= \frac{12a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{8a^4\sqrt{\cos(c+dx)}\sin(c}{3d}$$

$$= \frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{8a^4\sqrt{\cos(c+dx)}\sin(c}{3d}$$

Mathematica [C] time = 6.14523, size = 245, normalized size = 2.06

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(-336\cos(c)\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\csc\left(\tan^{-1}(\tan(c))+dx\right)\text{Hyp}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(-273*Cos[d*x]*Csc[c] - 399*Co
s[2*c + d*x]*Csc[c] + (168*(3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x +
 ArcTan[Tan[c]]])*Csc[c]*Sec[c])/Sqrt[Sec[c]^2] - 640*Cos[c + d*x]*Sqrt[Cos
[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4
}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + 80*Sin[2
*(c + d*x)] + 6*Sin[3*(c + d*x)] - 336*Cos[c]*Csc[d*x + ArcTan[Tan[c]]]*Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sqrt[Sec[c
]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(960*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.33, size = 194, normalized size = 1.6

$$-\frac{8a^4}{15d}\left(-6\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2) + 26\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2) + 20\sqrt{2}\left(\sin(1/2\,dx+c/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(3/2),x)
```

```
[Out] -8/15*a^4*(-6*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+26*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+20*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(3/2), x)
```

3.171 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{40a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (40*a^4*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^4*Sin[c + d*x])/(3*d*Cos[c
+ d*x]^(3/2)) + (8*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^4*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.122476, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2757, 2636, 2641, 2639, 2635}

$$\frac{40a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(5/2),x]
```


```
[Out] (40*a^4*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^4*Sin[c + d*x])/(3*d*Cos[c
+ d*x]^(3/2)) + (8*a^4*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (2*a^4*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\cos^{\frac{3}{2}}(c+dx)} + \frac{6a^4}{\sqrt{\cos(c+dx)}} + 4a^4\sqrt{\cos(c+dx)} + a^4\cos^{\frac{3}{2}}(c+dx)\right) dx$$

$$= a^4\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + a^4\int \cos^{\frac{3}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \left(4a^4\right)\int \sqrt{\cos(\ldots}$$

$$= \frac{8a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{12a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^4\sqrt{\text{co}\ldots}}{}$$

$$= \frac{40a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{2a^4\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.313448, size = 70, normalized size = 0.71

$$\frac{a^4\left(5\sin(c+dx)+24\sin(2(c+dx))+\sin(3(c+dx))+80\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^4*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*Sin[c + d*x] + 24
*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 3.161, size = 292, normalized size = 3.

$$\frac{8a^4}{3d}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\left(\sin\left(1/2\,dx+c/2\right)\right)^6\cos\left(1/2\,dx+c/2\right)+10\sqrt{2\left(\sin\left(1/2\,dx+c\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(5/2),x)
```

```
[Out] 8/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+7
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(5/2), x)
```

3.172 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{66a^4\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (8*a^4*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (66*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.146417, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2639, 2641}

$$\frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{66a^4\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-56*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (32*a^4*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (8*a^4*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (66*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\cos^{\frac{7}{2}}(c+dx)} + \frac{4a^4}{\cos^{\frac{5}{2}}(c+dx)} + \frac{6a^4}{\cos^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\cos(c+dx)}} + a^4\sqrt{\cos(c+dx)} \right) dx$$

$$= a^4 \int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + a^4 \int \sqrt{\cos(c+dx)}\,dx + \left(4a^4\right) \int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \left(4a^4\right) \int \frac{}{\sqrt{c}}$$

$$= \frac{2a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{12a^4}{d\sqrt{c}}$$

$$= -\frac{10a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6}{5}$$

$$= -\frac{56a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{32a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6}{5}$$

Mathematica [C] time = 4.34984, size = 283, normalized size = 2.34

$$a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(-\frac{168\sec(c)\cos^2(c+dx)\left(\csc(c)\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(3\cos\left(c-\tan^{-1}(\tan(c))-dx\right)+\cos\left(c+\tan^{-1}(\tan\right.\right.\right.}{\sqrt{\sec^2(c)}\sqrt{\sin^2(\tan}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*((363*Cos[d*x] + 141*Cos[2*c +
 d*x] + 40*Cos[c + 2*d*x] - 40*Cos[3*c + 2*d*x] + 183*Cos[2*c + 3*d*x] - 15
*Cos[4*c + 3*d*x])*Csc[c] - 640*Cos[c + d*x]^3*Sqrt[Cos[d*x - ArcTan[Cot[c]
]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] - (168*Cos[c + d*x]^2*Sec[c]*(
-2*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]] + (3*Cos[c - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + Ar
cTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Sec[c]^2]*S
qrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(960*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 4.058, size = 386, normalized size = 3.2

$$-32\,\frac{\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^2}a^4}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1}d}\left(-\frac{7\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}}{20\sqrt{-2\,\left(\sin\left(1/\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(7/2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(-7/20*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+41/60*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/24*cos(1/2*d*x+1/2*c)
```

```
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
-1/2)^2-33/40*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2
*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/320*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(7/2), x)
```

3.173 $\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=147

$$\frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{94a^4\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{64a^4\sin(c}{5d\sqrt{\cos(c}}$$

```
[Out] (-64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2,
 2])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Cos[c + d*x]^(5/2)) + (94*a^4*Sin[c + d*x])/(21*d*Cos[c + d*x]
^(3/2)) + (64*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.166529, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2757, 2636, 2641, 2639}

$$\frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{94a^4\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{64a^4\sin(c}{5d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(9/2),x]
```


```
[Out] (-64*a^4*EllipticE[(c + d*x)/2, 2])/(5*d) + (136*a^4*EllipticF[(c + d*x)/2,
 2])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Cos[c + d*x]^(5/2)) + (94*a^4*Sin[c + d*x])/(21*d*Cos[c + d*x]
^(3/2)) + (64*a^4*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```


Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \int \left(\frac{a^4}{\cos^{\frac{9}{2}}(c+dx)} + \frac{4a^4}{\cos^{\frac{7}{2}}(c+dx)} + \frac{6a^4}{\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\cos^{\frac{3}{2}}(c+dx)} + \frac{a^4}{\sqrt{\cos(c+dx)}}\right) dx$$

$$= a^4 \int \frac{1}{\cos^{\frac{9}{2}}(c+dx)}\,dx + a^4 \int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(4a^4\right) \int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + \left(4a^4\right) \int \frac{}{\cos^{\frac{3}{2}}(}$$

$$= \frac{2a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^4\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+}{d\sqrt{\cos(c+d}}$$

$$= -\frac{8a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{6a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{94a^4\,\mathrm{s}}{21d\,\mathrm{co}}$$

$$= -\frac{64a^4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{136a^4F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2a^4\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{94}{21}$$

Mathematica [C] time = 5.12433, size = 298, normalized size = 2.03

$$\frac{a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1344\sec(c)\cos^3(c+dx)\left(\csc(c)\sqrt{\sin^2\left(\tan^{-1}(\tan(c))+dx\right)}\left(3\cos\left(c-\tan^{-1}(\tan(c))-dx\right)+\cos\left(c+\tan^{-1}(\tan(c\right.\right.\right.}{\sqrt{\sec^2(c)}\sqrt{\sin^2\left(\tan^{-1}\right.}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Cos[c + d*x]^(9/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*((2016*Cos[c] + 295*Cos[d*x] -
 295*Cos[2*c + d*x] + 2184*Cos[c + 2*d*x] + 504*Cos[3*c + 2*d*x] + 235*Cos[
2*c + 3*d*x] - 235*Cos[4*c + 3*d*x] + 672*Cos[3*c + 4*d*x])*Csc[c] - 2720*C
os[c + d*x]^4*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hypergeometr
icPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[
c]]]*Sin[c] - (1344*Cos[c + d*x]^3*Sec[c]*(-2*HypergeometricPFQ[{-1/2, -1/4
}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]] + (3*Cos[c
 - d*x - ArcTan[Tan[c]]] + Cos[c + d*x + ArcTan[Tan[c]]])*Csc[c]*Sqrt[Sin[d
*x + ArcTan[Tan[c]]]^2]))/(Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]
)))/(6720*d*Cos[c + d*x]^(7/2))
```

Maple [B] time = 4.355, size = 439, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4/cos(d*x+c)^(9/2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(253/420*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-47/672*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
```

```
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-1/896*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/cos(d*x + c)^(9/2), x)
```

3.174 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{\cos(c}}{3ad}$$

```
[Out] (21*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + (7*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*a*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rubi [A] time = 0.109611, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2767, 2748, 2635, 2641, 2639}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5\sin(c+dx)\sqrt{\cos(c}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (21*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) + (7*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(5*a*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-\frac{7}{2}a\cos(c+dx)\right)dx}{a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{5\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{7\int \cos^{\frac{5}{2}}(c+dx)\,dx}{2a}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{5\int$$

$$= \frac{21E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} + \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c}{5ad}$$

Mathematica [C] time = 1.79916, size = 315, normalized size = 2.46

$$\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\csc(c)\sqrt{\cos(c+dx)}\left(5\sin(2c)\sin(dx)+10\sin^2(c)\cos(dx)-6\cos(c)\left(\sin^2(c)\cos(2dx)-8\right)-3\sin(c)\cos(2c)\sin(2dx)+30\sin\left(\frac{c}{2}\right)\sin\left(\frac{d}{2}\right.}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - (2*Sqrt[Cos[c + d*x]]*Csc[c]*(15 + 10*Cos[d*x]*Sin[c]^2 -
6*Cos[c]*(-8 + Cos[2*d*x]*Sin[c]^2) + 30*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x
)/2] + 5*Sin[2*c]*Sin[d*x] - 3*Cos[2*c]*Sin[c]*Sin[2*d*x]))/d))/(15*a*(1 +
Cos[c + d*x]))
```

Maple [A] time = 2.244, size = 229, normalized size = 1.8

$$-\frac{1}{15da}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4
+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{7}{2}}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a), x)
```

3.175 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d
) + (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) - (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.101456, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2767, 2748, 2639, 2635, 2641}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d
) + (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d) - (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \sqrt{\cos(c+dx)}\left(\frac{3a}{2}-\frac{5}{2}a\cos(c+dx)\right)dx}{a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{3\int \sqrt{\cos(c+dx)}\,dx}{2a} + \frac{5\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}}\,d}{6a}$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3ad} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{d(a+a\cos(c+dx))}$$

Mathematica [C] time = 1.25831, size = 289, normalized size = 2.89

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\csc(c)\sqrt{\cos(c+dx)}\left(\sin(2c)\sin(dx)+2\sin^2(c)\cos(dx)+6\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+6\cos(c)+3\right)}{d} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i}}\right.}{}\right.}{3a(\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))
/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*
(c + d*x))]) + (2*Sqrt[Cos[c + d*x]]*Csc[c]*(3 + 6*Cos[c] + 2*Cos[d*x]*Sin[
c]^2 + 6*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x)/2] + Sin[2*c]*Sin[d*x]))/d))/(
3*a*(1 + Cos[c + d*x]))
```

Maple [A] time = 2.333, size = 215, normalized size = 2.2

$$-\frac{1}{3\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(5\,\mathrm{E}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-
8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
```

```
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

3.176 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=72

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) - (Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0863407, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2767, 2748, 2641, 2639}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) - (Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2767

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f
*x])^(n - 1))/(a*f*(a + b*Sin[e + f*x])), x] - Dist[d/(a*b), Int[(c + d*Sin
[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e +
 f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && E
qQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || EqQ
[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \frac{\frac{a}{2}-\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}}dx}{a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{2a} + \frac{3\int \sqrt{\cos(c+dx)}dx}{2a}$$

$$= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [C] time = 2.59413, size = 264, normalized size = 3.67

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\sqrt{\cos(c+dx)}\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+2\cot(c)+\csc(c)\right)}{d} + \frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4}\right.\right.}{}\right.}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d
*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c
+ d*x))]) - (2*Sqrt[Cos[c + d*x]]*(2*Cot[c] + Csc[c] + Sec[c/2]*Sec[(c + d*
x)/2]*Sin[(d*x)/2]))/d))/(a*(1 + Cos[c + d*x]))
```

Maple [A] time = 2.398, size = 199, normalized size = 2.8

$$\frac{1}{da}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\left(\text{Elli}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

3.177 $\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a*d)) + EllipticF[(c + d*x)/2, 2]/(a*d) + (Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0825385, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2769, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a*d)) + EllipticF[(c + d*x)/2, 2]/(a*d) + (Sqr
t[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2769

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a
*f*(a + b*Sin[e + f*x])), x] + Dist[(d*n)/(a*b), Int[(c + d*Sin[e + f*x])^(
n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n
] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{a+a\cos(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{a-a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{2a^2}$$
$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{2a}$$
$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [C] time = 1.00311, size = 256, normalized size = 3.66

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2\sqrt{\cos(c+dx)}\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c)\right)}{d} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(c+dx)}\right)+\right.}{\left(-1+e^{2ic}\right)d\sqrt{e^{-}}}\right.}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2
*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^
((2*I)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(
c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I
*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x
))]) + (2*Sqrt[Cos[c + d*x]]*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/
2]))/d))/(a*(1 + Cos[c + d*x]))
```

Maple [A] time = 1.99, size = 198, normalized size = 2.8

$$-\frac{1}{da}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\text{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\cos(c+dx)}}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(cos(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a), x)
```

3.178 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=70

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

```
[Out] EllipticE[(c + d*x)/2, 2]/(a*d) + EllipticF[(c + d*x)/2, 2]/(a*d) - (Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0851393, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2768, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a*d) + EllipticF[(c + d*x)/2, 2]/(a*d) - (Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\int \frac{-\frac{a}{2}-\frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\cos(c+dx)}\,dx}{2a}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d(a+a\cos(c+dx))}$$

Mathematica [C] time = 1.00402, size = 257, normalized size = 3.67

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\sqrt{\cos(c+dx)}\left(\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c)\right)}{d} + \frac{2i\sqrt{2}e^{-i(c+dx)}\left(\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-e^{2i(\cdots}\right.\right.}{\left(-1+e^{2ic\cdots}\right.}\right.}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(1 + E^((2*I)*(c + d*x)) + (-1 + E^((2*
I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(
(2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*
(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)
)]) - (2*Sqrt[Cos[c + d*x]]*(Csc[c] + Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2
]))/d))/(a*(1 + Cos[c + d*x]))
```

Maple [A] time = 2.057, size = 200, normalized size = 2.9

$$\frac{1}{da}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\left(\mathrm{E}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")

[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")

[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^2 + a*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\cos^{\frac{3}{2}}\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)

[Out] Integral(1/(cos(c + d*x)**(3/2) + sqrt(cos(c + d*x))), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.179 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} dx$

Optimal. Leaf size=96

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) + (3
*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]
]*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0987303, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2768, 2748, 2636, 2639, 2641}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (-3*EllipticE[(c + d*x)/2, 2])/(a*d) - EllipticF[(c + d*x)/2, 2]/(a*d) + (3
*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]
]*(a + a*Cos[c + d*x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} - \frac{\int \frac{-\frac{3a}{2}+\frac{1}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \frac{3\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a}$$

$$= -\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))} - \frac{3\int$$

$$= -\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+}{d\sqrt{\cos(c+dx)}(a+}$$

Mathematica [C] time = 2.06666, size = 297, normalized size = 3.09

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(2\cos\left(\frac{1}{2}(c-dx)\right)+\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{2d\sqrt{\cos(c+dx)}} - \frac{2i\sqrt{2}e^{-i(c+dx)}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\text{Hypergeom}}{}\right.}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] - E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(
d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c
 + d*x))]) + ((2*Cos[(c - d*x)/2] + Cos[(3*c + d*x)/2] + 3*Cos[(c + 3*d*x)/
2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/(2*d*Sqrt[Cos[c + d*x]])))/(a*(1 +
Cos[c + d*x]))
```

Maple [A] time = 2.2, size = 253, normalized size = 2.6

$$-\frac{1}{da}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\text{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
```

```
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/cos
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^3+a\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^3 + a*cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(1/(cos(c + d*x)**(5/2) + cos(c + d*x)**(3/2)), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.180 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$

Optimal. Leaf size=124

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d)
 + (5*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (3*Sin[c + d*x])/(a*d*Sqrt
[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rubi [A] time = 0.112024, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2768, 2748, 2636, 2641, 2639}

$$\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}+\frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}-\frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (3*EllipticE[(c + d*x)/2, 2])/(a*d) + (5*EllipticF[(c + d*x)/2, 2])/(3*a*d)
 + (5*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (3*Sin[c + d*x])/(a*d*Sqrt
[Cos[c + d*x]]) - Sin[c + d*x]/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 2768

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n
+ 1))/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x])), x] + Dist[d/(a*(b*c - a*d)),
Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx = -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{\int \frac{-\frac{5a}{2}+\frac{3}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{3\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{5\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \cdots$$

$$= \frac{3E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{5\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{3\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} \cdots$$

Mathematica [C] time = 3.57571, size = 332, normalized size = 2.68

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(10\cos\left(\frac{1}{2}(c-dx)\right)+8\cos\left(\frac{1}{2}(3c+dx)\right)+4\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+9\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{4d\cos^{\frac{3}{2}}(c+dx)} + \cdots\right)}{3a(\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/
(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(
c + d*x))]) - ((10*Cos[(c - d*x)/2] + 8*Cos[(3*c + d*x)/2] + 4*Cos[(c + 3*d
*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 9*Cos[(3*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]
*Sec[(c + d*x)/2])/(4*d*Cos[c + d*x]^(3/2))))/(3*a*(1 + Cos[c + d*x]))
```

Maple [B] time = 4.016, size = 413, normalized size = 3.3

$$\frac{1}{3da}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(10\sqrt{(\sin(1/2\,dx \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*x
+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
```

```
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*cos(1/2*d*x+1/2*c)+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*cos(1/2*d
*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a\cos(dx+c)^4+a\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a*cos(d*x + c)^4 + a*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.181 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=160

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)\sqrt{\cos}}{a^2d}$$

```
[Out] (56*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(a
^2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d) + (56*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(15*a^2*d) - (3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1
+ Cos[c + d*x])) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*
x])^2)
```

Rubi [A] time = 0.219112, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2748, 2635, 2641, 2639}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{56\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5\sin(c+dx)\sqrt{\cos}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (56*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(a
^2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d) + (56*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(15*a^2*d) - (3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1
+ Cos[c + d*x])) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*
x])^2)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2}-\frac{11}{2}a\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{45a^2}{2}-28a^2\cos\right.}{3a^4}$$

$$= -\frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{15\int \cos^{\frac{3}{2}}(c+dx)\,dx}{2a^2} + \frac{28\int c}{}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15a^2d} - \frac{3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))}$$

$$= \frac{56E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d} + \frac{56\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{15a^2d}$$

Mathematica [C] time = 2.50988, size = 367, normalized size = 2.29

$$\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\csc(c)\sqrt{\cos(c+dx)}\left(40\sin^2(c)\cos(dx)-6\sin(c)\sin(2c)\cos(2dx)+8\cos(c)(5\sin(c)\sin(dx)+27)-6\sin(c)\cos(2c)\sin(2dx)-10\sin\left(\frac{c}{2}\right.\right.}{3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(56*(1 + E^((2*I)*(c + d*x))) + 56*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - (2*Sqrt[Cos[c + d*x]]*Csc[c]*(120 + 40*Cos[d*x]*Sin[c]^2 -
 6*Cos[2*d*x]*Sin[c]*Sin[2*c] + 240*Sec[(c + d*x)/2]*Sin[c/2]*Sin[(d*x)/2]
```

```
- 10*Sec[(c + d*x)/2]^3*Sin[c/2]*Sin[(d*x)/2] + 8*Cos[c]*(27 + 5*Sin[c]*Sin
[d*x]) - 6*Cos[2*c]*Sin[c]*Sin[2*d*x] - 5*Sec[(c + d*x)/2]^2*Sin[c]*Tan[c/2
]))/(3*d)))/(5*a^2*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 2.619, size = 283, normalized size = 1.8

$$-\frac{1}{30\,a^2 d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(96\,(\cos(1/2\,dx+c/2))^{10}-352\,(\cos(1/2\,dx+c/2))^8+120\,(\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d
*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{9}{2}}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(9/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^2, x)
```

3.182 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c}{3d(a\cos(c+dx)+}$$

```
[Out] (-7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*
a^2*d) + (10*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) - (7*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^(5/2)*Sin[c
 + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.201635, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2748, 2639, 2635, 2641}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{7\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{10\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c}{3d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (-7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*
a^2*d) + (10*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) - (7*Cos[c + d*x]^(
3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (Cos[c + d*x]^(5/2)*Sin[c
 + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-\frac{9}{2}a\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \sqrt{\cos(c+dx)}\left(\frac{21a^2}{2}-15a^2 c\right.}{3a^4}$$

$$= -\frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{7\int \sqrt{\cos(c+dx)}\,dx}{2a^2} + \frac{5\int \cos}{}$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{5}{2}}(c}{3d(a}$$

$$= -\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d} - \frac{7\cos^{\frac{3}{2}}(c+dx)}{3a^2d(1+\cos}$$

Mathematica [C] time = 1.85242, size = 337, normalized size = 2.44

$$\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc(c)\sqrt{\cos(c+dx)}\left(72\cos\left(\frac{1}{2}(c-dx)\right)+54\cos\left(\frac{1}{2}(3c+dx)\right)+33\cos\left(\frac{1}{2}(c+3dx)\right)+9\cos\left(\frac{1}{2}(5c+3dx)\right)+\cos\left(\frac{1}{2}(3c+5dx)\right)-\cos\left(\frac{1}{2}(7c+5dx)\right)\right)}{2d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^
(I*(c + d*x))]) + (Sqrt[Cos[c + d*x]]*(72*Cos[(c - d*x)/2] + 54*Cos[(3*c +
d*x)/2] + 33*Cos[(c + 3*d*x)/2] + 9*Cos[(5*c + 3*d*x)/2] + Cos[(3*c + 5*d*x
)/2] - Cos[(7*c + 5*d*x)/2])*Csc[c]*Sec[(c + d*x)/2]^3)/(2*d)))/(3*a^2*(1 +
```

```
Cos[c + d*x])^2)
```

Maple [A] time = 2.396, size = 270, normalized size = 2.

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(16\left(\cos\left(1/2\,dx+c/2\right)\right)^8+12\left(\cos\left(1/2\,dx+c/2\right)\right)^6+20\sqrt{(\sin(1/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d*
x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x
+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*d*x+
1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(7/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^2, x)
```

3.183 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] (4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a^
2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.183698, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2765, 2977, 2748, 2641, 2639}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```


```
[Out] (4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a^
2*d) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x])) - (
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-\frac{7}{2}a\cos(c+dx)\right)}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\frac{5a^2}{2}-6a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{3a^4}$$

$$= -\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{6a^2} + \frac{2\int \sqrt{\cos(\ldots}}{a\ldots}$$

$$= \frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(\ldots}{3d(a+a\cos(c+\ldots}$$

Mathematica [C] time = 2.36648, size = 319, normalized size = 2.85

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(20\cos\left(\frac{1}{2}(c-dx)\right)+16\cos\left(\frac{1}{2}(3c+dx)\right)+9\cos\left(\frac{1}{2}(c+3dx)\right)+3\cos\left(\frac{1}{2}(5c+3dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{2d} + \frac{4i\sqrt{2}e^{-}\ldots}{\ldots}\right.}{3a^2(\cos\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
)/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I
*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*(20*Cos[(c - d*x)/2] + 16*Cos[(3*c + d*
x)/2] + 9*Cos[(c + 3*d*x)/2] + 3*Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Se
c[(c + d*x)/2]^3)/(2*d)))/(3*a^2*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 2.42, size = 257, normalized size = 2.3

$$\frac{1}{6a^2d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\left(\cos(1/2\,dx+c/2)\right)^6+10\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x
+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d*x+1
/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{5}{2}}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

3.184 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a^2*d)) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2
*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - (Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.187834, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2765, 2978, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(EllipticE[(c + d*x)/2, 2]/(a^2*d)) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2
*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - (Sqrt[
Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{\frac{a}{2}-\frac{5}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{\int \frac{-a^2+\frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{3a^4}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{2a^2}$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+a}$$

Mathematica [C] time = 6.30429, size = 640, normalized size = 5.87

$$-\frac{4\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\mathrm{si}}}{3d\sqrt{\cot^2(c)+1}(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] ((-I/2)*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hypergeome
tric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
 E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt
[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((
2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1
[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2
*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E
^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*
x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^2 - (4*C
os[c/2 + (d*x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - A
rcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])
]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(a + a*Cos[c + d*x])^2*Sqrt[1 +
 Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*((4*Csc[c])/d + (4*S
ec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]
^3*Sin[(d*x)/2])/(3*d) - (2*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(a + a*C
os[c + d*x])^2
```

Maple [A] time = 2.409, size = 257, normalized size = 2.4

$$-\frac{1}{6\,a^2 d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^6+4\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\,d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x+1/2
*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

3.185 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.0544056, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2764, 21, 2641}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] EllipticF[(c + d*x)/2, 2]/(3*a^2*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*
d*(a + a*Cos[c + d*x])^2)
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^2}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{a}{2}+\frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}dx}{3a^2}$$
$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{6a^2}$$
$$= \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

Mathematica [A] time = 0.20425, size = 63, normalized size = 1.11

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 2.062, size = 188, normalized size = 3.3

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\,dx+c/2)\right)^2+1}\text{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

3.186 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

```
[Out] EllipticE[(c + d*x)/2, 2]/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - (Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.183858, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2766, 2978, 2748, 2641, 2639}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*a^2*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])) - (Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{5a}{2}-\frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{a^2+\frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx}{3a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3a^2} + \int$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{\sqrt{\cos(c+}}{3d(a+a}$$

Mathematica [C] time = 1.99924, size = 304, normalized size = 2.79

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(7\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+3\cos\left(\frac{1}{2}(c+3dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{2d} + \frac{4i\sqrt{2}e^{-i(c+dx)}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c}}}{}\right.}{3a^2(\cos(c+dx)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] - 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/
(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(
c + d*x))]) - (Sqrt[Cos[c + d*x]]*(7*Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2
] + 3*Cos[(c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^3)/(2*d)))/(3*
a^2*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 2.322, size = 257, normalized size = 2.4

$$\frac{1}{6a^2d}\sqrt{\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12(\cos(1/2dx+c/2))^6-4\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{-2(\cos(1/2dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^3+2\,a^2\cos(dx+c)^2+a^2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 + a^
2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.187 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-\frac{\sin(c+}{3d\sqrt{\cos(c+dx)}(a\,\mathrm{cc}}$$

```
[Out] (-4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (4*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (5*Sin[c + d*x])/(3*a
^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[Cos[c
+ d*x]]*(a + a*Cos[c + d*x])^2)
```

Rubi [A] time = 0.20795, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2978, 2748, 2636, 2639, 2641}

$$-\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(\cos(c+dx)+1)}-\frac{\sin(c+}{3d\sqrt{\cos(c+dx)}(a\,\mathrm{cc}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```


```
[Out] (-4*EllipticE[(c + d*x)/2, 2])/(a^2*d) - (5*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (4*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) - (5*Sin[c + d*x])/(3*a
^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[Cos[c
+ d*x]]*(a + a*Cos[c + d*x])^2)
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{7a}{2}-\frac{3}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}$$

$$= -\frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}(1+\cos(c+dx$$

$$= -\frac{4E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{4\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{5}{3a^2d\sqrt{\cos(c}$$

Mathematica [C] time = 1.87504, size = 334, normalized size = 2.46

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(29\cos\left(\frac{1}{2}(c-dx)\right)+19\cos\left(\frac{1}{2}(3c+dx)\right)+31\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+12\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)}{4d\sqrt{\cos(c+dx)}} - \right.}{3a^2(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((-4*I)*Sqrt[2]*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) + ((29*Cos[(c - d*x)/2] + 19*Cos[(3*c + d*x)/2] + 31*Cos[(c
+ 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 12*Cos[(3*c + 5*d*x)/2])*Csc[c/2]*Se
```

```
c[c/2]*Sec[(c + d*x)/2]^3)/(4*d*Sqrt[Cos[c + d*x]])))/(3*a^2*(1 + Cos[c + d
*x])^2)
```

Maple [B] time = 2.519, size = 405, normalized size = 3.

$$-\frac{1}{6a^2d}\left(2\sqrt{2(\sin(1/2dx+c/2))^2-1}\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{-2(\sin(1/2dx+c/2))^4+(\sin(1/2dx+c/2))^2}\left(5\,\mathrm{EllipticF}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 + a^
2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.188 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-$$

```
[Out] (7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) - (7*Sin[c + d*x])/(
a^2*d*Sqrt[Cos[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1
 + Cos[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x
])^2)
```

Rubi [A] time = 0.239074, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2978, 2748, 2636, 2641, 2639}

$$\frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}+\frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}-\frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (7*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (10*EllipticF[(c + d*x)/2, 2])/(3*a
^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)) - (7*Sin[c + d*x])/(
a^2*d*Sqrt[Cos[c + d*x]]) - (7*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1
 + Cos[c + d*x])) - Sin[c + d*x]/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x
])^2)
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx = -\frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{9a}{2}-\frac{5}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} - \frac{\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}$$

$$= \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))}$$

$$= \frac{7E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{10F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{10\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{7\sin(c+}{a^2d\sqrt{\cos(c+}}$$

Mathematica [C] time = 5.49328, size = 364, normalized size = 2.25

$$\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(82\cos\left(\frac{1}{2}(c-dx)\right)+65\cos\left(\frac{1}{2}(3c+dx)\right)+68\cos\left(\frac{1}{2}(c+3dx)\right)+37\cos\left(\frac{1}{2}(5c+3dx)\right)+53\cos\left(\frac{1}{2}(3c+5dx)\right)+10\cos\left(\frac{1}{2}(7c+5\right.\right.}{8d\cos^{\frac{3}{2}}(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^2),x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(((4*I)*Sqrt[2]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1
+ E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3
```

```
/4, -E^((2*I)*(c + d*x))] - 10*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - ((82*Cos[(c - d*x)/2] + 65*Cos[(3*c + d*x)/2] + 68*Cos[(c
+ 3*d*x)/2] + 37*Cos[(5*c + 3*d*x)/2] + 53*Cos[(3*c + 5*d*x)/2] + 10*Cos[(7
*c + 5*d*x)/2] + 21*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2
]^3)/(8*d*Cos[c + d*x]^(3/2))))/(3*a^2*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 4.146, size = 413, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(6*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)+14*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-22/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)
*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^2\cos\left(dx+c\right)^5+2\,a^2\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^2*cos(d*x + c)^5 + 2*a^2*cos(d*x + c)^4 + a^
2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.189 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{63\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c+dx)}{2a}$$

```
[Out] (231*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (21*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) - (21*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) + (77*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(10*a^3*d) - (Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d
*(a + a*Cos[c + d*x])^3) - (4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*a*d*(a +
a*Cos[c + d*x])^2) - (63*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Cos[c + d*x]))
```

Rubi [A] time = 0.335191, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2748, 2635, 2641, 2639}

$$-\frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{63\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{77\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10a^3d}-\frac{21\sin(c+dx)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (231*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (21*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) - (21*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) + (77*Cos[c + d
*x]^(3/2)*Sin[c + d*x])/(10*a^3*d) - (Cos[c + d*x]^(9/2)*Sin[c + d*x])/(5*d
*(a + a*Cos[c + d*x])^3) - (4*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*a*d*(a +
a*Cos[c + d*x])^2) - (63*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(10*d*(a^3 + a^3*
Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{7}{2}}(c+dx)\left(\frac{9a}{2}-\frac{15}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(42a^2-\frac{105}{2}a^2\cos(c+dx)\right)}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)} -$$

$$= -\frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{4\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\cos(c+dx))^2} - \frac{63\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)} -$$

$$= -\frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{10a^3d} - \frac{\cos^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} -$$

$$= \frac{231E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{21F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{21\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} + \frac{77\cos^{\frac{3}{2}}(c+d}{10a}$$

Mathematica [C] time = 2.67888, size = 388, normalized size = 1.87

$$2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\sqrt{\cos(c+dx)}\left(\frac{1}{16}\sec\left(\frac{c}{2}\right)\left(-770\sin\left(c+\frac{dx}{2}\right)+840\sin\left(c+\frac{3dx}{2}\right)-150\sin\left(2c+\frac{3dx}{2}\right)+238\sin\left(\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((42*I)*Sqrt[2]*(11*(1 + E^((2*I)*(c + d*x))) + 11*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(
I*(c + d*x))]) - Sqrt[Cos[c + d*x]]*(264*Cot[c] + 198*Csc[c] + (Sec[c/2]*Se
c[(c + d*x)/2]^5*(1210*Sin[(d*x)/2] - 770*Sin[c + (d*x)/2] + 840*Sin[c + (3
*d*x)/2] - 150*Sin[2*c + (3*d*x)/2] + 238*Sin[2*c + (5*d*x)/2] + 40*Sin[3*c
 + (5*d*x)/2] + 5*Sin[3*c + (7*d*x)/2] + 5*Sin[4*c + (7*d*x)/2] - Sin[4*c +
 (9*d*x)/2] - Sin[5*c + (9*d*x)/2]))/16)))/(5*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 2.644, size = 296, normalized size = 1.4

$$-\frac{1}{20\,a^3d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(64\,(\cos(1/2\,dx+c/2))^{12}-288\,(\cos(1/2\,dx+c/2))^{10}-76\,(\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*cos(1/2*d
*x+1/2*c)^12-288*cos(1/2*d*x+1/2*c)^10-76*cos(1/2*d*x+1/2*c)^8-210*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-462*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+530*cos(1/2*d*x+1/2*c)^6-248*cos(1/2*d*x+1/2*c)^4+19*cos(1/2
*d*x+1/2*c)^2-1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{11}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/(a*cos(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos(dx+c)^{\frac{11}{2}}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(11/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3
*a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{11}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/(a*cos(d*x + c) + a)^3, x)
```

3.190 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3} dx$

Optimal. Leaf size=181

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{119\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{\sin(c+dx)\cos}{5d(a\cos(c+}$$

```
[Out] (-119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2]
)/(2*a^3*d) + (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) - (Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*Cos[c + d*x]^(5/2)*
Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - (119*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.322797, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2748, 2639, 2635, 2641}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{119\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{2a^3d}-\frac{\sin(c+dx)\cos}{5d(a\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2]
)/(2*a^3*d) + (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a^3*d) - (Cos[c + d*x
]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*Cos[c + d*x]^(5/2)*
Sin[c + d*x])/(3*a*d*(a + a*Cos[c + d*x])^2) - (119*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(30*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2}-\frac{13}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(25a^2-\frac{69}{2}a^2\cos(c+dx)\right)}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30d\left(a^3+a^3\cos(c+dx)\right)} -$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3ad(a+a\cos(c+dx))^2} - \frac{119\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{30d\left(a^3+a^3\cos(c+dx)\right)} -$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{2\cos^{\frac{5}{2}}}{3ad(}$$

$$= -\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{2a^3d} - \frac{\cos^{\frac{7}{2}}(c+dx}{5d(a+a\cos}$$

Mathematica [C] time = 1.86659, size = 369, normalized size = 2.04

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc(c)\sqrt{\cos(c+dx)}\left(1961\cos\left(\frac{1}{2}(c-dx)\right)+1609\cos\left(\frac{1}{2}(3c+dx)\right)+1165\cos\left(\frac{1}{2}(c+3dx)\right)+620\cos\left(\frac{1}{2}(5c+3dx)\right)+292\cos\left(\frac{1}{2}(3c+5dx)\right)+65\,\mathrm{c}}{12d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x))) + 119*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] + 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/
E^(I*(c + d*x))]) + (Sqrt[Cos[c + d*x]]*(1961*Cos[(c - d*x)/2] + 1609*Cos[(
3*c + d*x)/2] + 1165*Cos[(c + 3*d*x)/2] + 620*Cos[(5*c + 3*d*x)/2] + 292*Co
s[(3*c + 5*d*x)/2] + 65*Cos[(7*c + 5*d*x)/2] + 5*Cos[(5*c + 7*d*x)/2] - 5*C
os[(9*c + 7*d*x)/2])*Csc[c]*Sec[(c + d*x)/2]^5)/(12*d)))/(5*a^3*(1 + Cos[c
+ d*x])^3)
```

Maple [A] time = 2.313, size = 283, normalized size = 1.6

$$-\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{10}+468\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+330\,\sqrt{(s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*
d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(
1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{9}{2}}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(9/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^3, x)
```

3.191 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{8\sin(c+dx)\cos^{\frac{3}{2}}}{15ad(a\cos(c+d}$$

```
[Out] (49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/
(6*a^3*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3)
- (8*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (13
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.298601, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2765, 2977, 2748, 2641, 2639}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{13\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{8\sin(c+dx)\cos^{\frac{3}{2}}}{15ad(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])/
(6*a^3*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3)
- (8*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (13
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-\frac{11}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(12a^2-\frac{41}{2}a^2\cos(c+dx)\right)}{a+a\cos(c+dx)}}{15a^4}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\cos(c+dx)\right)} -$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{6d\left(a^3+a^3\cos(c+dx)\right)} -$$

$$= \frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{8\cos^{\frac{3}{2}}(c+dx)\sin}{15ad(a+a\cos(c}$$

Mathematica [C] time = 4.12932, size = 349, normalized size = 2.25

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(806\cos\left(\frac{1}{2}(c-dx)\right)+664\cos\left(\frac{1}{2}(3c+dx)\right)+470\cos\left(\frac{1}{2}(c+3dx)\right)+265\cos\left(\frac{1}{2}(5c+3dx)\right)+117\cos\left(\frac{1}{2}(3c+5dx)\right)\right.}{8d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-
1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - (Sqrt[Cos[c + d*x]]*(806*Cos[(c - d*x)/2] + 664*Cos[(3*c
 + d*x)/2] + 470*Cos[(c + 3*d*x)/2] + 265*Cos[(5*c + 3*d*x)/2] + 117*Cos[(3
*c + 5*d*x)/2] + 30*Cos[(7*c + 5*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2
]^5)/(8*d)))/(15*a^3*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 2.372, size = 270, normalized size = 1.7

$$\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(348\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+130\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d
*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2
*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{7}{2}}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(7/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^3, x)
```

3.192 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{2\sin(c+dx)\sqrt{\cos(c+d}}{5ad(a\cos(c+dx)+a)^2}$$

```
[Out] (-9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^
3*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^2) + (9*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.307742, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2765, 2977, 2978, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{2\sin(c+dx)\sqrt{\cos(c+d}}{5ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^
3*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*
Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^2) + (9*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx &= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-\frac{9}{2}a\cos(c+dx)\right)}{(a+a\cos(c+dx))^2}\,dx}{5a^2}\\
&= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}-\frac{\int \frac{3a^2-\frac{21}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{15a^4}\\
&= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}+\frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}-\\
&= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}+\frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)}+\\
&= -\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c}{5ad(a+a\cos(c+}\end{aligned}$$

Mathematica [C] time = 6.34664, size = 705, normalized size = 4.55

$$-\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\mathrm{si}}}{d\sqrt{\cot^2(c)+1}(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (((-9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyper
geometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2
*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]
*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
 E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometr
ic2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 +
E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[
1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*
I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
 (2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[
1 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*((36*Csc[c])/(5*d
) + (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (12*Sec[c/2]*Sec[
c/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sin
[(d*x)/2])/(5*d) - (12*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) + (2*Sec[c/2 +
(d*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A] time = 2.378, size = 270, normalized size = 1.7

$$-\frac{1}{20\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(36\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2*d*x
+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^3, x)
```

3.193 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] -EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (4*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.303611, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2765, 2978, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] -EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
 - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (4*Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) + (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{\int \frac{\frac{a}{2}-\frac{7}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\int \frac{-\frac{a^2}{2}-2a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{15a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)} - \ldots$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)} - \ldots$$

$$= -\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{4\sqrt{\cos(c+dx)}\sin(c \ldots}{15ad(a+a\cos(c+\ldots}$$

Mathematica [C] time = 3.59772, size = 334, normalized size = 2.15

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(14\cos\left(\frac{1}{2}(c-dx)\right)+16\cos\left(\frac{1}{2}(3c+dx)\right)+20\cos\left(\frac{1}{2}(c+3dx)\right)-5\cos\left(\frac{1}{2}(5c+3dx)\right)+3\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec^5\left(\frac{1}{2}\ldots\right.}{8d}\ldots}{15\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))
/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*
(c + d*x))]) + (Sqrt[Cos[c + d*x]]*(14*Cos[(c - d*x)/2] + 16*Cos[(3*c + d*x
)/2] + 20*Cos[(c + 3*d*x)/2] - 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)
/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d)))/(15*a^3*(1 + Cos[c + d*x
])^3)
```

Maple [A] time = 2.277, size = 270, normalized size = 1.7

$$-\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

3.194 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3} \, dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

```
[Out] EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
+ (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.305266, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2764, 2978, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```


```
[Out] EllipticE[(c + d*x)/2, 2]/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(6*a^3*d)
+ (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) + (Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Cos[c + d*x])^2) - (Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{a}{2}+\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} + \frac{\int \frac{2a^2+\frac{1}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))}\,dx}{15a^4}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)} + \int$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{15ad(a+a\cos(c+dx))^2} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\cos(c+dx)\right)} + \int \sqrt{}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\sqrt{\cos(c+dx)}\sin(c+}{15ad(a+a\cos(c+d}$$

Mathematica [C] time = 3.06659, size = 334, normalized size = 2.15

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sqrt{\cos(c+dx)}\left(4\cos\left(\frac{1}{2}(c-dx)\right)+26\cos\left(\frac{1}{2}(3c+dx)\right)+10\cos\left(\frac{1}{2}(c+3dx)\right)+5\cos\left(\frac{1}{2}(5c+3dx)\right)+3\cos\left(\frac{1}{2}(3c+5dx)\right)\right)\sec^5\left(\frac{1}{2}\right.}{8d}\right.}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((4*I)*Sqrt[2]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 +
E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4
, -E^((2*I)*(c + d*x))] - 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/
(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(
c + d*x))]) - (Sqrt[Cos[c + d*x]]*(4*Cos[(c - d*x)/2] + 26*Cos[(3*c + d*x)/
2] + 10*Cos[(c + 3*d*x)/2] + 5*Cos[(5*c + 3*d*x)/2] + 3*Cos[(3*c + 5*d*x)/2
])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(8*d)))/(15*a^3*(1 + Cos[c + d*x])
^3)
```

Maple [A] time = 2.42, size = 270, normalized size = 1.7

$$\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8-10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

3.195 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=155

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{5ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos(c+d}{5d(a\cos(c+dx)+a)}$$

```
[Out] (9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3
*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^2) - (9*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.311909, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2766, 2978, 2748, 2641, 2639}

$$\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{5ad(a\cos(c+dx)+a)^2}-\frac{\sin(c+dx)\sqrt{\cos(c+d}{5d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (9*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + EllipticF[(c + d*x)/2, 2]/(2*a^3
*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Cos[c + d*x])^3) - (2*S
qrt[Cos[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Cos[c + d*x])^2) - (9*Sqrt[Co
s[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{\int \frac{\frac{9a}{2}-\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}+\frac{\int \frac{\frac{21a^2}{2}-3a^2\cos}{\sqrt{\cos(c+dx)}(a+a\,c}}{15a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}-\frac{9\sqrt{\cos(c+dx)}\,s}{10d\left(a^3+a^3\cos\right.}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{5ad(a+a\cos(c+dx))^2}-\frac{9\sqrt{\cos(c+dx)}\,s}{10d\left(a^3+a^3\cos\right.}$$

$$= \frac{9E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{2\sqrt{\cos(}}{5ad(a}$$

Mathematica [C] time = 6.32288, size = 705, normalized size = 4.55

$$-\frac{2\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{1-\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\sin(c)\left(-\sqrt{\cot^2(c)+1}\right)\sin\left(dx-\tan^{-1}(\cot(c))\right)}\sqrt{\mathrm{si}}}{d\sqrt{\cot^2(c)+1}(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (((9*I)/10)*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*((2*E^((2*I)*d*x)*Hyperg
eometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*
(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*
Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 +
E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometri
c2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E
^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1
 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I
)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(a + a*Cos[c + d*x])^3 -
(2*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d
*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(a + a*Cos[c + d*x])^3*Sqrt[1
 + Cot[c]^2]) + (Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*((-36*Csc[c])/(5*d
```

```
) - (36*Sec[c/2]*Sec[c/2 + (d*x)/2]*Sin[(d*x)/2])/(5*d) - (8*Sec[c/2]*Sec[c
/2 + (d*x)/2]^3*Sin[(d*x)/2])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*Sin[
(d*x)/2])/(5*d) - (8*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(5*d) - (2*Sec[c/2 + (d
*x)/2]^4*Tan[c/2])/(5*d)))/(a + a*Cos[c + d*x])^3
```

Maple [A] time = 2.48, size = 268, normalized size = 1.7

$$\frac{1}{20\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(36\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8-10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+a^3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 + 3*
a^3*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.196 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=181

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}-\frac{8}{15ad\sqrt{\cos(c+}}$$

```
[Out] (-49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])
/(6*a^3*d) + (49*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]
/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d
*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) - (13*Sin[c + d*x])/(6*d*Sqrt[C
os[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.347249, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2978, 2748, 2636, 2639, 2641}

$$-\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}-\frac{13\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\left(a^3\cos(c+dx)+a^3\right)}-\frac{8}{15ad\sqrt{\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```


```
[Out] (-49*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - (13*EllipticF[(c + d*x)/2, 2])
/(6*a^3*d) + (49*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]
/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3) - (8*Sin[c + d*x])/(15*a*d
*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) - (13*Sin[c + d*x])/(6*d*Sqrt[C
os[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{11a}{2}-\frac{5}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^3} - \frac{8\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx}$$

$$= -\frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\cos(c+dx)}(a+a\cos(c+}$$

$$= -\frac{49E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{13F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{49\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{}{5d\sqrt{\cos(}}$$

Mathematica [C] time = 1.89995, size = 364, normalized size = 2.01

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}(c+3dx)\right)+374\cos\left(\frac{1}{2}(5c+3dx)\right)+670\cos\left(\frac{1}{2}(3c+5dx)\right)+65\cos\left(\frac{1}{2}\right.\right.}{16d\sqrt{\cos(c+dx)}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((-4*I)*Sqrt[2]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(
-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2
, 3/4, -E^((2*I)*(c + d*x))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x
))]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/
E^(I*(c + d*x))]) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 1243
*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2] +
 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[
(c + d*x)/2]^5)/(16*d*Sqrt[Cos[c + d*x]])))/(15*a^3*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 2.584, size = 555, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{5}+3\,a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{3}+a^{3}\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 3*
a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

3.197 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=207

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{119\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+}{10a^3d\sqrt{\cos(c}}$$

```
[Out] (119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Cos[c + d*x]^(3/2)) - (119*Sin[c +
d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*
(a + a*Cos[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a
*Cos[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*
Cos[c + d*x]))
```

Rubi [A] time = 0.359069, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2766, 2978, 2748, 2636, 2641, 2639}

$$\frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{119\sin(c+dx)}{30d\cos^{\frac{3}{2}}(c+dx)\left(a^3\cos(c+dx)+a^3\right)}+\frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)}-\frac{119\sin(c+}{10a^3d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```


```
[Out] (119*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + (11*EllipticF[(c + d*x)/2, 2])
/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Cos[c + d*x]^(3/2)) - (119*Sin[c +
d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) - Sin[c + d*x]/(5*d*Cos[c + d*x]^(3/2)*
(a + a*Cos[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)*(a + a
*Cos[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Cos[c + d*x]^(3/2)*(a^3 + a^3*
Cos[c + d*x]))
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx = -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{13a}{2}-\frac{7}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= -\frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}$$

$$= \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} - \frac{\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= \frac{119E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{11F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} + \frac{11\sin(c+dx)}{2a^3d\cos^{\frac{3}{2}}(c+dx)} - \frac{119\sin(}{10a^3d\sqrt{\text{co}}}$$

Mathematica [C] time = 2.45878, size = 394, normalized size = 1.9

$$\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(5134\cos\left(\frac{1}{2}(c-dx)\right)+4148\cos\left(\frac{1}{2}(3c+dx)\right)+4664\cos\left(\frac{1}{2}(c+3dx)\right)+2476\cos\left(\frac{1}{2}(5c+3dx)\right)+3340\cos\left(\frac{1}{2}(3c+5dx)\right)+94}{96d\cos^{\frac{3}{2}}(c+dx)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^3),x]
```

```
[Out] (Cos[(c + d*x)/2]^6*(((4*I)*Sqrt[2]*(119*(1 + E^((2*I)*(c + d*x))) + 119*(-
1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
 3/4, -E^((2*I)*(c + d*x))] - 55*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x)
)]))/(d*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E
^(I*(c + d*x))]) - ((5134*Cos[(c - d*x)/2] + 4148*Cos[(3*c + d*x)/2] + 4664
*Cos[(c + 3*d*x)/2] + 2476*Cos[(5*c + 3*d*x)/2] + 3340*Cos[(3*c + 5*d*x)/2]
 + 944*Cos[(7*c + 5*d*x)/2] + 1620*Cos[(5*c + 7*d*x)/2] + 165*Cos[(9*c + 7*
d*x)/2] + 357*Cos[(7*c + 9*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5)/(
96*d*Cos[c + d*x]^(3/2))))/(5*a^3*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 4.527, size = 453, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^3*(118/5*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)+238/
5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-4/3*cos(1/2*d*x+1/2*c)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2
-128/5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))+48*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2
*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/5*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5+32/15*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{a^3\cos\left(dx+c\right)^6+3\,a^3\cos\left(dx+c\right)^5+3\,a^3\cos\left(dx+c\right)^4+a^3\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/(a^3*cos(d*x + c)^6 + 3*a^3*cos(d*x + c)^5 + 3*
a^3*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.198 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{5a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (5*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d) +
 (5*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (5*
a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Cos
[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.233018, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2770, 2774, 216}

$$\frac{a\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{5a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{5\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{5a\sin(c+dx)\sqrt{\cos(c+dx)}}{8d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (5*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d) +
 (5*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) + (5*
a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) + (a*Cos
[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{6}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{5a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{5}{8}\int \sqrt{\cos(c+dx}$$

$$= \frac{5a\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{3d\sqrt{a+a\cos}}$$

$$= \frac{5a\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{3d\sqrt{a+a\cos}}$$

$$= \frac{5\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{5a\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{5a\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{12d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.327381, size = 105, normalized size = 0.68

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(14\sin\left(\frac{1}{2}(c+dx)\right)+3\sin\left(\frac{3}{2}(c+dx)\right)+2\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(15*Sqrt[2]*ArcSin[Sqrt[2]*Sin
[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(14*Sin[(c + d*x)/2] + 3*Sin[(3*(c +
d*x))/2] + 2*Sin[(5*(c + d*x))/2])))/(48*d)
```

Maple [A] time = 0.497, size = 196, normalized size = 1.3

$$-\frac{(-1+\cos(dx+c))^3}{24\,d\,(\sin(dx+c))^6}(\cos(dx+c))^{\frac{5}{2}}\sqrt{a(1+\cos(dx+c))}\left(8\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+10\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/24/d*cos(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*(8*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+10*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+15*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+15*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
)))/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [B] time = 2.39984, size = 2593, normalized size = 16.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/96*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1/
2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arct
an2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*s
in(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*
```

```
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))/d
```

Fricas [A] time = 1.73026, size = 317, normalized size = 2.06

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}\left(8\,\cos\left(dx+c\right)^{2}+10\,\cos\left(dx+c\right)+15\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-15\,\sqrt{a}\left(\cos\left(dx+c\right)+1\right)\operatorname{arcta}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 10*cos(d*x + c) + 15)*sq
rt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x +
 c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

3.199 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (3*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d) +
 (3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.174306, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2770, 2774, 216}

$$\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d) +
 (3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (a*
Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx = \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{3}{8}\int \frac{\sqrt{a+a\cos(}{\sqrt{\cos(c+}}$$

$$= \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{}}}\right.}{}$$

$$= \frac{3\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a\cos^{\frac{3}{2}}(c+dx)\sin}{2d\sqrt{a+a\cos(c}}$$

Mathematica [A] time = 0.200166, size = 91, normalized size = 0.78

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\cos(c}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x)
)/2])))/(8*d)
```

Maple [A] time = 0.45, size = 161, normalized size = 1.4

$$\frac{(-1+\cos(dx+c))^2}{4\,d(\sin(dx+c))^4}(\cos(dx+c))^{\frac{3}{2}}\sqrt{a(1+\cos(dx+c))}\left(2\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)+3\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/4/d*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(2*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)+3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
/cos(d*x+c)))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

Maxima [B] time = 2.05207, size = 1430, normalized size = 12.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
```

```
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 1)))/d
```

Fricas [A] time = 1.74785, size = 286, normalized size = 2.47

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}(2\,\cos\left(dx+c\right)+3)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\,\sqrt{a}(\cos\left(dx+c\right)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d
*x + c) - 3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out
```

3.200 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.116369, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2770, 2774, 216}

$$\frac{\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a*Sq
rt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx = \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}dx$$
$$= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.0920397, size = 77, normalized size = 1.07

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c
 + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

Maple [A] time = 0.432, size = 123, normalized size = 1.7

$$-\frac{-1+\cos(dx+c)}{d(\sin(dx+c))^2}\sqrt{\cos(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos}{1+c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)/cos(d*x+c)))/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] time = 2.01626, size = 1068, normalized size = 14.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
 - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

```
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

Fricas [A] time = 1.70276, size = 251, normalized size = 3.49

$$-\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a(\cos(c+dx)+1)}\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.201 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
```

Rubi [A] time = 0.058023, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2774, 216}

$$\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0426432, size = 50, normalized size = 1.35

$$\frac{\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c
 + d*x)/2])/d
```

Maple [B] time = 0.356, size = 80, normalized size = 2.2

$$2\,\frac{\sqrt{a\,(1+\cos(dx+c))}}{d\sqrt{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(1+cos(d*x+c)))^(
1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))
```

Maxima [B] time = 1.78194, size = 197, normalized size = 5.32

$$\frac{\sqrt{a}\arctan\left(\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

Fricas [A] time = 1.73402, size = 325, normalized size = 8.78

$$\left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{a\cos(dx+c)+a}\sqrt{-a}\sqrt{\cos(dx+c)}\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d}, -\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sqr
t(cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))/d, -
2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.202 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.0567769, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2771}

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.0475808, size = 39, normalized size = 1.08

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.415, size = 42, normalized size = 1.2

$$-2\,\frac{(-1+\cos(dx+c))\sqrt{a(1+\cos(dx+c))}}{d\sin(dx+c)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*(-1+cos(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [B] time = 1.5278, size = 132, normalized size = 3.67

$$\frac{2\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

Fricas [A] time = 1.63248, size = 130, normalized size = 3.61

$$\frac{2\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^
2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a(\cos(c+dx)+1)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.203 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.108944, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2772, 2771}

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (4*a
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2}{3}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0842057, size = 51, normalized size = 0.66

$$\frac{2(2\cos(c+dx)+1)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 2*Cos[c + d*x])*Tan[(c + d*x)/2])/(3*d*C
os[c + d*x]^(3/2))
```

Maple [A] time = 0.408, size = 54, normalized size = 0.7

$$-\frac{4\left(\cos\left(dx+c\right)\right)^{2}-2\cos\left(dx+c\right)-2}{3\,d\sin\left(dx+c\right)}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\cos\left(dx+c\right)\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*(2*cos(d*x+c)^2-cos(d*x+c)-1)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/co
s(d*x+c)^(3/2)
```

Maxima [B] time = 1.56495, size = 257, normalized size = 3.34

$$\frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2
)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 1))
```

Fricas [A] time = 1.61592, size = 163, normalized size = 2.12

$$\frac{2\sqrt{a\cos\left(dx+c\right)+a}\left(2\cos\left(dx+c\right)+1\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.204 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a
*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*S
in[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.165283, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2772, 2771}

$$\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (8*a
*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*S
in[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{4}{5}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{8}{15}\int \frac{\sqrt{a+a\,\mathrm{co}}}{\cos^{\frac{3}{2}}(}\\ &= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{16a\,\mathrm{s}}{15d\sqrt{\cos(c+d}}\end{aligned}$$

Mathematica [A] time = 0.0915232, size = 66, normalized size = 0.57

$$\frac{2\left(5\sin\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{5}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(5*Sin[(c + d*x)/2] + 2*Sin[
(5*(c + d*x))/2]))/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.434, size = 64, normalized size = 0.6

$$-\frac{16\,(\cos(dx+c))^{3}-8\,(\cos(dx+c))^{2}-2\,\cos(dx+c)-6}{15\,d\sin(dx+c)}\sqrt{a\,(1+\cos(dx+c))}\,(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(8*cos(d*x+c)^3-4*cos(d*x+c)^2-cos(d*x+c)-3)*(a*(1+cos(d*x+c)))^(1/
2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 1.58081, size = 320, normalized size = 2.78

$$\frac{2\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

Fricas [A] time = 1.63991, size = 190, normalized size = 1.65

$$\frac{2\sqrt{a\cos(dx+c)+a}\left(8\cos(dx+c)^{2}+4\cos(dx+c)+3\right)\sqrt{\cos(dx+c)}\sin(dx+c)}{15\left(d\cos(dx+c)^{4}+d\cos(dx+c)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt(
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.205 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{16a\sin(c+dx)}{35d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{}{3}$$

```
[Out] (2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (12*
a*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*
Sin[c + d*x])/(35*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (32*a*Si
n[c + d*x])/(35*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.227599, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2772, 2771}

$$\frac{16a\sin(c+dx)}{35d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (12*
a*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (16*a*
Sin[c + d*x])/(35*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (32*a*Si
n[c + d*x])/(35*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{6}{7}\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{24}{35}\int \frac{\sqrt{a+a\,\mathrm{c}}}{\cos^{\frac{5}{2}}(}$$
$$= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{16a\,\mathrm{s}}{35d\cos^{\frac{3}{2}}(c+d.}$$
$$= \frac{2a\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{12a\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{16a\,\mathrm{s}}{35d\cos^{\frac{3}{2}}(c+d.}$$

Mathematica [A] time = 0.121079, size = 66, normalized size = 0.43

$$\frac{2\left(7\sin\left(\frac{3}{2}(c+dx)\right)+2\sin\left(\frac{7}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{35d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(7*Sin[(3*(c + d*x))/2] + 2*
Sin[(7*(c + d*x))/2]))/(35*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.422, size = 74, normalized size = 0.5

$$-\frac{32\,(\cos(dx+c))^4-16\,(\cos(dx+c))^3-4\,(\cos(dx+c))^2-2\,\cos(dx+c)-10}{35\,d\sin(dx+c)}\sqrt{a\,(1+\cos(dx+c))}\,(\cos(dx+c))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/35/d*(16*cos(d*x+c)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)*(a*(1+
cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 1.59773, size = 382, normalized size = 2.5

$$\frac{2\left(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{58\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{35\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)
```

```
^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^4/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

Fricas [A] time = 1.65976, size = 217, normalized size = 1.42

$$\frac{2\left(16\,\cos\left(dx+c\right)^{3}+8\,\cos\left(dx+c\right)^{2}+6\,\cos\left(dx+c\right)+5\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*
x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{\cos\left(dx+c\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

3.206 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=160

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{11a^2\sin(c+dx)\sqrt{\cos(c+d}}{8d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (11*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (11*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) +
 (11*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
 (a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.248081, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2763, 21, 2770, 2774, 216}

$$\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{11a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{11a^2\sin(c+dx)\sqrt{\cos(c+d}}{8d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (11*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) +
 (11*a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
 (a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
```

```
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx = \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{11a^2}{2}+\frac{11}{2}a^2\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{6}(11a)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{11a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(11a)\int \sqrt{}$$

$$= \frac{11a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{5}{2}}(c}{3d\sqrt{a}}$$

$$= \frac{11a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{5}{2}}(c}{3d\sqrt{a}}$$

$$= \frac{11a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{11a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{11a^2\cos^{\frac{3}{2}}(c}{12d\sqrt{a+}}$$

Mathematica [A] time = 0.358705, size = 106, normalized size = 0.66

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(33\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(26\sin\left(\frac{1}{2}(c+dx)\right)+9\sin\left(\frac{3}{2}(c+dx)\right)+\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(33*Sqrt[2]*ArcSin[Sqrt[2]*S
in[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(26*Sin[(c + d*x)/2] + 9*Sin[(3*(c
+ d*x))/2] + 2*Sin[(5*(c + d*x))/2])))/(48*d)
```

Maple [A] time = 0.426, size = 197, normalized size = 1.2

$$\frac{a(-1+\cos(dx+c))^2}{24d(\sin(dx+c))^4}\left(8\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+22\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/24/d*a*(-1+cos(d*x+c))^2*(8*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)^2+22*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+33*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(
3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

Maxima [B] time = 2.53786, size = 2622, normalized size = 16.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x
+ 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
1*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a)
 + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - a
*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
 cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
```

```
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arctan2
((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^
(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
 cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))/
d
```

Fricas [A] time = 1.71423, size = 328, normalized size = 2.05

$$\frac{\left(8\,a\cos\left(dx+c\right)^2+22\,a\cos\left(dx+c\right)+33\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-33\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((8*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 33*a)*sqrt(a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c))*sin(d*x + c) - 33*(a*cos(d*x + c) + a)*sqrt(a)*arcta
n(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*c
os(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.207 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=120

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (7*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d) +
 (7*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (
a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.18491, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2763, 21, 2770, 2774, 216}

$$\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (7*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d) +
 (7*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) + (
a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx = \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{2}\int \frac{\sqrt{\cos(c+dx)}\left(\frac{7a^2}{2}+\frac{7}{2}a^2\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{4}(7a)\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{7a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{1}{8}(7a)\int \frac{\sqrt{a+\cdots}}{\sqrt{\cdots}}$$

$$= \frac{7a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{(7a)\,\mathrm{Subst}\left(\int \cdots\right)}{\cdots}$$

$$= \frac{7a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{7a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\cdots}{2d\sqrt{a+a\cos\cdots}}$$

Mathematica [A] time = 0.219042, size = 92, normalized size = 0.77

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(7\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(6\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\cos(c+\cdots}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(7*Sqrt[2]*ArcSin[Sqrt[2]*Si
n[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(6*Sin[(c + d*x)/2] + Sin[(3*(c + d*
x))/2])))/(8*d)
```

Maple [A] time = 0.378, size = 160, normalized size = 1.3

$$-\frac{a(-1+\cos(dx+c))}{4d(\sin(dx+c))^2}\left(2\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)+7\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\arctan\left(\frac{\sin\cdots}{\cos\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/d*a*(-1+cos(d*x+c))*(2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)+7*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2
```

Maxima [B] time = 2.04382, size = 1458, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.70305, size = 294, normalized size = 2.45

$$\frac{(2\,a\cos(dx+c)+7\,a)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)-7\,(a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{c}}{\sqrt{a}\sin(dx+}\right.}{4\,(d\cos(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*cos(d*x + c) + 7*a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*s
in(d*x + c) - 7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.208 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a^
2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.119702, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2763, 21, 2774, 216}

$$\frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (a^
2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\int \frac{\frac{3a^2}{2}+\frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx \\
&= \frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}+\frac{1}{2}(3a)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} \\
&= \frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}+\frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.104089, size = 79, normalized size = 1.05

$$
\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\right)}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Si
n[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Sin[(c + d*x)/2]))/(2*d)
```

Maple [B] time = 0.362, size = 168, normalized size = 2.2

$$
\frac{a}{d\left(1+\cos\left(dx+c\right)\right)}\left(3\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)+3\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\,\text{ar}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*a*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+cos(d*x
+c)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/(1+cos(d*x+c))
```

Maxima [B] time = 1.97065, size = 1084, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.65784, size = 258, normalized size = 3.44

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - 3*(a*cos(d*x
+ c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.209 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*
a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.124207, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2762, 21, 2774, 216}

$$\frac{2a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (2*
a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - (2a)\int \frac{-\frac{a}{2}-\frac{1}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + a\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}$$
$$= \frac{2a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.147204, size = 85, normalized size = 1.12

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/(d*Sqrt[Cos[c + d*x
]])
```

Maple [B] time = 0.354, size = 249, normalized size = 3.3

$$-2\,\frac{a\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\sin\left(dx+c\right)\right)^2}{d\left(-1+\cos\left(dx+c\right)\right)\left(1+\cos\left(dx+c\right)\right)^2\left(\cos\left(dx+c\right)\right)^{3/2}}\left(\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{3/2}\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*a*((cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+2*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)/cos(d*x+c))+cos(d*x+c)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [B] time = 1.96692, size = 1346, normalized size = 17.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*d)
```

Fricas [A] time = 1.73334, size = 298, normalized size = 3.92

$$\frac{2\left(\sqrt{a\cos\left(dx+c\right)+a}a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c}}{\sqrt{a}\sin(dx+c)}\right.\right.}{d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - (a*cos(d*x
+ c)^2 + a*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(3/2)/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.210 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=81

$$\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (1
0*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.118035, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2762, 21, 2771}

$$\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (1
0*a^2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{1}{3}(2a)\int \frac{-\frac{5a}{2}-\frac{5}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}(5a)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{10a^2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.111688, size = 52, normalized size = 0.64

$$\frac{2a(5\cos(c+dx)+1)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 5*Cos[c + d*x])*Tan[(c + d*x)/2])/(3*d
*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.331, size = 55, normalized size = 0.7

$$-\frac{2a\left(5(\cos(dx+c))^2-4\cos(dx+c)-1\right)}{3d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*a*(5*cos(d*x+c)^2-4*cos(d*x+c)-1)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c
)/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.58303, size = 169, normalized size = 2.09

$$\frac{4\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

Fricas [A] time = 1.65885, size = 166, normalized size = 2.05

$$\frac{2 \left(5 \, a \cos \left(dx+c\right)+a\right) \sqrt{a \cos \left(dx+c\right)+a} \sqrt{\cos \left(dx+c\right)} \sin \left(dx+c\right)}{3 \left(d \cos \left(dx+c\right)^{3}+d \cos \left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \cos \left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos \left(dx+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

3.211 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (6
*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (12*
a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.173241, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2762, 21, 2772, 2771}

$$\frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{12a^2\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```


```
[Out] (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (6
*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (12*
a^2*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}dx = \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{1}{5}(2a)\int \frac{-\frac{9a}{2}-\frac{9}{2}a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}dx$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}(9a)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}dx$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}(6a)\int \frac{\sqrt{}}{}$$

$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{6a^2\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{12}{5d\sqrt{\cos(c}}$$

Mathematica [A] time = 0.150755, size = 62, normalized size = 0.51

$$\frac{2a(3\cos(c+dx)+3\cos(2(c+dx))+4)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*T
an[(c + d*x)/2])/(5*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.338, size = 65, normalized size = 0.5

$$-\frac{2a\left(6(\cos(dx+c))^3-3(\cos(dx+c))^2-2\cos(dx+c)-1\right)}{5d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d*a*(6*cos(d*x+c)^3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*(a*(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 1.57754, size = 293, normalized size = 2.42

$$\frac{4\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{5d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 4/5*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

Fricas [A] time = 1.61888, size = 194, normalized size = 1.6

$$\frac{2\left(6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

3.212 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{104a^2\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
6*a^2*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (1
04*a^2*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) +
(208*a^2*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.234618, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2762, 21, 2772, 2771}

$$\frac{104a^2\sin(c+dx)}{105d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```


```
[Out] (2*a^2*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) + (2
6*a^2*Sin[c + d*x])/(35*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (1
04*a^2*Sin[c + d*x])/(105*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) +
(208*a^2*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{1}{7}(2a)\int \frac{-\frac{13a}{2}-\frac{13}{2}a\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}(13a)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{1}{35}(52a)\int \frac{\sqrt{}}{}$$
$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{104}{105d\cos^{\frac{3}{2}}(c}$$
$$= \frac{2a^2\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{104}{105d\cos^{\frac{3}{2}}(c}$$

Mathematica [A] time = 0.220819, size = 72, normalized size = 0.45

$$\frac{2a(117\cos(c+dx)+26\cos(2(c+dx))+26\cos(3(c+dx))+41)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{105d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(41 + 117*Cos[c + d*x] + 26*Cos[2*(c + d*x)
] + 26*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(105*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.336, size = 75, normalized size = 0.5

$$-\frac{2a\left(104(\cos(dx+c))^4-52(\cos(dx+c))^3-13(\cos(dx+c))^2-24\cos(dx+c)-15\right)}{105d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a*(104*cos(d*x+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-
15)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [A] time = 1.61708, size = 355, normalized size = 2.2

$$\frac{4\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{3}}{105\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 4/105*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^
(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 1))
```

Fricas [A] time = 1.65688, size = 235, normalized size = 1.46

$$\frac{2\left(104\,a\cos(dx+c)^{3}+52\,a\cos(dx+c)^{2}+39\,a\cos(dx+c)+15\,a\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{105\left(d\cos(dx+c)^{5}+d\cos(dx+c)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*
a)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)
^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

3.213 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=200

$$\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{163a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+$$

```
[Out] (163*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(64*d
) + (163*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]
]) + (163*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x
]]) + (17*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.358123, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2763, 2981, 2770, 2774, 216}

$$\frac{17a^3\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{24d\sqrt{a\cos(c+dx)+a}}+\frac{163a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{96d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (163*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(64*d
) + (163*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*d*Sqrt[a + a*Cos[c + d*x]
]) + (163*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(96*d*Sqrt[a + a*Cos[c + d*x
]]) + (17*a^3*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(24*d*Sqrt[a + a*Cos[c + d*x
]]) + (a^2*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx = \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos}$$

$$= \frac{17a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx}{4d}$$

$$= \frac{163a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{17a^3\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{5}{2}}(c+}{}$$

$$= \frac{163a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{163a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{17a^3\cos^{\frac{5}{2}}}{24d\sqrt{a}}$$

$$= \frac{163a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{163a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\cos(c+dx)}} + \frac{17a^3\cos^{\frac{5}{2}}}{24d\sqrt{a}}$$

$$= \frac{163a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{64d} + \frac{163a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{64d\sqrt{a+a\cos(c+dx)}} + \frac{163a^3\cos^{\frac{3}{2}}(}{96d\sqrt{a-}}$$

Mathematica [C] time = 4.30042, size = 182, normalized size = 0.91

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{5/2}\left(-6\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{-\frac{1}{2},\frac{3}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] +
 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-3/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2
] - 24*(3 + Cos[c + d*x])*Hypergeometric2F1[-1/2, 3/2, 9/2, 2*Sin[(c + d*x)
/2]^2]*Sin[c + d*x]^2 - 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{-1/2, 3/2,
2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*
d)
```

Maple [A] time = 0.441, size = 234, normalized size = 1.2

$$\frac{a^2(-1+\cos(dx+c))^2}{192\,d(\sin(dx+c))^4}\left(48\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)(\cos(dx+c))^3+184\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/192/d*a^2*(-1+cos(d*x+c))^2*(48*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)*cos(d*x+c)^3+184*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)^2+326*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+489*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^(3/2)/(
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

Maxima [B] time = 3.41485, size = 10058, normalized size = 50.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/768*(10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d
*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*co
s(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x + 4*c)^3
 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(2*a^2*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*
d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4
*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x +
 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d
*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3
2*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c)
 + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(16*a^2*
cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*cos(4*d*x + 4*c) +
3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a^2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 19*a^2
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*si
n(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 12*(4*a^2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a^2*sin(4*d*x
 + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 - 8
*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 14*a^2*cos(4*d*x +
4*c)^2 + 19*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d
*x + 4*c)^2 - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
```

```
 + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x
 + 4*c)^3 - 2*a^2*cos(4*d*x + 4*c)^2 - 13*a^2*cos(4*d*x + 4*c) + (3*a^2*cos
(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*
x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(
4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^
2 - 19*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))*sin(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 11*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*cos
(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*(2*a^2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)
 - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(3
*a^2*cos(4*d*x + 4*c) - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^
(1/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^
3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x +
4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d
*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4
*(3*a^2*sin(4*d*x + 4*c)^3 + 3*(a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x +
4*c) + a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
+ 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(3*a^2*sin(4*d*x + 4*c)^3 + 160*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c)^2 + 6*a^2*cos(
4*d*x + 4*c) + 43*a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 2*(6*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 6*(a^2*cos(4*d*x + 4*c)^2 - a
^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (320*a^2*cos(4*d*x + 4*c)^2 + 320*a
^2*sin(4*d*x + 4*c)^2 - 317*a^2*cos(4*d*x + 4*c) - 3*a^2)*sin(1/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) - 2*(20*a^2*cos(4*d*x + 4*c)^2 + 26*a^2*sin(4*d*x + 4*c)^2 -
 317*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x
 + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*(
10*a^2*cos(4*d*x + 4*c)^2 + 13*a^2*sin(4*d*x + 4*c)^2 - 160*a^2*sin(4*d*x +
 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 10*a^2*cos(4*d
*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 3*(a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (160*a^2*cos(4*d*x
+ 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*sin(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 + 120*a^2*cos(4*d*x + 4*c
)^2 - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*
x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 - 3*
```

```
a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 4*(3*a^2*cos(4*d*x + 4*c)^3 + 74*a^2*cos(4*d*x + 4*c)^2 - 197*a^2*cos(4*d*
x + 4*c) + (3*a^2*cos(4*d*x + 4*c) + 80*a^2)*sin(4*d*x + 4*c)^2 + 120*a^2 -
 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*
c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(a^2*cos(4*d*x + 4*c) + 40*
a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 + 126*a^2*cos(4*d*x +
 4*c)^2 + 243*a^2*cos(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(
4*d*x + 4*c)^2 + 120*a^2 - 40*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c
)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2
*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(6*a^2*cos(4
*d*x + 4*c)^3 + 214*a^2*cos(4*d*x + 4*c)^2 - 3*a^2*sin(4*d*x + 4*c)*sin(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 240*a^2*cos(4*d*x + 4*c) +
2*(3*a^2*cos(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c)^2 - (160*a^2*cos(4*d*
x + 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 - 157*a^2*cos(4*d*x + 4*c) - 3*a^2)
*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) - (80*a^2*cos(4*d*x + 4*c)^2 + 80*a^2*sin(
4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 2*(320*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2*sin(4*d*x + 4*c) + 157*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(80*a^2*cos(1/4*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (3*a^2*cos(4*d*x + 4*c) + 110*a^
2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 6*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*
c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d
*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x +
4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2
*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))
*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(si
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 -
2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(
4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c)
)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*a
```

```
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*
x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*cos(4*d*x + 4*c)
^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*
a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*
x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*
sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arc
tan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d
*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4
*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c)
+ a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(
4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)) - 1))*sqrt(a))/((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*
d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*d)
```

Fricas [A] time = 1.79196, size = 382, normalized size = 1.91

$$\frac{\left(48\,a^2\cos\left(dx+c\right)^3+184\,a^2\cos\left(dx+c\right)^2+326\,a^2\cos\left(dx+c\right)+489\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right.}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*((48*a^2*cos(d*x + c)^3 + 184*a^2*cos(d*x + c)^2 + 326*a^2*cos(d*x +
c) + 489*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 48
9*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.214 $\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{25a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{25a^3\sin}{8d\sqrt{}}$$

```
[Out] (25*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (25*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) +
 (13*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
 (a^2*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.295291, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2763, 2981, 2770, 2774, 216}

$$\frac{13a^3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{12d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d}+\frac{25a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{8d}+\frac{25a^3\sin}{8d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (25*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(8*d)
+ (25*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(8*d*Sqrt[a + a*Cos[c + d*x]]) +
 (13*a^3*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(12*d*Sqrt[a + a*Cos[c + d*x]]) +
 (a^2*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
```

```
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx = \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int \sqrt{\cos(c+dx)}\sqrt{a+a}$$

$$= \frac{13a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\sin(c+}{3d}$$

$$= \frac{25a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(}{}$$

$$= \frac{25a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\cos^{\frac{3}{2}}(}{}$$

$$= \frac{25a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{8d} + \frac{25a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{8d\sqrt{a+a\cos(c+dx)}} + \frac{13a^3\cos^{\frac{3}{2}}(}{12d\sqrt{a}}$$

Mathematica [C] time = 4.13667, size = 182, normalized size = 1.14

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{5/2}\left(-2\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{\frac{1}{2},\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] +
 3*Cos[2*(c + d*x)])*Hypergeometric2F1[-1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2
] - 8*(3 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2
]^2]*Sin[c + d*x]^2 - 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{1/2, 3/2, 2},
 {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

Maple [A] time = 0.395, size = 197, normalized size = 1.2

$$-\frac{a^2(-1+\cos(dx+c))}{24\,d\,(\sin(dx+c))^2}\left(8\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+34\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/24/d*a^2*(-1+cos(d*x+c))*(8*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)^2+34*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+75
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
(1/2)/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] time = 2.47858, size = 2651, normalized size = 16.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*
x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d
```

```
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))
- 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.75091, size = 342, normalized size = 2.14

$$\frac{\left(8\,a^2\cos\left(dx+c\right)^2+34\,a^2\cos\left(dx+c\right)+75\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-75\,\left(a^2\cos\left(dx+c\right)+a\right.}{24\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*((8*a^2*cos(d*x + c)^2 + 34*a^2*cos(d*x + c) + 75*a^2)*sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))*sin(d*x + c) - 75*(a^2*cos(d*x + c) + a^2)*sqr
t(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.215 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=120

$$\frac{19a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

```
[Out] (19*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (9*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) +
(a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.231562, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2763, 2981, 2774, 216}

$$\frac{19a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (19*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*d)
+ (9*a^3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c + d*x]]) +
(a^2*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{a^2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\sqrt{a+a\cos(c+dx)}\left(\frac{5a^2}{2}+\frac{9}{2}a^2\cos(c\right.}{\sqrt{\cos(c+dx)}}$$

$$= \frac{9a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{8}\left(19a^2\right)$$

$$= \frac{9a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d} - \frac{\left(19a^2\right)\text{Su}}{}$$

$$= \frac{19a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4d} + \frac{9a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{a^2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c}}{2d}$$

Mathematica [C] time = 3.93352, size = 182, normalized size = 1.52

$$\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{5/2}\left(2\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},\frac{3}{2},2\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] +
 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2]
 + 8*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]
^2]*Sin[c + d*x]^2 + 2*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 3/2, 2},
{1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

Maple [A] time = 0.382, size = 188, normalized size = 1.6

$$\frac{a^2}{4d(1+\cos(dx+c))}\left(2(\cos(dx+c))^2\sin(dx+c)+19\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*a^2*(2*cos(d*x+c)^2*sin(d*x+c)+19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)+1
1*cos(d*x+c)*sin(d*x+c)+19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(a*(1+cos(d*x+c)))^(1/2)
/cos(d*x+c)^(1/2)/(1+cos(d*x+c))
```

Maxima [B] time = 2.06429, size = 1493, normalized size = 12.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
+ 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*x
 + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
 + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.71037, size = 308, normalized size = 2.57

$$\frac{\left(2\,a^2\cos\left(dx+c\right)+11\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-19\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(d}}{\sqrt{}}\right.}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2*cos(d*x + c) + 11*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c) - 19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*
x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

3.216 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{5a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (5*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a^
3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.224958, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2762, 2981, 2774, 216}

$$\frac{5a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (5*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d - (a^
3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sq
rt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (2a)\int \frac{\left(-\frac{3a}{2}+\frac{1}{2}a\cos(c+dx)\right)\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= -\frac{a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\left(5a^2\right)\int \frac{\sqrt{a+a\cos}}{\sqrt{\cos(c}}$$

$$= -\frac{a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(5a^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x}{}}}\right.}{}$$

$$= \frac{5a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} - \frac{a^3\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+}{d\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 3.95516, size = 182, normalized size = 1.6

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{5/2}\left(6\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,\frac{5}{2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*(7*(89 + 28*Cos[c + d*x] +
 3*Cos[2*(c + d*x)])*Hypergeometric2F1[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2]
 + 24*(3 + Cos[c + d*x])*Hypergeometric2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2
]^2]*Sin[c + d*x]^2 + 6*Csc[(c + d*x)/2]^2*HypergeometricPFQ[{3/2, 2, 5/2},
 {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

Maple [B] time = 0.384, size = 269, normalized size = 2.4

$$-\frac{a^2(\sin(dx+c))^2}{d(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(5\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos(dx+c))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*a^2*(5*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2+10*(cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*c
os(d*x+c)+5*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+cos(d*x+c)^2*sin(d*x+c)+2*cos(d*x+c)*sin
```

```
(d*x+c))*sin(d*x+c)^2*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c
))^2/cos(d*x+c)^(3/2)
```

Maxima [B] time = 2.0153, size = 1314, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)
 + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*d)
```

Fricas [A] time = 1.76355, size = 338, normalized size = 2.96

$$\frac{\left(a^2 \cos(dx+c) + 2\,a^2\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c) - 5\left(a^2\cos(dx+c)^2 + a^2\cos(dx+c)\right)\sqrt{a}\,\mathrm{arctan}}{d\cos(dx+c)^2 + d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] ((a^2*cos(d*x + c) + 2*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin
(d*x + c) - 5*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a
*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x +
 c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

3.217 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (14
*a^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a
^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.225688, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2762, 2980, 2774, 216}

$$\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/d + (14
*a^3*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2*a
^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
```

```
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{3}(2a)\int \frac{\left(-\frac{7a}{2}-\frac{3}{2}a\cos(c+dx)\right)\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)} d$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + a^2\int \frac{\sqrt{a+a\cos}}{\sqrt{\cos(}}$$

$$= \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{\left(2a^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d} + \frac{14a^3\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+d}$$

Mathematica [C] time = 9.81616, size = 356, normalized size = 3.02

$$\frac{\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)(a(\cos(c+dx)+1))^{5/2}\left(256\sin^6\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,\frac{7}{2}\right\}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^5*(25
6*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[c/2 +
 (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*
Sin[c/2 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6*(2 - 3*Sin[c/2 + (d*x)/2]^2 + Si
n[c/2 + (d*x)/2]^4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[c/2 + (d*x)/2]^2]
]*(15 - 10*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4))/Sqrt[Sin[c/2 + (
d*x)/2]^2] - 14*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(45 + 30*Sin[c/2 + (d*x)/2
]^2 - 31*Sin[c/2 + (d*x)/2]^4 + 12*Sin[c/2 + (d*x)/2]^6)))/(672*d)
```

Maple [B] time = 0.368, size = 333, normalized size = 2.8

$$\frac{2a^2(\sin(dx+c))^4}{3d(-1+\cos(dx+c))^2(1+\cos(dx+c))^3}\left(3(\cos(dx+c))^3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3/d*a^2*(3*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+9*cos(d*x+c)^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/
cos(d*x+c))+9*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+3*(cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*
cos(d*x+c)^2*sin(d*x+c)+cos(d*x+c)*sin(d*x+c))*(a*(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)^4/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.05085, size = 1883, normalized size = 15.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.77647, size = 348, normalized size = 2.95

$$\frac{2\left(\left(8\,a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{a}\,\mathrm{arcta}}{3\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*((8*a^2*cos(d*x + c) + a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
*sin(d*x + c) - 3*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*sqrt(a)*arctan(
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(d*cos
(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

3.218 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{86a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (22*a^3*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) +
(86*a^3*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) +
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.224588, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2762, 2980, 2771}

$$\frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{86a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (22*a^3*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) +
(86*a^3*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) +
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{1}{5}(2a)\int \frac{\left(-\frac{11a}{2}-\frac{7}{2}a\cos(c+dx)\right)\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{15}\left(43a^2\right)\int$$

$$= \frac{22a^3\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{86a^3\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\,\mathrm{c}}}{5d}$$

Mathematica [A] time = 0.171155, size = 64, normalized size = 0.53

$$\frac{a^2(28\cos(c+dx)+43\cos(2(c+dx))+49)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(49 + 28*Cos[c + d*x] + 43*Cos[2*(c + d*x)]
)*Tan[(c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.342, size = 67, normalized size = 0.6

$$-\frac{2a^2\left(43(\cos(dx+c))^3-29(\cos(dx+c))^2-11\cos(dx+c)-3\right)}{15d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*a^2*(43*cos(d*x+c)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*(a*(1+cos(d*x
+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [A] time = 1.55885, size = 204, normalized size = 1.69

$$\frac{8\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1} - \frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{15d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
```

```
7)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2))
```

Fricas [A] time = 1.76355, size = 209, normalized size = 1.73

$$\frac{2\left(43\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+3\,a^2\right)\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x +
 c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)
^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)
```

3.219 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{}{21d\sqrt{c}}$$

```
[Out] (6*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4
6*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (9
2*a^3*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2
*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.286009, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2762, 2980, 2772, 2771}

$$\frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{}{21d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (6*a^3*Sin[c + d*x])/(7*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]) + (4
6*a^3*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) + (9
2*a^3*Sin[c + d*x])/(21*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]) + (2
*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}-\frac{1}{7}(2a)\int \frac{\left(-\frac{15a}{2}-\frac{11}{2}a\cos(c+dx)\right)\sqrt{a+a\cos(c+\cdots}{\cos^{\frac{7}{2}}(c+dx)} \\ &= \frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+\frac{1}{7}\left(23a^2\right)\int \cdots \\ &= \frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{2a^2\sqrt{a+\cdots}}{7\cdots} \\ &= \frac{6a^3\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{46a^3\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}+\frac{\cdots}{21d\sqrt{\cos(\cdots}}\end{aligned}$$

Mathematica [A] time = 5.2457, size = 74, normalized size = 0.46

$$\frac{a^2(93\cos(c+dx)+23\cos(2(c+dx))+23\cos(3(c+dx))+29)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{21d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(29 + 93*Cos[c + d*x] + 23*Cos[2*(c + d*x)]
 + 23*Cos[3*(c + d*x)])*Tan[(c + d*x)/2])/(21*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.36, size = 77, normalized size = 0.5

$$-\frac{2a^2\left(46(\cos(dx+c))^4-23(\cos(dx+c))^3-11(\cos(dx+c))^2-9\cos(dx+c)-3\right)}{21d\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}(\cos(dx\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d*a^2*(46*cos(d*x+c)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3
)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [A] time = 1.62213, size = 328, normalized size = 2.04

$$\frac{8\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{21\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 8/21*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

Fricas [A] time = 1.96982, size = 242, normalized size = 1.5

$$\frac{2\left(46\,a^2\cos(dx+c)^3+23\,a^2\cos(dx+c)^2+12\,a^2\cos(dx+c)+3\,a^2\right)\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{21\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c) +
 3*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x
 + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)
```

3.220 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{584a^3\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (38*a^3*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) +
(146*a^3*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]])
+ (584*a^3*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]
) + (1168*a^3*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*
x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2
))
```

Rubi [A] time = 0.350675, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2762, 2980, 2772, 2771}

$$\frac{584a^3\sin(c+dx)}{315d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```



```
[Out] (38*a^3*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]) +
(146*a^3*Sin[c + d*x])/(105*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]])
+ (584*a^3*Sin[c + d*x])/(315*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]
) + (1168*a^3*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*
x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2
))
```


Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}dx = \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} - \frac{1}{9}(2a)\int \frac{\left(-\frac{19a}{2}-\frac{15}{2}a\cos(c+dx)\right)\sqrt{a+a\cos(c+dx}}{\cos^{\frac{9}{2}}(c+dx)}$$
$$= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{1}{21}\left(73a^2\right)\int$$
$$= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a}}{9}$$
$$= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{5}{315d\cos^{\frac{3}{2}}(}$$
$$= \frac{38a^3\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\sin(c+dx)}{105d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{5}{315d\cos^{\frac{3}{2}}(}$$

Mathematica [A] time = 5.3282, size = 84, normalized size = 0.42

$$\frac{a^2(698\cos(c+dx)+803\cos(2(c+dx))+146\cos(3(c+dx))+146\cos(4(c+dx))+727)\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx}}{315d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(727 + 698*Cos[c + d*x] + 803*Cos[2*(c + d*
x)] + 146*Cos[3*(c + d*x)] + 146*Cos[4*(c + d*x)])*Tan[(c + d*x)/2])/(315*d
*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.368, size = 87, normalized size = 0.4

$$-\frac{2a^2\left(584(\cos(dx+c))^5-292(\cos(dx+c))^4-73(\cos(dx+c))^3-89(\cos(dx+c))^2-95\cos(dx+c)-35\right)}{315d\sin(dx+c)}\sqrt{a(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)/cos(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a^2*(584*cos(d*x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+
c)^2-95*cos(d*x+c)-35)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [A] time = 1.59456, size = 390, normalized size = 1.94

$$\frac{8\left(\frac{315\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1449\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1287\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{572\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{104\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right.}{315\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] 8/315*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^
(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*s
qrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 1))
```

Fricas [A] time = 1.96677, size = 282, normalized size = 1.4

$$\frac{2\left(584\,a^2\cos(dx+c)^4+292\,a^2\cos(dx+c)^3+219\,a^2\cos(dx+c)^2+130\,a^2\cos(dx+c)+35\,a^2\right)\sqrt{a\cos(dx+c)+a}}{315\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
 + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)
```

3.221 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (4*a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(1/4)*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.0561667, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2762, 8}

$$\frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/4),x]
```

```
[Out] (4*a^2*Sin[c + d*x])/(d*Cos[c + d*x]^(1/4)*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx &= \frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}-(4a)\int 0\,dx\\ &= \frac{4a^2\sin(c+dx)}{d\sqrt[4]{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0850375, size = 51, normalized size = 1.34

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{3/2}}{d\sqrt[4]{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/4),x]
```

```
[Out] (2*(a*(1 + Cos[c + d*x]))^(3/2)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(d*Cos
[c + d*x]^(1/4))
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int (a+\cos(dx+c)\,a)^{\frac{3}{2}}\,(\cos(dx+c))^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/4),x)
```

```
[Out] int((a+cos(d*x+c)*a)^(3/2)/cos(d*x+c)^(5/4),x)
```

Maxima [B] time = 1.5125, size = 163, normalized size = 4.29

$$\frac{4\left(\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] 4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^(1/4))
```

Fricas [A] time = 2.04949, size = 132, normalized size = 3.47

$$\frac{4\sqrt{a\cos(dx+c)+a}\,a\cos(dx+c)^{\frac{3}{4}}\sin(dx+c)}{d\cos(dx+c)^2+d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4*sqrt(a*cos(d*x + c) + a)*a*cos(d*x + c)^(3/4)*sin(d*x + c)/(d*cos(d*x + c
)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.222 $\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a\cos(e+fx)+a}}\right)}{f}$$

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + a*Cos[e + f*x]]])/f
```

Rubi [A] time = 0.0597373, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2774, 216}

$$\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a\cos(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[e + f*x]]/Sqrt[Cos[e + f*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a + a*Cos[e + f*x]]])/f
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cos(e+fx)}}{\sqrt{\cos(e+fx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(e+fx)}{\sqrt{a+a\cos(e+fx)}}\right)}{f}\\ &= \frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a+a\cos(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [A] time = 0.0643995, size = 50, normalized size = 1.35

$$\frac{\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(e+fx)\right)\right)\sec\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\cos(e+fx)+1)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[e + f*x]]/Sqrt[Cos[e + f*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(e + f*x)/2]]*Sqrt[a*(1 + Cos[e + f*x])]*Sec[(e
 + f*x)/2])/f
```

Maple [B] time = 0.392, size = 80, normalized size = 2.2

$$2\,\frac{\sqrt{a\left(\cos\left(fx+e\right)+1\right)}}{f\sqrt{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\arctan\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x)
```

```
[Out] 2/f/cos(f*x+e)^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(a*(cos(f*x+e)+1))^(
1/2)*arctan(sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)/cos(f*x+e))
```

Maxima [B] time = 1.79942, size = 197, normalized size = 5.32

$$\sqrt{a}\arctan\left(\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}+2\,\cos\left(2\,fx+2\,e\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(2\,fx+2\,e\right),\cos\left(2\,f\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*
e) + 1)^(1/4)*sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + si
n(f*x + e), (cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + cos(f*
x + e))/f
```

Fricas [A] time = 2.09459, size = 325, normalized size = 8.78

$$\left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos\left(fx+e\right)^{2}-2\,\sqrt{a\cos\left(fx+e\right)+a}\sqrt{-a}\sqrt{\cos\left(fx+e\right)}\sin\left(fx+e\right)+a\cos\left(fx+e\right)-a}{\cos\left(fx+e\right)+1}\right)}{f},-\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)+a}\sqrt{\cos\left(fx+e\right)}}{\sqrt{a}\sin\left(fx+e\right)}\right)}{f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(f*x + e)^2 - 2*sqrt(a*cos(f*x + e) + a)*sqrt(-a)*sqr
t(cos(f*x + e))*sin(f*x + e) + a*cos(f*x + e) - a)/(cos(f*x + e) + 1))/f, -
2*sqrt(a)*arctan(sqrt(a*cos(f*x + e) + a)*sqrt(cos(f*x + e))/(sqrt(a)*sin(f
*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cos\left(e+fx\right)+1\right)}}{\sqrt{\cos\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))**(1/2)/cos(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(e + f*x) + 1))/sqrt(cos(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos\left(fx+e\right)+a}}{\sqrt{\cos\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(f*x+e))^(1/2)/cos(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(f*x + e) + a)/sqrt(cos(f*x + e)), x)
```

3.223 $\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}$$

```
[Out] (-2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a - a*Cos[e + f*x]]])/f
```

Rubi [A] time = 0.0721399, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2774, 216}

$$-\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[e + f*x]]/Sqrt[-Cos[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[e + f*x])/Sqrt[a - a*Cos[e + f*x]]])/f
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-a\cos(e+fx)}}{\sqrt{-\cos(e+fx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, \frac{a\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}\\ &= -\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(e+fx)}{\sqrt{a-a\cos(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [C] time = 3.46982, size = 188, normalized size = 4.95

$$\frac{\sqrt{\cos(e)-i\sin(e)}\sqrt{-\cos(e+fx)}\left(\cot\left(\frac{1}{2}(e+fx)\right)+i\right)\sqrt{a-a\cos(e+fx)}\left(\tanh^{-1}\left(\frac{e^{ifx}}{\sqrt{\cos(e)-i\sin(e)}\sqrt{e^{2ifx}(\cos(e)+i\sin(e))-i\sin}}\right.\right.}{\sqrt{2}f\sqrt{\cos(e+fx)(\cos(fx)+i\sin(fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[e + f*x]]/Sqrt[-Cos[e + f*x]],x]
```

```
[Out] ((ArcTanh[E^(I*f*x)/(Sqrt[Cos[e] - I*Sin[e]]*Sqrt[Cos[e] + E^((2*I)*f*x)*(C
os[e] + I*Sin[e]) - I*Sin[e]])] + ArcTanh[Sqrt[Cos[e] + E^((2*I)*f*x)*(Cos[
e] + I*Sin[e]) - I*Sin[e]]/Sqrt[Cos[e] - I*Sin[e]]])*Sqrt[-Cos[e + f*x]]*Sq
rt[a - a*Cos[e + f*x]]*(I + Cot[(e + f*x)/2])*Sqrt[Cos[e] - I*Sin[e]])/(Sqr
t[2]*f*Sqrt[Cos[e + f*x]*(Cos[f*x] + I*Sin[f*x])])
```

Maple [B] time = 0.336, size = 91, normalized size = 2.4

$$-\frac{\sin\left(fx+e\right)}{f\left(-1+\cos\left(fx+e\right)\right)}\sqrt{-2\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\sqrt{-2a\left(-1+\cos\left(fx+e\right)\right)}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\right)\frac{1}{\sqrt{-\cos\left(fx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(-2*a*(-1+cos(f*x+e)))^(1/2)*sin(
f*x+e)*arctan(1/2*2^(1/2)*(-2*cos(f*x+e)/(cos(f*x+e)+1))^(1/2))/(-cos(f*x+e
))^(1/2)/(-1+cos(f*x+e))
```

Maxima [B] time = 1.84935, size = 567, normalized size = 14.92

$$\sqrt{-a}\left(\log\left(4\sqrt{\cos\left(2fx+2e\right)^2+\sin\left(2fx+2e\right)^2+2\cos\left(2fx+2e\right)+1}\cos\left(\frac{1}{2}\arctan\left(\sin\left(2fx+2e\right),\cos\left(2fx+2\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a)*(log(4*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*
f*x + 2*e) + 1)*cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2
+ 4*sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)*
sin(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + 8*(cos(2*f*x +
 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*cos(1/2*arctan
2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + 4) - log(cos(f*x + e)^2 + sin(
f*x + e)^2 + sqrt(cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2
*e) + 1)*(cos(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2 + sin(
1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1))^2) + 2*(cos(2*f*x + 2*
e)^2 + sin(2*f*x + 2*e)^2 + 2*cos(2*f*x + 2*e) + 1)^(1/4)*(cos(f*x + e)*cos
(1/2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)) + sin(f*x + e)*sin(1/
2*arctan2(sin(2*f*x + 2*e), cos(2*f*x + 2*e) + 1)))))/f
```

Fricas [A] time = 2.2619, size = 435, normalized size = 11.45

$$\left[\frac{\sqrt{-a}\log\left(\frac{4\sqrt{-a\cos\left(fx+e\right)+a}\left(2\cos\left(fx+e\right)^{2}+3\cos\left(fx+e\right)+1\right)\sqrt{-a}\sqrt{-\cos\left(fx+e\right)}-\left(8a\cos\left(fx+e\right)^{2}+8a\cos\left(fx+e\right)+a\right)\sin\left(fx+e\right)}{\sin\left(fx+e\right)}\right)}{2f}, \frac{\sqrt{a}\arctan\left(\cdots\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a)*log((4*sqrt(-a*cos(f*x + e) + a)*(2*cos(f*x + e)^2 + 3*cos(f*
x + e) + 1)*sqrt(-a)*sqrt(-cos(f*x + e)) - (8*a*cos(f*x + e)^2 + 8*a*cos(f*
x + e) + a)*sin(f*x + e))/sin(f*x + e))/f, sqrt(a)*arctan(1/2*sqrt(-a*cos(f
*x + e) + a)*sqrt(-cos(f*x + e))*(2*cos(f*x + e) + 1)/(sqrt(a)*cos(f*x + e)
*sin(f*x + e)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\left(\cos\left(e+fx\right)-1\right)}}{\sqrt{-\cos\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(f*x+e))**(1/2)/(-cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(cos(e + f*x) - 1))/sqrt(-cos(e + f*x)), x)
```

Giac [B] time = 2.23476, size = 188, normalized size = 4.95

$$\frac{\sqrt{2}\left(\frac{a^{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-a}}{2\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{|a|}-\frac{\sqrt{2}\left(a^{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a}}{2\sqrt{a}}\right)-a^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\sqrt{a}|a|}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(f*x+e))^(1/2)/(-cos(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(a^2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*f*x + 1/2*e)^2 - a)
/sqrt(a))/sqrt(a) - sqrt(2)*arctan(sqrt(-a)/sqrt(a))/sqrt(a))*sgn(tan(1/2*f
*x + 1/2*e))/abs(a) - sqrt(2)*(a^2*arctan(1/2*sqrt(2)*sqrt(-a)/sqrt(a)) - a
^2*arctan(sqrt(-a)/sqrt(a)))*sgn(tan(1/2*f*x + 1/2*e))/(sqrt(a)*abs(a)))/f
```

3.224 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=171

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*Sqrt[a]*d) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])])/(Sqrt[a]*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*S
qrt[a + a*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a +
a*Cos[c + d*x]])
```

Rubi [A] time = 0.420348, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2778, 2983, 2982, 2782, 205, 2774, 216}

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a\cos(c+dx)+a}}+\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4\sqrt{a}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(4*Sqrt[a]*d) -
 (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Cos[c + d*x]])])/(Sqrt[a]*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*S
qrt[a + a*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a +
a*Cos[c + d*x]])
```


Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\sqrt{\cos(c+dx)}(-3a+a\cos(c+dx))}{\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\frac{a^2}{2}-\frac{7}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} + \frac{7\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{8a} - \int \frac{}{\sqrt{\cos(}}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin}{\sqrt{a+a\,c}}\right.}{4ad}$$

$$= \frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx}{4d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 1.19256, size = 289, normalized size = 1.69

$$\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(-4\sqrt{1+e^{2i(c+dx)}}\sin\left(\frac{1}{2}(c+dx)\right)+2\sqrt{1+e^{2i(c+dx)}}\sin\left(\frac{3}{2}(c+dx)\right)-7\sin\left(\frac{1}{2}(c+dx)\right)\text{ta}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*((7*I)*ArcTanh[Sqrt[1 + E^((2*I)*(c +
d*x))]]*Cos[(c + d*x)/2] - 4*Sqrt[1 + E^((2*I)*(c + d*x))]*Sin[(c + d*x)/2]
 - 7*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]*Sin[(c + d*x)/2] + 7*ArcSinh[E^
(I*(c + d*x))]*((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 8*Sqrt[2]*ArcTa
nh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]*((-I)*Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Sin[(3*
(c + d*x))/2]))/(4*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a*(1 + Cos[c + d*x]
)])
```

Maple [A] time = 0.432, size = 196, normalized size = 1.2

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^3}{8\,da\,(\sin(dx+c))^6}(\cos(dx+c))^{\frac{5}{2}}\sqrt{a\,(1+\cos(dx+c))}\left(2\sqrt{2}\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/8/d*2^(1/2)/a*cos(d*x+c)^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^
3*(2*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+7*2^(1/2)*arctan(sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+8*arcsin((-1+cos(d*x+c))/sin(
d*x+c)))/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)
```

Fricas [A] time = 2.64071, size = 459, normalized size = 2.68

$$\frac{\sqrt{a\cos(dx+c)+a}(2\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)-7\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right.}{4\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d
*x + c) - 7*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
```

```
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 4*sqrt(2)*(a*cos(d*x + c) + a)*arc
tan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x +
c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.225 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -(ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]/(Sqrt[a]*d)) + (S
qrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a
 + a*Cos[c + d*x]])
```

Rubi [A] time = 0.279974, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2778, 2982, 2782, 205, 2774, 216}

$$-\frac{\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -(ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]/(Sqrt[a]*d)) + (S
qrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a
 + a*Cos[c + d*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{2a}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2+a \cdots}\right.}{\cdots}$$

$$= -\frac{\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 1.2273, size = 227, normalized size = 1.77

$$-\frac{ie^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(-\sqrt{2}e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-4e^{i(c+dx)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left(\sqrt{1+e^{2}\cdots}\right.\right.}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] ((-I)*(-(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))]) - 4*E^(I*(c + d*
x))*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
+ Sqrt[2]*((-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c +
 d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2]*Sqrt[Cos[c
 + d*x]])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt
[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.405, size = 159, normalized size = 1.2

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))^2}{2\,da\,(\sin(dx+c))^4}(\cos(dx+c))^{\frac{3}{2}}\sqrt{a\,(1+\cos(dx+c))}\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}-\sqrt{2}\arctan\left(\frac{\sin(dx+}{\cos(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)/a*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2
*(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-2^(1/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))-2*arcsin((-1+cos(d*x+c)
)/sin(d*x+c)))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)
```

Fricas [A] time = 2.39317, size = 420, normalized size = 3.28

$$\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\sqrt{a\cos(dx+c)+a}}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a)
+ sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.226 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.169815, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2777, 2774, 216, 2782, 205}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```


```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(Sqrt[a]*d) - (
Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
a*Cos[c + d*x]])])/(Sqrt[a]*d)
```


Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a} - \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [C] time = 0.398151, size = 161, normalized size = 1.69

$$\frac{i\left(1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(-\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (I*(1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*(-
ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/
(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.405, size = 125, normalized size = 1.3

$$-\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)}{da\left(\sin\left(dx+c\right)\right)^2}\sqrt{\cos\left(dx+c\right)}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(\sqrt{2}\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)+\arcsin\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/d*2^(1/2)/a*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(2^(1/2)*arctan(si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+arcsin((-1+cos(d*x+c
))/sin(d*x+c)))*(-1+cos(d*x+c))/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.37281, size = 263, normalized size = 2.77

$$\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(a*cos(d*x + c) + a), x)
```

3.227 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.0617989, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2782, 205}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [A] time = 0.05071, size = 51, normalized size = 0.91

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2])/(d*Sqrt[a*
(1 + Cos[c + d*x])])
```

Maple [A] time = 0.283, size = 69, normalized size = 1.2

$$-\frac{\sqrt{2}}{da}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{a(1+\cos(dx+c))}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/d*2^(1/2)/a/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(1+cos
(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.16797, size = 451, normalized size = 8.05

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}}\right.}{\sqrt{a}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(-1/a)
*sqrt(cos(d*x + c))*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(
cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*
x + c))*sqrt(a)))/(sqrt(a)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a(\cos(c+dx)+1)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.228 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.127829, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2779, 12, 2782, 205}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{a}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin}{\sqrt{\cos(c+dx)}\sqrt{\ldots}}\right.}{d}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 2.23491, size = 180, normalized size = 1.94

$$2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\cos(c+dx)(\cos(c+dx)+2)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\ldots}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc[
(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x
)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (Hypergeometric2F1[2,
 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/1
0))/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.382, size = 206, normalized size = 2.2

$$-\frac{\sqrt{2}(\sin(dx+c))^2}{da(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left((\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+2\cos(d\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/d*2^(1/2)/a*(cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))+(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+
cos(d*x+c))/sin(d*x+c))+cos(d*x+c)*sin(d*x+c)*2^(1/2))*(a*(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32743, size = 373, normalized size = 4.01

$$-\frac{\frac{\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}}\right)}{\sqrt{a}}-2\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{ad\cos(dx+c)^2+ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^2 + cos(d*x +
 c))*sqrt(a)))/sqrt(a) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(
d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a(\cos(c+dx)+1)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*cos(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.229 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=131

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*
Sqrt[a + a*Cos[c + d*x]]) - (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])
```

Rubi [A] time = 0.235723, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2779, 2984, 12, 2782, 205}

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```


```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])])/(Sqrt[a]*d) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*
Sqrt[a + a*Cos[c + d*x]]) - (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])
```


Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{2\int}{}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} + \int$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}} - \frac{(2a)}{}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{}{3a}$$

Mathematica [C] time = 7.51377, size = 473, normalized size = 3.61

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{2}\right\},\left\{1,\frac{9}{2}\right\},\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(12*Cos[(c + d*x)/2]^4*Hypergeo
metricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*
x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin
[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c
/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2
```

```
))))/(63*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

Maple [B] time = 0.391, size = 274, normalized size = 2.1

$$-\frac{\sqrt{2}\left(\sin\left(dx+c\right)\right)^{4}}{3\,da\left(-1+\cos\left(dx+c\right)\right)^{2}\left(1+\cos\left(dx+c\right)\right)^{3}}\left(3\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{5/2}\left(\cos\left(dx+c\right)\right)^{3}\arcsin\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}\right)+9\,\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/3/d*2^(1/2)/a*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^3*arcsin((
-1+cos(d*x+c))/sin(d*x+c))+9*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2
*arcsin((-1+cos(d*x+c))/sin(d*x+c))+9*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos
(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+3*(cos(d*x+c)/(1+cos(d*x+c)))^(5
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+2^(1/2)*cos(d*x+c)^2*sin(d*x+c)-cos(
d*x+c)*sin(d*x+c)*2^(1/2))*sin(d*x+c)^4*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*
x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.28813, size = 412, normalized size = 3.15

$$-\frac{2\sqrt{a\cos\left(dx+c\right)+a}\left(\cos\left(dx+c\right)-1\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{3\sqrt{2}\left(a\cos(dx+c)^{3}+a\cos(dx+c)^{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(}}{2\left(\cos(dx+c)^{2}+\cos(\right.}\right.}{\sqrt{a}}}{3\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(
d*x + c) - 3*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*arctan(1/2*sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c)/((cos(d*x + c)^
2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)
^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.230 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=169

$$-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/
2)*Sqrt[a + a*Cos[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])
```

Rubi [A] time = 0.364864, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2779, 2984, 12, 2782, 205}

$$-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/
2)*Sqrt[a + a*Cos[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sq
rt[a + a*Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\int}{}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{15}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{15}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} + \frac{}{15}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}} - \frac{}{1}$$

Mathematica [C] time = 9.81551, size = 1540, normalized size = 9.11

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a + a*Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48
825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (
d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11
/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^
10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 -
 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 22665
6*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d
```

```
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c
/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*S
qrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh
[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/
2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c
+ d*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x
)/2]^2)))/(675*d*Sqrt[a*(1 + Cos[c + d*x])]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7
/2)*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

Maple [B] time = 0.404, size = 341, normalized size = 2.

$$-\frac{\sqrt{2}\,(\sin(dx+c))^6}{15\,da\,(-1+\cos(dx+c))^3\,(1+\cos(dx+c))^4}\left(15\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^4\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{7/2}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/15/d*2^(1/2)/a*(15*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4*(cos(
d*x+c)/(1+cos(d*x+c)))^(7/2)+60*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+90*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+60*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+15*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+13*2^(1/2)*cos(d*x+c)
^3*sin(d*x+c)-2^(1/2)*cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c)*2^(1/
2))*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^6/(-1+cos(d*x+c))^3/(1+cos(d*x+c))^
4/cos(d*x+c)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22822, size = 440, normalized size = 2.6

$$\frac{2 \sqrt{a\cos(dx+c)+a}\left(13 \cos(dx+c)^2 - \cos(dx+c)+3\right)\sqrt{\cos(dx+c)}\sin(dx+c) - \frac{15\sqrt{2}\left(a\cos(dx+c)^4+a\cos(dx+c)^3\right)\arctan\left(\frac{\sqrt{}}{}\right.}{\sqrt{a}}}{15\left(ad\cos(dx+c)^4+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqr
t(cos(d*x + c))*sin(d*x + c) - 15*sqrt(2)*(a*cos(d*x + c)^4 + a*cos(d*x + c
)^3)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x
 + c)/((cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)))/sqrt(a))/(a*d*cos(d*x + c)
^4 + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*cos(d*x + c)^(7/2)), x)
```

3.231 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{7\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{4d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{\cos(c+dx)+1}}$$

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (7*ArcSin[Sin[c +
d*x]/Sqrt[1 + Cos[c + d*x]]])/(4*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*
d*Sqrt[1 + Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 +
 Cos[c + d*x]])
```

Rubi [A] time = 0.272263, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2778, 2983, 2982, 2781, 216, 2774}

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{7\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{4d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (7*ArcSin[Sin[c +
d*x]/Sqrt[1 + Cos[c + d*x]]])/(4*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*
d*Sqrt[1 + Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 +
 Cos[c + d*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
```

```
x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}} - \frac{1}{4}\int \frac{(-3+\cos(c+dx))\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$$
$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}} - \frac{1}{4}\int \frac{\frac{1}{2}-\frac{7}{2}\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,d$$
$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}} + \frac{7}{8}\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx - \int \frac{}{\sqrt{\cos}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1+\cos(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right.}{4d}$$
$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{7\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1+\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin}{2d\sqrt{1+\cos(}}$$

Mathematica [C] time = 0.824201, size = 286, normalized size = 2.27

$$\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(-4\sqrt{1+e^{2i(c+dx)}}\sin\left(\frac{1}{2}(c+dx)\right)+2\sqrt{1+e^{2i(c+dx)}}\sin\left(\frac{3}{2}(c+dx)\right)-7\sin\left(\frac{1}{2}(c+dx)\right)\tanh^{-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((7*I)*ArcTanh[Sqrt
[1 + E^((2*I)*(c + d*x))]]*Cos[(c + d*x)/2] - 4*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Sin[(c + d*x)/2] - 7*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]*Sin[(c + d*x
)/2] + 7*ArcSinh[E^(I*(c + d*x))]*((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
) + 8*Sqrt[2]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
 d*x))])]*((-I)*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 2*Sqrt[1 + E^((2*I)*
```

```
(c + d*x))]*Sin[(3*(c + d*x))/2]))/(4*d*Sqrt[1 + E^((2*I)*(c + d*x))])
```

Maple [A] time = 0.344, size = 187, normalized size = 1.5

$$-\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{3}}{8d\left(\sin\left(dx+c\right)\right)^{6}}\left(\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{2+2\cos\left(dx+c\right)}\left(2\sin\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\cos\left(dx+c\right)+4\sqrt{2}\,\mathrm{ar}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/8/d*2^(1/2)*cos(d*x+c)^(5/2)*(2+2*cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^3*(2
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+4*2^(1/2)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*a
rctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/(cos(d*x+c)
/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)
```

Fricas [A] time = 2.11419, size = 402, normalized size = 3.19

$$\frac{\left(2\cos\left(dx+c\right)-1\right)\sqrt{\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+4\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos}}{\sin(dx+c)}\right.}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*cos(d*x + c) - 1)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x
 + c) + 4*(sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c)
 + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - 7*(cos(d*x + c) + 1)*arctan(sqrt(c
os(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)
```

3.232 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d - ArcSin[Sin[c + d*x]/S
qrt[1 + Cos[c + d*x]]]/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Co
s[c + d*x]])
```

Rubi [A] time = 0.187322, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2778, 2982, 2781, 216, 2774}

$$\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d - ArcSin[Sin[c + d*x]/S
qrt[1 + Cos[c + d*x]]]/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Co
s[c + d*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} - \frac{1}{2}\int \frac{-1+\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx \\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} - \frac{1}{2}\int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx \\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\right.}{d} \\ &= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} - \frac{\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.784782, size = 224, normalized size = 2.64

$$-\frac{ie^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(-\sqrt{2}e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-4e^{i(c+dx)}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left(\sqrt{1+e^{2i(c+}}\right.\right.}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] ((-I)*(-(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d*x))]) - 4*E^(I*(c + d*
x))*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]
+ Sqrt[2]*((-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))] + E^(I*(c +
 d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2]*Sqrt[Cos[c
 + d*x]/(1 + Cos[c + d*x])])/(Sqrt[2]*d*E^((I/2)*(c + d*x))*Sqrt[1 + E^((2*
I)*(c + d*x))])
```

Maple [A] time = 0.305, size = 151, normalized size = 1.8

$$-\frac{\sqrt{2}(-1+\cos(dx+c))^2}{2d(\sin(dx+c))^4}(\cos(dx+c))^{\frac{3}{2}}\sqrt{2+2\cos(dx+c)}\left(\sqrt{2}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-\sin(dx+c)\sqrt{\frac{\cos(dx}{1+\cos(d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*2^(1/2)*cos(d*x+c)^(3/2)*(2+2*cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(2
^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
```

```
+c)))^(1/2)+arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))
)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```

Fricas [A] time = 1.99741, size = 365, normalized size = 4.29

$$-\frac{\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\sqrt{\cos}}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sq
rt(cos(d*x + c))/sin(d*x + c)) - (cos(d*x + c) + 1)*arctan(sqrt(cos(d*x + c
) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - sqrt(cos(d*x + c) + 1)*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{\cos(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(cos(c + d*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```

3.233 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*ArcSin[Sin[c +
d*x]/Sqrt[1 + Cos[c + d*x]]])/d
```

Rubi [A] time = 0.116653, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2777, 2774, 216, 2781}

$$\frac{2\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*ArcSin[Sin[c +
d*x]/Sqrt[1 + Cos[c + d*x]]])/d
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx = -\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx + \int \frac{\sqrt{1+\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} + \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}$$
$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d}$$

Mathematica [C] time = 0.247385, size = 135, normalized size = 2.5

$$-\frac{i\left(1+e^{i(c+dx)}\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d\sqrt{1+e^{2i(c+dx)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] ((-I)*(1 + E^(I*(c + d*x)))*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(-1
 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - ArcTanh[Sqrt
[1 + E^((2*I)*(c + d*x))]])*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])])/(d*Sqrt[
1 + E^((2*I)*(c + d*x))])
```

Maple [B] time = 0.296, size = 124, normalized size = 2.3

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))}{2\,d\,(\sin(dx+c))^2}\sqrt{2+2\,\cos(dx+c)}\sqrt{\cos(dx+c)}\left(\sqrt{2}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+2\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*2^(1/2)*(2+2*cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(2^(
1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/cos(d*x+c)))/sin(d*x+c)^2/(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.9667, size = 204, normalized size = 3.78

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x +
 c)) - 2*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos{(c+dx)}}}{\sqrt{\cos{(c+dx)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(cos(c + d*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos{(dx+c)}}}{\sqrt{\cos{(dx+c)}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(cos(d*x + c) + 1), x)
```

3.234 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d
```

Rubi [A] time = 0.0428342, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2781, 216}

$$\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx &= -\frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}\\ &= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} \end{aligned}$$

Mathematica [A] time = 0.0386087, size = 49, normalized size = 1.81

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2])/(d*Sqrt[1
+ Cos[c + d*x]])
```

Maple [B] time = 0.178, size = 63, normalized size = 2.3

$$-\frac{1}{d}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2+2\cos(dx+c)}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/cos(d*x+c)^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2+2*cos(d*x+c))^(1
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.05092, size = 159, normalized size = 5.89

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^2+\cos(dx+c)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*
x + c)/(cos(d*x + c)^2 + cos(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\cos(c+dx)+1}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))), x)
```

3.235 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rubi [A] time = 0.0845289, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2779, 2781, 216}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}dx = \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} - \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}dx$$
$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} + \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}$$
$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}$$

Mathematica [C] time = 1.7732, size = 178, normalized size = 2.87

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{2}\cos(c+dx)(\cos(c+dx)+2)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-\cos(c+dx)+\cos(c+dx)\sqrt{2-2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 + Cos[c + d*x])*Csc[
(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x
)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (Hypergeometric2F1[2,
 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d*x]*Tan[c + d*x])/1
0))/(d*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]])
```

Maple [B] time = 0.289, size = 210, normalized size = 3.4

$$-\frac{\sqrt{2}(\sin(dx+c))^2}{2d(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(\sqrt{2}(\cos(dx+c))^2\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/2/d*2^(1/2)*(2^(1/2)*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*2^(1/2)*cos(d*x+c)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2^(1/2)*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+2*cos(d*x+c)*sin(d*x+c))*si
n(d*x+c)^2*(2+2*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+
c)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.13109, size = 343, normalized size = 5.53

$$-\frac{\left(\sqrt{2}\cos\left(dx+c\right)^{2}+\sqrt{2}\cos\left(dx+c\right)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(\cos(dx+c)^{2}+\cos(dx+c)\right)}\right)-2\sqrt{\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*arctan(1/2*sqrt(2)*sqrt(c
os(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c)^2 + cos(d*x
+ c))) - 2*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 + d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos\left(c+dx\right)+1}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*cos(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos\left(dx+c\right)+1}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(3/2)), x)
```

3.236 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}+\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d + (2*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) - (2*Sin[c + d*x])/(3*d*Sqrt[Co
s[c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rubi [A] time = 0.165176, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2779, 2984, 12, 2781, 216}

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}+\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d + (2*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) - (2*Sin[c + d*x])/(3*d*Sqrt[Co
s[c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}dx = \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{1}{3}\int \frac{1-2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}dx$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} - \frac{2}{3}\int -\frac{}{2\surd}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} + \int \frac{}{\sqrt{\cos}}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}} - \frac{\sqrt{2}\,\mathrm{Subs}}{}$$

$$= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+}{3d\sqrt{\cos(c+dx)}\sqrt{1-}}$$

Mathematica [C] time = 6.62099, size = 471, normalized size = 4.81

$$-\frac{2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{2}\right\},\left\{1,\frac{9}{2}\right\},\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*(12*Cos[(c + d*x)/2]^4*Hypergeo
metricPFQ[{2, 2, 7/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*
x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 + 12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin
[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2
)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c
/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c
/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2
))))/(63*d*Sqrt[1 + Cos[c + d*x]]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

Maple [B] time = 0.306, size = 278, normalized size = 2.8

$$-\frac{\sqrt{2}(\sin(dx+c))^4}{6d(-1+\cos(dx+c))^2(1+\cos(dx+c))^3}\left(3(\cos(dx+c))^3\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}\sqrt{2}+9\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/6/d*2^(1/2)*(3*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+9*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+9*cos(d*x+c)*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+3*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+2*cos(
d*x+c)^2*sin(d*x+c)-2*cos(d*x+c)*sin(d*x+c))*sin(d*x+c)^4*(2+2*cos(d*x+c))^
(1/2)/(-1+cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.15851, size = 382, normalized size = 3.9

$$-\frac{2\sqrt{\cos(dx+c)+1}(\cos(dx+c)-1)\sqrt{\cos(dx+c)}\sin(dx+c)-3\left(\sqrt{2}\cos(dx+c)^3+\sqrt{2}\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}}{\cdots}\right.}{3\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(cos(d*x + c) + 1)*(cos(d*x + c) - 1)*sqrt(cos(d*x + c))*sin(d*
x + c) - 3*(sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2)*arctan(1/2*sqr
t(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x + c)/(cos(d*x + c)^2
 + cos(d*x + c))))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(5/2)), x)
```

3.237 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=134

$$-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26\sin(}{15d\sqrt{\cos(c+dx)}}$$

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*Sin[c + d*x])/(
5*d*Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Cos
[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[
c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rubi [A] time = 0.242472, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2779, 2984, 12, 2781, 216}

$$-\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26\sin(}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])])/d) + (2*Sin[c + d*x])/(
5*d*Cos[c + d*x]^(5/2)*Sqrt[1 + Cos[c + d*x]]) - (2*Sin[c + d*x])/(15*d*Cos
[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[
c + d*x]]*Sqrt[1 + Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{1}{5}\int \frac{1-4\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2}{15}\int -$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} + \frac{}{15d\sqrt{\cos}}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} + \frac{}{15d\sqrt{\cos}}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} + \frac{}{15d\sqrt{\cos}}$$

$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}} - \frac{2\sin(c-}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

Mathematica [C] time = 7.95131, size = 1538, normalized size = 11.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[1 + Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*(4725*Sin[c/2 + (d*x)/2]^2 - 48
825*Sin[c/2 + (d*x)/2]^4 + 210105*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (
d*x)/2]^8 + 655812*Sin[c/2 + (d*x)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11
/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^
10 - 40*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 -
 518760*Sin[c/2 + (d*x)/2]^12 + 1770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 22665
6*Sin[c/2 + (d*x)/2]^14 - 1500*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d
*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c
/2 + (d*x)/2]^16 + 440*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]
```

```
^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*
x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/
2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[S
in[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*S
qrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^
14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh
[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/
2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c
+ d*x)/2]^4*HypergeometricPFQ[{2, 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x
)/2]^2)))/(675*d*Sqrt[1 + Cos[c + d*x]]*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2)*
(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

Maple [B] time = 0.316, size = 344, normalized size = 2.6

$$-\frac{\sqrt{2}\left(\sin\left(dx+c\right)\right)^{6}}{30\,d\left(-1+\cos\left(dx+c\right)\right)^{3}\left(1+\cos\left(dx+c\right)\right)^{4}}\left(15\sqrt{2}\left(\cos\left(dx+c\right)\right)^{4}\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{7/2}\arcsin\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] -1/30/d*2^(1/2)*(15*2^(1/2)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))+60*2^(1/2)*cos(d*x+c)^3*(cos(d*x+c)/(1+c
os(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+90*2^(1/2)*cos(d*x+c)^
2*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+60*2
^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))+15*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))+26*cos(d*x+c)^3*sin(d*x+c)-2*cos(d*x+c)^2*sin(d*x+c)+6*cos
(d*x+c)*sin(d*x+c))*sin(d*x+c)^6*(2+2*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))^3/(
1+cos(d*x+c))^4/cos(d*x+c)^(7/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.13795, size = 410, normalized size = 3.06

$$\frac{2\left(13\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+3\right)\sqrt{\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-15\left(\sqrt{2}\cos\left(dx+c\right)^{4}+\sqrt{2}\cos\left(dx+\right.\right.}{15\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(cos(d*x + c) + 1)*sqrt(
cos(d*x + c))*sin(d*x + c) - 15*(sqrt(2)*cos(d*x + c)^4 + sqrt(2)*cos(d*x +
 c)^3)*arctan(1/2*sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))*sin(d*x
 + c)/(cos(d*x + c)^2 + cos(d*x + c))))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^
3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos\left(dx+c\right)+1}\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*cos(d*x + c)^(7/2)), x)
```

3.238 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{3\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (-3*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
(9*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*
Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.424541, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2765, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{3\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-3*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) +
(9*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2)) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*d*
Sqrt[a + a*Cos[c + d*x]])
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-3a\cos(c+dx)\right)}{\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{\int \frac{-\frac{3a^2}{2}+3a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{2a^3}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{3\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a^2} + \frac{9\int \frac{}{\sqrt{\cos(}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{9\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{}{\sqrt{\cos(}}\right.}{2d}$$

$$= -\frac{3\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} + \frac{9\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [C] time = 5.24788, size = 229, normalized size = 1.32

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\frac{\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)}{d}+\frac{3ie^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(2\sinh^{-1}\left(e^{i(c+dx)}\right)+3\sqrt{2}\tanh^{-1}\left(\right.\right.}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((3*I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x
)))/E^(I*(c + d*x))]*(2*ArcSinh[E^(I*(c + d*x))] + 3*Sqrt[2]*ArcTanh[(1 - E
^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 2*ArcTanh[Sqrt[1
 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]) + (Sqr
t[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/
2]))/d))/(a*(1 + Cos[c + d*x]))^(3/2)
```

Maple [A] time = 0.414, size = 227, normalized size = 1.3

$$\frac{\sqrt{2}\left(-1+\cos(dx+c)\right)^3}{4a^2d\left(\sin(dx+c)\right)^7}\left(\cos(dx+c)\right)^{\frac{5}{2}}\sqrt{a\left(1+\cos(dx+c)\right)}\left(2\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\left(\cos(dx+c)\right)^2+6\arctan\left(\frac{\sin}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/d*2^(1/2)/a^2*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^3*(a*(1+cos(d*x+c)))^(1/
2)*(2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+6*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+2^(1
/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+9*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*sin(d*x+c)-3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 3.04479, size = 552, normalized size = 3.17

$$-\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos(dx+c)+a}(2\cos(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*s
qrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d*x + c
) - 12*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x
+ c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2
 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.239 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) - (
5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.29224, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2765, 2982, 2782, 205, 2774, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(3/2)*d) - (
5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[
c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d
*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\frac{a}{2}-2a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a^2} - \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{4a}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d} - \frac{2\,\text{Subst}}{}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{3/2}d} - \frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [C] time = 3.59941, size = 215, normalized size = 1.6

$$\frac{\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)}{d} - \frac{ie^{\frac{1}{2}i(c+dx)}\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(4\sinh^{-1}\left(e^{i(c+dx)}\right)+5\sqrt{2}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-4\tan\right.}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}}\right.}{(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^3*(((-I)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)
))/E^(I*(c + d*x))]*(4*ArcSinh[E^(I*(c + d*x))] + 5*Sqrt[2]*ArcTanh[(1 - E^
(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 4*ArcTanh[Sqrt[1
+ E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]) - (Sqrt
[Cos[c + d*x]]*Sec[(c + d*x)/2]*Tan[(c + d*x)/2])/d))/(a*(1 + Cos[c + d*x])
)^(3/2)
```

Maple [A] time = 0.374, size = 195, normalized size = 1.5

$$\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{2}}{4\,a^{2}d\left(\sin\left(dx+c\right)\right)^{5}}\left(\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(4\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\sqrt{2}\sin\left(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/d*2^(1/2)/a^2*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^2*(a*(1+cos(d*x+c)))^(1/
2)*(4*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/
2)*sin(d*x+c)+2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 3.0719, size = 521, normalized size = 3.89

$$\frac{5\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-8\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 8*(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)
*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
 + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.240 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
+ d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a
 + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.130338, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2764, 12, 2782, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
+ d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a
 + a*Cos[c + d*x])^(3/2))
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\int \frac{a}{2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{4a}\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx,x,-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.324653, size = 118, normalized size = 1.22

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)+1}\left(\sqrt{\cos(c+dx)+1}\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\right)}{2d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[1 + Cos[c + d*x]]*(ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[
(c + d*x)/2]^2]]*Sqrt[1 + Cos[c + d*x]] + 2*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sin[(c + d*x)/2]))/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 0.373, size = 146, normalized size = 1.5

$$\frac{\sqrt{2}(-1+\cos(dx+c))}{4a^2d(\sin(dx+c))^3}\sqrt{\cos(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(\sqrt{2}\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/d*2^(1/2)/a^2*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))
*(2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*
x+c)^3/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 2.26915, size = 401, normalized size = 4.13

$$\frac{\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)+2\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\cos\left(c\right.}}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(
2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(
d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c
))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\left(a\left(\cos\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

3.241 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.12948, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2766, 12, 2782, 205}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*
d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{3a}{2\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}dx}{2a^2}$$
$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}dx}{4a}$$
$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c}}\right.}{2d}$$
$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}}$$

Mathematica [A] time = 0.550759, size = 106, normalized size = 1.09

$$-\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(3\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\mathrm{s}}\right.\right.}{2d(a(\cos(c+dx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*(2 + 3*ArcTanh[Sqrt[-(Sec[c + d*x]*Si
n[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])*Sin[(c + d
*x)/2])/(2*d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [B] time = 0.3, size = 170, normalized size = 1.8

$$-\frac{\sqrt{2}}{4a^2d(1+\cos(dx+c))\sin(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*(3*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)+3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-cos(d
*x+c)^2*2^(1/2)+cos(d*x+c)*2^(1/2))/(1+cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x
+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

Fricas [A] time = 2.31778, size = 404, normalized size = 4.16

$$\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqr
t(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*co
s(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a(\cos(c+dx)+1))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a*(cos(c + d*x) + 1))**(3/2)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.242 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (-7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sqrt[Cos[c + d*x]]
*(a + a*Cos[c + d*x])^(3/2)) + (5*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.246947, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2766, 2984, 12, 2782, 205}

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (-7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Sqrt[Cos[c + d*x]]
*(a + a*Cos[c + d*x])^(3/2)) + (5*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Cos[c + d*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{5a}{2}-a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}} + \frac{5\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}$$

Mathematica [C] time = 7.16074, size = 456, normalized size = 3.33

$$2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{2,2,\frac{5}{2}\right\},\left\{1,\frac{9}{2}\right\},\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{70\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-35} - \frac{1}{6}\left(1-2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]*((4*Cos[(c +
d*x)/2]^4*HypergeometricPFQ[{2, 2, 5/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(-35 + 70*Sin[c/2 + (d*x)
/2]^2) - (Csc[c/2 + (d*x)/2]^6*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-3*ArcTanh[Sqrt[Sin[c/2 + (d*x
)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*(-25 + 91*Sin[c/2 + (d*x)/2]^2 - 100
*Sin[c/2 + (d*x)/2]^4 + 34*Sin[c/2 + (d*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/2]^
2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-75 + 298*Sin[c/2 + (d*x)/2]^2 - 350*Sin[
```

```
c/2 + (d*x)/2]^4 + 124*Sin[c/2 + (d*x)/2]^6)))/6))/(d*(a*(1 + Cos[c + d*x])
)^(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

Maple [B] time = 0.382, size = 245, normalized size = 1.8

$$\frac{\sqrt{2}\sin(dx+c)}{4\,a^2d(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(-7\,(\cos(dx+c))^2\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arcsin\left(\frac{-1+\cos(dx}{\sin(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/d*2^(1/2)/a^2*(-7*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-14*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-7*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+5*cos(d*x
+c)^3*2^(1/2)-cos(d*x+c)^2*2^(1/2)-4*cos(d*x+c)*2^(1/2))*sin(d*x+c)*(a*(1+c
os(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

Fricas [A] time = 2.50175, size = 471, normalized size = 3.44

$$-\frac{7\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2\sqrt{a\cos(dx}}{4\left(a^2d\cos(dx+c)^3+2\,a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(7*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*
arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(
d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*
(5*cos(d*x + c) + 4)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3
 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.243 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{11\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-$$

```
[Out] (11*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)
*(a + a*Cos[c + d*x])^(3/2)) + (7*Sin[c + d*x])/(6*a*d*Cos[c + d*x]^(3/2)*S
qrt[a + a*Cos[c + d*x]]) - (19*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.388123, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2766, 2984, 12, 2782, 205}

$$\frac{11\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```


```
[Out] (11*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*Cos[c + d*x]^(3/2)
*(a + a*Cos[c + d*x])^(3/2)) + (7*Sin[c + d*x])/(6*a*d*Cos[c + d*x]^(3/2)*S
qrt[a + a*Cos[c + d*x]]) - (19*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + a*Cos[c + d*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx = -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\frac{7a}{2}-2a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} + \frac{7\sin(c+dx)}{6ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{11\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}} +$$

Mathematica [C] time = 9.12399, size = 589, normalized size = 3.33

$$\cot^3\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-80\sin^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{2,2,2,\frac{7}{2}\right\},\left\{1\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (Cot[c/2 + (d*x)/2]^3*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^2*(-80*Cos[(c +
 d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 + 120*Cos[(c + d*
```

```
x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]
^2) + 21*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]]*(-392 + 2347*Sin[c/2 + (d*x)/2]^2 - 5391*Sin[c/2 + (d*x)/
2]^4 + 5972*Sin[c/2 + (d*x)/2]^6 - 3232*Sin[c/2 + (d*x)/2]^8 + 696*Sin[c/2
+ (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(
-5880 + 37165*Sin[c/2 + (d*x)/2]^2 - 89856*Sin[c/2 + (d*x)/2]^4 + 103992*Si
n[c/2 + (d*x)/2]^6 - 58336*Sin[c/2 + (d*x)/2]^8 + 12960*Sin[c/2 + (d*x)/2]^
10))))/(945*d*(a*(1 + Cos[c + d*x]))^(3/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/
2))
```

Maple [B] time = 0.432, size = 313, normalized size = 1.8

$$\frac{\sqrt{2}\,(\sin(dx+c))^3}{12\,a^2 d\,(-1+\cos(dx+c))^2\,(1+\cos(dx+c))^3}\left(-33\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3\sin(dx+c)\left(\frac{\cos(dx}{1+\cos(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/12/d*2^(1/2)/a^2*(-33*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-99*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-99*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(5/2)-33*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(5/2)+19*2^(1/2)*cos(d*x+c)^4-7*cos(d*x+c)^3*2^(1/2)-16*cos(d*x+c)
^2*2^(1/2)+4*cos(d*x+c)*2^(1/2))*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^3/(-1+
cos(d*x+c))^2/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Fricas [A] time = 2.56882, size = 506, normalized size = 2.86

$$\frac{33\sqrt{2}\left(\cos(dx+c)^4+2\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2\sqrt{a\cos(dx}{12\left(a^2 d\cos(dx+c)^4+2\,a^2 d\cos(dx+c)^3+a^2 d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(33*sqrt(2)*(cos(d*x + c)^4 + 2*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(
a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c) +
a)*(19*cos(d*x + c)^2 + 12*cos(d*x + c) - 4)*sqrt(cos(d*x + c))*sin(d*x + c
))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.244 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=214

$$-\frac{5\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{115\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+}{4d(a\cos(c+dx)+}$$

```
[Out] (-5*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
(115*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)) - (15*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16
*a*d*(a + a*Cos[c + d*x])^(3/2)) + (35*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16
*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.576522, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2765, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{5\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)\sqrt{\cos(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}+\frac{115\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+}{4d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-5*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) +
(115*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/
(4*d*(a + a*Cos[c + d*x])^(5/2)) - (15*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16
*a*d*(a + a*Cos[c + d*x])^(3/2)) + (35*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16
*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-5a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{45a^2}{4}-\frac{35}{2}a^2\cos(c+d\right.}{\sqrt{a+a\cos(c+dx)}}}{8a^4}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{15\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{35\sqrt{\cos(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{5\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} + \frac{115\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+}{4d(a+a\cos(c+dx}$$

Mathematica [C] time = 6.6648, size = 385, normalized size = 1.8

$$\frac{\sqrt{\cos(c+dx)}\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{8\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}+\frac{8\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}-\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}+\frac{23\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}-\frac{\tan(\ldots}{}\right.}{(a(\cos(c+dx)+1))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((5*I)/2)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x
))]*(8*ArcSinh[E^(I*(c + d*x))] + (23*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 8*ArcTanh[Sqrt[1 + E^((2*I)*(c
 + d*x))]])*Cos[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*
(a*(1 + Cos[c + d*x]))^(5/2)) + (Cos[c/2 + (d*x)/2]^5*Sqrt[Cos[c + d*x]]*((
8*Cos[(d*x)/2]*Sin[c/2])/d + (8*Cos[c/2]*Sin[(d*x)/2])/d + (23*Sec[c/2]*Sec
[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(4*d) - (Sec[c/2]*Sec[c/2 + (d*x)/2]^4*Sin[
(d*x)/2])/(2*d) + (23*Sec[c/2 + (d*x)/2]*Tan[c/2])/(4*d) - (Sec[c/2 + (d*x)
/2]^3*Tan[c/2])/(2*d)))/(a*(1 + Cos[c + d*x]))^(5/2)
```

Maple [A] time = 0.418, size = 344, normalized size = 1.6

$$\frac{\sqrt{2}(-1+\cos(dx+c))^5}{32\,da^3(\sin(dx+c))^{11}}(\cos(dx+c))^{\frac{7}{2}}\sqrt{a(1+\cos(dx+c))}\left(16\,(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\sqrt{2}\sqrt{\frac{\cos\ldots}{1+\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*cos(d*x+c)^(7/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c)
)^5*(16*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*2^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+80*cos(d*x+c)*2^(1/2)*arctan(
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+115*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-20*2^(1/2)*cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+80*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+115*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*sin(d*x+c)-35*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(cos(d*x
+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^11
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 4.60882, size = 672, normalized size = 3.14

$$-\frac{115\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos(dx+c)+}}{32\left(a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(16*cos(d*x + c)^2 + 55*cos(d
*x + c) + 35)*sqrt(cos(d*x + c))*sin(d*x + c) - 160*(cos(d*x + c)^3 + 3*cos
(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*
cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.245 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=174

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) - (
43*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) - (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.439644, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2765, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\cos(c+dx)+a)^{5/2}}-\frac{11\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(5/2)*d) - (
43*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) - (11*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-4a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int \frac{\frac{11a^2}{4}-8a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{8a^4}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a^3} - \frac{43\int \frac{}{\sqrt{}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{11\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{}{\sqrt{}}\right.}{a^3d}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{5/2}d} - \frac{43\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+d}{4d(a+a\cos(c+dx))^{5}}$$

Mathematica [C] time = 6.5825, size = 349, normalized size = 2.01

$$\frac{\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\left(\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{15\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}+\frac{\tan\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{15\tan\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}\right)}{(a(\cos(c+dx)+1))^{5/2}}-$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((-I/4)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]
*(32*ArcSinh[E^(I*(c + d*x))] + 43*Sqrt[2]*ArcTanh[(1 - E^(I*(c + d*x)))/(S
qrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 32*ArcTanh[Sqrt[1 + E^((2*I)*(c +
d*x))]])*Cos[c/2 + (d*x)/2]^5)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*(a*
(1 + Cos[c + d*x]))^(5/2)) + (Cos[c/2 + (d*x)/2]^5*Sqrt[Cos[c + d*x]]*((-15
*Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(4*d) + (Sec[c/2]*Sec[c/2 + (d
*x)/2]^4*Sin[(d*x)/2])/(2*d) - (15*Sec[c/2 + (d*x)/2]*Tan[c/2])/(4*d) + (Se
c[c/2 + (d*x)/2]^3*Tan[c/2])/(2*d)))/(a*(1 + Cos[c + d*x]))^(5/2)
```

Maple [B] time = 0.369, size = 312, normalized size = 1.8

$$\frac{\sqrt{2}\left(-1+\cos(dx+c)\right)^4}{32\,da^3\left(\sin(dx+c)\right)^9}\left(\cos(dx+c)\right)^{\frac{5}{2}}\sqrt{a\left(1+\cos(dx+c)\right)}\left(32\,\cos(dx+c)\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^4*(a*(1+cos(d*x+c)))^(1
/2)*(32*cos(d*x+c)*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)/cos(d*x+c))*sin(d*x+c)+15*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos
(d*x+c)^2+43*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+32*ar
ctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d
*x+c)-4*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+43*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-11*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 4.35628, size = 641, normalized size = 3.68

$$\frac{43\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos\left(dx+c\right)\cdots}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(15*cos(d*x + c) + 11)*sqrt(cos
(d*x + c))*sin(d*x + c) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt
(a)*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.246 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + (7*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*
d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.258875, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2765, 2978, 12, 2782, 205}

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{7\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + (7*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*
d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\frac{a}{2}-3a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{\int -\frac{3a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,d}{8a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,d}{32a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\right.}{16ad}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{7\sqrt{\cos(c+dx)}\sin(c}{16ad(a+a\cos(c+d}$$

Mathematica [A] time = 0.798077, size = 149, normalized size = 1.09

$$\frac{\cos^5\left(\frac{1}{2}(c+dx)\right)\left(3\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(5-2\tan^2\left(\frac{1}{2}(c+dx)\right.\right.\right.}{4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^5*(3*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sq
rt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[
(c + d*x)/2]*(5 - 2*Tan[(c + d*x)/2]^2)))/(4*d*Sqrt[Cos[(c + d*x)/2]^2]*(a*
(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 0.349, size = 214, normalized size = 1.6

$$\frac{\sqrt{2}(-1+\cos(dx+c))^3}{32\,da^3(\sin(dx+c))^7}(\cos(dx+c))^{\frac{3}{2}}\sqrt{a(1+\cos(dx+c))}\left(7\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2-4\sqrt{2}\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^3*(a*(1+cos(d*x+c)))^(1
/2)*(7*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-4*2^(1/2)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*cos(d*x+c)*sin(d*x+c)-3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/
2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 2.18415, size = 493, normalized size = 3.6

$$\frac{3\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right)+2\sqrt{a\cos\left(dx\right.}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c)
 + a)*(7*cos(d*x + c) + 3)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.247 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*
(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.249835, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2764, 2978, 12, 2782, 205}

$$\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*d*(a + a*Cos[c + d*x])^(5/2)) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*
(a + a*Cos[c + d*x])^(3/2))
```


Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{a}{2}+a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{5a^2}{4\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{8a^4}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{32a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{\cdots}{\sqrt{\mathrm{co}\cdots}}\right.}{16ad}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+\cdots}{16ad(a+a\cos(c+d\cdots}$$

Mathematica [A] time = 0.990611, size = 122, normalized size = 0.89

$$\frac{\sin^3\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\left(6\csc^2\left(\frac{1}{2}(c+dx)\right)-5\cot^4\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\cdots}\right.\right.}{8d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*(-2 + 6*Csc[(c + d*x)/2]^2 - 5*ArcTanh
[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]^4*Sqrt[2 - 2*Se
c[c + d*x]])*Sin[(c + d*x)/2]^3)/(8*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 0.344, size = 213, normalized size = 1.6

$$-\frac{\sqrt{2}(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\sqrt{\cos(dx+c)}\sqrt{a(1+\cos(dx+c))}\left(\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+4\sqrt{2}\cos(dx+c\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c
))^2*(2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+4*2^(1/2)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*cos(d*x+c)*sin(d*x+c)-5*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/sin(d*x+c)^5/(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 2.24168, size = 490, normalized size = 3.58

$$\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)+2\sqrt{a\cos(dx}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sq
rt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) + 2*sqrt(a*cos(d*x + c)
 + a)*(cos(d*x + c) + 5)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

3.248 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (19*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*d*(a + a*Cos[c + d*x])^(5/2)) - (9*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.260541, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2766, 2978, 12, 2782, 205}

$$\frac{19\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{3/2}}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (19*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
4*d*(a + a*Cos[c + d*x])^(5/2)) - (9*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{7a}{2}-a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\int \frac{19a}{4\sqrt{\cos(c+dx)}\sqrt{a}}}{8a}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{19\int \frac{}{\sqrt{\cos(c+dx)}}}{32}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} - \frac{19\,\mathrm{Subst}\left(\int \frac{}{2a}\right.}{}$$

$$= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{9\sqrt{\cos}}{16ad(a}$$

Mathematica [A] time = 1.26198, size = 134, normalized size = 0.98

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)(9\cos(c+dx)+13)\sqrt{2-2\sec(c+dx)}-76\cos^4\left(\frac{1}{2}(c+dx)\right)\tanh^{-1}\left(\right.\right.}{32\sqrt{2}a^2d\sqrt{\cos(c+dx)-1}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -(Sec[(c + d*x)/2]^2*(-76*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]
*Cos[(c + d*x)/2]^4 + Cos[c + d*x]*(13 + 9*Cos[c + d*x])*Sqrt[2 - 2*Sec[c +
 d*x]])*Tan[(c + d*x)/2])/(32*Sqrt[2]*a^2*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*
(1 + Cos[c + d*x])])
```

Maple [B] time = 0.332, size = 245, normalized size = 1.8

$$\frac{\sqrt{2}(-1+\cos(dx+c))}{32da^3(1+\cos(dx+c))(\sin(dx+c))^3}\sqrt{a(1+\cos(dx+c))}\left(19\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*(19*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(
d*x+c)+38*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*sin(d*x+c)*cos(d*x+c)-9*cos(d*x+c)^3*2^(1/2)+19*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-4*cos(d*x+c)^2*2^(
1/2)+13*cos(d*x+c)*2^(1/2))/(1+cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

Fricas [A] time = 2.29245, size = 495, normalized size = 3.61

$$\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+c)}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right)-2\sqrt{a\cos(d}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt(a*cos(d*x + c
) + a)*(9*cos(d*x + c) + 13)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.249 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=177

$$\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{75\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}-$$

```
[Out] (-75*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Sqrt[Cos[c + d*x
]]*(a + a*Cos[c + d*x])^(5/2)) - (13*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x
]]*(a + a*Cos[c + d*x])^(3/2)) + (49*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.402734, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2766, 2978, 2984, 12, 2782, 205}

$$\frac{49\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{75\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```


```
[Out] (-75*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Sqrt[Cos[c + d*x
]]*(a + a*Cos[c + d*x])^(5/2)) - (13*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x
]]*(a + a*Cos[c + d*x])^(3/2)) + (49*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d
*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{9a}{2}-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+$$

$$= -\frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}} - \frac{13\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a+a\cos(c+$$

$$= -\frac{75\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a+a\cos(c+dx)$$

Mathematica [C] time = 7.68358, size = 506, normalized size = 2.86

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{8\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{2,2,2,\frac{5}{2}\right\},\left\{1,1,\frac{11}{2}\right\},\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{315\left(2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}+\frac{1}{120}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((8*Cos[(c +
d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(315*(-1 + 2*Sin[c
/2 + (d*x)/2]^2)) + (Csc[c/2 + (d*x)/2]^8*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sq
rt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Si
n[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-343
 + 1465*Sin[c/2 + (d*x)/2]^2 - 2021*Sin[c/2 + (d*x)/2]^4 + 824*Sin[c/2 + (d
*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5145
 + 33980*Sin[c/2 + (d*x)/2]^2 - 87764*Sin[c/2 + (d*x)/2]^4 + 109737*Sin[c/2
 + (d*x)/2]^6 - 66122*Sin[c/2 + (d*x)/2]^8 + 15344*Sin[c/2 + (d*x)/2]^10)))
/120))/(d*(a*(1 + Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

Maple [B] time = 0.345, size = 303, normalized size = 1.7

$$\frac{\sqrt{2}}{32\,da^3\sin(dx+c)(1+\cos(dx+c))^2}\left(75\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^3\sin(dx+c)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(75*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*cos(d*x+c)^3*sin(d*x+c)+225*cos(d*x+c)^2*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+225*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))+75*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+
c))/sin(d*x+c))-49*2^(1/2)*cos(d*x+c)^4-36*cos(d*x+c)^3*2^(1/2)+53*cos(d*x+
c)^2*2^(1/2)+32*cos(d*x+c)*2^(1/2))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(1+
cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.30193, size = 563, normalized size = 3.18

$$-\frac{75\sqrt{2}\left(\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx+}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right.}{32\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(75*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + c
os(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*sq
rt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*sqrt
(a*cos(d*x + c) + a)*(49*cos(d*x + c)^2 + 85*cos(d*x + c) + 32)*sqrt(cos(d*
x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.250 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{95\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{163\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}\cdot$$

```
[Out] (163*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/
2)*(a + a*Cos[c + d*x])^(5/2)) - (17*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/
2)*(a + a*Cos[c + d*x])^(3/2)) + (95*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(
3/2)*Sqrt[a + a*Cos[c + d*x]]) - (299*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.546145, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2766, 2978, 2984, 12, 2782, 205}

$$\frac{95\sin(c+dx)}{48a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{299\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{163\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (163*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*Cos[c + d*x]^(3/
2)*(a + a*Cos[c + d*x])^(5/2)) - (17*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/
2)*(a + a*Cos[c + d*x])^(3/2)) + (95*Sin[c + d*x])/(48*a^2*d*Cos[c + d*x]^(
3/2)*Sqrt[a + a*Cos[c + d*x]]) - (299*Sin[c + d*x])/(48*a^2*d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx = -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{11a}{2}-3a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= -\frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}} - \frac{17\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx)}$$

$$= \frac{163\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}$$

Mathematica [C] time = 10.6955, size = 639, normalized size = 2.94

$$-\frac{\cot^5\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(640\sin^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^8\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{2,2,2,2,\frac{7}{2}\right\}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -(Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^4*(640*Cos[(c
+ d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 13/2}, Sin[c/2 +
 (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 - 1280*Cos
[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2, 2, 7/2}, {1, 1, 13/2}, Sin[c/2 + (
d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[
c/2 + (d*x)/2]^2) + 33*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2
/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-10935 + 72902*Sin[c/2
 + (d*x)/2]^2 - 188110*Sin[c/2 + (d*x)/2]^4 + 234156*Sin[c/2 + (d*x)/2]^6 -
 140732*Sin[c/2 + (d*x)/2]^8 + 33208*Sin[c/2 + (d*x)/2]^10) + Sqrt[Sin[c/2
+ (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-1148175 + 10333785*Sin[c/2 +
(d*x)/2]^2 - 38990350*Sin[c/2 + (d*x)/2]^4 + 79946462*Sin[c/2 + (d*x)/2]^6
- 96281836*Sin[c/2 + (d*x)/2]^8 + 68243596*Sin[c/2 + (d*x)/2]^10 - 26448512
*Sin[c/2 + (d*x)/2]^12 + 4344400*Sin[c/2 + (d*x)/2]^14))))/(41580*d*(a*(1 +
 Cos[c + d*x]))^(5/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(7/2))
```

Maple [B] time = 0.355, size = 377, normalized size = 1.7

$$\frac{\sqrt{2}\sin(dx+c)}{96\,da^3(-1+\cos(dx+c))(1+\cos(dx+c))^3}\left(489\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}(\cos(dx+c))^4\sin(dx+c)\arcsin\left(\frac{-1+\cos(}{\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/96/d*2^(1/2)/a^3*(489*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^4*sin(
d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+1956*arcsin((-1+cos(d*x+c))/sin(d
*x+c))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+2934*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)+1956*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+489*arcsin((-1+cos(d*x+c))/sin(d*x+c))*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-299*cos(d*x+c)^5*2^(1/2)-204*2^(
1/2)*cos(d*x+c)^4+343*cos(d*x+c)^3*2^(1/2)+192*cos(d*x+c)^2*2^(1/2)-32*cos(
d*x+c)*2^(1/2))*sin(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/(1+cos(
d*x+c))^3/cos(d*x+c)^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.37963, size = 599, normalized size = 2.76

$$\frac{489\sqrt{2}\left(\cos(dx+c)^5+3\cos(dx+c)^4+3\cos(dx+c)^3+\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin(dx}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right.}{96\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(489*sqrt(2)*(cos(d*x + c)^5 + 3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 + c
os(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(a)*
sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) - 2*(2
99*cos(d*x + c)^3 + 503*cos(d*x + c)^2 + 160*cos(d*x + c) - 32)*sqrt(a*cos(
d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^
3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.251 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{259\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{189\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}+\frac{637\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

```
[Out] (-7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) +
(637*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/
(6*d*(a + a*Cos[c + d*x])^(7/2)) - (7*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(16*
a*d*(a + a*Cos[c + d*x])^(5/2)) - (259*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(19
2*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + (189*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.748577, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2765, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{259\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{189\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}+\frac{637\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) +
(637*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(7/2)*Sin[c + d*x])/
(6*d*(a + a*Cos[c + d*x])^(7/2)) - (7*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(16*
a*d*(a + a*Cos[c + d*x])^(5/2)) - (259*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(19
2*a^2*d*(a + a*Cos[c + d*x])^(3/2)) + (189*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
```

```
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(\frac{7a}{2}-7a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}dx}{6a^2}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{105a^2}{4}-\frac{77}{2}a^2\cos(c+dx\right.}{(a+a\cos(c+dx))^{3/2}}}{24a^4}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{7\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{259\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} + \frac{637\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+}{6d(a+a\cos(c+dx}$$

Mathematica [C] time = 6.72162, size = 448, normalized size = 1.76

$$\frac{\sqrt{\cos(c+dx)}\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{16\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}+\frac{16\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}+\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}-\frac{15\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}+523\ldots\right.}{(a(\cos(c+dx)+1))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (((7*I)/8)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x
))]*(64*ArcSinh[E^(I*(c + d*x))] + 91*Sqrt[2]*ArcTanh[(1 - E^(I*(c + d*x)))
/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - 64*ArcTanh[Sqrt[1 + E^((2*I)*(c
 + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*
(a*(1 + Cos[c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Cos[c + d*x]]*((
16*Cos[(d*x)/2]*Sin[c/2])/d + (16*Cos[c/2]*Sin[(d*x)/2])/d + (523*Sec[c/2]*
Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(24*d) - (15*Sec[c/2]*Sec[c/2 + (d*x)/2]
^4*Sin[(d*x)/2])/(4*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*Sin[(d*x)/2])/(3*d)
 + (523*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - (15*Sec[c/2 + (d*x)/2]^3*Tan[
c/2])/(4*d) + (Sec[c/2 + (d*x)/2]^5*Tan[c/2])/(3*d)))/(a*(1 + Cos[c + d*x])
)^(7/2)
```

Maple [B] time = 0.408, size = 464, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*cos(d*x+c)^(9/2)*(-1+cos(d*x+c))^7*(a*(1+cos(d*x+c)))^(
1/2)*(192*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+907*cos(d*
x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1344*arctan(sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+
1911*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+343*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+2688*cos(d*x+c)*2^(1/2)*arc
tan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+382
2*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-875*2^(1/2)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1344*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+1911*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*sin(d*x+c)-567*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^15
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

Fricas [A] time = 6.25466, size = 797, normalized size = 3.14

$$-\frac{1911\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(1911*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2
+ 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(192*cos(d*x + c)^3 + 1099*cos(d*
x + c)^2 + 1442*cos(d*x + c) + 567)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2688*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c
)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(
d*x + c))/(sqrt(a)*sin(d*x + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x
+ c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{9}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.252 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{49\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{177\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) - (
177*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - (17*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(48*
a*d*(a + a*Cos[c + d*x])^(5/2)) - (49*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*
a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.601126, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2765, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{49\sin(c+dx)\sqrt{\cos(c+dx)}}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{177\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]])/(a^(7/2)*d) - (
177*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - (17*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(48*
a*d*(a + a*Cos[c + d*x])^(5/2)) - (49*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(64*
a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-6a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}dx}{6a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{51a^2}{4}-24a^2\cos(c+dx)\right)}{(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{49\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} -$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{49\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{17\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{49\sqrt{\cos(c+dx)}\sin(c+dx)}{64a^2d(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{a^{7/2}d} - \frac{177\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}$$

Mathematica [C] time = 6.70821, size = 412, normalized size = 1.93

$$\frac{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\left(-\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d}+\frac{11\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}-\frac{247\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{24d}-\frac{\tan\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}\right.}{3d}\right.}{(a(\cos(c+dx)+1))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] ((-I/4)*E^((I/2)*(c + d*x))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]
*(64*ArcSinh[E^(I*(c + d*x))] + (177*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 64*ArcTanh[Sqrt[1 + E^((2*I)*(c
 + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*
(a*(1 + Cos[c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Cos[c + d*x]]*((
-247*Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(24*d) + (11*Sec[c/2]*Sec[
c/2 + (d*x)/2]^4*Sin[(d*x)/2])/(4*d) - (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*Sin[(
d*x)/2])/(3*d) - (247*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) + (11*Sec[c/2 + (
d*x)/2]^3*Tan[c/2])/(4*d) - (Sec[c/2 + (d*x)/2]^5*Tan[c/2])/(3*d)))/(a*(1 +
 Cos[c + d*x]))^(7/2)
```

Maple [B] time = 0.379, size = 432, normalized size = 2.

$$\frac{\sqrt{2}(-1+\cos(dx+c))^6}{384\,da^4(\sin(dx+c))^{13}}(\cos(dx+c))^{\frac{7}{2}}\sqrt{a(1+\cos(dx+c))}\left(247(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+384\arctan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*cos(d*x+c)^(7/2)*(-1+cos(d*x+c))^6*(a*(1+cos(d*x+c)))^(
1/2)*(247*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+384*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(
d*x+c)*2^(1/2)+531*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+
c)+115*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+768*cos(d*x+c
)*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*s
in(d*x+c)+1062*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-215
*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+384*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+531*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-147*2^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^13
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^{\frac{7}{2}}}{(a\cos (dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

Fricas [A] time = 5.09586, size = 761, normalized size = 3.56

$$\frac{531\,\sqrt{2}\left(\cos (dx+c)^4+4\,\cos (dx+c)^3+6\,\cos (dx+c)^2+4\,\cos (dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(531*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(247*cos(d
*x + c)^2 + 362*cos(d*x + c) + 147)*sqrt(cos(d*x + c))*sin(d*x + c) - 768*(
cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x
 + c))))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x +
 c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.253 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=177

$$\frac{67\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}-\frac{13\sin(c+dx)\sqrt{\cos(c}}{48ad(a\cos(c+dx)+}$$

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(6
*d*(a + a*Cos[c + d*x])^(7/2)) - (13*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a
*d*(a + a*Cos[c + d*x])^(5/2)) + (67*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*
a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.403644, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2765, 2977, 2978, 12, 2782, 205}

$$\frac{67\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{6d(a\cos(c+dx)+a)^{7/2}}-\frac{13\sin(c+dx)\sqrt{\cos(c}}{48ad(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(6
*d*(a + a*Cos[c + d*x])^(7/2)) - (13*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(48*a
*d*(a + a*Cos[c + d*x])^(5/2)) + (67*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*
a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-5a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{\frac{13a^2}{4}-\frac{27}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{24a^4}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{67\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} -$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{67\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{67\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{13\sqrt{\cos(c+dx)}\sin(c+d}{48ad(a+a\cos(c+dx))^5}$$

Mathematica [A] time = 2.68773, size = 176, normalized size = 0.99

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(8\,\mathrm{t}\right.\right.}{24a^4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(\cos(c+dx)+1)^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*Sqrt[a*(1 + Cos[c + d*x])]*(15*ArcSin[Sin[(c + d*x)/2]/
Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]*(33 - 26*Tan[(c + d*x)/2]^2 + 8*Ta
n[(c + d*x)/2]^4)))/(24*a^4*d*Sqrt[Cos[(c + d*x)/2]^2]*(1 + Cos[c + d*x])^4
)
```

Maple [A] time = 0.378, size = 280, normalized size = 1.6

$$\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{5}}{384\,da^{4}\left(\sin\left(dx+c\right)\right)^{11}}\left(\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(67\left(\cos\left(dx+c\right)\right)^{3}\sqrt{2}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}+15\arcsin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^5*(a*(1+cos(d*x+c)))^(
1/2)*(67*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-17*2^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+30*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos
(d*x+c)*sin(d*x+c)-35*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+
15*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-15*2^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^11
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

Fricas [A] time = 2.3279, size = 585, normalized size = 3.31

$$\frac{15\sqrt{2}\left(\cos\left(dx+c\right)^{4}+4\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)^{2}+4\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin}{2\left(a\cos(dx+c)^{2}+a\cos(dx+c)\right)}\right.}{384\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4
*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
```

```
(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) +
2*sqrt(a*cos(d*x + c) + a)*(67*cos(d*x + c)^2 + 50*cos(d*x + c) + 15)*sqrt(
cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3
+ 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.254 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=177

$$\frac{17\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)\sqrt{\cos(c}}{6d(a\cos(c+dx)+}$$

```
[Out] (7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6
*d*(a + a*Cos[c + d*x])^(7/2)) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*
d*(a + a*Cos[c + d*x])^(5/2)) + (17*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a
^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.402715, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2765, 2978, 12, 2782, 205}

$$\frac{17\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)\sqrt{\cos(c}}{6d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6
*d*(a + a*Cos[c + d*x])^(7/2)) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*
d*(a + a*Cos[c + d*x])^(5/2)) + (17*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a
^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\frac{a}{2}-4a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{\int \frac{-\frac{a^2}{4}-\frac{9}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{24a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} -$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{17\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} -$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{3\sqrt{\cos(c+dx)}\sin(c+d}{16ad(a+a\cos(c+dx))^{5}}$$

Mathematica [A] time = 1.83357, size = 148, normalized size = 0.84

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left((135\cos(c+dx)+140\cos(2(c+dx))+17\cos(3(c+dx))+140)\sqrt{2-2\sec(c+dx)}+6\right.}{3072\sqrt{2}a^3d\sqrt{\cos(c+dx)-1}\sqrt{a(\cos(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(672*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*
Cos[(c + d*x)/2]^6 + (140 + 135*Cos[c + d*x] + 140*Cos[2*(c + d*x)] + 17*Co
s[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a
^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.379, size = 280, normalized size = 1.6

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^4}{384\,da^4\,(\sin(dx+c))^9}(\cos(dx+c))^{\frac{3}{2}}\sqrt{a\,(1+\cos(dx+c))}\left(17\,(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+21\arcsin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*cos(d*x+c)^(3/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+
c))^4*(17*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+53*2^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+42*arcsin((-1+cos(d*x+c))/sin(d*x+c))*co
s(d*x+c)*sin(d*x+c)-49*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+21*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-21*2^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

Fricas [A] time = 2.2687, size = 585, normalized size = 3.31

$$\frac{21\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right.}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(21*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4
*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) +
2*sqrt(a*cos(d*x + c) + a)*(17*cos(d*x + c)^2 + 70*cos(d*x + c) + 21)*sqrt(
cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3
+ 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.255 $\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{13\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)}$$

```
[Out] (13*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d
*(a + a*Cos[c + d*x])^(5/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2
*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.403176, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2764, 2978, 12, 2782, 205}

$$-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{13\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{6d(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```


```
[Out] (13*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d
*(a + a*Cos[c + d*x])^(5/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192*a^2
*d*(a + a*Cos[c + d*x])^(3/2))
```


Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{a}{2}+2a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} + \frac{\int \frac{\frac{11a^2}{4}+\frac{3}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx}{24a^4}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \ldots$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} + \ldots$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} - \ldots$$

$$= \frac{13\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} + \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\sqrt{\cos(c+dx)}\sin(c+d\ldots}{16ad(a+a\cos(c+dx))\ldots}$$

Mathematica [A] time = 2.68266, size = 149, normalized size = 0.84

$$\frac{\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(4\cos(c+dx)-5\cos(2(c+dx))-156\cos^4\left(\frac{1}{2}(c+dx)\right.\ldots\right.}{192a^4d(\cos(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*(73 + 4*Cos
[c + d*x] - 5*Cos[2*(c + d*x)] - 156*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d
*x)/2]^2)]]*Cos[(c + d*x)/2]^4*Cot[(c + d*x)/2]^2*Sqrt[2 - 2*Sec[c + d*x]])
*Sin[(c + d*x)/2])/(192*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.397, size = 280, normalized size = 1.6

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^3}{384\,da^4\,(\sin(dx+c))^7}\sqrt{\cos(dx+c)}\sqrt{a\,(1+\cos(dx+c))}\left(5\,(\cos(dx+c))^3\,\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-39\,\arcsin\left(\frac{-1}{\quad}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+
c))^3*(5*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-39*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-7*2^(1/2)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-78*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(
d*x+c)*sin(d*x+c)-37*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3
9*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+39*2^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

Fricas [A] time = 2.11669, size = 582, normalized size = 3.29

$$\frac{39\,\sqrt{2}\left(\cos(dx+c)^4+4\,\cos(dx+c)^3+6\,\cos(dx+c)^2+4\,\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\sin}{2\left(a\cos(dx+c)^2+a\cos(dx+c)\right)}\right.}{384\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(39*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 + 4
*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt
(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) -
2*sqrt(a*cos(d*x + c) + a)*(5*cos(d*x + c)^2 - 2*cos(d*x + c) - 39)*sqrt(co
s(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 +
6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

3.256 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{103\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{63\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)\sqrt{\cos}}{6d(a\cos(c+dx}$$

```
[Out] (63*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(5/2)) - (103*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192
*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.410082, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2766, 2978, 12, 2782, 205}

$$-\frac{103\sin(c+dx)\sqrt{\cos(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}+\frac{63\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{5\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\cos(c+dx)+a)^{5/2}}-\frac{\sin(c+dx)\sqrt{\cos}}{6d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] (63*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
6*d*(a + a*Cos[c + d*x])^(7/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a
*d*(a + a*Cos[c + d*x])^(5/2)) - (103*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(192
*a^2*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}+\frac{\int \frac{\frac{11a}{2}-2a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}-\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}+\frac{\int \frac{\frac{73a^2}{4}-\frac{15}{2}a^2\cos(}{\sqrt{\cos(c+dx)}(a+a\cos}}{24a^4}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}-\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}-\frac{103\sqrt{\cos(c+dx)}\,s}{192a^2d(a+a\cos(}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}-\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}-\frac{103\sqrt{\cos(c+dx)}\,s}{192a^2d(a+a\cos(}$$

$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}-\frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}}-\frac{103\sqrt{\cos(c+dx)}\,s}{192a^2d(a+a\cos(}$$

$$= \frac{63\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}-\frac{5\sqrt{\cos(c+}}{16ad(a+}$$

Mathematica [A] time = 2.1354, size = 148, normalized size = 0.84

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left((1089\cos(c+dx)+532\cos(2(c+dx))+103\cos(3(c+dx))+532)\sqrt{2-2\sec(c+dx}\right.}{3072\sqrt{2}a^3d\sqrt{\cos(c+dx)-1}\sqrt{a(\cos(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] -(Sec[(c + d*x)/2]^4*(-6048*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)
]]*Cos[(c + d*x)/2]^6 + (532 + 1089*Cos[c + d*x] + 532*Cos[2*(c + d*x)] + 1
03*Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(3072*Sqrt
[2]*a^3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [B] time = 0.382, size = 313, normalized size = 1.8

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^2}{384\,da^4\,(1+\cos(dx+c))\,(\sin(dx+c))^5}\sqrt{a\,(1+\cos(dx+c))}\left(189\,\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\mathrm{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^2*(189*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3
*sin(d*x+c)+567*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-103*2^(1/2)*cos(d*x+c)^4+567*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*cos(d*x
+c)-163*cos(d*x+c)^3*2^(1/2)+189*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)+71*cos(d*x+c)^2*2^(1/2)+195*cos(d*x+c
)*2^(1/2))/(1+cos(d*x+c))/cos(d*x+c)^(1/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c))), x)
```

Fricas [A] time = 2.36065, size = 590, normalized size = 3.33

$$\frac{189\,\sqrt{2}\left(\cos(dx+c)^4+4\,\cos(dx+c)^3+6\,\cos(dx+c)^2+4\,\cos(dx+c)+1\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{a}\sqrt{\cos(dx+c)}\,\mathrm{s}}{2\left(a\cos(dx+c)^2+a\cos(dx+c\right.}\right.}{384\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(189*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqr
t(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*cos(d*x + c))) -
 2*sqrt(a*cos(d*x + c) + a)*(103*cos(d*x + c)^2 + 266*cos(d*x + c) + 195)*s
qrt(cos(d*x + c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c
)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(cos(d*x + c))), x)
```

3.257 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=217

$$\frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{363\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

```
[Out] (-363*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*Sqrt[Cos[c + d*
x]]*(a + a*Cos[c + d*x])^(7/2)) - (19*Sin[c + d*x])/(48*a*d*Sqrt[Cos[c + d*
x]]*(a + a*Cos[c + d*x])^(5/2)) - (199*Sin[c + d*x])/(192*a^2*d*Sqrt[Cos[c
+ d*x]]*(a + a*Cos[c + d*x])^(3/2)) + (691*Sin[c + d*x])/(192*a^3*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.546912, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2766, 2978, 2984, 12, 2782, 205}

$$\frac{691\sin(c+dx)}{192a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}-\frac{199\sin(c+dx)}{192a^2d\sqrt{\cos(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{363\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```


```
[Out] (-363*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*Sqrt[Cos[c + d*
x]]*(a + a*Cos[c + d*x])^(7/2)) - (19*Sin[c + d*x])/(48*a*d*Sqrt[Cos[c + d*
x]]*(a + a*Cos[c + d*x])^(5/2)) - (199*Sin[c + d*x])/(192*a^2*d*Sqrt[Cos[c
+ d*x]]*(a + a*Cos[c + d*x])^(3/2)) + (691*Sin[c + d*x])/(192*a^3*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx = -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{13a}{2}-3a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx}\cdots$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx}\cdots$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx}\cdots$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx}\cdots$$

$$= -\frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}} - \frac{19\sin(c+dx)}{48ad\sqrt{\cos(c+dx)}(a+a\cos(c+dx}\cdots$$

$$= -\frac{363\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/}}\cdots$$

Mathematica [C] time = 8.28609, size = 559, normalized size = 2.58

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{16\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^8\left(\frac{1}{2}(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{2,2,2,2,\frac{5}{2}\right\},\left\{1,1,1,\frac{13}{2}\right\},\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{3465\left(2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((16*Cos[(c +
 d*x)/2]^8*HypergeometricPFQ[{2, 2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2 +
(d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-1 +
 2*Sin[c/2 + (d*x)/2]^2)) - (Csc[c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*x)/2
]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*ArcTan
h[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2
]^6*(2187 - 12908*Sin[c/2 + (d*x)/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4 - 26380
*Sin[c/2 + (d*x)/2]^6 + 8752*Sin[c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 + (d*x)/2
]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-229635 + 2120790*Sin[c/2 + (d*x)/2]^2
- 8267707*Sin[c/2 + (d*x)/2]^4 + 17646926*Sin[c/2 + (d*x)/2]^6 - 22251094*S
in[c/2 + (d*x)/2]^8 + 16548816*Sin[c/2 + (d*x)/2]^10 - 6712984*Sin[c/2 + (d
*x)/2]^12 + 1144608*Sin[c/2 + (d*x)/2]^14)))/1680))/(d*(a*(1 + Cos[c + d*x]
))^(7/2)*(1 - 2*Sin[c/2 + (d*x)/2]^2)^(3/2))
```

Maple [B] time = 0.388, size = 377, normalized size = 1.7

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))}{384\,da^4\,(\sin(dx+c))^3\,(1+\cos(dx+c))^2}\left(-1089\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^4\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*(-1+cos(d*x+c))*(-1089*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2
)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^4*sin(d*x+c)-4356*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^3*si
n(d*x+c)-6534*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))+691*cos(d*x+c)^5*2^(1/2)-4356*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c
))+1183*2^(1/2)*cos(d*x+c)^4-1089*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3
/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-275*cos(d*x+c)^3*2^(1/2)-1215*cos(d*
x+c)^2*2^(1/2)-384*cos(d*x+c)*2^(1/2))*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^
3/(1+cos(d*x+c))^2/cos(d*x+c)^(3/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.30733, size = 662, normalized size = 3.05

$$-\frac{1089 \sqrt{2}\left(\cos\left(dx+c\right)^{5}+4 \cos\left(dx+c\right)^{4}+6 \cos\left(dx+c\right)^{3}+4 \cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{}}{2\left(a\cos(dx+c\right.}\right.}{384\left(a^{4}d\cos\left(dx+c\right)^{5}+4\,a^{4}d\cos\left(dx+c\right)^{4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(1089*sqrt(2)*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3
+ 4*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d*
x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*c
os(d*x + c))) - 2*(691*cos(d*x + c)^3 + 1874*cos(d*x + c)^2 + 1599*cos(d*x
+ c) + 384)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^4*
d*cos(d*x + c)^5 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*
d*cos(d*x + c)^2 + a^4*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(3/2)), x)
```

3.258 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=257

$$\frac{193\sin(c+dx)}{64a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{109\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{629\sin(c+dx)}{64a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}}$$

```
[Out] (1015*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*Cos[c + d*x]^(3
/2)*(a + a*Cos[c + d*x])^(7/2)) - (23*Sin[c + d*x])/(48*a*d*Cos[c + d*x]^(3
/2)*(a + a*Cos[c + d*x])^(5/2)) - (109*Sin[c + d*x])/(64*a^2*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + (193*Sin[c + d*x])/(64*a^3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (629*Sin[c + d*x])/(64*a^3*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.703447, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2766, 2978, 2984, 12, 2782, 205}

$$\frac{193\sin(c+dx)}{64a^3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}-\frac{109\sin(c+dx)}{64a^2d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{629\sin(c+dx)}{64a^3d\sqrt{\cos(c+dx)}\sqrt{a\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```



```
[Out] (1015*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*Cos[c + d*x]^(3
/2)*(a + a*Cos[c + d*x])^(7/2)) - (23*Sin[c + d*x])/(48*a*d*Cos[c + d*x]^(3
/2)*(a + a*Cos[c + d*x])^(5/2)) - (109*Sin[c + d*x])/(64*a^2*d*Cos[c + d*x]
^(3/2)*(a + a*Cos[c + d*x])^(3/2)) + (193*Sin[c + d*x])/(64*a^3*d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Cos[c + d*x]]) - (629*Sin[c + d*x])/(64*a^3*d*Sqrt[Cos
[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])
```


Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}dx = -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} + \frac{\int \frac{\frac{15a}{2}-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}dx}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= -\frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}} - \frac{23\sin(c+dx)}{48ad\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+$$

$$= \frac{1015\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx$$

Mathematica [C] time = 8.30114, size = 273, normalized size = 1.06

$$\frac{ie^{-\frac{3}{2}i(c+dx)}\cos^7\left(\frac{1}{2}(c+dx)\right)\left(3045\sqrt{2}\left(1+e^{i(c+dx)}\right)^6\left(1+e^{2i(c+dx)}\right)^{3/2}\tanh^{-1}\left(\frac{1-e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)-2\left(8277e^{i(c+dx)}+14388e^{2i(}\right.\right.}{96d\left(1+e^{i(c+dx)}\right)^6\cos^{\frac{3}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + a*Cos[c + d*x])^(7/2)),x]
```

```
[Out] ((I/96)*(-2*(1887 + 8277*E^(I*(c + d*x)) + 14388*E^((2*I)*(c + d*x)) + 1310
8*E^((3*I)*(c + d*x)) + 5622*E^((4*I)*(c + d*x)) - 5622*E^((5*I)*(c + d*x))
 - 13108*E^((6*I)*(c + d*x)) - 14388*E^((7*I)*(c + d*x)) - 8277*E^((8*I)*(c
 + d*x)) - 1887*E^((9*I)*(c + d*x))) + 3045*Sqrt[2]*(1 + E^(I*(c + d*x)))^6
*(1 + E^((2*I)*(c + d*x)))^(3/2)*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])*Cos[(c + d*x)/2]^7)/(d*E^(((3*I)/2)*(c + d*x)
)*(1 + E^(I*(c + d*x)))^6*Cos[c + d*x]^(3/2)*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [B] time = 0.408, size = 435, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*(-3045*cos(d*x+c)^5*sin(d*x+c)*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-15225*(cos(d*x+c)/(1+cos(d*x+c
)))^(5/2)*cos(d*x+c)^4*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-30450*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)-30450*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-15225*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+1887*cos(d*
x+c)^6*2^(1/2)-3045*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)+3195*cos(d*x+c)^5*2^(1/2)-831*2^(1/2)*cos(d*x+c)^4
-3355*cos(d*x+c)^3*2^(1/2)-1024*cos(d*x+c)^2*2^(1/2)+128*cos(d*x+c)*2^(1/2)
)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(1+cos(d*x+c))^3/cos(d*x+c)^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.4316, size = 695, normalized size = 2.7

$$\frac{3045\,\sqrt{2}\left(\cos\left(dx+c\right)^{6}+4\,\cos\left(dx+c\right)^{5}+6\,\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{}}{2\left(a\cos(dx+c\right.}\right.}{384\left(a^{4}d\cos\left(dx+c\right)^{6}+4\,a^{4}d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(3045*sqrt(2)*(cos(d*x + c)^6 + 4*cos(d*x + c)^5 + 6*cos(d*x + c)^4 +
 4*cos(d*x + c)^3 + cos(d*x + c)^2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*cos(d
*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 + a*
cos(d*x + c))) - 2*(1887*cos(d*x + c)^4 + 5082*cos(d*x + c)^3 + 4251*cos(d*
x + c)^2 + 896*cos(d*x + c) - 128)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*a^4*d*cos(d*x + c)^5 + 6*a^4*d*
cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^4*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*cos(d*x + c)^(5/2)), x)
```

3.259 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=217

$$\frac{853\sin(c+dx)\sqrt{\cos(c+dx)}}{3072a^3d(a\cos(c+dx)+a)^{3/2}}-\frac{187\sin(c+dx)\sqrt{\cos(c+dx)}}{768a^2d(a\cos(c+dx)+a)^{5/2}}+\frac{35\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}a^{9/2}d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(}{8d(a\cos(c+dx)}$$

```
[Out] (35*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(1024*Sqrt[2]*a^(9/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(8*d*(a + a*Cos[c + d*x])^(9/2)) - (19*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*a*d*(a + a*Cos[c + d*x])^(7/2)) - (187*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
768*a^2*d*(a + a*Cos[c + d*x])^(5/2)) + (853*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3072*a^3*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.556924, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2765, 2977, 2978, 12, 2782, 205}

$$\frac{853\sin(c+dx)\sqrt{\cos(c+dx)}}{3072a^3d(a\cos(c+dx)+a)^{3/2}}-\frac{187\sin(c+dx)\sqrt{\cos(c+dx)}}{768a^2d(a\cos(c+dx)+a)^{5/2}}+\frac{35\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}a^{9/2}d}-\frac{\sin(c+dx)\cos^{\frac{5}{2}}(}{8d(a\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

```
[Out] (35*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(1024*Sqrt[2]*a^(9/2)*d) - (Cos[c + d*x]^(5/2)*Sin[c + d*x])
/(8*d*(a + a*Cos[c + d*x])^(9/2)) - (19*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(9
6*a*d*(a + a*Cos[c + d*x])^(7/2)) - (187*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
768*a^2*d*(a + a*Cos[c + d*x])^(5/2)) + (853*Sqrt[Cos[c + d*x]]*Sin[c + d*x
])/(3072*a^3*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a}{2}-7a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{7/2}}\,dx}{8a^2}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{57a^2}{4}-\frac{65}{2}a^2\cos(c+d\right.}{(a+a\cos(c+dx))^{5/2}}}{48a^4}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx}{768a^2d(a+a\cos(c+dx))^{5/}}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx}{768a^2d(a+a\cos(c+dx))^{5/}}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx}{768a^2d(a+a\cos(c+dx))^{5/}}$$

$$= -\frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}} - \frac{187\sqrt{\cos(c+dx)}\sin(c+dx}{768a^2d(a+a\cos(c+dx))^{5/}}$$

$$= \frac{35\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024\sqrt{2}a^{9/2}d} - \frac{\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{19\cos^{\frac{3}{2}}(c+dx)\sin(c}{96ad(a+a\cos(c+a}$$

Mathematica [A] time = 6.03563, size = 347, normalized size = 1.6

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^9\left(\frac{c}{2}+\frac{dx}{2}\right)\left(1-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^{9/2}\left(\frac{1}{8}\left(\frac{1}{1-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}+\frac{7}{6\left(1-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx\right.\right.}\right.\right.}{d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(a(co}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^9*Sin[c/2 + (d*x)/2]*(1 - Sec[(c + d*x)/2]^2*Sin[c/2
+ (d*x)/2]^2)^(9/2)*((35*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]
]*Sqrt[Cos[(c + d*x)/2]^2]*Csc[c/2 + (d*x)/2])/(128*(1 - Sec[(c + d*x)/2]^2
*Sin[c/2 + (d*x)/2]^2)^(9/2)) + (35/(16*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (
d*x)/2]^2)^4) + 35/(24*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2)^3) + 7
/(6*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2)^2) + (1 - Sec[(c + d*x)/2
]^2*Sin[c/2 + (d*x)/2]^2)^(-1))/8))/(d*Sqrt[Cos[(c + d*x)/2]^2]*(a*(1 + Cos
[c + d*x]))^(9/2))
```

Maple [A] time = 0.386, size = 346, normalized size = 1.6

$$\frac{\sqrt{2}(-1+\cos(dx+c))^7}{6144\,da^5(\sin(dx+c))^{15}}(\cos(dx+c))^{\frac{7}{2}}\sqrt{a(1+\cos(dx+c))}\left(853\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^4-34\,(\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(9/2),x)
```

```
[Out] 1/6144/d*2^(1/2)/a^5*cos(d*x+c)^(7/2)*(-1+cos(d*x+c))^7*(a*(1+cos(d*x+c)))^
(1/2)*(853*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4-34*cos(d*
x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+105*arcsin((-1+cos(d*x+c))
/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-364*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)^2+315*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*si
n(d*x+c)-350*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+315*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-105*2^(1/2)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)+105*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(
cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^15
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

Fricas [A] time = 2.27238, size = 684, normalized size = 3.15

$$\frac{105\sqrt{2}\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos}}{2}\right.}{6144\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/6144*(105*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a
*cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^
2 + a*cos(d*x + c))) + 2*(853*cos(d*x + c)^3 + 819*cos(d*x + c)^2 + 455*cos
(d*x + c) + 105)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/
(a^5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 +
10*a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(a\cos(dx+c)+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

3.260 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$

Optimal. Leaf size=217

$$\frac{73\sin(c+dx)\sqrt{\cos(c+dx)}}{1024a^3d(a\cos(c+dx)+a)^{3/2}}+\frac{33\sin(c+dx)\sqrt{\cos(c+dx)}}{256a^2d(a\cos(c+dx)+a)^{5/2}}+\frac{45\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}a^{9/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(\ldots}{8d(a\cos(c+dx)\ldots}$$

```
[Out] (45*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(1024*Sqrt[2]*a^(9/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(8*d*(a + a*Cos[c + d*x])^(9/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(32
*a*d*(a + a*Cos[c + d*x])^(7/2)) + (33*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(25
6*a^2*d*(a + a*Cos[c + d*x])^(5/2)) + (73*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(1024*a^3*d*(a + a*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 0.570841, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2765, 2977, 2978, 12, 2782, 205}

$$\frac{73\sin(c+dx)\sqrt{\cos(c+dx)}}{1024a^3d(a\cos(c+dx)+a)^{3/2}}+\frac{33\sin(c+dx)\sqrt{\cos(c+dx)}}{256a^2d(a\cos(c+dx)+a)^{5/2}}+\frac{45\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{1024\sqrt{2}a^{9/2}d}-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(\ldots}{8d(a\cos(c+dx)\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

```
[Out] (45*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])])/(1024*Sqrt[2]*a^(9/2)*d) - (Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(8*d*(a + a*Cos[c + d*x])^(9/2)) - (5*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(32
*a*d*(a + a*Cos[c + d*x])^(7/2)) + (33*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(25
6*a^2*d*(a + a*Cos[c + d*x])^(5/2)) + (73*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(1024*a^3*d*(a + a*Cos[c + d*x])^(3/2))
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
```

```
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a}{2}-6a\cos(c+dx)\right)}{(a+a\cos(c+dx))^{7/2}}\,dx}{8a^2}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} - \frac{\int \frac{\frac{15a^2}{4}-21a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx}{48a^4}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}} + \frac{33\sqrt{\cos(c+dx)}\sin(c+dx)}{256a^2d(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{45\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{1024\sqrt{2}a^{9/2}d} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}} - \frac{5\sqrt{\cos(c+dx)}\sin(c}{32ad(a+a\cos(c+d}$$

Mathematica [A] time = 2.13667, size = 158, normalized size = 0.73

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left((2466\cos(c+dx)+1072\cos(2(c+dx))+702\cos(3(c+dx))+73\cos(4(c+dx))+99\right.}{65536\sqrt{2}a^4d\sqrt{\cos(c+dx)-1}\sqrt{a(\cos(c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(9/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^6*(5760*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]
*Cos[(c + d*x)/2]^8 + (999 + 2466*Cos[c + d*x] + 1072*Cos[2*(c + d*x)] + 70
2*Cos[3*(c + d*x)] + 73*Cos[4*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c
+ d*x)/2])/(65536*Sqrt[2]*a^4*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Cos[c +
 d*x])])
```

Maple [A] time = 0.387, size = 346, normalized size = 1.6

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^6}{2048\,da^5\,(\sin(dx+c))^{13}}\,(\cos(dx+c))^{\frac{5}{2}}\,\sqrt{a\,(1+\cos(dx+c))}\left(73\,\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos(dx+c))^4+45\,\arcsin\left(\frac{-1}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(9/2),x)
```

```
[Out] -1/2048/d*2^(1/2)/a^5*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^6*(a*(1+cos(d*x+c)))
^(1/2)*(73*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+45*arcsin
((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+278*cos(d*x+c)^3*2^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+135*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)^2*sin(d*x+c)-156*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(
d*x+c)^2+135*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-150*2
^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+45*arcsin((-1+cos(d*x+c
))/sin(d*x+c))*sin(d*x+c)-45*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^13
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

Fricas [A] time = 2.23223, size = 680, normalized size = 3.13

$$\frac{45\sqrt{2}\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+}}{2\left(a\cos\right.}\right.}{2048\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="fricas")
```

```
[Out] 1/2048*(45*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3 +
 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(a*
cos(d*x + c) + a)*sqrt(a)*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2
 + a*cos(d*x + c))) + 2*(73*cos(d*x + c)^3 + 351*cos(d*x + c)^2 + 195*cos(d
*x + c) + 45)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^
5*d*cos(d*x + c)^5 + 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*
a^5*d*cos(d*x + c)^2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(9/2), x)
```

3.261 $\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx$

Optimal. Leaf size=16

$$\sqrt{2}\sin^{-1}\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

```
[Out] Sqrt[2]*ArcSin[Sin[x]/(1 + Cos[x])]
```

Rubi [A] time = 0.0433364, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2781, 216}

$$\sqrt{2}\sin^{-1}\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[x]]*Sqrt[1 + Cos[x]]),x]
```

```
[Out] Sqrt[2]*ArcSin[Sin[x]/(1 + Cos[x])]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(x)}\sqrt{1+\cos(x)}}\,dx &= -\left(\sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(x)}{1+\cos(x)}\right)\right)\\ &= \sqrt{2}\sin^{-1}\left(\frac{\sin(x)}{1+\cos(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.0249956, size = 30, normalized size = 1.88

$$\frac{2\cos\left(\frac{x}{2}\right)\tan^{-1}\left(\frac{\sin\left(\frac{x}{2}\right)}{\sqrt{\cos(x)}}\right)}{\sqrt{\cos(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[x]]*Sqrt[1 + Cos[x]]),x]
```

```
[Out] (2*ArcTan[Sin[x/2]/Sqrt[Cos[x]]]*Cos[x/2])/Sqrt[1 + Cos[x]]
```

Maple [B] time = 0.096, size = 36, normalized size = 2.3

$$-\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\sqrt{2+2\,\cos(x)}\,\arcsin\left(\frac{-1+\cos(x)}{\sin(x)}\right)\frac{1}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^(1/2)/(cos(x)+1)^(1/2),x)
```

```
[Out] -1/cos(x)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*(2+2*cos(x))^(1/2)*arcsin((-1+cos
(x))/sin(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 1.93107, size = 116, normalized size = 7.25

$$\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(x)+1}\sqrt{\cos(x)}\sin(x)}{2\left(\cos(x)^2+\cos(x)\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*sqrt(cos(x) + 1)*sqrt(cos(x))*sin(x)/(cos(x)^2 +
 cos(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\cos(x)+1}\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**(1/2)/(1+cos(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(x) + 1)*sqrt(cos(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(x)+1}\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(1+cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(x) + 1)*sqrt(cos(x))), x)
```

3.262 $\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{\cos(x)}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]])]
)/Sqrt[a]
```

Rubi [A] time = 0.060865, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2782, 205}

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{\cos(x)}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]]),x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]])]
)/Sqrt[a]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\,dx &= -\left((2a)\,\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(x)}{\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\right)\right)\\ &= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{\cos(x)}\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.0193084, size = 32, normalized size = 0.78

$$\frac{2\cos\left(\frac{x}{2}\right)\tan^{-1}\left(\frac{\sin\left(\frac{x}{2}\right)}{\sqrt{\cos(x)}}\right)}{\sqrt{a(\cos(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[x]]*Sqrt[a + a*Cos[x]]),x]
```

```
[Out] (2*ArcTan[Sin[x/2]/Sqrt[Cos[x]]]*Cos[x/2])/Sqrt[a*(1 + Cos[x])]
```

Maple [A] time = 0.13, size = 42, normalized size = 1.

$$-\frac{\sqrt{2}}{a}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\sqrt{a(\cos(x)+1)}\arcsin\left(\frac{-1+\cos(x)}{\sin(x)}\right)\frac{1}{\sqrt{\cos(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x)
```

```
[Out] -2^(1/2)/a/cos(x)^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*(a*(cos(x)+1))^(1/2)*arcs
in((-1+cos(x))/sin(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07373, size = 346, normalized size = 8.44

$$\left[\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(x)}\sin(x)-3\cos(x)^2-2\cos(x)+1}{\cos(x)^2+2\cos(x)+1}\right), \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{\cos(x)}}{2\left(\cos(x)^2+\cos(x)\right)\sqrt{}}\right.}{\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(-1/a)*sqrt(
cos(x))*sin(x) - 3*cos(x)^2 - 2*cos(x) + 1)/(cos(x)^2 + 2*cos(x) + 1)), sqr
t(2)*arctan(1/2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(cos(x))*sin(x)/((cos(x)^2 +
 cos(x))*sqrt(a)))/sqrt(a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a(\cos(x)+1)}\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**(1/2)/(a+a*cos(x))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(x) + 1))*sqrt(cos(x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(x) + a}\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^(1/2)/(a+a*cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(x) + a)*sqrt(cos(x))), x)
```

3.263 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx$

Optimal. Leaf size=129

$$-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4d}$$

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*C
os[c + d*x]])])/(4*d) + (3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a -
 a*Cos[c + d*x]]) - (a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a - a*Cos
[c + d*x]])
```

Rubi [A] time = 0.191758, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2770, 2775, 207}

$$-\frac{a\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}+\frac{3a\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*C
os[c + d*x]])])/(4*d) + (3*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a -
 a*Cos[c + d*x]]) - (a*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[a - a*Cos
[c + d*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}\,dx = -\frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} - \frac{3}{4}\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx$$

$$= \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} - \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} + \frac{3}{8}\int \frac{\sqrt{a-a\cos(}}{\sqrt{\cos(c+}}$$

$$= \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} - \frac{a\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} + \frac{(3a)\,\mathrm{Subst}\left(\int \frac{}{-a}\right.}{}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4d} + \frac{3a\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} - \frac{a\cos}{2d}$$

Mathematica [C] time = 4.04623, size = 289, normalized size = 2.24

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\left(2\sqrt{2}\left(\cos\left(\frac{3}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]*(3*ArcTanh[E^(I*d*x)/(Sqrt[Co
s[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]
])]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + 3*ArcTanh[Sqrt[Cos[c]
+ E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]]*(I
 + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + 2*Sqrt[2]*(-2*Cos[(c + d*x)/
2] + Cos[(3*(c + d*x))/2])*Csc[(c + d*x)/2]*Sqrt[Cos[c + d*x]*(Cos[d*x] + I
*Sin[d*x])]))/(8*d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))
*Sin[c]])
```

Maple [A] time = 0.404, size = 165, normalized size = 1.3

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))}{8\,d\,(\sin(dx+c))^3}\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/8/d*2^(1/2)*(-1+cos(d*x+c))*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+3*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(-2*a*(-1+cos(d*x+
c)))^(1/2)*cos(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^3
```

Maxima [B] time = 2.06477, size = 1435, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(-a) + 3*sqrt(-a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) + arctan2((cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) + 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - 1)))/d
```

Fricas [A] time = 2.21998, size = 417, normalized size = 3.23

$$\frac{3\sqrt{a}\log\left(\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-4\sqrt{-a\cos}}{16d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(a)*log((4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(
d*x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*a*cos(d*
x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*sqrt(-a*cos(d*x +
c) + a)*(2*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt(cos(d*x + c)))/(d*sin(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.264 $\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[
c + d*x]])])/d - (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a - a*Cos[c +
d*x]])
```

Rubi [A] time = 0.121755, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2770, 2775, 207}

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[
c + d*x]])])/d - (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a - a*Cos[c +
d*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\,dx = -\frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{1}{2}\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= -\frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}} - \frac{a\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right.}{d}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}}$$

Mathematica [C] time = 0.720696, size = 264, normalized size = 3.11

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}\left(-2\sqrt{2}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+i\sin(dx))} + \sqrt{\cos(c)-i\sin(c)}\left(\cot\left(\frac{1}{2}\right.\right.\right.}{2d\sqrt{i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c
] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])]
*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + ArcTanh[Sqrt[Cos[c] + E^(
(2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]]*(I + Co
t[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] - 2*Sqrt[2]*Cot[(c + d*x)/2]*Sqrt[C
os[c + d*x]*(Cos[d*x] + I*Sin[d*x])]))/(2*d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c]
 + I*(-1 + E^((2*I)*d*x))*Sin[c]])
```

Maple [A] time = 0.354, size = 95, normalized size = 1.1

$$\frac{\sqrt{2}(1+\cos(dx+c))}{2d\sin(dx+c)}\left(\text{Artanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}} - \cos(dx+c)\right)\sqrt{-2a(-1+\cos(dx+c))}\frac{}{\sqrt{\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)*(1+cos(d*x+c))*(arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-cos(d*x+c))*(-2*a*(-1+cos(d*x+c)))^(1/2)/co
s(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 2.01623, size = 1073, normalized size = 12.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c
```

```
) - (cos(d*x + c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)))*sqrt(-a) + sqrt(-a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

Fricas [A] time = 2.25367, size = 389, normalized size = 4.58

$$\frac{\sqrt{a}\log\left(-\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}+\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-4\sqrt{-a\cos}}{4d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(a)*log(-(4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*
x + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) + (8*a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*sqrt(-a*cos(d*x + c)
 + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-a\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.265 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*C
os[c + d*x]])])/d
```

Rubi [A] time = 0.0661367, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2775, 207}

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*C
os[c + d*x]])])/d
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}\\ &= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [C] time = 0.514349, size = 278, normalized size = 5.79

$$\frac{2e^{idx}\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\sqrt{\cos(c)-i\sin(c)}\sqrt{a-a\cos(c+dx)}\sqrt{e^{-idx}\left(i\sin(c)\left(-1+e^{2idx}\right)+\cos(c)\left(1+e^{2idx}\right)\right)}\left(\tanh^{-1}\left(\right.\right.}{d\left(i\cos\left(\frac{c}{2}\right)\left(-1+e^{idx}\right)-\sin\left(\frac{c}{2}\right)\left(1+e^{idx}\right)\right)\sqrt{2i\sin(c)\left(-1+\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*E^(I*d*x)*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((
2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])] + ArcTanh[Sqrt[Cos[c] + E^((2*I
)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]])*Sqrt[a - a
*Cos[c + d*x]]*(Cos[c/2] + I*Sin[c/2])*Sqrt[Cos[c] - I*Sin[c]]*Sqrt[((1 + E
^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)])/(d*(I*(-1
 + E^(I*d*x))*Cos[c/2] - (1 + E^(I*d*x))*Sin[c/2])*Sqrt[2*(1 + E^((2*I)*d*x
))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]])
```

Maple [B] time = 0.252, size = 84, normalized size = 1.8

$$\frac{\sqrt{2}\sin(dx+c)}{d(-1+\cos(dx+c))}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{-2a(-1+\cos(dx+c))}\mathrm{Artanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2*a*(-1+cos(d*x+c)))^(1/2)*
sin(d*x+c)*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(-1+
cos(d*x+c))
```

Maxima [B] time = 1.79409, size = 200, normalized size = 4.17

$$\frac{\sqrt{-a}\arctan\left(\left(\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + s
in(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d
*x + c))/d
```

Fricas [A] time = 2.28595, size = 417, normalized size = 8.69

$$\left[\frac{\sqrt{a}\log\left(\frac{4\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\sqrt{\cos(dx+c)}-\left(8a\cos(dx+c)^2+8a\cos(dx+c)+a\right)\sin(dx+c)}{\sin(dx+c)}\right)}{2d}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a\cos}}{2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a)*log((4*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*sqrt(a)*sqrt(cos(d*x + c)) - (8*a*cos(d*x + c)^2 + 8*a*cos(d*x +
 c) + a)*sin(d*x + c))/sin(d*x + c))/d, sqrt(-a)*arctan(1/2*sqrt(-a*cos(d*x
 + c) + a)*sqrt(-a)*(2*cos(d*x + c) + 1)/(a*sqrt(cos(d*x + c))*sin(d*x + c)
))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))/sqrt(cos(c + d*x)), x)
```

Giac [B] time = 2.28903, size = 193, normalized size = 4.02

$$\frac{\sqrt{2}\left(\frac{a^2\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{|a|}-\frac{\sqrt{2}\left(a^2\arctan\left(\frac{\sqrt{2}\sqrt{a}}{2\sqrt{-a}}\right)-a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sqrt{-a}|a|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(a^2*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)/sqrt(-a))/sqrt(-a) - sqrt(2)*arctan(sqrt(a)/sqrt(-a))/sqrt(-a))*sgn(tan(1
/2*d*x + 1/2*c))/abs(a) - sqrt(2)*(a^2*arctan(1/2*sqrt(2)*sqrt(a)/sqrt(-a))
 - a^2*arctan(sqrt(a)/sqrt(-a)))*sgn(tan(1/2*d*x + 1/2*c))/(sqrt(-a)*abs(a)
))/d
```

3.266 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rubi [A] time = 0.0566217, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2771}

$$\frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Mathematica [A] time = 0.0444338, size = 40, normalized size = 1.08

$$\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a - a*Cos[c + d*x]]*Cot[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.332, size = 46, normalized size = 1.2

$$-\frac{\sqrt{2}\sin(dx+c)}{d(-1+\cos(dx+c))}\sqrt{-2a(-1+\cos(dx+c))}\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*2^(1/2)*sin(d*x+c)*(-2*a*(-1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*
x+c)^(1/2)
```

Maxima [B] time = 1.57563, size = 111, normalized size = 3.

$$\frac{2\left(\sqrt{2}\sqrt{a}-\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a) - sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(
d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(3/2))
```

Fricas [A] time = 1.84738, size = 113, normalized size = 3.05

$$\frac{2\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)}{d\sqrt{\cos(dx+c)}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)/(d*sqrt(cos(d*x + c))*sin(d*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a(\cos(c+dx)-1)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(-a*(cos(c + d*x) - 1))/cos(c + d*x)**(3/2), x)
```

Giac [B] time = 2.41281, size = 126, normalized size = 3.41

$$-\frac{\sqrt{2}\left(a^2\left(\frac{\sqrt{2}}{\sqrt{a}|a|}-\frac{2}{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a|a|}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{\sqrt{2}\left(\sqrt{2}a^2-a^2\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sqrt{a}|a|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(a^2*(sqrt(2)/(sqrt(a)*abs(a)) - 2/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a)*abs(a)))*sgn(tan(1/2*d*x + 1/2*c)) + sqrt(2)*(sqrt(2)*a^2 - a^2)*sgn(
tan(1/2*d*x + 1/2*c))/(sqrt(a)*abs(a)))/d
```

3.267 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

```
[Out] (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) - (4*a
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rubi [A] time = 0.115855, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2772, 2771}

$$\frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) - (4*a
*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{2}{3}\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4a\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.112201, size = 52, normalized size = 0.66

$$-\frac{2(2\cos(c+dx)-1)\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(-1 + 2*Cos[c + d*x])*Sqrt[a - a*Cos[c + d*x]]*Cot[(c + d*x)/2])/(3*d*C
os[c + d*x]^(3/2))
```

Maple [A] time = 0.323, size = 56, normalized size = 0.7

$$\frac{\sqrt{2}\,(2\cos(dx+c)-1)\sin(dx+c)}{3d(-1+\cos(dx+c))}\sqrt{-2a(-1+\cos(dx+c))}\,(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/3/d*2^(1/2)*(2*cos(d*x+c)-1)*(-2*a*(-1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(-1+
cos(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [B] time = 1.5577, size = 235, normalized size = 2.97

$$-\frac{2\left(\sqrt{2}\sqrt{a}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(2)*sqrt(a) - 4*sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 3*sqrt(2)*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

Fricas [A] time = 1.9009, size = 143, normalized size = 1.81

$$-\frac{2\sqrt{-a\cos(dx+c)+a}\left(2\cos(dx+c)^2+\cos(dx+c)-1\right)}{3d\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-a*cos(d*x + c) + a)*(2*cos(d*x + c)^2 + cos(d*x + c) - 1)/(d*cos
(d*x + c)^(3/2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.39757, size = 177, normalized size = 2.24

$$\frac{\sqrt{2}\left(2\,a^2\left(\frac{\sqrt{2}}{\sqrt{a}|a|}-\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a|a|}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{\sqrt{2}\left(\sqrt{2}a^2-2\,a^2\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\sqrt{a}|a|}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(2*a^2*(sqrt(2)/(sqrt(a)*abs(a)) - (3*a*tan(1/2*d*x + 1/2*c)^2
- a)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*ab
s(a)))*sgn(tan(1/2*d*x + 1/2*c)) + sqrt(2)*(sqrt(2)*a^2 - 2*a^2)*sgn(tan(1/
2*d*x + 1/2*c))/(sqrt(a)*abs(a)))/d
```

3.268 $\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=118

$$-\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

```
[Out] (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]) - (8*a
*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) + (16*a*S
in[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rubi [A] time = 0.172602, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2772, 2771}

$$-\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{16a\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]) - (8*a
*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]) + (16*a*S
in[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{4}{5}\int \frac{\sqrt{a-a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{8}{15}\int \frac{\sqrt{a-a}}{\cos}\\
&= \frac{2a\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}-\frac{8a\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{16}{15d\sqrt{\cos(c+}}
\end{aligned}$$

Mathematica [A] time = 0.147521, size = 62, normalized size = 0.53

$$\frac{2(-4\cos(c+dx)+4\cos(2(c+dx))+7)\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{a-a\cos(c+dx)}}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[a - a*Cos[c + d*x]]*(7 - 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Cot[(
c + d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.34, size = 66, normalized size = 0.6

$$-\frac{\sqrt{2}\left(8\,(\cos(dx+c))^2-4\,\cos(dx+c)+3\right)\sin(dx+c)}{15\,d\,(-1+\cos(dx+c))}\sqrt{-2\,a\,(-1+\cos(dx+c))}\,(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-cos(d*x+c)*a)^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/15/d*2^(1/2)*(8*cos(d*x+c)^2-4*cos(d*x+c)+3)*(-2*a*(-1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/(-1+cos(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [B] time = 1.54642, size = 298, normalized size = 2.53

$$\frac{2\left(7\sqrt{2}\sqrt{a}-\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(7*sqrt(2)*sqrt(a) - 17*sqrt(2)*sqrt(a)*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 25*sqrt(2)*sqrt(a)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 15*sqrt(2)
*sqrt(a)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 1))
```

Fricas [A] time = 1.87124, size = 169, normalized size = 1.43

$$\frac{2\left(8\cos(dx+c)^3+4\cos(dx+c)^2-\cos(dx+c)+3\right)\sqrt{-a\cos(dx+c)+a}}{15\,d\cos(dx+c)^{\frac{5}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(-a*cos(d
*x + c) + a)/(d*cos(d*x + c)^(5/2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.20359, size = 219, normalized size = 1.86

$$-\frac{\sqrt{2}\left(2\,a^{2}\left(\frac{4\,\sqrt{2}}{\sqrt{a}|a|}-\frac{15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}+20\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)a+12\,a^{2}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}|a|}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{\sqrt{2}\left(7\,\sqrt{2}a^{2}-8\,a^{2}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{\sqrt{a}|a|}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(2)*(2*a^2*(4*sqrt(2)/(sqrt(a)*abs(a)) - (15*(a*tan(1/2*d*x + 1/2
*c)^2 - a)^2 + 20*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 12*a^2)/((a*tan(1/2*d*
x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*abs(a)))*sgn(tan(1/
2*d*x + 1/2*c)) + sqrt(2)*(7*sqrt(2)*a^2 - 8*a^2)*sgn(tan(1/2*d*x + 1/2*c))
/(sqrt(a)*abs(a)))/d
```

3.269 $\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=114

$$-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}-\frac{3\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/(4*d
) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x]]) - (Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 - Cos[c + d*x]])
```

Rubi [A] time = 0.153723, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2770, 2775, 207}

$$-\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}-\frac{3\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/(4*d
) + (3*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x]]) - (Cos
[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 - Cos[c + d*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)\,dx = -\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} - \frac{3}{4}\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$$
$$= \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} + \frac{3}{8}\int \frac{\sqrt{1-\cos(c+dx}}{\sqrt{\cos(c+dx)}}$$
$$= \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} - \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,\right.}{}$$
$$= -\frac{3\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} + \frac{3\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} - \frac{\cos^{\frac{3}{2}}(c+dx}{2d\sqrt{1-c}}$$

Mathematica [C] time = 0.477127, size = 284, normalized size = 2.49

$$-\frac{\sqrt{-(\cos(c+dx)-1)\cos(c+dx)}\left(2\sqrt{2}\left(\cos\left(\frac{3}{2}(c+dx)\right)-2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2),x]
```

```
[Out] -(Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])]*(3*ArcTanh[E^(I*d*x)/(Sqrt[Cos[
c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])
]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + 3*ArcTanh[Sqrt[Cos[c] +
E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]]*(I +
 Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] + 2*Sqrt[2]*(-2*Cos[(c + d*x)/2]
 + Cos[(3*(c + d*x))/2])*Csc[(c + d*x)/2]*Sqrt[Cos[c + d*x]*(Cos[d*x] + I*S
in[d*x])]))/(8*d*Sqrt[(1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*S
in[c]])
```

Maple [A] time = 0.338, size = 164, normalized size = 1.4

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))}{8\,d\,(\sin(dx+c))^3}\left(2\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2-\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-3\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x)
```

```
[Out] 1/8/d*2^(1/2)*(-1+cos(d*x+c))*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)+3*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))*(2-2*cos(d*x+c))^(
1/2)*cos(d*x+c)^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^3
```

Maxima [B] time = 2.00639, size = 1762, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)) - (cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d
*x + 2*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) - sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))) + 3*log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - 3*log(sqrt(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) + 1) + 3*log(((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 2*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 3*lo
g(((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))^2 + (cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) - 2*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1))/d
```

Fricas [A] time = 2.1249, size = 332, normalized size = 2.91

$$-\frac{2\left(2\cos(dx+c)^2-\cos(dx+c)-3\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-3\log\left(-\frac{2(\cos(dx+c)+1)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(2}{\sin(dx+c)}\right.}{8d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*cos(d*x + c)^2 - cos(d*x + c) - 3)*sqrt(-cos(d*x + c) + 1)*sqrt(
cos(d*x + c)) - 3*log(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(c
os(d*x + c)) - (2*cos(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c
))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)*cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.270 $\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

```
[Out] ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])
```

Rubi [A] time = 0.0859657, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2770, 2775, 207}

$$\frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]],x]
```

```
[Out] ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt
[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx = -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{1}{2}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= -\frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$
$$= \frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}}$$

Mathematica [C] time = 0.589676, size = 252, normalized size = 3.5

$$\frac{\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)}\left(-2\sqrt{2}\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(\cos(dx)+i\sin(dx))}+\sqrt{\cos(c)-i\sin(c)}\left(\cot\left(\frac{1}{2}(\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((2*I)*d*x)*(C
os[c] + I*Sin[c]) - I*Sin[c]])]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[
c]] + ArcTanh[Sqrt[Cos[c] + E^((2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/S
qrt[Cos[c] - I*Sin[c]]]*(I + Cot[(c + d*x)/2])*Sqrt[Cos[c] - I*Sin[c]] - 2*
Sqrt[2]*Cot[(c + d*x)/2]*Sqrt[Cos[c + d*x]*(Cos[d*x] + I*Sin[d*x])])*Sqrt[C
os[c + d*x]*Sin[(c + d*x)/2]^2])/(2*d*Sqrt[Cos[c + d*x]*(Cos[d*x] + I*Sin[d
*x])])
```

Maple [A] time = 0.312, size = 94, normalized size = 1.3

$$\frac{\sqrt{2}(1+\cos(dx+c))}{2d\sin(dx+c)}\left(\text{Artanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-\cos(dx+c)\right)\sqrt{2-2\cos(dx+c)}\frac{1}{\sqrt{\cos(dx\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)*(1+cos(d*x+c))*(arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)-cos(d*x+c))*(2-2*cos(d*x+c))^(1/2)/cos(d*x+
c)^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.87976, size = 1304, normalized size = 18.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*((cos(d*x + c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
```

```
 + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))) - log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sqrt(co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + 2*(cos(2*d*x + 2*c)^2 + s
in(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + log(sqrt(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))^2 + sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))^2 - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) - log(
((cos(d*x + c)^2 + sin(d*x + c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))^2 + (cos(d*x + c)^2 + sin(d*x + c)^2)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))) + 1) + log(((cos(d*x + c)^2 + sin(d*x + c)^
2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(d*x +
c)^2 + sin(d*x + c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1))^2)*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1) - 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))
+ 1))/d
```

Fricas [A] time = 2.17102, size = 304, normalized size = 4.22

$$-\frac{2(\cos(dx+c)+1)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)} - \log\left(-\frac{2(\cos(dx+c)+1)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}+(2\cos(dx+c)+1)\sin(dx+c)}{\sin(dx+c)}\right)}{2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - log
(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*cos
(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)), x)
```

3.271 $\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] (-2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d
```

Rubi [A] time = 0.0433369, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2775, 207}

$$-\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [C] time = 0.488938, size = 277, normalized size = 7.49

$$\frac{2e^{idx}\left(\cos\left(\frac{c}{2}\right)+i\sin\left(\frac{c}{2}\right)\right)\sqrt{\cos(c)-i\sin(c)}\sqrt{1-\cos(c+dx)}\sqrt{e^{-idx}\left(i\sin(c)\left(-1+e^{2idx}\right)+\cos(c)\left(1+e^{2idx}\right)\right)}\left(\tanh^{-1}\left(\cdots\right.\right.}{d\left(i\cos\left(\frac{c}{2}\right)\left(-1+e^{idx}\right)-\sin\left(\frac{c}{2}\right)\left(1+e^{idx}\right)\right)\sqrt{2i\sin(c)\left(-1+e\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*E^(I*d*x)*(ArcTanh[E^(I*d*x)/(Sqrt[Cos[c] - I*Sin[c]]*Sqrt[Cos[c] + E^((
2*I)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]])] + ArcTanh[Sqrt[Cos[c] + E^((2*I
)*d*x)*(Cos[c] + I*Sin[c]) - I*Sin[c]]/Sqrt[Cos[c] - I*Sin[c]]])*Sqrt[1 - C
os[c + d*x]]*(Cos[c/2] + I*Sin[c/2])*Sqrt[Cos[c] - I*Sin[c]]*Sqrt[((1 + E^(
(2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)])/(d*(I*(-1 +
 E^(I*d*x))*Cos[c/2] - (1 + E^(I*d*x))*Sin[c/2])*Sqrt[2*(1 + E^((2*I)*d*x))
*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]])
```

Maple [B] time = 0.201, size = 83, normalized size = 2.2

$$\frac{\sqrt{2}\sin(dx+c)}{d(-1+\cos(dx+c))}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2-2\cos(dx+c)}\operatorname{Artanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-2*cos(d*x+c))^(1/2)*sin(d*
x+c)*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(-1+cos(d*
x+c))
```

Maxima [B] time = 1.7449, size = 298, normalized size = 8.05

$$2\operatorname{arsinh}(1)+\log\left(\cos(dx+c)^2+\sin(dx+c)^2+\sqrt{\cos(2dx+2c)^2+\sin(2dx+2c)^2+2\cos(2dx+2c)+1}\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arcsinh(1) + log(cos(d*x + c)^2 + sin(d*x + c)^2 + sqrt(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))^2) + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)))))/d
```

Fricas [A] time = 2.12918, size = 167, normalized size = 4.51

$$\frac{\log\left(-\frac{2(\cos(dx+c)+1)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(2\cos(dx+c)+1)\sin(dx+c)}{\sin(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (2*
cos(d*x + c) + 1)*sin(d*x + c))/sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [B] time = 2.07705, size = 149, normalized size = 4.03

$$-\frac{\sqrt{2}\left(\left(\sqrt{2}\log\left(\sqrt{2}+\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}\right)-\sqrt{2}\log\left(\sqrt{2}-\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}\right)\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-(\sqrt{}\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*((sqrt(2)*log(sqrt(2) + sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1)) - s
qrt(2)*log(sqrt(2) - sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1)))*sgn(tan(1/2*d*x +
1/2*c)) - (sqrt(2)*log(sqrt(2) + 1) - sqrt(2)*log(sqrt(2) - 1))*sgn(tan(1/2
*d*x + 1/2*c)))/d
```

3.272 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

```
[Out] (2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0424697, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2771}

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.0436007, size = 39, normalized size = 1.11

$$\frac{2\sqrt{1-\cos(c+dx)}\cot\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[1 - Cos[c + d*x]]*Cot[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.279, size = 45, normalized size = 1.3

$$-\frac{\sqrt{2}\sin(dx+c)}{d(-1+\cos(dx+c))}\sqrt{2-2\cos(dx+c)}\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*2^(1/2)*(2-2*cos(d*x+c))^(1/2)*sin(d*x+c)/(-1+cos(d*x+c))/cos(d*x+c)^(
1/2)
```

Maxima [B] time = 1.53573, size = 101, normalized size = 2.89

$$\frac{2\left(\sqrt{2}-\frac{\sqrt{2}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2) - sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(d*(sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

Fricas [A] time = 1.85173, size = 111, normalized size = 3.17

$$\frac{2\left(\cos\left(dx+c\right)+1\right)\sqrt{-\cos\left(dx+c\right)+1}}{d\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)/(d*sqrt(cos(d*x + c))*sin(d*x
+ c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1-\cos\left(c+dx\right)}}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(1 - cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

Giac [A] time = 2.12484, size = 84, normalized size = 2.4

$$-\frac{\sqrt{2}\left(\sqrt{2}\left(\sqrt{2}-1\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\left(\sqrt{2}-\frac{2}{\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*(sqrt(2) - 1)*sgn(tan(1/2*d*x + 1/2*c)) + (sqrt(2) - 2/sq
rt(-tan(1/2*d*x + 1/2*c)^2 + 1))*sgn(tan(1/2*d*x + 1/2*c)))/d
```

3.273 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

```
[Out] (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) - (4*Sin[c
 + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0845189, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2772, 2771}

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) - (4*Sin[c
 + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{2}{3}\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}-\frac{4\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.095005, size = 51, normalized size = 0.68

$$-\frac{2\sqrt{1-\cos(c+dx)}(2\cos(c+dx)-1)\cot\left(\frac{1}{2}(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[1 - Cos[c + d*x]]*(-1 + 2*Cos[c + d*x])*Cot[(c + d*x)/2])/(3*d*Cos
[c + d*x]^(3/2))
```

Maple [A] time = 0.275, size = 55, normalized size = 0.7

$$\frac{\sqrt{2}\,(2\cos(dx+c)-1)\sin(dx+c)}{3d(-1+\cos(dx+c))}\sqrt{2-2\cos(dx+c)}\,(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/3/d*2^(1/2)*(2*cos(d*x+c)-1)*sin(d*x+c)*(2-2*cos(d*x+c))^(1/2)/(-1+cos(d*
x+c))/cos(d*x+c)^(3/2)
```

Maxima [B] time = 1.59445, size = 221, normalized size = 2.95

$$-\frac{2\left(\sqrt{2}-\frac{4\sqrt{2}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sqrt{2}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(2) - 4*sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sqrt(2)*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)^(5/2)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 1))
```

Fricas [A] time = 1.89588, size = 140, normalized size = 1.87

$$-\frac{2\left(2\cos(dx+c)^2+\cos(dx+c)-1\right)\sqrt{-\cos(dx+c)+1}}{3d\cos(dx+c)^{\frac{3}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*cos(d*x + c)^2 + cos(d*x + c) - 1)*sqrt(-cos(d*x + c) + 1)/(d*cos(d
*x + c)^(3/2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.11125, size = 126, normalized size = 1.68

$$\frac{\sqrt{2}\left(\sqrt{2}\left(\sqrt{2}-2\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2\left(\sqrt{2}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*(sqrt(2)*(sqrt(2) - 2)*sgn(tan(1/2*d*x + 1/2*c)) + 2*(sqrt(2) -
 (3*tan(1/2*d*x + 1/2*c)^2 - 1)/((tan(1/2*d*x + 1/2*c)^2 - 1)*sqrt(-tan(1/2
*d*x + 1/2*c)^2 + 1)))*sgn(tan(1/2*d*x + 1/2*c)))/d
```

3.274 $\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}+\frac{16\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

```
[Out] (2*Sin[c + d*x])/(5*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)) - (8*Sin[c
 + d*x])/(15*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) + (16*Sin[c + d*x
])/(15*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.131729, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2772, 2771}

$$-\frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}+\frac{16\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```


```
[Out] (2*Sin[c + d*x])/(5*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)) - (8*Sin[c
 + d*x])/(15*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)) + (16*Sin[c + d*x
])/(15*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```


Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{4}{5}\int \frac{\sqrt{1-\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{8}{15}\int \frac{\sqrt{1-\cos(c}{\cos^{\frac{3}{2}}(c+}\\
&= \frac{2\sin(c+dx)}{5d\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}-\frac{8\sin(c+dx)}{15d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{16\sin(c}{15d\sqrt{1-\cos(c+}
\end{aligned}$$

Mathematica [A] time = 0.117131, size = 61, normalized size = 0.54

$$\frac{2\sqrt{1-\cos(c+dx)}\left(8\cos^2(c+dx)-4\cos(c+dx)+3\right)\cot\left(\frac{1}{2}(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[1 - Cos[c + d*x]]*(3 - 4*Cos[c + d*x] + 8*Cos[c + d*x]^2)*Cot[(c +
d*x)/2])/(15*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.293, size = 65, normalized size = 0.6

$$-\frac{\sqrt{2}\left(8\left(\cos(dx+c)\right)^2-4\cos(dx+c)+3\right)\sin(dx+c)}{15d\left(-1+\cos(dx+c)\right)}\sqrt{2-2\cos(dx+c)}\left(\cos(dx+c)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/15/d*2^(1/2)*(8*cos(d*x+c)^2-4*cos(d*x+c)+3)*(2-2*cos(d*x+c))^(1/2)*sin(
d*x+c)/(-1+cos(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [B] time = 1.56646, size = 282, normalized size = 2.52

$$\frac{2\left(7\sqrt{2}-\frac{17\sqrt{2}\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{25\sqrt{2}\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\sqrt{2}\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(7*sqrt(2) - 17*sqrt(2)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 25*sqrt(
2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 15*sqrt(2)*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/
(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*
(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

Fricas [A] time = 1.83888, size = 166, normalized size = 1.48

$$\frac{2\left(8\cos(dx+c)^3+4\cos(dx+c)^2-\cos(dx+c)+3\right)\sqrt{-\cos(dx+c)+1}}{15d\cos(dx+c)^{\frac{5}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(8*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - cos(d*x + c) + 3)*sqrt(-cos(d*x
 + c) + 1)/(d*cos(d*x + c)^(5/2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.16339, size = 154, normalized size = 1.38

$$-\frac{\sqrt{2}\left(\sqrt{2}(7\sqrt{2}-8)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+2\left(4\sqrt{2}-\frac{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2+20\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(2)*(sqrt(2)*(7*sqrt(2) - 8)*sgn(tan(1/2*d*x + 1/2*c)) + 2*(4*sqr
t(2) - (15*(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 20*tan(1/2*d*x + 1/2*c)^2 - 8)/
((tan(1/2*d*x + 1/2*c)^2 - 1)^2*sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1)))*sgn(tan
(1/2*d*x + 1/2*c)))/d
```

3.275 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=185

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a}}\right.}{\sqrt{a}d}$$

```
[Out] (7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*
x]])])/(4*Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*Sqrt[a - a*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*d*Sqrt[a - a*Cos[c + d*x]])
```

Rubi [A] time = 0.445111, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2778, 2983, 2982, 2782, 208, 2775, 207}

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{a-a\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{a-a\cos(c+dx)}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*
x]])])/(4*Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(4*d*Sqrt[a - a*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*d*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx = \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{\sqrt{\cos(c+dx)}(3a+a\cos(c+dx))}{\sqrt{a-a\cos(c+dx)}}\,dx}{4a}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} - \frac{\int \frac{-\frac{a^2}{2}-\frac{7}{2}a^2\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx}{4a^2}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} - \frac{7\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{8a} + \int \frac{}{\sqrt{\cos(c}}$$

$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{a-a\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a-a\cos(c+dx)}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{a\sin}{\sqrt{\cos(c+dx)}}\right.}{4d}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{4\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{\sqrt{\cos(c+d}}{4d\sqrt{a-a}}$$

Mathematica [C] time = 1.18719, size = 256, normalized size = 1.38

$$-\frac{ie^{-2i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\left(7\sqrt{2}e^{2i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-16e^{2i(c+dx)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left(\sqrt{1+e\right.\right.}{8\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] ((-I/8)*(-1 + E^(I*(c + d*x)))*(7*Sqrt[2]*E^((2*I)*(c + d*x))*ArcSinh[E^(I*
(c + d*x))] - 16*E^((2*I)*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(Sqrt[1 + E^((2*I)*(c + d*x))]*(
1 + 2*E^(I*(c + d*x)) + 2*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x))) + 7*E^
((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt[Cos[c + d*x
]])/(Sqrt[2]*d*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a
*Cos[c + d*x]])
```

Maple [A] time = 0.385, size = 195, normalized size = 1.1

$$-\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{3}}{4\,d\left(\sin\left(dx+c\right)\right)^{5}}\left(\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(2\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\left(\cos\left(dx+c\right)\right)^{2}+3\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\cos\left(dx+c\right)-4\,\mathrm{Art}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^3*(2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*cos(d*x+c)^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*a
rctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+7*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)/(-2*a*(-1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{-a\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(-a*cos(d*x + c) + a), x)
```

Fricas [A] time = 2.11052, size = 626, normalized size = 3.38

$$\frac{4\sqrt{2}\sqrt{a}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+7\sqrt{a}\log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)}{\cdots}\right.}{8\,ad\,\mathrm{si}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x +
 c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((
cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 7*sqrt(a)*log(-(2*sqrt(-a*c
os(d*x + c) + a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d
*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*sqrt(-a*cos(d*x +
 c) + a)*(2*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*s
in(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.21496, size = 203, normalized size = 1.1

$$-\frac{\sqrt{2}\left(\frac{7\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{8\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{2\left(\left(-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^{\frac{3}{2}}+2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^2a}\right)|a|}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*(7*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a)/sqrt(-a))/(sqrt(-a)*a) - 8*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/s
qrt(-a))/(sqrt(-a)*a) - 2*((-a*tan(1/2*d*x + 1/2*c)^2 + a)^(3/2) + 2*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)*a)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^2*a))*abs(
a)/d
```

3.276 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]
)]/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[
c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[a - a*Cos[c + d*x]])
```

Rubi [A] time = 0.295414, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2778, 2982, 2782, 208, 2775, 207}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]
)]/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[
c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d) + (Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a-a\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}}+\frac{\int \frac{a+a\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx}{2a}\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}}-\frac{\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx\\ &= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{a-a\cos(c+dx)}}-\frac{\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx,x,\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}-\frac{(2a)\,\text{Subst}\left(\int\right.}{}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}+\frac{\sqrt{\cos(c+dx)}}{d\sqrt{a-a\cos}}\end{aligned}$$

Mathematica [C] time = 0.869351, size = 228, normalized size = 1.62

$$-\frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\left(\sqrt{2}e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-4e^{i(c+dx)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left(\sqrt{1+e^{2i(c+d}}\right.\right.}{2\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] ((-I/2)*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d
*x))] - 4*E^(I*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E
^((2*I)*(c + d*x))])] + Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c
 + d*x))] + E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt[C
os[c + d*x]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt
[a - a*Cos[c + d*x]])
```

Maple [A] time = 0.364, size = 162, normalized size = 1.2

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))^2}{d\,(\sin(dx+c))^3}(\cos(dx+c))^{\frac{3}{2}}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-\operatorname{Artanh}\left(\frac{\sqrt{2}}{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}+\operatorname{Artanh}\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^2*((cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*cos(d*x+c)-arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^
(1/2)+arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(-2*a*(-1+cos(d*x+c)))^(1/2)/sin
(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{-a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(-a*cos(d*x + c) + a), x)
```

Fricas [A] time = 2.00281, size = 593, normalized size = 4.21

$$\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+\sqrt{a}\log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)+1)}{\sin}\right.}{2\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c
) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((co
s(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + sqrt(a)*log(-(2*sqrt(-a*cos(d
*x + c) + a)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d*x +
 c) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*sqrt(-a*cos(d*x + c)
+ a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{-a(\cos(c+dx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(-a*(cos(c + d*x) - 1)), x)
```

Giac [A] time = 2.19494, size = 171, normalized size = 1.21

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{2\,\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{2\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)a}\right)|a|}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
)/sqrt(-a))/(sqrt(-a)*a) - 2*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqr
t(-a))/(sqrt(-a)*a) - 2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/((a*tan(1/2*d*x
 + 1/2*c)^2 + a)*a))*abs(a)/d
```

3.277 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*
x]])])/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.180415, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2777, 2775, 207, 2782, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a - a*Cos[c + d*x]],x]
```



```
[Out] (2*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*
x]])])/(Sqrt[a]*d) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])])/(Sqrt[a]*d)
```


Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a-a\cos(c+dx)}}\,dx = -\frac{\int \frac{\sqrt{a-a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx}{a} + \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, \frac{a\sin(}{\sqrt{\cos(c+dx)}\sqrt{}}\right.}{d}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [C] time = 0.374269, size = 161, normalized size = 1.5

$$-\frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a - a*Cos[c + d*x]],x]
```

```
[Out] ((-I)*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))
]*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])
)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])
```

Maple [A] time = 0.262, size = 116, normalized size = 1.1

$$\frac{\sqrt{2}\left(-1+\cos(dx+c)\right)}{d\sin(dx+c)}\sqrt{\cos(dx+c)}\left(\text{Artanh}\left(\frac{\sqrt{2}}{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}-2\,\text{Artanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\frac{1}{\sqrt{-2a(-1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(arctanh(1/2*2^(1/2)/(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)-2*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)))/(-2*a*(-1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02515, size = 451, normalized size = 4.21

$$\frac{\sqrt{2}\sqrt{a}\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)+2\sqrt{a}\log\left(-\frac{2\sqrt{-a\cos(dx+c)+a}\sqrt{a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}+(}{\sin(dx+c)}\right.}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c
) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((co
s(d*x + c) - 1)*sin(d*x + c))) + 2*sqrt(a)*log(-(2*sqrt(-a*cos(d*x + c) + a
)*sqrt(a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + (2*a*cos(d*x + c) + a)*si
n(d*x + c))/sin(d*x + c)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-a(\cos(c+dx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(-a*(cos(c + d*x) - 1)), x)
```

Giac [A] time = 2.38153, size = 113, normalized size = 1.06

$$-\frac{\sqrt{2}\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{2\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)|a|}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sq
rt(-a))/sqrt(-a) - arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/sqr
t(-a))*abs(a)/(a*d)
```

3.278 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d))
```

Rubi [A] time = 0.0670286, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2782, 208}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d))
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx &= -\frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, \frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}\end{aligned}$$

Mathematica [C] time = 0.328336, size = 118, normalized size = 2.03

$$\frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] (I*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))]*A
rcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(d*S
qrt[1 + E^((2*I)*(c + d*x))]*Sqrt[a - a*Cos[c + d*x]])
```

Maple [A] time = 0.329, size = 77, normalized size = 1.3

$$-2\,{\frac{\sin \left(dx+c \right) }{d\sqrt{-2\,a \left(-1+\cos \left(dx+c \right) \right) }\sqrt{\cos \left(dx+c \right) }}\sqrt{{\frac{\cos \left(dx+c \right) }{1+\cos \left(dx+c \right) }}}{\rm Artanh} \left(1/2\,\sqrt{2}{\frac{1}{\sqrt{{\frac{\cos \left(dx+c \right) }{1+\cos \left(dx+c \right) }}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2))*sin(d*x+c)/(-2*a*(-1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.25284, size = 400, normalized size = 6.9

$$\left[\frac{\sqrt{2} \log\left(-\frac{\frac{2 \, \sqrt{2} \sqrt{-a \cos\left(d x + c\right) + a} {\left(\cos\left(d x + c\right) + 1\right)} \sqrt{\cos\left(d x + c\right)}}{\sqrt{a}} - {\left(3 \, \cos\left(d x + c\right) + 1\right)} \sin\left(d x + c\right)}{{\left(\cos\left(d x + c\right) - 1\right)} \sin\left(d x + c\right)}\right)}{2 \, \sqrt{a} d}, \frac{\sqrt{2} \sqrt{-\frac{1}{a}} \arctan\left(\frac{\sqrt{2} \sqrt{-a \cos\left(d x + c\right) + a} \sqrt{-\frac{1}{a}} \sqrt{\cos\left(d x + c\right)}}{\sin\left(d x + c\right)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)*(cos(d*x + c) + 1)*s
qrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x +
c) - 1)*sin(d*x + c)))/(sqrt(a)*d), sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(
-a*cos(d*x + c) + a)*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a\left(\cos\left(c+dx\right)-1\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*sqrt(cos(c + d*x))), x)
```

Giac [B] time = 2.15811, size = 185, normalized size = 3.19

$$-\frac{\sqrt{2}\left(\frac{a^2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}-\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}\right)}{|a|\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{\left(a\arctan\left(\frac{\sqrt{2}\sqrt{a}}{\sqrt{-a}}\right)-a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\right)\operatorname{sgn}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\sqrt{-a}|a|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*(a^2*(arctan(sqrt(2)*sqrt(a)/sqrt(-a))/(sqrt(-a)*a) - arctan(sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a))/(abs(a)*sgn(tan(1/2*
d*x + 1/2*c))) - (a*arctan(sqrt(2)*sqrt(a)/sqrt(-a)) - a*arctan(sqrt(a)/sqr
t(-a)))*sgn(tan(1/2*d*x + 1/2*c))/(sqrt(-a)*abs(a)))/d
```

3.279 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[a - a*Cos[c + d*x]])
```

Rubi [A] time = 0.13275, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2779, 12, 2782, 208}

$$\frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]
]*Sqrt[a - a*Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{a}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx}{a}$$
$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} + \int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\,dx$$
$$= \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{2a^2-ax^2}\,dx, x, \frac{a\sin(c+}{\sqrt{\cos(c+dx)}\sqrt{a-}}\right.}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Mathematica [C] time = 0.375236, size = 157, normalized size = 1.65

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)-\frac{e^{-\frac{1}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] (2*(-(((1 + E^((2*I)*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])/(Sqrt[2]*E^((I/2)*(c + d*x)))) + 2*Sqrt[1 + E
^((2*I)*(c + d*x))]*Cos[(c + d*x)/2])*Sin[(c + d*x)/2])/(d*Sqrt[1 + E^((2*I
)*(c + d*x))]*Sqrt[Cos[c + d*x]]*Sqrt[a - a*Cos[c + d*x]])
```

Maple [A] time = 0.346, size = 160, normalized size = 1.7

$$\frac{\sqrt{2}(\sin(dx+c))^3}{d\left((\cos(dx+c))^2-1\right)}\left(\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\text{Artanh}\left(\frac{\sqrt{2}}{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}+\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\text{Artan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*sin(d*x+c)^3/cos(d*x+c)^(3/2)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)
+(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2))*2^(1/2)-2*cos(d*x+c))/(-2*a*(-1+cos(d*x+c)))^(1/2)/(cos(d*x+
c)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22723, size = 417, normalized size = 4.39

$$\frac{\sqrt{2}\sqrt{a}\cos(dx+c)\log\left(-\frac{\frac{2\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\sqrt{-a\cos(dx+c)+a}(\cos}{2\,ad\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*cos(d*x + c)*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c) + a)
*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*sin(d
*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*sqrt(-a*cos(d*
x + c) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)*sin(d*
x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-a(\cos(c+dx)-1)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a*(cos(c + d*x) - 1))*cos(c + d*x)**(3/2)), x)
```

Giac [A] time = 2.35726, size = 92, normalized size = 0.97

$$\frac{\sqrt{2}a\left(\frac{\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}|a|}+\frac{2}{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}|a|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*a*(arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt(-a)*a
bs(a)) + 2/(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*abs(a)))/d
```

3.280 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3
/2)*Sqrt[a - a*Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sq
rt[a - a*Cos[c + d*x]])
```

Rubi [A] time = 0.250542, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2779, 2984, 12, 2782, 208}

$$\frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3
/2)*Sqrt[a - a*Cos[c + d*x]]) + (2*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sq
rt[a - a*Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{a+2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} - \frac{2\int$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} + \int$$

$$= \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}} - \frac{(2a)}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} +$$

Mathematica [C] time = 0.314834, size = 171, normalized size = 1.27

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+1)-\frac{3e^{-\frac{3}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{2\sqrt{2}}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] (2*((-3*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))])])/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))) + 2*Sq
rt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x])*Sin[(c + d
*x)/2])/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^(3/2)*Sqrt[a - a*Co
s[c + d*x]])
```

Maple [A] time = 0.358, size = 171, normalized size = 1.3

$$-\frac{\sqrt{2}\left(\sin\left(dx+c\right)\right)^{5}}{3\,d\left(-1+\cos\left(dx+c\right)\right)^{2}\left(1+\cos\left(dx+c\right)\right)}\left(3\,\cos\left(dx+c\right)\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{5/2}\mathrm{Artanh}\left(1/2\,\sqrt{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -1/3/d*2^(1/2)*sin(d*x+c)^5*(3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+3*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))*2^(1/2)-2*cos(d*x+c))/cos(d*x+c)^(5/2)/(-1+cos(d*x+c))^2/(-2*a*(-1+cos
(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33819, size = 451, normalized size = 3.34

$$\frac{3\,\sqrt{2}\sqrt{a}\cos\left(dx+c\right)^{2}\log\left(-\frac{\frac{2\,\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\,\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+4\,\sqrt{-a\cos\left(dx+c\right)+}}{6\,ad\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*sqrt(a)*cos(d*x + c)^2*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c)
+ a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)*s
in(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*sqrt(-a*co
s(d*x + c) + a)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(
a*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a-a*cos(d*x+c))**(1/2),x)
```

[Out] Timed out

Giac [A] time = 2.1637, size = 122, normalized size = 0.9

$$\frac{\sqrt{2}a\left(\frac{3\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}|a|}-\frac{4a}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}|a|}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*a*(3*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sqrt
(-a)*abs(a)) - 4*a/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)*abs(a)))/d
```

3.281 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx$

Optimal. Leaf size=173

$$\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\ \sqrt{}$$

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5
/2)*Sqrt[a - a*Cos[c + d*x]]) + (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*S
qrt[a - a*Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])
```

Rubi [A] time = 0.40089, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2779, 2984, 12, 2782, 208}

$$\frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}+\frac{26\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}-\ \sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```


```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])])/(Sqrt[a]*d)) + (2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5
/2)*Sqrt[a - a*Cos[c + d*x]]) + (2*Sin[c + d*x])/(15*d*Cos[c + d*x]^(3/2)*S
qrt[a - a*Cos[c + d*x]]) + (26*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[
a - a*Cos[c + d*x]])
```


Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx = \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{\int \frac{a+4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}\,dx}{5a}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} - \frac{2\int}{}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{}{15a}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{}{15a}$$

$$= \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{2\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} + \frac{}{15a}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a-a\cos(c+dx)}}\right)}{\sqrt{a}d} + \frac{2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}} +$$

Mathematica [C] time = 0.653393, size = 218, normalized size = 1.26

$$\frac{e^{-\frac{5}{2}i(c+dx)}\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\left(15e^{i(c+dx)}+40e^{2i(c+dx)}+40e^{3i(c+dx)}+15e^{4i(c+dx)}+13e^{5i(c+dx)}+13\right)-15\sqrt{2}\left(1+\right.\right.}{60d\sqrt{1+e^{2i(c+dx)}}\cos^{\frac{5}{2}}(c+dx)\sqrt{a-a\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[a - a*Cos[c + d*x]]),x]
```

```
[Out] ((2*Sqrt[1 + E^((2*I)*(c + d*x))]*(13 + 15*E^(I*(c + d*x)) + 40*E^((2*I)*(c
 + d*x)) + 40*E^((3*I)*(c + d*x)) + 15*E^((4*I)*(c + d*x)) + 13*E^((5*I)*(c
 + d*x))) - 15*Sqrt[2]*(1 + E^((2*I)*(c + d*x)))^3*ArcTanh[(1 + E^(I*(c + d
*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])*Sin[(c + d*x)/2])/(60*d*E^(
((5*I)/2)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^(5/2)*Sqrt[
a - a*Cos[c + d*x]])
```

Maple [B] time = 0.366, size = 305, normalized size = 1.8

$$\frac{\sqrt{2}\,(\sin(dx+c))^7}{15\,d\,(-1+\cos(dx+c))^3\,(1+\cos(dx+c))^3}\left(15\,(\cos(dx+c))^3\,\sqrt{2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{7/2}\mathrm{Artanh}\left(1/2\,\sqrt{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a-cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/15/d*2^(1/2)*sin(d*x+c)^7*(15*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(7/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+45*cos(d
*x+c)^2*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*2^(1/2)/(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2))+45*cos(d*x+c)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(7/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+15*2^(1/2
)*(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2))-26*cos(d*x+c)^3-2*cos(d*x+c)^2-6*cos(d*x+c))/cos(d*x+c)^(7/
2)/(-1+cos(d*x+c))^3/(-2*a*(-1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.37675, size = 485, normalized size = 2.8

$$\frac{15\,\sqrt{2}\sqrt{a}\cos(dx+c)^3\log\left(-\frac{\frac{2\,\sqrt{2}\sqrt{-a\cos(dx+c)+a}(\cos(dx+c)+1)\sqrt{\cos(dx+c)}}{\sqrt{a}}-(3\,\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\left(13\,\cos(dx+c)^3\right.}{30\,ad\cos(dx+c)^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(2)*sqrt(a)*cos(d*x + c)^3*log(-(2*sqrt(2)*sqrt(-a*cos(d*x + c
) + a)*(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sqrt(a) - (3*cos(d*x + c) + 1)
*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(13*cos(
d*x + c)^3 + 14*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sqrt(-a*cos(d*x + c) +
 a)*sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a-a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.25681, size = 184, normalized size = 1.06

$$\frac{\sqrt{2}a\left(\frac{15\arctan\left(\frac{\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}|a|}+\frac{2\left(15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{2}+10\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)a+12a^{2}\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}|a|}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a-a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*sqrt(2)*a*(15*arctan(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)/sqrt(-a))/(sq
rt(-a)*abs(a)) + 2*(15*(a*tan(1/2*d*x + 1/2*c)^2 - a)^2 + 10*(a*tan(1/2*d*x
 + 1/2*c)^2 - a)*a + 12*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a)*abs(a)))/d
```

3.282 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}+\frac{7\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+d\cdots}{\sqrt{2}\sqrt{1-\cos(c+dx)}\cdots}\right.}{d}$$

```
[Out] (7*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/(4*d)
 - (Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
 + d*x]])])/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x
]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 - Cos[c + d*x]])
```

Rubi [A] time = 0.301612, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2778, 2983, 2982, 2782, 206, 2775, 207}

$$\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{1-\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{4d\sqrt{1-\cos(c+dx)}}+\frac{7\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+d\cdots}{\sqrt{2}\sqrt{1-\cos(c+dx)}\cdots}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/(4*d)
 - (Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
 + d*x]])])/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[1 - Cos[c + d*x
]]) + (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[1 - Cos[c + d*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
```

```
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx &= \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} + \frac{1}{4}\int \frac{\sqrt{\cos(c+dx)}(3+\cos(c+dx))}{\sqrt{1-\cos(c+dx)}}\,dx \\
&= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} - \frac{1}{4}\int \frac{-\frac{1}{2}-\frac{7}{2}\cos(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} - \frac{7}{8}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{}{\sqrt{1-c}} \\
&= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{4d\sqrt{1-\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{1-\cos(c+dx)}} - \frac{7\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos}}\right.}{4d} \\
&= \frac{7\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{4d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)}\sin(c}{4d\sqrt{1-\cos(c+}}
\end{aligned}$$

Mathematica [C] time = 0.41225, size = 255, normalized size = 1.58

$$-\frac{ie^{-2i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\left(7\sqrt{2}e^{2i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-16e^{2i(c+dx)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left(\sqrt{1+e^{2i(c}}\right.\right.}{8\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] ((-I/8)*(-1 + E^(I*(c + d*x)))*(7*Sqrt[2]*E^((2*I)*(c + d*x))*ArcSinh[E^(I*
(c + d*x))] - 16*E^((2*I)*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]
*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(Sqrt[1 + E^((2*I)*(c + d*x))]*(
1 + 2*E^(I*(c + d*x)) + 2*E^((2*I)*(c + d*x)) + E^((3*I)*(c + d*x))) + 7*E^
((2*I)*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt[Cos[c + d*x
]])/(Sqrt[2]*d*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - C
os[c + d*x]])
```

Maple [A] time = 0.329, size = 194, normalized size = 1.2

$$-\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{3}}{4d\left(\sin\left(dx+c\right)\right)^{5}}\left(\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(2\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\left(\cos\left(dx+c\right)\right)^{2}+3\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\cos\left(dx+c\right)-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)*cos(d*x+c)^(5/2)*(-1+cos(d*x+c))^3*(2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*cos(d*x+c)^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4*a
rctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+7*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)))/(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^5/(2-2*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{-\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(-cos(d*x + c) + 1), x)
```

Fricas [A] time = 2.00554, size = 653, normalized size = 4.06

$$4\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+2\left(2\cos\left(dx+c\right)^{2}+3\cos\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c)
+ 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c)
 - 1)*sin(d*x + c)))*sin(d*x + c) + 2*(2*cos(d*x + c)^2 + 3*cos(d*x + c) +
```

```
1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) + 7*log(2*(sqrt(-cos(d*x + c)
 + 1)*sqrt(cos(d*x + c)) + sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 7*log
(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - sin(d*x + c))/sin(d*x + c)
)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(1-cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.0876, size = 219, normalized size = 1.36

$$-\frac{\sqrt{2}\left(7\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{\sqrt{2}+\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}\right)-\frac{4\left(\left(-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^{\frac{3}{2}}+2\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}+8\log\left(\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}+1+\right.\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*(7*sqrt(2)*log((sqrt(2) - sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))/
(sqrt(2) + sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))) - 4*((-tan(1/2*d*x + 1/2*c)^
2 + 1)^(3/2) + 2*sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))/(tan(1/2*d*x + 1/2*c)^2
 + 1)^2 + 8*log(sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1) - 8*log(-sqrt(-tan(1
/2*d*x + 1/2*c)^2 + 1) + 1))/d
```

3.283 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt
[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]
)])/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])
```

Rubi [A] time = 0.213549, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2778, 2982, 2782, 206, 2775, 207}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{1-\cos(c+dx)}}+\frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])]/d - (Sqrt
[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]
)])/d + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]])
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} + \frac{1}{2}\int \frac{1+\cos(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{1}{2}\int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,\right.}{}$$
$$= \frac{\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{\sqrt{\cos(c+dx)}\sin(c+}{d\sqrt{1-\cos(c+dx}}$$

Mathematica [C] time = 0.158419, size = 227, normalized size = 1.92

$$-\frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\left(\sqrt{2}e^{i(c+dx)}\sinh^{-1}\left(e^{i(c+dx)}\right)-4e^{i(c+dx)}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\left(\sqrt{1+e^{2i(c+dx)}}\right.\right.}{2\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] ((-I/2)*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*E^(I*(c + d*x))*ArcSinh[E^(I*(c + d
*x))] - 4*E^(I*(c + d*x))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E
^((2*I)*(c + d*x))])] + Sqrt[2]*((1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c
 + d*x))] + E^(I*(c + d*x))*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Sqrt[C
os[c + d*x]])/(Sqrt[2]*d*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt
[1 - Cos[c + d*x]])
```

Maple [A] time = 0.31, size = 161, normalized size = 1.4

$$\frac{\sqrt{2}\left(-1+\cos(dx+c)\right)^2}{d\left(\sin(dx+c)\right)^3}\left(\cos(dx+c)\right)^{\frac{3}{2}}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)-\text{Artanh}\left(\frac{\sqrt{2}}{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}+\text{Artanh}\left(\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*cos(d*x+c)^(3/2)*(-1+cos(d*x+c))^2*((cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*cos(d*x+c)-arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^
(1/2)+arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2))/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(2-2*cos(d*x+c))^(1/2)/sin(d*x+c
)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{-\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(-cos(d*x + c) + 1), x)
```

Fricas [B] time = 2.02748, size = 617, normalized size = 5.23

$$\frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+2(\cos(dx+c)+1)\sqrt{-\cos(dx+}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) +
1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) -
 1)*sin(d*x + c)))*sin(d*x + c) + 2*(cos(d*x + c) + 1)*sqrt(-cos(d*x + c) +
 1)*sqrt(cos(d*x + c)) + log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c))
+ sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - log(2*(sqrt(-cos(d*x + c) + 1)
*sqrt(cos(d*x + c)) - sin(d*x + c))/sin(d*x + c))*sin(d*x + c))/(d*sin(d*x
+ c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1-\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(1-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(1 - cos(c + d*x)), x)
```

Giac [A] time = 2.14402, size = 190, normalized size = 1.61

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{\sqrt{2}+\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}\right)-\frac{4\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\,\log\left(\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+1\right)-2\,\log\left(-\sqrt{-\tan\left(\frac{1}{2}\right.}\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))/(sq
rt(2) + sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))) - 4*sqrt(-tan(1/2*d*x + 1/2*c)^
2 + 1)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*log(sqrt(-tan(1/2*d*x + 1/2*c)^2 +
1) + 1) - 2*log(-sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1))/d
```

3.284 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] (2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d - (
Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d
*x]])])/d
```

Rubi [A] time = 0.129491, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2777, 2775, 207, 2782, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sin[c + d*x]/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])])/d - (
Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d
*x]])])/d
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx - \int \frac{\sqrt{1-\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Mathematica [C] time = 0.105018, size = 160, normalized size = 1.88

$$-\frac{i\left(-1+e^{i(c+dx)}\right)\sqrt{e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)}\left(\sinh^{-1}\left(e^{i(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{2}d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[1 - Cos[c + d*x]],x]
```

```
[Out] ((-I)*(-1 + E^(I*(c + d*x)))*Sqrt[(1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))
]*(ArcSinh[E^(I*(c + d*x))] - Sqrt[2]*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2
]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]])
)/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]])
```

Maple [A] time = 0.217, size = 117, normalized size = 1.4

$$-\frac{\sqrt{2}(-1+\cos(dx+c))}{d\sin(dx+c)}\sqrt{\cos(dx+c)}\left(-\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}+2\,\mathrm{Artanh}\left(\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\frac{1}{\sqrt{2-2\cos(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(-arctanh(1/2*2^(1/2)/(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+2*arctanh((cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)))/(2-2*cos(d*x+c))^(1/2)/sin(d*x+c)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-cos(d*x + c) + 1), x)
```

Fricas [B] time = 1.93228, size = 460, normalized size = 5.41

$$\frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)+2\log\left(\frac{2\left(\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}+\sin(dx+c)\right)}{\sin(dx+c)}\right)-}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) +
1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) -
 1)*sin(d*x + c))) + 2*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) +
sin(d*x + c))/sin(d*x + c)) - 2*log(2*(sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x
 + c)) - sin(d*x + c))/sin(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1-\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(1-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(1 - cos(c + d*x)), x)
```

Giac [A] time = 2.16918, size = 142, normalized size = 1.67

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}{\sqrt{2}+\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}}\right)+\log\left(\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}+1\right)-\log\left(-\sqrt{-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}+1\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(1-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))/(sq
rt(2) + sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1))) + log(sqrt(-tan(1/2*d*x + 1/2*c
)^2 + 1) + 1) - log(-sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1))/d
```

3.285 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d)
```

Rubi [A] time = 0.0482254, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2782, 206}

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d)
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\\ &= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}\end{aligned}$$

Mathematica [C] time = 0.138234, size = 110, normalized size = 2.34

$$\frac{ie^{-i(c+dx)}\left(-1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}d\sqrt{-(\cos(c+dx)-1)\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (I*(-1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 + E^(I*(
c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/(Sqrt[2]*d*E^(I*(c + d
*x))*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])
```

Maple [B] time = 0.289, size = 84, normalized size = 1.8

$$4\,{\frac { \left(-1+\cos \left(dx+c \right) \right) \sin \left(dx+c \right) }{d\sqrt {\cos \left(dx+c \right) } \left(2-2\,\cos \left(dx+c \right) \right) ^{3/2}}\sqrt {{\frac {\cos \left(dx+c \right) }{1+\cos \left(dx+c \right) }}}{\rm Artanh} \left(1/2\,\sqrt {2}{\frac {1}{\sqrt {{\frac {\cos \left(dx+c \right) }{1+\cos \left(dx+c \right) }}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 4/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))*sin(d*x+c)*arctanh(1/
2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/cos(d*x+c)^(1/2)/(2-2*cos(d*x+
c))^(3/2)
```

Maxima [C] time = 1.91575, size = 335, normalized size = 7.13

$$\sqrt{2}\log\left(\frac{4\left(\left|i\,e^{(i\,dx+i\,c)}-i\right|^{2}+2\,\sqrt{\cos(2\,dx+2\,c)^{2}+\sin(2\,dx+2\,c)^{2}+2\,\cos(2\,dx+2\,c)+1}\left(\cos\left(\frac{1}{2}\,\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c)+1)\right)^{2}+\sin\left(\frac{1}{2}\,\arctan(\sin(2\,d\cdots\right.\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(4*(abs(I*e^(I*d*x + I*c) - I)^2 + 2*sqrt(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x + I*c) - I)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*sqrt(2)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(I*d*x + I*c) - I)^2)/d
```

Fricas [B] time = 2.13078, size = 231, normalized size = 4.91

$$\frac{\sqrt{2}\log\left(-\frac{2\,\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\,\sqrt{\cos(dx+c)}-(3\,\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-cos(d*x + c) + 1
)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) -
1)*sin(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(1 - cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [A] time = 2.04332, size = 107, normalized size = 2.28

$$\frac{\sqrt{2}\left(\log\left(\sqrt{2}+1\right)-\log\left(\sqrt{2}-1\right)-\log\left(\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+1\right)+\log\left(-\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+1\right)\right)}{2\,d\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(log(sqrt(2) + 1) - log(sqrt(2) - 1) - log(sqrt(-tan(1/2*d*x +
1/2*c)^2 + 1) + 1) + log(-sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1))/(d*sgn(ta
n(1/2*d*x + 1/2*c)))
```

3.286 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d) + (2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x
]])
```

Rubi [A] time = 0.0925313, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2779, 2782, 206}

$$\frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d) + (2*Sin[c + d*x])/(d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x
]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} + \int \frac{1}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} - \frac{2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{\sin(c+dx)}{\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$
$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2\sin(c+dx)}{d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Mathematica [C] time = 0.138378, size = 152, normalized size = 1.83

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right) - \frac{e^{-\frac{1}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{\sqrt{2}}\right)}{d\sqrt{1+e^{2i(c+dx)}}\sqrt{-(\cos(c+dx)-1)\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(3/2)),x]
```

```
[Out] (2*(-(((1 + E^((2*I)*(c + d*x)))*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqr
t[1 + E^((2*I)*(c + d*x))])])/(Sqrt[2]*E^((I/2)*(c + d*x)))) + 2*Sqrt[1 + E
^((2*I)*(c + d*x))]*Cos[(c + d*x)/2])*Sin[(c + d*x)/2])/(d*Sqrt[1 + E^((2*I
)*(c + d*x))]*Sqrt[-((-1 + Cos[c + d*x])*Cos[c + d*x])])
```

Maple [B] time = 0.305, size = 159, normalized size = 1.9

$$\frac{\sqrt{2}(\sin(dx+c))^3}{d\left((\cos(dx+c))^2-1\right)}\left(\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\text{Artanh}\left(\frac{\sqrt{2}}{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}+\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\text{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*2^(1/2)*sin(d*x+c)^3/cos(d*x+c)^(3/2)*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)
+(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2))*2^(1/2)-2*cos(d*x+c))/(2-2*cos(d*x+c))^(1/2)/(cos(d*x+c)^2-1
)
```

Maxima [C] time = 1.9527, size = 540, normalized size = 6.51

$$-\frac{\sqrt{2}\left(2\sqrt{2}\sin(dx+c)\sin\left(\frac{1}{2}\arctan(\sin(2dx+2c),\cos(2dx+2c)+1)\right)+2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\cos\left(\frac{1}{2}\arctan(s\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*(2*sqrt(2)*sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)) + 2*(sqrt(2)*cos(d*x + c) + sqrt(2))*cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*log(4*(abs(I*e^(I*d*x + I*c) - I)^2 +
 2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x +
 I*c) - I)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*sqr
t(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(
I*d*x + I*c) - I)^2))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*d)
```

Fricas [A] time = 2.2223, size = 396, normalized size = 4.77

$$\frac{\sqrt{2}\cos\left(dx+c\right)\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+4\left(\cos\left(dx+c\right)+1\right)\sqrt{-\cos\left(dx+c\right)+1}\sqrt{\cos\left(dx+c\right)}}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*cos(d*x + c)*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt(-co
s(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((c
os(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(cos(d*x + c) + 1)*sqrt(-c
os(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-\cos\left(c+dx\right)}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(1 - cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Giac [A] time = 1.94227, size = 97, normalized size = 1.17

$$\frac{\sqrt{2}\left(\frac{4}{\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}-\log\left(\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+1\right)+\log\left(-\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(4/sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) - log(sqrt(-tan(1/2*d*x +
1/2*c)^2 + 1) + 1) + log(-sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1))/d
```

3.287 $\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=122

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d) + (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(
3/2)) + (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.182391, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2779, 2984, 12, 2782, 206}

$$\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[Sin[c + d*x]/(Sqrt[2]*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c
+ d*x]])])/d) + (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(
3/2)) + (2*Sin[c + d*x])/(3*d*Sqrt[1 - Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\cos(c+dx)}\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\int \frac{1+2\cos(c+dx)}{\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} - \frac{2}{3}\int -\frac{}{2\sqrt{}}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} + \int \frac{}{\sqrt{1-}}$$

$$= \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}} - \frac{2\,\mathrm{Subst}}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sin(c+dx)}{\sqrt{2}\sqrt{1-\cos(c+dx)}\sqrt{\cos(c+dx)}}\right)}{d} + \frac{2\sin(c+dx)}{3d\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)} + \frac{}{3d\sqrt{}}$$

Mathematica [C] time = 0.286337, size = 170, normalized size = 1.39

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+1)-\frac{3e^{-\frac{3}{2}i(c+dx)}\left(1+e^{2i(c+dx)}\right)^2\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)}{2\sqrt{2}}\right)}{3d\sqrt{1+e^{2i(c+dx)}}\sqrt{1-\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - Cos[c + d*x]]*Cos[c + d*x]^(5/2)),x]
```

```
[Out] (2*((-3*(1 + E^((2*I)*(c + d*x)))^2*ArcTanh[(1 + E^(I*(c + d*x)))/(Sqrt[2]*
Sqrt[1 + E^((2*I)*(c + d*x))])])/(2*Sqrt[2]*E^(((3*I)/2)*(c + d*x))) + 2*Sq
rt[1 + E^((2*I)*(c + d*x))]*Cos[(c + d*x)/2]*(1 + Cos[c + d*x]))*Sin[(c + d
*x)/2])/(3*d*Sqrt[1 + E^((2*I)*(c + d*x))]*Sqrt[1 - Cos[c + d*x]]*Cos[c + d
*x]^(3/2))
```

Maple [A] time = 0.321, size = 170, normalized size = 1.4

$$-\frac{\sqrt{2}(\sin(dx+c))^5}{3d(-1+\cos(dx+c))^2(1+\cos(dx+c))}\left(3\cos(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}\mathrm{Artanh}\left(1/2\sqrt{2}\frac{1}{\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}}\right)\sqrt{2}+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/3/d*2^(1/2)*sin(d*x+c)^5*(3*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)+3*(cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*arctanh(1/2*2^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))*2^(1/2)-2*cos(d*x+c))/cos(d*x+c)^(5/2)/(-1+cos(d*x+c))^2/(2-2*cos(d*x+
c))^(1/2)/(1+cos(d*x+c))
```

Maxima [C] time = 1.90369, size = 760, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*l
og(4*(abs(I*e^(I*d*x + I*c) - I)^2 + 2*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))^2) - 2*(sqrt(2)*abs(I*e^(I*d*x + I*c) - I)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)) - 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)^(1/4) + 4)/abs(I*e^(I*d*x + I*c) - I)^2) - 2*(sqrt(2)*sin(d
*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (sqrt(2)
*cos(d*x + c) + 3*sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4) - 4*(sqrt(2)*sin(d*x + c)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) + (sqrt(2)*cos(d*x + c) - sqrt(2))*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4))/((sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin
(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*d)
```

Fricas [A] time = 2.1414, size = 429, normalized size = 3.52

$$\frac{3\sqrt{2}\cos(dx+c)^2\log\left(-\frac{2\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\sqrt{-\cos(dx+c)+1}\sqrt{\cos(dx+c)}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+4\left(\cos(dx+c)^2\right.}{6\,d\cos(dx+c)^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*cos(d*x + c)^2*log(-(2*(sqrt(2)*cos(d*x + c) + sqrt(2))*sqrt
(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)) - (3*cos(d*x + c) + 1)*sin(d*x + c))
/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) + 4*(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)*sqrt(-cos(d*x + c) + 1)*sqrt(cos(d*x + c)))/(d*cos(d*x + c)
^2*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.81366, size = 120, normalized size = 0.98

$$-\frac{\sqrt{2}\left(\frac{8}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}+3\,\log\left(\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+1\right)-3\,\log\left(-\sqrt{-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(8/((tan(1/2*d*x + 1/2*c)^2 - 1)*sqrt(-tan(1/2*d*x + 1/2*c)^2
+ 1)) + 3*log(sqrt(-tan(1/2*d*x + 1/2*c)^2 + 1) + 1) - 3*log(-sqrt(-tan(1/2
*d*x + 1/2*c)^2 + 1) + 1))/d
```

3.288 $\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{2^{5/6}\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}F_1\left(\frac{1}{2};-\frac{4}{3},\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}$$

```
[Out] (2^(5/6)*AppellF1[1/2, -4/3, 1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*x])
/2]*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(5/6))
```

Rubi [A] time = 0.10798, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2787, 2785, 133}

$$\frac{2^{5/6}\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}F_1\left(\frac{1}{2};-\frac{4}{3},\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (2^(5/6)*AppellF1[1/2, -4/3, 1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*x])
/2]*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx = \frac{\sqrt[3]{a+a\cos(c+dx)}\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{1+\cos(c+dx)}\,dx}{\sqrt[3]{1+\cos(c+dx)}}$$
$$= \frac{\left(\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{4/3}}{\sqrt[6]{2-x}\sqrt{x}}\,dx, x, 1-\cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{5/6}}$$
$$= \frac{2^{5/6}F_1\left(\frac{1}{2};-\frac{4}{3},\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)\sqrt[3]{a+a\cos(c+dx)}\sin(c}{d(1+\cos(c+dx))^{5/6}}$$

Mathematica [F] time = 15.3061, size = 0, normalized size = 0.

$$\int \cos^{\frac{4}{3}}(c+dx)\sqrt[3]{a+a\cos(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(1/3), x]
```

Maple [F] time = 0.159, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{4}{3}}\sqrt[3]{a+\cos(dx+c)\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(1/3),x)
```

```
[Out] int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{1}{3}}\cos(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\cos(dx+c)+a)^{\frac{1}{3}}\cos(dx+c)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(4/3)*(a+a*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{1}{3}}\cos(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(1/3)*cos(d*x + c)^(4/3), x)
```

3.289 $\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[6]{2}\sin(c+dx)(a\cos(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{4}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

```
[Out] (2*2^(1/6)*AppellF1[1/2, -4/3, -1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*
x])/2]*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/6)
)
```

Rubi [A] time = 0.121835, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2787, 2785, 133}

$$\frac{2\sqrt[6]{2}\sin(c+dx)(a\cos(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{4}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (2*2^(1/6)*AppellF1[1/2, -4/3, -1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*
x])/2]*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/6)
)
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx = \frac{(a+a\cos(c+dx))^{2/3}\int \cos^{\frac{4}{3}}(c+dx)(1+\cos(c+dx))^{2/3}\,dx}{(1+\cos(c+dx))^{2/3}}$$
$$= \frac{\left((a+a\cos(c+dx))^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{4/3}\sqrt[6]{2-x}}{\sqrt{x}}\,dx,x,1-\cos(c+\right.}{d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{7/6}}$$
$$= \frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};-\frac{4}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)(a+a\cos(c+dx)}{d(1+\cos(c+dx))^{7/6}}$$

Mathematica [F] time = 3.28333, size = 0, normalized size = 0.

$$\int \cos^{\frac{4}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] Integrate[Cos[c + d*x]^(4/3)*(a + a*Cos[c + d*x])^(2/3), x]
```

Maple [F] time = 0.168, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{4}{3}}(a+\cos(dx+c)a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(2/3),x)
```

```
[Out] int(cos(d*x+c)^(4/3)*(a+cos(d*x+c)*a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(4/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(4/3)*(a+a*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

3.290 $\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=79

$$\frac{2\sqrt[6]{2}\sin(c+dx)(a\cos(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{5}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

```
[Out] (2*2^(1/6)*AppellF1[1/2, -5/3, -1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*
x])/2]*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/6)
)
```

Rubi [A] time = 0.120858, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2787, 2785, 133}

$$\frac{2\sqrt[6]{2}\sin(c+dx)(a\cos(c+dx)+a)^{2/3}F_1\left(\frac{1}{2};-\frac{5}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)}{d(\cos(c+dx)+1)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3),x]
```


```
[Out] (2*2^(1/6)*AppellF1[1/2, -5/3, -1/6, 3/2, 1 - Cos[c + d*x], (1 - Cos[c + d*
x])/2]*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 + Cos[c + d*x])^(7/6)
)
```


Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx = \frac{(a+a\cos(c+dx))^{2/3}\int \cos^{\frac{5}{3}}(c+dx)(1+\cos(c+dx))^{2/3}\,dx}{(1+\cos(c+dx))^{2/3}}$$
$$= \frac{\left((a+a\cos(c+dx))^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{(1-x)^{5/3}\sqrt[6]{2-x}}{\sqrt{x}}dx, x, 1-\cos(c+dx)\right)}{d\sqrt{1-\cos(c+dx)}(1+\cos(c+dx))^{7/6}}$$
$$= \frac{2\sqrt[6]{2}F_1\left(\frac{1}{2};-\frac{5}{3},-\frac{1}{6};\frac{3}{2};1-\cos(c+dx),\frac{1}{2}(1-\cos(c+dx))\right)(a+a\cos(c+dx))^{2/3}}{d(1+\cos(c+dx))^{7/6}}$$

Mathematica [F] time = 2.65374, size = 0, normalized size = 0.

$$\int \cos^{\frac{5}{3}}(c+dx)(a+a\cos(c+dx))^{2/3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] Integrate[Cos[c + d*x]^(5/3)*(a + a*Cos[c + d*x])^(2/3), x]
```

Maple [F] time = 0.165, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{5}{3}}(a+\cos(dx+c)\,a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/3)*(a+cos(d*x+c)*a)^(2/3),x)
```

```
[Out] int(cos(d*x+c)^(5/3)*(a+cos(d*x+c)*a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\cos(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)^{\frac{5}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/3)*(a+a*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{2}{3}}\cos(dx+c)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)*(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(2/3)*cos(d*x + c)^(5/3), x)
```

3.291 $\int (a + a\cos(c + dx))\sec^{\frac{7}{2}}(c + dx)\,dx$

Optimal. Leaf size=151

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

```
[Out] (-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.110528, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3768, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{6a\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$

$$= a\int \sec^{\frac{5}{2}}(c+dx)\,dx + a\int \sec^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}a\int \sqrt{\sec(c+}$$

$$= \frac{6a\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a\sec^{\frac{5}{2}}(c+dx)}{5d}$$

$$= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{6a\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}$$

$$= -\frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{3d}$$

Mathematica [C] time = 1.60632, size = 268, normalized size = 1.77

$$a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(9\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+5\left(-\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/E^(I*(c +
d*x)) + (1 - E^((2*I)*c))*Sqrt[Sec[c + d*x]]*(9*Cos[d*x]*Csc[c] + (5 + 3*Se
c[c + d*x])*Tan[c + d*x])))/(15*(d - d*E^((2*I)*c)))
```

Maple [B] time = 4.081, size = 384, normalized size = 2.5

$$-4\frac{\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin(1/2\,dx+c/2)\right)^2a}}{\sin(1/2\,dx+c/2)\sqrt{2\left(\cos(1/2\,dx+c/2)\right)^2-1}d}\left(-1/12\frac{\cos(1/2\,dx+c/2)\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+(\sin(}}{\left(\left(\cos(1/2\,dx+c/2)\right)^2-1/2\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^(7/2),x)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-1/12*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-1/40*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-3/5*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)-3/10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.292 $\int (a + a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\sqrt{\cos(c+}}{}$$

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rubi [A] time = 0.097052, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3768, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))\,dx$$
$$= a\int \sec^{\frac{3}{2}}(c+dx)\,dx + a\int \sec^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}a\int \sqrt{\sec(c+}$$
$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\Big(a\sqrt{\cos(c+dx}$$
$$= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{3d}$$

Mathematica [C] time = 1.10846, size = 255, normalized size = 2.07

$$\frac{a(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+\left(-1\right.\right.\right.}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(
1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c
))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I
)*(c + d*x))] + E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/E^(I*(c + d*
x)) - (-1 + E^((2*I)*c))*Sqrt[Sec[c + d*x]]*(3*Cos[d*x]*Csc[c] + Tan[c + d*
x])))/(3*(d - d*E^((2*I)*c)))
```

Maple [B] time = 3.5, size = 369, normalized size = 3.

$$\frac{2a}{3d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\Bigg(2\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\,\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a/(4*sin(1/2*
d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(2*sin(1/2*d
```

```
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.293 $\int (a + a\cos(c + dx)) \sec^{\frac{3}{2}}(c + dx)\, dx$

Optimal. Leaf size=97

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0872682, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3771, 2641, 3768, 2639}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx &= \int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))\,dx\\ &= a\int \sqrt{\sec(c+dx)}\,dx + a\int \sec^{\frac{3}{2}}(c+dx)\,dx\\ &= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.\\ &= \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - (a\\ &= -\frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|2}{d}\end{aligned}$$

Mathematica [C] time = 1.21317, size = 124, normalized size = 1.28

$$-\frac{2iae^{-i(c+dx)}\left(\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)-1\right)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((-2*I)*a*(-1 + Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*x]])
/(d*E^(I*(c + d*x)))
```

Maple [A] time = 2.437, size = 146, normalized size = 1.5

$$-2\,\frac{a\left(\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{(\sin(1/2\,dx+c/2))^2}\,\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)+\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\mathrm{s}\right.}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*a*((2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.294 $\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.0757447, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3238, 3787, 3771, 2639, 2641}

$$\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{a+a\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx$$
$$= a\int \frac{1}{\sqrt{\sec(c+dx)}}dx + a\int \sqrt{\sec(c+dx)}\,dx$$
$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$
$$= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{d}$$

Mathematica [C] time = 1.01724, size = 141, normalized size = 1.88

$$-\frac{2ia\left(-2\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+2e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)+e^{2i(c+dx)}+1\right)}{d\left(1+e^{2i(c+dx)}\right)\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((-2*I)*a*(1 + E^((2*I)*(c + d*x)) - 2*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*
x))]))/(d*(1 + E^((2*I)*(c + d*x)))*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.89, size = 150, normalized size = 2.

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\,a\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(\text{EllipticF}\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\,\sin(1/2\,dx+c/2)\sqrt{2\,(\text{co}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\cos(dx+c)+a)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int\sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x
)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.295 $\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0899282, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*a*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx &= \int \frac{a+a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.\\ &= \frac{2a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}\end{aligned}$$

Mathematica [C] time = 1.22555, size = 140, normalized size = 1.39

$$\frac{ae^{-2ic}(\sin(2c)-i\cos(2c))\left(-\frac{12\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+2i\sin(c+dx)+6\right.}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*((-I)*Cos[2*c] + Sin[2*c])*(6 - (12*Hypergeometric2F1[-1/4, 1/2, 3/4, -E
^((2*I)*(c + d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] + 2*Sqrt[1 + E^((2*I)*(c
 + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*
x] + (2*I)*Sin[c + d*x]))/(3*d*E^((2*I)*c)*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 1.986, size = 225, normalized size = 2.2

$$-\frac{2a}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+1/2*
```

```
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{a\cos(dx+c)+a}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx+\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d
*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.296 $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{5d}$$

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.100378, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2639, 2641}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{6a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```


```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{a+a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx \\
&= \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{3d}
\end{aligned}$$

Mathematica [C] time = 0.894369, size = 224, normalized size = 1.76

$$-\frac{iae^{-3i(c+dx)}\left(-72e^{2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+40e^{3i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)-10e}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((-I/120)*a*(1 + Cos[c + d*x])*(-3 - 10*E^(I*(c + d*x)) + 33*E^((2*I)*(c +
d*x)) + 39*E^((4*I)*(c + d*x)) + 10*E^((5*I)*(c + d*x)) + 3*E^((6*I)*(c + d
*x)) - 72*E^((2*I)*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2
F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + 40*E^((3*I)*(c + d*x))*Sqrt[1 +
E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]
)*Sec[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(d*E^((3*I)*(c + d*x)))
```

Maple [A] time = 2.144, size = 219, normalized size = 1.7

$$-\frac{2a}{15d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\left(\cos(1/2\,dx+c/2)\right)^7-28\left(\cos(1/2\,dx+c/2)\right)^5+5\sqrt{(\sin(1/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(24*cos(1/2
*d*x+1/2*c)^7-28*cos(1/2*d*x+1/2*c)^5+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))+4*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(
-3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.297 $\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a\sqrt{\cos(c+d}}{}$$

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.112773, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2641, 2639}

$$\frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a\sqrt{\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (6*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*a*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*a*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{a+a\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx\\
&= a\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3a)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}(5a)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\
&= \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5a)\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}(3\\
&= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10a}{21d}\\
&= \frac{6a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{21d}\end{aligned}$$

Mathematica [C] time = 2.15109, size = 198, normalized size = 1.31

$$\frac{ae^{-4i(c+dx)}\sqrt{\sec(c+dx)}(\cos(4(c+dx))+i\sin(4(c+dx)))\left(504ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-200i\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(Cos[4*(c + d*x)] + I*Sin[4*(c + d*x)])*((-504*I)*Cos
[c + d*x] + ((504*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4,
1/2, 3/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) - (200*I)*Sqrt[1 + E^((2*I
)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + 42*S
in[c + d*x] + 130*Sin[2*(c + d*x)] + 42*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*
x)]))/(420*d*E^((4*I)*(c + d*x)))
```

Maple [A] time = 2.421, size = 270, normalized size = 1.8

$$-\frac{2a}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8-528\left(\sin\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-528*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+25*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-122*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\cos(dx+c)+a}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

3.298 $\int (a+a\cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{16a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

```
[Out] (-16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (16*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d)
```

Rubi [A] time = 0.146726, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{4a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{16a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (16*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (4*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx &= \int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\,dx\\
&= \left(2a^2\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx+\int \sec^{\frac{3}{2}}(c+dx)\left(a^2+a^2\sec^2(c+dx)\right)\,dx\\
&= \frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{3}\left(2a^2\right)\int \sqrt{\text{s}}\\
&= \frac{16a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{4a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2a^2\sec^{\frac{5}{2}}(c+}{}\\
&= \frac{4a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{16a^2\sqrt{\sec(c+dx)}\sin(c+}{5d}\\
&= -\frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{3}
\end{aligned}$$

Mathematica [C] time = 1.85274, size = 261, normalized size = 1.62

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}(24\csc(c)\cos(dx)+\tan(c+dx)(3\sec(c+dx)+10))-\frac{2i\sqrt{2}e^{-i(c+dx)}}{}\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(12*(1 + E^((2*I)*(c + d*x))) + 12*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))
/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(24*Cos[d*x]*Csc
[c] + (10 + 3*Sec[c + d*x])*Tan[c + d*x])))/(30*d)
```

Maple [B] time = 4.154, size = 386, normalized size = 2.4

$$-8\frac{\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}a^{2}}}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(-1/12\frac{\cos\left(1/2\,dx+c/2\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\left(\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1/2\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(7/2),x)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-1/12*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+17/30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(7/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

3.299 $\int (a+a\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2\sqrt{\cos}}{}$$

```
[Out] (-4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (4*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.133578, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (4*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx &= \int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2\,dx\\
&= \left(2a^2\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx+\int \sqrt{\sec(c+dx)}\left(a^2+a^2\sec^2(c+dx)\right)dx\\
&= \frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(4a^2\right)\int \sqrt{\mathrm{s}\ldots}\\
&= \frac{4a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(4a^2\sqrt{\cos(c\ldots}\right.\\
&= -\frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\ldots\right.}{3d}
\end{aligned}$$

Mathematica [C] time = 1.30895, size = 250, normalized size = 1.91

$$\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}(\tan(c+dx)+6\csc(c)\cos(dx))-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(-1+e^{2ic}\right)\sqrt{1\ldots}\right.}{\ldots}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E
^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -E^((2*I)*(c + d*x))] + 2*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(
E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c]
 + Tan[c + d*x])))/(6*d)
```

Maple [B] time = 3.681, size = 371, normalized size = 2.8

$$\frac{4a^2}{3d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(4*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+7*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\cos(dx+c)^2 + 2\,a^2\cos(dx+c) + a^2\right)\sec(dx+c)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.300 $\int (a + a\cos(c + dx))^2 \sec^{\frac{3}{2}}(c + dx)\,dx$

Optimal. Leaf size=64

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.107616, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3788, 3771, 2641, 4043}

$$\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4043

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] /;
 FreeQ[{b, e, f, A, C, m}, x] && EqQ[C*m + A*(m + 1), 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(2a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+$$
$$= \frac{4a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+d}{d}$$

Mathematica [A] time = 0.142289, size = 48, normalized size = 0.75

$$\frac{2a^2\sqrt{\sec(c+dx)}\left(\sin(c+dx)+2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[Sec[c + d*x]]*(2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
 Sin[c + d*x]))/d
```

Maple [A] time = 2.31, size = 104, normalized size = 1.6

$$-4\frac{a^2\left(\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-\left(\sin(1/2\,dx+c/2)\right)^2\cos}{\sin(1/2\,dx+c/2)\sqrt{2\left(\cos(1/2\,dx+c/2)\right)^2-1}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(3/2),x)
```

```
[Out] -4*a^2*((2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.301 $\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.122386, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3788, 3771, 2639, 4045, 2641}

$$\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx=\int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\left(2a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(4a^2\right)\int \sqrt{\sec(c+dx)}\,dx+\left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$
$$=\frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(4a^2\sqrt{\text{co}}\right.$$
$$=\frac{4a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{8a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{3d}$$

Mathematica [C] time = 1.00238, size = 127, normalized size = 1.19

$$\frac{a^2\left(\frac{24i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-4i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+\sin(c+dx)-6i\right)\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*(((24*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt
[1 + E^((2*I)*(c + d*x))] + 2*(-6*I - (4*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c
+ d*x])))/(3*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 2.292, size = 228, normalized size = 2.1

$$-\frac{4a^2}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+2\sqrt{2\left(\sin(1/2\,dx+c/2)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(2*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d*x+
```

```
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\cos(c+dx)\sqrt{\sec(c+dx)}\,dx+\int \cos^2(c+dx)\sqrt{\sec(c+dx)}\,dx+\int \sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2*sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)*sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x
)**2*sqrt(sec(c + d*x)), x) + Integral(sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.302 $\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{5d}$$

```
[Out] (16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.137919, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (16*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a^2*Sin[c + d
*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx &= \int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \left(2a^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}\left(8a^2\right)\int \frac{1}{\sqrt{\sec(c+
\end{aligned}$$

$$\begin{aligned}
&= \frac{2a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx}} \\
&= \frac{16a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{3d}
\end{aligned}$$

Mathematica [C] time = 1.46604, size = 136, normalized size = 1.01

$$\frac{a^2\left(\frac{192i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 40i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 40\sin(c+dx) + 6\sin(2(c+dx)) - 9}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^2*(-96*I + ((192*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (40*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 40*Sin
[c + d*x] + 6*Sin[2*(c + d*x)]))/(30*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 2.125, size = 250, normalized size = 1.9

$$-\frac{4a^2}{15d}\sqrt{\left(2\left(\cos\left(1/2dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-12\left(\sin\left(1/2dx+c/2\right)\right)^6\cos\left(1/2dx+c/2\right)+32\left(\sin\left(1/2dx+c/2\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+32*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-13*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^2}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^2\cos\left(dx+c\right)^2+2a^2\cos\left(dx+c\right)+a^2}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/sqrt(sec(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{2\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{\cos^2\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{1}{\sqrt{\sec\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x
)**2/sqrt(sec(c + d*x)), x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.303 $\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2\sqrt{\cos(c+}{}$$

```
[Out] (12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]
])
```

Rubi [A] time = 0.150534, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{8a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{7d}+\frac{12a^2\sqrt{\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```


```
[Out] (12*a^2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (8*a^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(7*d) + (2*a^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*a^2*Sin[c + d
*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^2*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]
])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \left(2a^2\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{a^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}\left(6a^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}\left(12a^2\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+}$$
$$= \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^2\sin(c+dx)}{7d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(4a^2\right)\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}$$
$$= \frac{12a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} +$$
$$= \frac{12a^2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{7d}$$

Mathematica [C] time = 1.66868, size = 149, normalized size = 0.93

$$\frac{a^2\left(\frac{672i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} + 2\left(-80i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 85\sin(c+dx) + 28\sin(2(c+d\right.\right.}{140d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^2*(((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-168*I - (80*I)*Sqrt[1 + E^((2*I)*(c + d*x)
)]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 85
*Sin[c + d*x] + 28*Sin[2*(c + d*x)] + 5*Sin[3*(c + d*x)])))/(140*d*Sqrt[Sec
```

```
[c + d*x]])
```

Maple [A] time = 2.04, size = 272, normalized size = 1.7

$$-\frac{4a^2}{35d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(40\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8-116\left(\sin\left(1/2\,dx+c/2\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/35*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(40*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-116*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+126*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+10*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-39*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)/sec(d*x + c)^(3/2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{2\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{\cos^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(2*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(cos(c + d*
x)**2/sec(c + d*x)**(3/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.304 $\int (a+a\cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=187

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)\sqrt{\sec(c+$$

```
[Out] (-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.233406, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3768, 3771, 2639, 2641}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{52a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{28a^3\sin(c+dx)\sqrt{\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (28*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (52*a^3*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(5*d) + (2*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3\,dx$$
$$= \int \left(a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+3a^3\sec^{\frac{7}{2}}(c+dx)+a^3\sec^{\frac{9}{2}}(c+dx)\right.$$
$$= a^3\int \sec^{\frac{3}{2}}(c+dx)\,dx+a^3\int \sec^{\frac{9}{2}}(c+dx)\,dx+\left(3a^3\right)\int \sec^{\frac{5}{2}}(c+dx)\,dx+\left(\right.$$
$$= \frac{2a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}+\frac{6a^3\sec^{\frac{5}{2}}(c+}{}$$
$$= \frac{28a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{52a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{6a^3\sec^{\frac{5}{2}}(c}{}$$
$$= -\frac{2a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$
$$= -\frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{52a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{2}$$

Mathematica [C] time = 2.71249, size = 279, normalized size = 1.49

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\left(294\csc(c)\cos(dx)+(63\cos(c+dx)+65\cos(2(c+dx))+80)\,\mathrm{ta}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(294*Cos[d*x]
*Csc[c] + (80 + 63*Cos[c + d*x] + 65*Cos[2*(c + d*x)])*Sec[c + d*x]^2*Tan[c
 + d*x])))/(420*d)
```

Maple [B] time = 4.434, size = 439, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(9/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-13/168*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(co
s(1/2*d*x+1/2*c)^2-1/2)^2+53/105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-7/20*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/448*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-3/16
0*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(
cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

3.305 $\int (a + a\cos(c + dx))^3 \sec^{\frac{7}{2}}(c + dx)\, dx$

Optimal. Leaf size=157

$$\frac{2a^3 \sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d} + \frac{2a^3 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d} + \frac{36a^3 \sin(c+dx)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}$$

```
[Out] (-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.205886, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3771, 2641, 3768, 2639}

$$\frac{2a^3 \sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d} + \frac{2a^3 \sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d} + \frac{36a^3 \sin(c+dx)\sqrt{\sec(c+dx)}}{5d} + \frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (36*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a^3*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx=\int \sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3\,dx$$
$$=\int \left(a^3\sqrt{\sec(c+dx)}+3a^3\sec^{\frac{3}{2}}(c+dx)+3a^3\sec^{\frac{5}{2}}(c+dx)+a^3\sec^{\frac{7}{2}}(c+dx)\right.$$
$$=a^3\int \sqrt{\sec(c+dx)}\,dx+a^3\int \sec^{\frac{7}{2}}(c+dx)\,dx+\left(3a^3\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx+($$
$$=\frac{6a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d}+\frac{2a^3\sec^{\frac{5}{2}}(c+}{5}$$
$$=\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{36a^3\sqrt{\sec(c+dx)}\sin(c+}{5d}$$
$$=-\frac{6a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$
$$=-\frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{d}$$

Mathematica [C] time = 1.76233, size = 259, normalized size = 1.65

$$\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}(18\csc(c)\cos(dx)+\tan(c+dx)(\sec(c+dx)+5))-\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{}{1}}}{}\right.}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E
^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4,
 -E^((2*I)*(c + d*x))] + 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(
E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c
] + (5 + Sec[c + d*x])*Tan[c + d*x])))/(20*d)
```

Maple [B] time = 3.892, size = 386, normalized size = 2.5

$$-16\frac{\sqrt{-\left(-2(\cos(1/2dx+c/2))^2+1\right)(\sin(1/2dx+c/2))^2}a^3}{\sin(1/2dx+c/2)\sqrt{2(\cos(1/2dx+c/2))^2-1}d}\left(\frac{7\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{-2(\cos(1/2dx+c/2))^2+1}\mathrm{E}}{10\sqrt{-2(\sin(1/2dx+c/2))^4+(\sin(1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(7/2),x)
```

```
[Out] -16*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(7/10*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/16*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-9/10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-9/20*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/160*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

3.306 $\int (a + a\cos(c + dx))^3 \sec^{\frac{5}{2}}(c + dx)\, dx$

Optimal. Leaf size=131

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d} + \frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{4a^3\sqrt{\mathrm{co}}\ldots}{\ldots}$$

```
[Out] (-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.17981, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3771, 2639, 2641, 3768}

$$\frac{2a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d} + \frac{6a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{4a^3\sqrt{\mathrm{co}}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(3*d) + (6*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^3*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \int \left(\frac{a^3}{\sqrt{\sec(c+dx)}} + 3a^3\sqrt{\sec(c+dx)} + 3a^3\sec^{\frac{3}{2}}(c+dx) + a^3\sec^{\frac{5}{2}}(c+dx)\right)$$
$$= a^3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^3\int \sec^{\frac{5}{2}}(c+dx)\,dx + \left(3a^3\right)\int \sqrt{\sec(c+dx)}\,dx +$$
$$= \frac{6a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}a^3\int \sqrt{\sec(}$$
$$= \frac{2a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$
$$= -\frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{20a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{3d}$$

Mathematica [C] time = 0.952797, size = 157, normalized size = 1.2

$$-\frac{ia^3\sec^{\frac{3}{2}}(c+dx)\left(6e^{-2i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+20\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\cos(c\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

```
[Out] ((-I/3)*a^3*Sec[c + d*x]^(3/2)*(-6 - 6*Cos[2*(c + d*x)] + (6*(1 + E^((2*I)*
(c + d*x)))^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/
E^((2*I)*(c + d*x)) + 20*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))] + (2*I)*Sin[c + d*x] + (9*I
)*Sin[2*(c + d*x)]))/d
```

Maple [B] time = 3.743, size = 371, normalized size = 2.8

$$\frac{4a^3}{3d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(10\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(5/2),x)
```

```
[Out] 4/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(10*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2-18*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))+10*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

3.307 $\int (a + a\cos(c + dx))^3 \sec^{\frac{3}{2}}(c + dx)\,dx$

Optimal. Leaf size=131

$$\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^3\sqrt{\cos(c+dx)}\sqrt{s}}{}$$

```
[Out] (4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.176438, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{2a^3\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{20a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4a^3\sqrt{\cos(c+dx)}\sqrt{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```


```
[Out] (4*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (20*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (2*a^3*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^3*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/d
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx &= \int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\
&= \int \left(\frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+3a^3\sqrt{\sec(c+dx)}+a^3\sec^{\frac{3}{2}}(c+dx)\right)dx\\
&= a^3\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+a^3\int \sec^{\frac{3}{2}}(c+dx)\,dx+\left(3a^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\\
&= \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{1}{3}a^3\int \sqrt{\sec(c+dx)}\,dx-\\
&= \frac{6a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{6a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}\\
&= \frac{4a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}
\end{aligned}$$

Mathematica [C] time = 1.28915, size = 135, normalized size = 1.03

$$\frac{a^3\left(\frac{24i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-10i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+\sin(c+dx)+3\tan(c+dx)-6i\right.\right.}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(((24*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqrt
[1 + E^((2*I)*(c + d*x))] + 2*(-6*I - (10*I)*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + Sin[c
 + d*x] + 3*Tan[c + d*x])))/(3*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 2.476, size = 172, normalized size = 1.3

$$-\frac{4a^3}{3d}\left(2\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+5\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\mathrm{EllipticF}\left(\cos(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(3/2),x)
```

```
[Out] -4/3*a^3*(2*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-4*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.308 $\int (a + a\cos(c + dx))^3 \sqrt{\sec(c + dx)}\, dx$

Optimal. Leaf size=131

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\big(}{5d}$$

```
[Out] (36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])
/(d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.178093, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2639, 2641}

$$\frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{4a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{36a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\big(}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```


```
[Out] (36*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (4*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a^3*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a^3*Sin[c + d*x])
/(d*Sqrt[Sec[c + d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx &= \int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \int \left(\frac{a^3}{\sec^{\frac{5}{2}}(c+dx)}+\frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)}+\frac{3a^3}{\sqrt{\sec(c+dx)}}+a^3\sqrt{\sec(c+dx)}\right)dx \\
&= a^3\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+a^3\int \sqrt{\sec(c+dx)}\,dx+\left(3a^3\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+ \\
&= \frac{2a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}}+\frac{1}{5}\left(3a^3\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+a^3\int \sqrt{\mathrm{s}} \\
&= \frac{6a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d} \\
&= \frac{36a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}
\end{aligned}$$

Mathematica [C] time = 1.25303, size = 137, normalized size = 1.05

$$\frac{a^3\left(\frac{144i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}}+2\left(-20i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx)+10\sin(c+dx)+\sin(2(c+dx))\right.\right.}{10d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*(((144*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))])/Sqr
t[1 + E^((2*I)*(c + d*x))] + 2*(-36*I - (20*I)*Sqrt[1 + E^((2*I)*(c + d*x))
]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 10*
Sin[c + d*x] + Sin[2*(c + d*x)])))/(10*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 2.009, size = 250, normalized size = 1.9

$$-\frac{4a^3}{5d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+14\left(\sin(1/2\,dx+c/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3*sec(d*x+c)^(1/2),x)
```

```
[Out] -4/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-4*sin(1/
2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+5*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-6*sin(1/2*d*x+1/2*c
)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.309 $\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=161

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3\sqrt{\mathrm{co}}}{}$$

```
[Out] (28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
d*x]])
```

Rubi [A] time = 0.208761, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2641, 2639}

$$\frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{52a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{52a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{28a^3\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```



```
[Out] (28*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (52*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(21*d) + (2*a^3*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (6*a^3*Sin[c +
 d*x])/(5*d*Sec[c + d*x]^(3/2)) + (52*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}dx = \int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}dx$$

$$= \int \left(\frac{a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{3}{2}}(c+dx)} + \frac{a^3}{\sqrt{\sec(c+dx)}}\right)dx$$

$$= a^3\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}dx + a^3\int \frac{1}{\sqrt{\sec(c+dx)}}dx + \left(3a^3\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}dx + \left(3a^3\right)\int \frac{}{\sec^{\frac{3}{2}}(}$$

$$= \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^3\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}dx + a^3\int$$

$$= \frac{2a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{52a}{21d}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{d}$$

$$= \frac{28a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{52a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{21d}$$

Mathematica [C] time = 1.76873, size = 146, normalized size = 0.91

$$\frac{a^3\left(\frac{4704i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 1040i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 1070\sin(c+dx) + 252\sin(2(c+dx}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^3*(-2352*I + ((4704*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (1040*I)*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 1
070*Sin[c + d*x] + 252*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqrt
[Sec[c + d*x]])
```

Maple [A] time = 2.339, size = 272, normalized size = 1.7

$$-\frac{4a^{3}}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(120\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-432\left(\sin\left(1/2\,dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-432*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1
/2*c)+602*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+65*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-208*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{3}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int \frac{3\cos\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{3\cos^{2}\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{\cos^{3}\left(c+dx\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int \frac{1}{\sqrt{\sec\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)/sqrt(sec(c + d*x)), x) + Integral(3*cos(c + d
*x)**2/sqrt(sec(c + d*x)), x) + Integral(cos(c + d*x)**3/sqrt(sec(c + d*x))
, x) + Integral(1/sqrt(sec(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.310 $\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{21d}$$

```
[Out] (68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
5*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]
^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.234638, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2639, 2641}

$$\frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{44a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```


```
[Out] (68*a^3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(1
5*d) + (44*a^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^3*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (6*a^3*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (68*a^3*Sin[c + d*x])/(45*d*Sec[c + d*x]
^(3/2)) + (44*a^3*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \int \left(\frac{a^3}{\sec^{\frac{9}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{7}{2}}(c+dx)} + \frac{3a^3}{\sec^{\frac{5}{2}}(c+dx)} + \frac{a^3}{\sec^{\frac{3}{2}}(c+dx)}\right) dx$$
$$= a^3 \int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^3 \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(3a^3\right) \int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(3a^3\right) \int \frac{1}{\sec^{\frac{5}{2}}(c}$$
$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a^3\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}a^3 \int \sqrt{\sec(c+}$$
$$= \frac{2a^3\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^3\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{68a^3\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{44a^3\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{15}\left(7a^3\right) \int$$
$$= \frac{18a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{3d}$$
$$= \frac{68a^3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{44a^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{21d}$$

Mathematica [C] time = 2.18728, size = 156, normalized size = 0.83

$$\frac{a^3\left(\frac{22848i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 5280i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 5820\sin(c+dx) + 2044\sin(2(c+\right.}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^3*(-11424*I + ((22848*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (5280*I)*Sqrt[1 + E^((2*I)*(c + d
*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
 5820*Sin[c + d*x] + 2044*Sin[2*(c + d*x)] + 540*Sin[3*(c + d*x)] + 70*Sin[
4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 2.276, size = 260, normalized size = 1.4

$$-\frac{4a^3}{315d}\sqrt{\left(2\left(\cos\left(1/2dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(560\left(\cos\left(1/2dx+c/2\right)\right)^{11}-600\left(\cos\left(1/2dx+c/2\right)\right)^9+212\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(560*cos
(1/2*d*x+1/2*c)^11-600*cos(1/2*d*x+1/2*c)^9+212*cos(1/2*d*x+1/2*c)^7+66*cos
(1/2*d*x+1/2*c)^5-430*cos(1/2*d*x+1/2*c)^3+165*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-35
7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))+192*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\cos\left(dx+c\right)+a\right)^3}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^3\cos\left(dx+c\right)^3+3a^3\cos\left(dx+c\right)^2+3a^3\cos\left(dx+c\right)+a^3}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int\frac{3\cos\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{3\cos^2\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{\cos^3\left(c+dx\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{1}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] a**3*(Integral(3*cos(c + d*x)/sec(c + d*x)**(3/2), x) + Integral(3*cos(c +
d*x)**2/sec(c + d*x)**(3/2), x) + Integral(cos(c + d*x)**3/sec(c + d*x)**(3
/2), x) + Integral(sec(c + d*x)**(-3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.311 $\int (a+a\cos(c+dx))^4 \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=187

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{\sec(}}{5d}$$

```
[Out] (-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.253234, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3771, 2641, 3768, 2639}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}+\frac{8a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{94a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{64a^4\sin(c+dx)\sqrt{\sec(}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(21*d) + (64*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (94*a^4*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (8*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*
x])/(5*d) + (2*a^4*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a + a\cos(c+dx))^4 \sec^{\frac{9}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}(a + a\sec(c+dx))^4\,dx$$
$$= \int \left(a^4\sqrt{\sec(c+dx)} + 4a^4\sec^{\frac{3}{2}}(c+dx) + 6a^4\sec^{\frac{5}{2}}(c+dx) + 4a^4\sec^{\frac{7}{2}}(c+dx) + \right.$$
$$= a^4\int \sqrt{\sec(c+dx)}\,dx + a^4\int \sec^{\frac{9}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sec^{\frac{3}{2}}(c+dx)\,dx + \left(4a^4\right.$$
$$= \frac{8a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{4a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{5}{2}}(c+dx)}{5d}$$
$$= \frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{64a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}$$
$$= -\frac{8a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{6a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{d}$$
$$= -\frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{21d}$$

Mathematica [C] time = 1.84398, size = 271, normalized size = 1.45

$$a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}\left(672\csc(c)\cos(dx) + \tan(c+dx)\left(15\sec^2(c+dx) + 84\sec(c+dx) + \right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(((-4*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(168*(1 + E^((2*I)*(c + d*x))) + 168*(-1
 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2,
3/4, -E^((2*I)*(c + d*x))] + 85*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))
]))/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(672*Cos[d*x]
*Csc[c] + (235 + 84*Sec[c + d*x] + 15*Sec[c + d*x]^2)*Tan[c + d*x])))/(840*
d)
```

Maple [B] time = 4.226, size = 439, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(9/2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(253/420*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-47/672*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-4/5*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-2/5*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-1/80*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-1/896*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^4)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4\right)\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(9/2), x)
```

3.312 $\int (a+a\cos(c+dx))^4 \sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{}$$

```
[Out] (-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d)
```

Rubi [A] time = 0.226644, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3771, 2639, 2641, 3768}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{8a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{66a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(3*d) + (66*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (8*a^4*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^4*Sec[c + d*x]^(5/2)*Sin[c + d*x])/
(5*d)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^{\frac{7}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \int \left(\frac{a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)} + 6a^4\sec^{\frac{3}{2}}(c+dx) + 4a^4\sec^{\frac{5}{2}}(c+dx) + a$$
$$= a^4\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + a^4\int \sec^{\frac{7}{2}}(c+dx)\,dx + \left(4a^4\right)\int \sqrt{\sec(c+dx)}\,dx + \left(4a\right.$$
$$= \frac{12a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{8a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^4\sec^{\frac{5}{2}}(c+dx}{5d}$$
$$= \frac{2a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{d}$$
$$= -\frac{10a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}$$
$$= -\frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}$$

Mathematica [C] time = 2.41061, size = 278, normalized size = 1.73

$$a^4(\cos(c+dx)+1)^4\sec^8\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sec(c+dx)}(30\cos(c)\sin(dx) - 3(5\cos(2c)-61)\csc(c)\cos(dx) + 2\tan(c+dx)(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*(((-8*I)*Sqrt[2]*Sqrt[E^(I*(c
+ d*x))/(1 + E^((2*I)*(c + d*x)))]*(21*(1 + E^((2*I)*(c + d*x))) + 21*(-1 +
 E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/
4, -E^((2*I)*(c + d*x))] + 20*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])
)/(E^(I*(c + d*x))*(-1 + E^((2*I)*c))) + Sqrt[Sec[c + d*x]]*(-3*(-61 + 5*Co
s[2*c])*Cos[d*x]*Csc[c] + 30*Cos[c]*Sin[d*x] + 2*(20 + 3*Sec[c + d*x])*Tan[
c + d*x])))/(240*d)
```

Maple [B] time = 3.829, size = 386, normalized size = 2.4

$$-32\frac{\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}a^{4}}{\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(-\frac{7\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{20\sqrt{-2\left(\sin\left(1/\right.\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(7/2),x)
```

```
[Out] -32*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(-7/20*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+41/60*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/24*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
-1/2)^2-33/40*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2
*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)-1/320*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(dx+c\right)+a\right)^{4}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{4}\cos\left(dx+c\right)^{4}+4\,a^{4}\cos\left(dx+c\right)^{3}+6\,a^{4}\cos\left(dx+c\right)^{2}+4\,a^{4}\cos\left(dx+c\right)+a^{4}\right)\sec\left(dx+c\right)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(7/2), x)
```

3.313 $\int (a + a\cos(c + dx))^4 \sec^{\frac{5}{2}}(c + dx)\,dx$

Optimal. Leaf size=118

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d} + \frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{40a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{3d}$$

```
[Out] (40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.210607, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3791, 3769, 3771, 2641, 2639, 3768}

$$\frac{2a^4\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d} + \frac{8a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{40a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(5/2),x]
```

```
[Out] (40*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*a^4*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (8*a^4*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d + (2*a^4*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4\sec^{\frac{5}{2}}(c+dx)\,dx=\int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\int \left(\frac{a^4}{\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^4}{\sqrt{\sec(c+dx)}}+6a^4\sqrt{\sec(c+dx)}+4a^4\sec^{\frac{3}{2}}(c+dx)+a^4\,\text{se}\right.$$
$$=a^4\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx+a^4\int \sec^{\frac{5}{2}}(c+dx)\,dx+\left(4a^4\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+\left(4a\right.$$
$$=\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^4\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}$$
$$=\frac{8a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{12a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{d}$$
$$=\frac{40a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{8a^4\sqrt{\sec(}}{}$$

Mathematica [A] time = 0.334052, size = 70, normalized size = 0.59

$$\frac{a^4\sec^{\frac{3}{2}}(c+dx)\left(5\sin(c+dx)+24\sin(2(c+dx))+\sin(3(c+dx))+80\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(5/2),x]
```

```
[Out] (a^4*Sec[c + d*x]^(3/2)*(80*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] +
5*Sin[c + d*x] + 24*Sin[2*(c + d*x)] + Sin[3*(c + d*x)]))/(6*d)
```

Maple [B] time = 3.509, size = 292, normalized size = 2.5

$$\frac{8a^4}{3d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\left(\sin(1/2\,dx+c/2)\right)^6\cos(1/2\,dx+c/2)+10\sqrt{2\left(\sin(1/2\,dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(5/2),x)
```

```
[Out] 8/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4/(4*sin(1/
2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)+10*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2-14*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-5*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+7
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4\cos(dx+c)^4+4\,a^4\cos(dx+c)^3+6\,a^4\cos(dx+c)^2+4\,a^4\cos(dx+c)+a^4\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(5/2), x)
```

3.314 $\int (a + a\cos(c + dx))^4 \sec^{\frac{3}{2}}(c + dx)\, dx$

Optimal. Leaf size=159

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] (56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.211547, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3791, 3769, 3771, 2639, 2641, 3768}

$$\frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(3/2),x]
```


```
[Out] (56*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*d) + (2*a^4*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (8*a^4*Sin[c +
d*x])/(3*d*Sqrt[Sec[c + d*x]]) + (2*a^4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\cos(c+dx))^4 \sec^{\frac{3}{2}}(c+dx)\,dx = \int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \int \left(\frac{a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{6a^4}{\sqrt{\sec(c+dx)}} + 4a^4\sqrt{\sec(c+dx)} + a^4\sec^{\frac{3}{2}}(\right.$$

$$= a^4\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + a^4\int \sec^{\frac{3}{2}}(c+dx)\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \left(4a\right.$$

$$= \frac{2a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2a^4\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{5}\left(3a^4\right)\int$$

$$= \frac{12a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{8a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{d}$$

$$= \frac{56a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{3d}$$

Mathematica [C] time = 1.45534, size = 150, normalized size = 0.94

$$\frac{a^4\left(\frac{672i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 320i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 80\sin(c+dx) + 63\tan(c+dx) + 3\,\mathrm{si}\right.}{30d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^4*(-336*I + ((672*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*
x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (320*I)*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] + 80*S
in[c + d*x] + 3*Sec[c + d*x]*Sin[3*(c + d*x)] + 63*Tan[c + d*x]))/(30*d*Sqr
t[Sec[c + d*x]])
```

Maple [A] time = 2.5, size = 194, normalized size = 1.2

$$-\frac{8a^4}{15d}\left(-6\left(\sin\left(1/2dx+c/2\right)\right)^6\cos\left(1/2dx+c/2\right)+26\left(\sin\left(1/2dx+c/2\right)\right)^4\cos\left(1/2dx+c/2\right)+20\sqrt{2\left(\sin\left(1/2dx+c/\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(3/2),x)
```

```
[Out] -8/15*a^4*(-6*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+26*sin(1/2*d*x+1/2*c)
^4*cos(1/2*d*x+1/2*c)+20*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-19*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^4\cos(dx+c)^4+4a^4\cos(dx+c)^3+6a^4\cos(dx+c)^2+4a^4\cos(dx+c)+a^4\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4 \sec(dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sec(d*x + c)^(3/2), x)
```

3.315 $\int (a+a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4\sqrt{\mathrm{c}}}{}$$

```
[Out] (64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]])
```

Rubi [A] time = 0.227094, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2641, 2639}

$$\frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{94a^4\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{136a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{64a^4\sqrt{\mathrm{c}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (64*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (136*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(21*d) + (2*a^4*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (8*a^4*Sin[c
+ d*x])/(5*d*Sec[c + d*x]^(3/2)) + (94*a^4*Sin[c + d*x])/(21*d*Sqrt[Sec[c +
 d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a + a\cos(c+dx))^4\sqrt{\sec(c+dx)}\,dx &= \int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{7}{2}}(c+dx)}\,dx \\
&= \int \left(\frac{a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4}{\sqrt{\sec(c+dx)}} + a^4\sqrt{\sec(c+\right. \\
&= a^4\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + a^4\int \sqrt{\sec(c+dx)}\,dx + \left(4a^4\right)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \left(4a\right. \\
&= \frac{2a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^4\sin(c+dx)}{d\sqrt{\sec(c+dx)}} + \frac{1}{7}\left(5a^4\right)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx}\right. \\
&= \frac{8a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{d} \\
&= \frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{6a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{d} \\
&= \frac{64a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{136a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{21d}
\end{aligned}$$

Mathematica [C] time = 1.55245, size = 146, normalized size = 0.91

$$\frac{a^4\left(\frac{10752i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 2720i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 1910\sin(c+dx) + 336\sin(2(c+d\right.}{420d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^4*(-5376*I + ((10752*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c +
 d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (2720*I)*Sqrt[1 + E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x] +
1910*Sin[c + d*x] + 336*Sin[2*(c + d*x)] + 30*Sin[3*(c + d*x)]))/(420*d*Sqr
t[Sec[c + d*x]])
```

Maple [A] time = 2.192, size = 272, normalized size = 1.7

$$-\frac{8a^4}{105d}\sqrt{\left(2\left(\cos\left(1/2dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(60\cos\left(1/2dx+c/2\right)\left(\sin\left(1/2dx+c/2\right)\right)^8-258\left(\sin\left(1/2dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*sec(d*x+c)^(1/2),x)
```

```
[Out] -8/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(60*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-258*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/
2*c)+448*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+85*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-168*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-167*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(dx+c\right)+a\right)^4\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^4\cos\left(dx+c\right)^4+4\,a^4\cos\left(dx+c\right)^3+6\,a^4\cos\left(dx+c\right)^2+4\,a^4\cos\left(dx+c\right)+a^4\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos (dx+c)+a)^4\sqrt {\sec (dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*sqrt(sec(d*x + c)), x)
```

3.316 $\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=187

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{7d}$$

```
[Out] (152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x
]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.255478, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3791, 3769, 3771, 2639, 2641}

$$\frac{122a^4\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{32a^4\sin(c+dx)}{7d\sqrt{\sec(c+dx)}}+\frac{32a^4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (152*a^4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
15*d) + (32*a^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(7*d) + (2*a^4*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (8*a^4*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2)) + (122*a^4*Sin[c + d*x])/(45*d*Sec[c + d*x
]^(3/2)) + (32*a^4*Sin[c + d*x])/(7*d*Sqrt[Sec[c + d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3791

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^4}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(a+a\sec(c+dx))^4}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \int \left(\frac{a^4}{\sec^{\frac{9}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{7}{2}}(c+dx)} + \frac{6a^4}{\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^4}{\sec^{\frac{3}{2}}(c+dx)} + \frac{a^4}{\sqrt{\sec(c+dx)}}\right) dx$$
$$= a^4 \int \frac{1}{\sec^{\frac{9}{2}}(c+dx)}\,dx + a^4 \int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(4a^4\right) \int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \left(4a^4\right) \int \frac{}{\sec^{\frac{3}{2}}(}$$
$$= \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{12a^4\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{9}\left(7a^4\right) \int \frac{}{\sec^{\frac{5}{2}}}$$
$$= \frac{2a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^4\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{8a^4\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{122}{45d}$$
$$= \frac{46a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{3d}$$
$$= \frac{152a^4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{32a^4\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{7d}$$

Mathematica [C] time = 2.12605, size = 156, normalized size = 0.83

$$\frac{a^4\left(\frac{51072i\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)}{\sqrt{1+e^{2i(c+dx)}}} - 11520i\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\sec(c+dx) + 12240\sin(c+dx) + 3556\sin(2(c\right.}{2520d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^4*(-25536*I + ((51072*I)*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c
+ d*x))])/Sqrt[1 + E^((2*I)*(c + d*x))] - (11520*I)*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]*Sec[c + d*x]
+ 12240*Sin[c + d*x] + 3556*Sin[2*(c + d*x)] + 720*Sin[3*(c + d*x)] + 70*Si
n[4*(c + d*x)]))/(2520*d*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 2.44, size = 260, normalized size = 1.4

$$-\frac{8 a^4}{315 d}\sqrt{\left(2 \left(\cos \left(1/2 dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(280 \left(\cos \left(1/2 dx+c/2\right)\right)^{11}-120 \left(\cos \left(1/2 dx+c/2\right)\right)^9+34 \left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4/sec(d*x+c)^(1/2),x)
```

```
[Out] -8/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^4*(280*cos
(1/2*d*x+1/2*c)^11-120*cos(1/2*d*x+1/2*c)^9+34*cos(1/2*d*x+1/2*c)^7+72*cos(
1/2*d*x+1/2*c)^5-485*cos(1/2*d*x+1/2*c)^3+180*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-399
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))+219*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \cos \left(dx+c\right)+a\right)^4}{\sqrt{\sec \left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{a^4 \cos \left(dx+c\right)^4+4 a^4 \cos \left(dx+c\right)^3+6 a^4 \cos \left(dx+c\right)^2+4 a^4 \cos \left(dx+c\right)+a^4}{\sqrt{\sec \left(dx+c\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^4}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4/sqrt(sec(d*x + c)), x)
```

3.317 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=164

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\big(}{3ad}$$

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d
) - (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (5*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*a*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
))
```

Rubi [A] time = 0.167667, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3818, 3787, 3768, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\big(}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```


```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*d
) - (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + (5*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(3*a*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]
))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{5}{2}a\sec(c+dx)\right)dx}{a^2}$$
$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a} + \frac{5\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a}$$
$$= -\frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{5\int \sqrt{\mathrm{s}}$$
$$= -\frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} + \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(5\sqrt{\mathrm{co}}\right.}{}$$
$$= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3ad} -$$

Mathematica [C] time = 3.17333, size = 285, normalized size = 1.74

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\sqrt{\sec(c+dx)}\left(18\csc(c)\cos(dx)+\sec(c+dx)\left(\tan\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\right)\right)\right.}{3ad(\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - 5
*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^((
2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*Cos[d*x]*Csc[c] + Sec[c + d*x]*(-5*Sec[(
c + d*x)/2]*Sin[(3*(c + d*x))/2] + Tan[(c + d*x)/2])))/(3*a*d*(1 + Cos[c +
```

```
d*x]))
```

Maple [B] time = 3.859, size = 413, normalized size = 2.5

$$\frac{1}{3da}\sqrt{-\left(-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(10\sqrt{\left(\sin\left(1/2\,dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] 1/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a/cos(1/2*d*x
+1/2*c)/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2
+1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-18*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-36*sin(1/2*d*x+1/2*c)^6-5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*cos(1/2*d*x+1/2*c)+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*cos(1/2*d
*x+1/2*c)+44*sin(1/2*d*x+1/2*c)^4-11*sin(1/2*d*x+1/2*c)^2)/(2*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a), x)
```

3.318 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3\sqrt{\cos(c+dx)}}{}$$

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.154284, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3818, 3787, 3771, 2641, 3768, 2639}

$$-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{3\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```


```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(d*(a + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$
$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{3}{2}a\sec(c+dx)\right)dx}{a^2}$$
$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a} + \frac{3\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a}$$
$$= \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\ldots\right.}{\ldots}$$
$$= -\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{3\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(\ldots}{d(a+a\sec(c+\ldots}$$
$$= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \ldots$$

Mathematica [C] time = 1.90738, size = 256, normalized size = 1.88

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\sqrt{\sec(c+dx)}\left(6\csc(c)\cos(dx)-2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\left(3\left(-1+e^{2ic}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-\left(-1+e^{2ic}\ldots\right.}{\left(-1+e^{2ic}\right)d}\right.}{a(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] -
E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^(
(2*I)*c))) + (Sqrt[Sec[c + d*x]]*(6*Cos[d*x]*Csc[c] - 2*Tan[(c + d*x)/2]))/
```

```
d))/(a*(1 + Cos[c + d*x]))
```

Maple [A] time = 2.668, size = 253, normalized size = 1.9

$$-\frac{1}{da}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\mathrm{El}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] -(-cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+6*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-5*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a/cos
(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a\cos\left(dx+c\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a), x)
```

3.319 $\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.144357, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3818, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x]),x]
```


```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) + (
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
 b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{a+a\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{a}{2}-\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a} + \left(\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.$$

$$= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \sqrt{}$$

Mathematica [C] time = 0.981239, size = 180, normalized size = 1.64

$$-\frac{4i\left(-\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)+e^{2}\right.}{ad\left(1+e^{i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x]),x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]^2*(1 + E^((2*I)*(c + d*x)) - (1 + E^(I*(c + d*x)))
*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*
(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c +
 d*x]])/(a*d*(1 + E^(I*(c + d*x)))^3)
```

Maple [A] time = 2.394, size = 200, normalized size = 1.8

$$\frac{1}{da}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\mathrm{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
```

```
cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a), x)
```

3.320 $\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=110

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d))
+ (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.138834, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3820, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```



```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d))
+ (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) +
 (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3820

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 1))/(a*f*(
a + b*Csc[e + f*x])), x] + Dist[(d*(n - 1))/(a*b), Int[(d*Csc[e + f*x])^(n
- 1)*(a - b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^
2 - b^2, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+a\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx &= \int \frac{\sqrt{\sec(c+dx)}}{a+a\sec(c+dx)}\,dx\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \frac{a-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{2a^2}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a}+\frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}\\ &= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}-\frac{(\\ &= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{ad}\end{aligned}$$

Mathematica [C] time = 0.952056, size = 181, normalized size = 1.65

$$-\frac{4i\left(\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)-e}{ad\left(1+e^{i(c+dx)}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] ((-4*I)*Cos[(c + d*x)/2]^2*(-1 - E^((2*I)*(c + d*x)) + (1 + E^(I*(c + d*x))
)*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)
*(c + d*x))] + E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c
+ d*x]])/(a*d*(1 + E^(I*(c + d*x)))^3)
```

Maple [A] time = 2.007, size = 198, normalized size = 1.8

$$-\frac{1}{da}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\,(\mathrm{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(1/2),x)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(
```

```
cos(1/2*d*x+1/2*c),2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1/
2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(cos(c + d*x)*sqrt(sec(c + d*x)) + sqrt(sec(c + d*x))), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.321 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=112

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.140751, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3819, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```


```
[Out] (3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
 (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \frac{-\frac{3a}{2}+\frac{1}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{a^2} \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \sqrt{\sec(c+dx)}\,dx}{2a}+\frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2a}+\frac{(3\cdots}{} \\
&= \frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\cdots}}{ad}
\end{aligned}
$$

Mathematica [C] time = 1.64768, size = 311, normalized size = 2.78

$$
\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\frac{\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(\cos\left(\frac{1}{2}(c-dx)\right)+2\cos\left(\frac{1}{2}(3c+dx)\right)+2\cos\left(\frac{1}{2}(c+3dx)\right)+\cos\left(\frac{1}{2}(5c+3dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{2d}+\frac{2i\sqrt{2}e^{-i(c+dx)}\sqrt{\frac{e\cdots}{1+\cdots}}\cdots}{\cdots}\right.}{a(\cos(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + E
^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeomet
ric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(d*E^(I*(c + d*x))*(-1 + E^((
2*I)*c))) - ((Cos[(c - d*x)/2] + 2*Cos[(3*c + d*x)/2] + 2*Cos[(c + 3*d*x)/2
] + Cos[(5*c + 3*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d
*x]])/(2*d)))/(a*(1 + Cos[c + d*x]))
```

Maple [A] time = 2.356, size = 199, normalized size = 1.8

$$
\frac{1}{da}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\left(\mathrm{Ellipti}\cdots\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(3/2),x)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)/a/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\cos(c+dx)\sec^{\frac{3}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(cos(c + d*x)*sec(c + d*x)**(3/2) + sec(c + d*x)**(3/2)), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.322 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3\sqrt{\cos(c+dx)}}{}$$

```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
+ (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*
d) + (5*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Sec
[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rubi [A] time = 0.161564, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3819, 3787, 3769, 3771, 2641, 2639}

$$\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3\sqrt{\cos(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (-3*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
+ (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a*
d) + (5*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sqrt[Sec
[c + d*x]]*(a + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{5a}{2}+\frac{3}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} - \frac{3\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{2a} + \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a}$$

$$= \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))} + \frac{5\int \sqrt{\sec(c+dx)}\,dx}{6a} -$$

$$= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{}{d\sqrt{\sec(c+}}$$

$$= -\frac{3\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}$$

Mathematica [C] time = 4.00192, size = 312, normalized size = 2.23

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{\sec(c+dx)}\left(\sin(2c)\cos(2dx)-6\cos(c)\sin(dx)+\cos(2c)\sin(2dx)+3(\cos(2c)+2)\csc(c)\cos(dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*(9*(1 + E^((2*I)*(c + d*x))) + 9*(-1 + E^((2*I)*c))*Sqrt[1 + E^((
2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] +
5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^(
(2*I)*c))) + 2*Sqrt[Sec[c + d*x]]*(3*(2 + Cos[2*c])*Cos[d*x]*Csc[c] + Cos[2
```

```
*d*x]*Sin[2*c] - 3*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 6*Cos[c]*Sin[d*
x] + Cos[2*c]*Sin[2*d*x] - 3*Tan[c/2])))/(3*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 2.188, size = 215, normalized size = 1.5

$$-\frac{1}{3\,da}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-
8*sin(1/2*d*x+1/2*c)^6+18*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x+1/2*c)^2)/a/co
s(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1
/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.323 $\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=168

$$-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{3ad}$$

```
[Out] (21*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a*d) + (7*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d
*x]))
```

Rubi [A] time = 0.170117, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3819, 3787, 3769, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)),x]
```


```
[Out] (21*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a*d) + (7*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])/(3*a*
d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x]/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d
*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3819

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e +
 f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$$

$$= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{\int \frac{-\frac{7a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{a^2}$$

$$= -\frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{5\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} + \frac{7\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{2a}$$

$$= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{5\int$$

$$= \frac{7\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))} - \frac{(5\sqrt{}$$

$$= \frac{21\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} - \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}$$

Mathematica [C] time = 2.69027, size = 341, normalized size = 2.03

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-\sqrt{\sec(c+dx)}\left(18(11\cos(2c)+17)\csc(c)\cos(dx)+4\left(10\sin(2c)\cos(2dx)-3\sin(3c)\cos(3dx)-99\,\mathrm{co}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(((8*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
 d*x)))]*(63*(1 + E^((2*I)*(c + d*x))) + 63*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] +
 25*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
```

```
eometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E
^((2*I)*c))) - Sqrt[Sec[c + d*x]]*(18*(17 + 11*Cos[2*c])*Cos[d*x]*Csc[c] +
4*(10*Cos[2*d*x]*Sin[2*c] - 3*Cos[3*d*x]*Sin[3*c] - 30*Sec[c/2]*Sec[(c + d*
x)/2]*Sin[(d*x)/2] - 99*Cos[c]*Sin[d*x] + 10*Cos[2*c]*Sin[2*d*x] - 3*Cos[3*
c]*Sin[3*d*x] - 30*Tan[c/2]))))/(60*a*d*(1 + Cos[c + d*x]))
```

Maple [A] time = 2.067, size = 229, normalized size = 1.4

$$-\frac{1}{15da}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))+48*sin(1/2*d*x+1/2*c)^8-56*sin(1/2*d*x+1/2*c)^6-30*sin(1/2*d*x+1/2*c)^4
+23*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)*sec(d*x + c)^(7/2)), x)
```

3.324 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=202

$$-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3a^2d}$$

```
[Out] (7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) - (7*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (10*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a^2*d) - (7*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(
1 + Sec[c + d*x])) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```

Rubi [A] time = 0.273034, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3816, 4019, 3787, 3768, 3771, 2639, 2641}

$$-\frac{7\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{10\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}-\frac{7\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) - (7*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) + (10*Sec[c + d*x]^(3/
2)*Sin[c + d*x])/(3*a^2*d) - (7*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(
1 + Sec[c + d*x])) - (Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c +
d*x])^2)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
```

```
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{21a^2}{2}-15a^2\sec(c\right.}{3a^4}$$

$$= -\frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{7\int \sec^{\frac{3}{2}}(c+dx)\,dx}{2a^2} + \frac{5\int \sec^{\frac{5}{2}}(c}{a^2}$$

$$= -\frac{7\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \cdots$$

$$= -\frac{7\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{10\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d} - \frac{7\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \cdots$$

$$= \frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{3a^2d}$$

Mathematica [C] time = 2.38324, size = 287, normalized size = 1.42

$$-\frac{\left(-1+e^{ic}\right)\csc\left(\frac{c}{2}\right)e^{-\frac{1}{2}i(4c+3dx)}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(7e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{3/2}\left(1+e^{i(c+dx)}\right)^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c}\right.\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -((-1 + E^(I*c))*Cos[(c + d*x)/2]*Csc[c/2]*(-10 - 37*E^(I*(c + d*x)) - 65*E
^((2*I)*(c + d*x)) - 82*E^((3*I)*(c + d*x)) - 68*E^((4*I)*(c + d*x)) - 53*E
^((5*I)*(c + d*x)) - 21*E^((6*I)*(c + d*x)) + (10*I)*(1 + E^(I*(c + d*x)))^
3*(1 + E^((2*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
7*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^3*(1 + E^((2*I)*(c + d*x)))^(3/2)*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sqrt[Sec[c + d*x]])/
(12*a^2*d*E^((I/2)*(4*c + 3*d*x))*(1 + E^((2*I)*(c + d*x)))*(1 + Cos[c + d*
x])^2)
```

Maple [A] time = 4.327, size = 413, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(6*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)+14*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2/3*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2-22/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)
*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/3*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{5}{2}}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^2, x)
```

3.325 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4\sqrt{\cos(c+}$$

```
[Out] (-4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - (5*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.250873, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{5\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{4\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{4\sqrt{\cos(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```



```
[Out] (-4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (4*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d) - (5*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - (Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
```

```
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \sqrt{\sec(c+dx)}\left(\frac{5a^2}{2} - 6a^2\sec(c+\right.}{3a^4}$$

$$= -\frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int \sqrt{\sec(c+dx)}\,dx}{6a^2} + \frac{2\int \sec^{\frac{3}{2}}(c}{a^2}$$

$$= \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{2\int \frac{}{\sqrt{s}}}{}$$

$$= -\frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} - \frac{5\sec^{\frac{3}{2}}(c+dx}{3a^2d(1+\text{se}}$$

$$= -\frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{3a^2d}$$

Mathematica [C] time = 1.26467, size = 252, normalized size = 1.43

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-4ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3{}_2F\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] -(Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-4*I)*(1 + E^(I*(c + d*x)))^3*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))])/E^(I*(c + d*x)) + 40*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(29 + 50*Cos[c
 + d*x] + 17*Cos[2*(c + d*x)] + (12*I)*Sin[c + d*x] + (7*I)*Sin[2*(c + d*x)
]))*(Cos[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Co
s[c + d*x])^2)
```

Maple [A] time = 2.704, size = 405, normalized size = 2.3

$$-\frac{1}{6a^2d}\left(2\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^4+\left(\sin(1/2\,dx+c/2)\right)^2}\left(5\,\mathrm{Ellipti}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] -1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(
1/2*d*x+1/2*c)-48*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^6+86*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*si
n(1/2*d*x+1/2*c)^4-37*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
sin(1/2*d*x+1/2*c)^2)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{a^{2}\cos\left(dx+c\right)^{2}+2\,a^{2}\cos\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^2, x)
```

3.326 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) +
 (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2
*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.239916, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3816, 4019, 3787, 3771, 2639, 2641}

$$-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d) +
 (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2
*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Sec[c
 + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^2}\,dx &= \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx \\
&= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{5}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2} \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^4} \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,d}{2a^2} \\
&= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{co}}}{3a^2} \\
&= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}
\end{aligned}$$

Mathematica [C] time = 1.19032, size = 242, normalized size = 1.62

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-ie^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\ {}_2F_1\left(\frac{1}{2},\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])/E^(I*(c + d*x)) + 16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(5 + 14*Cos[c + d
*x] + 5*Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin
```

```
[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 2.355, size = 257, normalized size = 1.7

$$\frac{1}{6\,a^2d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,(\cos(1/2\,dx+c/2))^6-4\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^2,x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*x
+1/2*c)^6-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-16*cos(1/2*d*x+1/2*c)^4+3*cos(1/2*d*x+1/2*
c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{\sec(dx+c)}}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^2, x)
```

3.327 $\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.0969191, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3815, 21, 3771, 2641}

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*d)
 + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{6a^2}$$
$$= \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{6a^2}$$
$$= \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} + \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

Mathematica [A] time = 0.362804, size = 98, normalized size = 1.27

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)+4\sqrt{\cos(c+dx)}\cos^3\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(4*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x
]]*EllipticF[(c + d*x)/2, 2] - Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3
*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [B] time = 2.233, size = 188, normalized size = 2.4

$$-\frac{1}{6a^2d}\sqrt{\left(2(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2(\cos(1/2\,dx+c/2))^2+1}\mathrm{EllipticF}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+2*cos(1/2*d*x+1/2*c)^4-3*cos(1/2*d*x+1/
2*c)^2+1)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2\right)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sqrt(sec(d*x +
c))), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\cos^2(c+dx)\sqrt{\sec(c+dx)}+2\cos(c+dx)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(cos(c + d*x)**2*sqrt(sec(c + d*x)) + 2*cos(c + d*x)*sqrt(sec(c
+ d*x)) + sqrt(sec(c + d*x))), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.328 $\int \frac{1}{(a+a\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-$$

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.23798, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3817, 4019, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*
a^2*d) + (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - (Se
c[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{5a}{2}+\frac{1}{2}a\sec(c+dx)\right)}{a+a\sec(c+dx)}\,dx}{3a^2}$$

$$= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\frac{3a^2}{2}-a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3a^4}$$

$$= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,dx}{3a^2} -$$

$$= \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d(1+\sec(c+dx))} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}$$

$$= -\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{3a^2d}$$

Mathematica [C] time = 1.34923, size = 239, normalized size = 1.6

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(e^{-i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3{}_2F_1\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(16*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*(
-7 - 10*Cos[c + d*x] - 7*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^3*Sqrt[1
 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])/E^(I*(c + d*x)) + I*Sin[2*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*Sin[[(c
```

```
+ 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 2.415, size = 257, normalized size = 1.7

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\left(\cos\left(1/2\,dx+c/2\right)\right)^6+4\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\left(\cos\left(1/2\,dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^6+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-20*cos(1/2*d*x+1/2*c)^4+9*cos(1/2*d*x+1/2
*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(3
/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.329 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=152

$$-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}$$

```
[Out] (4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a
^2*d) - (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.240454, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a
^2*d) - (5*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) -
(Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{7a}{2}+\frac{3}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{-6a^2+\frac{5}{2}a^2\sec(c+d}{\sqrt{\sec(c+dx)}}}{3a^4}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{5\int\sqrt{\sec(c+dx}}{6a^2}$$

$$= -\frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\left(5\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{4\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{3a^2d}$$

Mathematica [C] time = 1.95542, size = 259, normalized size = 1.7

$$-\frac{\sin(c)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(2ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -(Cos[(c + d*x)/2]*Csc[c/2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*Sin[c]*(Cos[d*x] +
I*Sin[d*x])*((-24*I)*Cos[(c + d*x)/2] - (18*I)*Cos[(3*(c + d*x))/2] - (6*I)
*Cos[(5*(c + d*x))/2] + 20*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2] + ((2*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d
```

```
*x)) + Sin[(c + d*x)/2] + 2*Sin[(3*(c + d*x))/2] + 3*Sin[(5*(c + d*x))/2]))
/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 2.485, size = 257, normalized size = 1.7

$$\frac{1}{6a^2d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(24\left(\cos\left(1/2\,dx+c/2\right)\right)^6+10\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\left(\cos\left(1/2\,d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1/2*d*x
+1/2*c)^6+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*cos(1/2*d*x+1/2*c)^4+15*cos(1/2*d*x+1
/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^2\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^2\cos\left(dx+c\right)^2+2\,a^2\cos\left(dx+c\right)+a^2\right)\sec\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(5
/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.330 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7\sqrt{\cos(c+dx}$$

```
[Out] (-7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (7*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rubi [A] time = 0.263604, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(\sec(c+dx)+1)}+\frac{10\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{7\sqrt{\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (-7*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d
) + (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a^2*d) + (10*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]) - (7*Sin[c + d*x])
/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sqrt[S
ec[c + d*x]]*(a + a*Sec[c + d*x])^2)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{9a}{2}+\frac{5}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= -\frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}$$

$$= \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{7\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}(1+\sec(c+dx))} - \frac{\sin(c}{3d\sqrt{\sec(c+dx)}(}$$

$$= -\frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{}{3a^2d}$$

$$= -\frac{7\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{10\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{3a^2d}$$

Mathematica [C] time = 1.68772, size = 257, normalized size = 1.44

$$e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(7ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+3$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-84*I)*Cos[(
c + d*x)/2] - (63*I)*Cos[(3*(c + d*x))/2] - (21*I)*Cos[(5*(c + d*x))/2] + 8
0*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + ((7*I)*
(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2
, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) + 3*Sin[(c + d*x)/2]
 + 10*Sin[(3*(c + d*x))/2] + 12*Sin[(5*(c + d*x))/2] + Sin[(7*(c + d*x))/2]
))/(6*a^2*d*E^(I*d*x)*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 2.381, size = 270, normalized size = 1.5

$$-\frac{1}{6a^2d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(16\left(\cos(1/2\,dx+c/2)\right)^8+12\left(\cos(1/2\,dx+c/2)\right)^6+20\sqrt{(\sin(1/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*cos(1/2*d*
x+1/2*c)^8+12*cos(1/2*d*x+1/2*c)^6+20*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x
+1/2*c)^3+42*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*cos(1/2*d*x+
1/2*c)^4+21*cos(1/2*d*x+1/2*c)^2-1)/a^2/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^3/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a\cos(dx+c)+a\right)^2\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(7
/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2)), x)
```

3.331 $\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=200

$$-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{a^2d}$$

```
[Out] (56*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^2
*d) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (56*Sin[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])
/(a^2*d*Sqrt[Sec[c + d*x]]) - (3*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^2)
```

Rubi [A] time = 0.280514, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3817, 4020, 3787, 3769, 3771, 2639, 2641}

$$-\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)),x]
```

```
[Out] (56*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*a^2
*d) - (5*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^2*d) + (56*Sin[c + d*x])/(15*a^2*d*Sec[c + d*x]^(3/2)) - (5*Sin[c + d*x])
/(a^2*d*Sqrt[Sec[c + d*x]]) - (3*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1
 + Sec[c + d*x])) - Sin[c + d*x]/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^2)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
```

```
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$$

$$= -\frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{11a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx}{3a^2}$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \cdots \int \cdots 15$$

$$= -\frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} - \cdots$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \cdots$$

$$= \frac{56\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{a^2d\sqrt{\sec(c+dx)}} - \frac{3\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \cdots$$

$$= \frac{56\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\cdots}{a^2d}$$

Mathematica [C] time = 1.79692, size = 271, normalized size = 1.36

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(-112ie^{-\frac{1}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^3 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^2*Sec[c + d*x]^(9/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((1344*I)*Cos[
(c + d*x)/2] + (1008*I)*Cos[(3*(c + d*x))/2] + (336*I)*Cos[(5*(c + d*x))/2]
 - 1200*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (
(112*I)*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometri
c2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((I/2)*(c + d*x)) - 34*Sin[(c
+ d*x)/2] - 148*Sin[(3*(c + d*x))/2] - 168*Sin[(5*(c + d*x))/2] - 11*Sin[(7
*(c + d*x))/2] + 3*Sin[(9*(c + d*x))/2]))/(60*a^2*d*E^(I*d*x)*(1 + Cos[c +
d*x])^2)
```

Maple [A] time = 2.322, size = 283, normalized size = 1.4

$$-\frac{1}{30\,a^2d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(96\,(\cos(1/2\,dx+c/2))^{10}-352\,(\cos(1/2\,dx+c/2))^8+120\,(\cos\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^2/sec(d*x+c)^(9/2),x)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*cos(1/2*d
*x+1/2*c)^10-352*cos(1/2*d*x+1/2*c)^8+120*cos(1/2*d*x+1/2*c)^6-150*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-336*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^3*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))+266*cos(1/2*d*x+1/2*c)^4-135*cos(1/2*d*x+1/2*c)^2+5)/a^2/cos
(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*sec(d*x + c)^(9
/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**2/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^2/sec(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate(1/((a*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2)), x)
```

3.332 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49\sqrt{\cos}}{}$$

```
[Out] (-49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(6*a^3*d) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) - (Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.372146, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3816, 4019, 3787, 3771, 2641, 3768, 2639}

$$-\frac{13\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{49\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (-49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a
^3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(6*a^3*d) + (49*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10*a^3*d) - (Sec[c + d*
x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (8*Sec[c + d*x]^(5/2)
*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (13*Sec[c + d*x]^(3/2)*Sin
[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
```

```
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5a}{2}-\frac{11}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(12a^2-\frac{41}{2}a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}d}{15a^4}$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} -$$

$$= -\frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d\left(a^3+a^3\sec(c+dx)\right)} -$$

$$= \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - 1$$

$$= -\frac{13\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} - \frac{\sec^{\frac{7}{2}}(c-}{5d(a+}$$

$$= -\frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{6a^3d}$$

Mathematica [C] time = 2.18322, size = 363, normalized size = 1.64

$$2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1284\cos\left(\frac{1}{2}(c-dx)\right)+921\cos\left(\frac{1}{2}(3c+dx)\right)+1243\cos\left(\frac{1}{2}(c+3dx)\right)+374\cos\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1
+ E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x
))] - 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-
1 + E^((2*I)*c))) + ((1284*Cos[(c - d*x)/2] + 921*Cos[(3*c + d*x)/2] + 1243
*Cos[(c + 3*d*x)/2] + 374*Cos[(5*c + 3*d*x)/2] + 670*Cos[(3*c + 5*d*x)/2] +
 65*Cos[(7*c + 5*d*x)/2] + 147*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[
(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [B] time = 3.016, size = 555, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] -1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(65*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)+588*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^8-1634*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+1488*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-439*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)
^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sec(dx+c)^{\frac{3}{2}}}{a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^3, x)
```

3.333 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=195

$$-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}$$

```
[Out] (9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3
*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*
a^3*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (9*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.360219, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3816, 4019, 3787, 3771, 2639, 2641}

$$-\frac{9\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3
*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2*
a^3*d) - (Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (
2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) - (9*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(10*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
```

```
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3a}{2}-\frac{9}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(3a^2-\frac{21}{2}a^2\sec(c+dx)\right)}{a+a\sec(c+dx)}\,d}{15a^4}$$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)} - $$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)} + $$

$$= -\frac{\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{9\sqrt{\sec(c+dx)}\sin(c+dx)}{10d\left(a^3+a^3\sec(c+dx)\right)} + $$

$$= \frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{2a^3d}$$

Mathematica [C] time = 2.30206, size = 274, normalized size = 1.41

$$e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(-3ie^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5 {}_2F$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(((-3*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^((2*I)*(c + d*x)) + 160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*(34 +
69*Cos[c + d*x] + 34*Cos[2*(c + d*x)] + 7*Cos[3*(c + d*x)] + (2*I)*Sin[c +
d*x] + (6*I)*Sin[2*(c + d*x)] + (2*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2
] + I*Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 2.56, size = 268, normalized size = 1.4

$$\frac{1}{20\,a^{3}d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(36\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{8}-10\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-46*cos(1/2*d*x+1/2*c)^6+8*cos(1/2*d*x+1
/2*c)^4+cos(1/2*d*x+1/2*c)^2+1)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{\sec\left(dx+c\right)}}{\left(a\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec\left(dx+c\right)}}{a^{3}\cos\left(dx+c\right)^{3}+3\,a^{3}\cos\left(dx+c\right)^{2}+3\,a^{3}\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/(a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*
a^3*cos(d*x + c) + a^3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{\sec(c+dx)}}{\cos^3(c+dx)+3\cos^2(c+dx)+3\cos(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(cos(c + d*x)**3 + 3*cos(c + d*x)**2 + 3*cos(c
+ d*x) + 1), x)/a**3
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^3, x)
```

3.334 $\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\text{si}}{5}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d
) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^
3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (4*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.350335, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3816, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{\text{si}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*d
) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a^
3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (4*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3816

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*C
sc[e + f*x])^(n - 2))/(f*(2*m + 1)), x] + Dist[d^2/(a*b*(2*m + 1)), Int[(a
+ b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n +
2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
 - n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
```

```
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}-\frac{7}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-2a^2-\frac{9}{2}a^2\sec(c+dx}{\sqrt{\sec(c+dx)}(a+a\sec(c+}}{15a^4}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+}{6d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+}{6d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{4\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c+}{6d\left(a^3+a^3\sec(c+\right.}$$

$$= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{6a^3d}$$

Mathematica [C] time = 2.01847, size = 363, normalized size = 1.86

$$2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+7\cos\left(\frac{1}{2}(c+3dx)\right)+26\cos\left(\frac{1}{2}(5c+3a\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] -
 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E^
((2*I)*c))) - ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 7*Cos[(c + 3*d
*x)/2] + 26*Cos[(5*c + 3*d*x)/2] + 10*Cos[(3*c + 5*d*x)/2] + 5*Cos[(7*c + 5
*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqr
t[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 2.411, size = 270, normalized size = 1.4

$$\frac{1}{60\,a^3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\left(\cos(1/2\,dx+c/2)\right)^8-10\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d*
x+1/2*c)^8-10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-22*cos(1/2*d*x+1/2*c)^6+6*cos(1/2*d*x+1/
2*c)^4+7*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.335 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-$$

```
[Out] -(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*
d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a
^3*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.345086, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3815, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```


```
[Out] -(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^3*
d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(6*a
^3*d) + (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - (S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3815

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Cs
c[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] - Dist[d/(a*b*(2*m + 1)), Int[(a +
 b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*(a*(n - 1) - b*(m + n)*Cs
c[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Lt
Q[m, -1] && LtQ[1, n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
```

```
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$$

$$=\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$=\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\int \frac{\frac{a^2}{2}+3a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c\cdots}}{15a^4}$$

$$=\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+dx)}\sin(c\cdots}{6d\left(a^3+a^3\sec(c+\cdots\right.}$$

$$=\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+dx)}\sin(c\cdots}{6d\left(a^3+a^3\sec(c+\cdots\right.}$$

$$=\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{\sqrt{\sec(c+dx)}\sin(c\cdots}{6d\left(a^3+a^3\sec(c+\cdots\right.}$$

$$=-\frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|\cdots\right.}{6a^3d}$$

Mathematica [C] time = 1.89485, size = 363, normalized size = 1.86

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(36\cos\left(\frac{1}{2}(c-dx)\right)+9\cos\left(\frac{1}{2}(3c+dx)\right)+17\cos\left(\frac{1}{2}(c+3dx)\right)+16\cos\left(\frac{1}{2}(5c+\cdots\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((-2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(
c + d*x)))]*(3*(1 + E^((2*I)*(c + d*x))) + 3*(-1 + E^((2*I)*c))*Sqrt[1 + E^
((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))]
+ 5*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hyperg
eometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1 + E
^((2*I)*c))) + ((36*Cos[(c - d*x)/2] + 9*Cos[(3*c + d*x)/2] + 17*Cos[(c + 3
*d*x)/2] + 16*Cos[(5*c + 3*d*x)/2] + 20*Cos[(3*c + 5*d*x)/2] - 5*Cos[(7*c +
 5*d*x)/2] + 3*Cos[(5*c + 7*d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*S
qrt[Sec[c + d*x]])/32))/(15*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 2.447, size = 270, normalized size = 1.4

$$-\frac{1}{60\,a^3d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(12\,(\cos(1/2\,dx+c/2))^8+10\,\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*cos(1/2*d*x+1/2*c)^6-24*cos(1/2*d*x+1
/2*c)^4+17*cos(1/2*d*x+1/2*c)^2-3)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.336 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=195

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-$$

```
[Out] (-9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2
*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.356466, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3817, 4019, 4020, 3787, 3771, 2639, 2641}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^3\sec(c+dx)+a^3\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}-\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (-9*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) + (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(2
*a^3*d) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^2) + (Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a^3 + a^3*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4019

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(d*(A*b
 - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1))/(a*f*
(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
```

```
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sqrt{\sec(c+dx)}\left(-\frac{9a}{2}+\frac{3}{2}a\sec(c+dx)\right)}{(a+a\sec(c+dx))^2}dx}{5a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} - \frac{\int \frac{3a^2-\frac{9}{2}a^2\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+}}{15a^4}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c-}{2d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c-}{2d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{5ad(a+a\sec(c+dx))^2} + \frac{\sqrt{\sec(c+dx)}\sin(c-}{2d\left(a^3+a^3\sec(c+\right.}$$

$$= -\frac{9\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}$$

Mathematica [C] time = 2.82389, size = 272, normalized size = 1.39

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)\left(i\left(3e^{-2i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5 {}_2F_1\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(160*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + I*
(-68 - 128*Cos[c + d*x] - 68*Cos[2*(c + d*x)] - 24*Cos[3*(c + d*x)] + (3*(1
 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2,
3/4, 7/4, -E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*x)) + (6*I)*Sin[c + d*x] +
 (8*I)*Sin[2*(c + d*x)] + (6*I)*Sin[3*(c + d*x)]))*(Cos[(c + 3*d*x)/2] + I*
Sin[(c + 3*d*x)/2]))/(40*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 2.234, size = 270, normalized size = 1.4

$$-\frac{1}{20a^3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(36\left(\cos(1/2\,dx+c/2)\right)^8+10\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\left(\cos(1/\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/20*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(36*cos(1/2*d
*x+1/2*c)^8+10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
```

```
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+18*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))-66*cos(1/2*d*x+1/2*c)^6+38*cos(1/2*d*x
+1/2*c)^4-9*cos(1/2*d*x+1/2*c)^2+1)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.337 $\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=195

$$-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{10a^3d}$$

```
[Out] (49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(6*a^3*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3)
 - (8*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.358869, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3817, 4020, 3787, 3771, 2639, 2641}

$$-\frac{13\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{49\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx\right.}{10a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (49*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*a^
3*d) - (13*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(6*a^3*d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3)
 - (8*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
```

```
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{11a}{2}+\frac{5}{2}a\sec(c+dx)}{\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{41a^2}{2}+12a^2\sec}{\sqrt{\sec(c+dx)}(a+a\sec}}{15a^4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)}\,s}{6d\left(a^3+a^3\sec\right.}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)}\,s}{6d\left(a^3+a^3\sec\right.}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{8\sqrt{\sec(c+dx)}\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{13\sqrt{\sec(c+dx)}\,s}{6d\left(a^3+a^3\sec\right.}$$

$$= \frac{49\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{13\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{6a^3d}$$

Mathematica [C] time = 2.15564, size = 378, normalized size = 1.94

$$\frac{2\cos^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{1}{32}\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\left(1134\cos\left(\frac{1}{2}(c-dx)\right)+1071\cos\left(\frac{1}{2}(3c+dx)\right)+923\cos\left(\frac{1}{2}(c+3dx)\right)+694\,\mathrm{co}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^6*(((2*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
 + d*x)))]*(147*(1 + E^((2*I)*(c + d*x))) + 147*(-1 + E^((2*I)*c))*Sqrt[1 +
 E^((2*I)*(c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x)
)] + 65*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hy
pergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)*(c + d*x))]))/(E^(I*(c + d*x))*(-1
 + E^((2*I)*c))) - ((1134*Cos[(c - d*x)/2] + 1071*Cos[(3*c + d*x)/2] + 923*
Cos[(c + 3*d*x)/2] + 694*Cos[(5*c + 3*d*x)/2] + 470*Cos[(3*c + 5*d*x)/2] +
265*Cos[(7*c + 5*d*x)/2] + 117*Cos[(5*c + 7*d*x)/2] + 30*Cos[(9*c + 7*d*x)/
2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2]^5*Sqrt[Sec[c + d*x]])/32))/(15*a^3*d
*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 2.625, size = 270, normalized size = 1.4

$$\frac{1}{60\,a^3d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(348\,\left(\cos\left(1/2\,dx+c/2\right)\right)^8+130\,\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{-2\,\left(\cos(1\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(7/2),x)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*cos(1/2*d
*x+1/2*c)^8+130*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*cos(1/2*d*x+1/2*c)^5
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-578*cos(1/2*d*x+1/2*c)^6+264*cos(1/2
*d*x+1/2*c)^4-37*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d
*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^3\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^3\cos\left(dx+c\right)^3+3\,a^3\cos\left(dx+c\right)^2+3\,a^3\cos\left(dx+c\right)+a^3\right)\sec\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sec(d*x + c)^(7/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2)), x)
```

3.338 $\int \frac{1}{(a+a\cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)} dx$

Optimal. Leaf size=221

$$\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)} + \frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{119\sqrt{\cos(c}{}$$

```
[Out] (-119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*
a^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x
]/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d
*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Sqrt
[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rubi [A] time = 0.381186, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3817, 4020, 3787, 3769, 3771, 2641, 2639}

$$\frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{119\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\left(a^3\sec(c+dx)+a^3\right)} + \frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d} - \frac{119\sqrt{\cos(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)),x]
```

```
[Out] (-119*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(10*
a^3*d) + (11*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(2*a^3*d) + (11*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c + d*x]]) - Sin[c + d*x
]/(5*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3) - (2*Sin[c + d*x])/(3*a*d
*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2) - (119*Sin[c + d*x])/(30*d*Sqrt
[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3817

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e
 + f*x])^n)/(f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m,
 -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

Rule 4020

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n)/(b*f*(2*m +
1)), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e
```

```
 + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e +
 f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0
] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{\int \frac{-\frac{13a}{2}+\frac{7}{2}a\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} -$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} -$$

$$= -\frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^2} -$$

$$= \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{\sin(c+dx)}{5d\sqrt{\sec(c+dx)}(a+a\sec(c+dx))^3} - \frac{2\sin(c}{3ad\sqrt{\sec(c+dx)}(a}$$

$$= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{}{5d\sqrt{se}}$$

$$= -\frac{119\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{11\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{2a^3d}$$

Mathematica [C] time = 2.29981, size = 285, normalized size = 1.29

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(119ie^{-\frac{3}{2}i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^5{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^3*Sec[c + d*x]^(9/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((-5355*I)*Cos
[(c + d*x)/2] - (3927*I)*Cos[(3*(c + d*x))/2] - (1785*I)*Cos[(5*(c + d*x))/
2] - (357*I)*Cos[(7*(c + d*x))/2] + 5280*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + ((119*I)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E
^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
/E^(((3*I)/2)*(c + d*x)) + 193*Sin[(c + d*x)/2] + 579*Sin[(3*(c + d*x))/2]
+ 555*Sin[(5*(c + d*x))/2] + 227*Sin[(7*(c + d*x))/2] + 10*Sin[(9*(c + d*x)
)/2]))/(120*a^3*d*E^(I*d*x)*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 2.506, size = 283, normalized size = 1.3

$$-\frac{1}{60a^3d}\sqrt{\left(2\left(\cos(1/2dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(160\left(\cos(1/2dx+c/2)\right)^{10}+468\left(\cos(1/2dx+c/2)\right)^8+330\sqrt{(s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^3/sec(d*x+c)^(9/2),x)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/2*
d*x+1/2*c)^10+468*cos(1/2*d*x+1/2*c)^8+330*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/
2*d*x+1/2*c)^5+714*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*cos(1/2*d*x+1/2*c)^5*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*cos(
1/2*d*x+1/2*c)^6+474*cos(1/2*d*x+1/2*c)^4-47*cos(1/2*d*x+1/2*c)^2+3)/a^3/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)^5/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c)
 + a^3)*sec(d*x + c)^(9/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**3/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^3/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2)), x)
```

3.339 $\int \sqrt{a + a\cos(c + dx)} \sec^{\frac{9}{2}}(c + dx)\, dx$

Optimal. Leaf size=153

$$\frac{2a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}} + \frac{12a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}} + \frac{16a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}} + \frac{32a\sin(c+dx)\sqrt{\sec(c+}}{35d\sqrt{a\cos(c+dx)+}}$$

```
[Out] (32*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (1
6*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (12*
a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*S
ec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.282498, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4222, 2772, 2771}

$$\frac{2a\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}} + \frac{12a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}} + \frac{16a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}} + \frac{32a\sin(c+dx)\sqrt{\sec(c+}}{35d\sqrt{a\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2),x]
```

```
[Out] (32*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (1
6*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (12*
a*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*S
ec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}\left(6\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+}}{\cos^{\frac{7}{2}}(c+d}$$
$$= \frac{12a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{35}\left(24\sqrt{\cos(c+d}\right.$$
$$= \frac{16a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{12a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{7}{2}}(c+dx)s}{7d\sqrt{a+a\cos}}$$
$$= \frac{32a\sqrt{\sec(c+dx)}\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{16a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{12a\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.216151, size = 71, normalized size = 0.46

$$\frac{2(18\cos(c+dx)+4\cos(2(c+dx))+4\cos(3(c+dx))+9)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(9 + 18*Cos[c + d*x] + 4*Cos[2*(c + d*x)] + 4
*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(35*d)
```

Maple [A] time = 0.471, size = 82, normalized size = 0.5

$$-\frac{\left(32\,(\cos(dx+c))^4-16\,(\cos(dx+c))^3-4\,(\cos(dx+c))^2-2\,\cos(dx+c)-10\right)\cos(dx+c)}{35\,d\sin(dx+c)}\left((\cos(dx+c))^{-1}\right)^{\frac{9}{2}}\sqrt{a(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/35/d*(16*cos(d*x+c)^4-8*cos(d*x+c)^3-2*cos(d*x+c)^2-cos(d*x+c)-5)*cos(d*
x+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.62818, size = 382, normalized size = 2.5

$$\frac{2\left(\frac{35\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{70\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{58\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\sqrt{2}\sqrt{a}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^4}{35\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 70*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 58*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 9*sqrt(2)*sqrt(a)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^4/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(4*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 6*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1))
```

Fricas [A] time = 1.59174, size = 220, normalized size = 1.44

$$\frac{2\left(16\cos\left(dx+c\right)^{3}+8\cos\left(dx+c\right)^{2}+6\cos\left(dx+c\right)+5\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{35\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*cos(d*x + c)^3 + 8*cos(d*x + c)^2 + 6*cos(d*x + c) + 5)*sqrt(a*cos
(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c)^3)*sqrt(cos
(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(9/2), x)
```

3.340 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{8a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (16*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (8
*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.22103, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4222, 2772, 2771}

$$\frac{2a\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{8a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{16a\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

```
[Out] (16*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (8
*a*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a*
Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\left(4\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos}}{\cos^{\frac{5}{2}}(c}$$

$$= \frac{8a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{15}\left(8\sqrt{\cos(c+}\right.$$

$$= \frac{16a\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{8a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{5}{2}}(c+dx}{5d\sqrt{a+a\cos}}$$

Mathematica [A] time = 0.128528, size = 61, normalized size = 0.53

$$\frac{2(4\cos(c+dx)+4\cos(2(c+dx))+7)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(7 + 4*Cos[c + d*x] + 4*Cos[2*(c + d*x)])*Sec
[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.46, size = 72, normalized size = 0.6

$$-\frac{\left(16\,(\cos(dx+c))^3-8\,(\cos(dx+c))^2-2\,\cos(dx+c)-6\right)\cos(dx+c)}{15\,d\sin(dx+c)}\left((\cos(dx+c))^{-1}\right)^{\frac{7}{2}}\sqrt{a\,(1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/15/d*(8*cos(d*x+c)^3-4*cos(d*x+c)^2-cos(d*x+c)-3)*cos(d*x+c)*(1/cos(d*x+
c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.60247, size = 320, normalized size = 2.78

$$\frac{2\left(\frac{15\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{17\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{7\sqrt{2}\sqrt{a}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{15\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sqrt(2)*sqrt(
a)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 17*sqrt(2)*sqrt(a)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 7*sqrt(2)*sqrt(a)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
```

```
7)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(3*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 1))
```

Fricas [A] time = 1.61622, size = 193, normalized size = 1.68

$$\frac{2\,\sqrt{a\cos\left(dx+c\right)+a}\left(8\,\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)+3\right)\sin\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(a*cos(d*x + c) + a)*(8*cos(d*x + c)^2 + 4*cos(d*x + c) + 3)*sin(d
*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.341 $\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=77

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.159009, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4222, 2772, 2771}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}+\frac{4a\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

```
[Out] (4*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a
*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}$$
$$= \frac{4a\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.100592, size = 51, normalized size = 0.66

$$\frac{2(2\cos(c+dx)+1)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 2*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Tan[(
c + d*x)/2])/(3*d)
```

Maple [A] time = 0.463, size = 62, normalized size = 0.8

$$-\frac{\left(4\left(\cos(dx+c)\right)^2-2\cos(dx+c)-2\right)\cos(dx+c)}{3d\sin(dx+c)}\left(\left(\cos(dx+c)\right)^{-1}\right)^{\frac{5}{2}}\sqrt{a\left(1+\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/3/d*(2*cos(d*x+c)^2-cos(d*x+c)-1)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+
cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.60486, size = 257, normalized size = 3.34

$$\frac{2\left(\frac{3\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sqrt{2}\sqrt{a}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sqrt{2}\sqrt{a}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sqrt(2)*sqrt(a)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sqrt(2)*sqrt(a)*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5)*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2
)*(2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1
```

```
)^4 + 1))
```

Fricas [A] time = 1.58491, size = 163, normalized size = 2.12

$$\frac{2\sqrt{a\cos(dx+c)+a}(2\cos(dx+c)+1)\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(a*cos(d*x + c) + a)*(2*cos(d*x + c) + 1)*sin(d*x + c)/((d*cos(d*x
+ c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.342 $\int \sqrt{a + a\cos(c + dx)}\sec^{\frac{3}{2}}(c + dx)\,dx$

Optimal. Leaf size=36

$$\frac{2a\sin(c + dx)\sqrt{\sec(c + dx)}}{d\sqrt{a\cos(c + dx) + a}}$$

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.103033, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {4222, 2771}

$$\frac{2a\sin(c + dx)\sqrt{\sec(c + dx)}}{d\sqrt{a\cos(c + dx) + a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a + a\cos(c + dx)}\sec^{\frac{3}{2}}(c + dx)\,dx &= \left(\sqrt{\cos(c + dx)}\sqrt{\sec(c + dx)}\right)\int \frac{\sqrt{a + a\cos(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)}\,dx\\ &= \frac{2a\sqrt{\sec(c + dx)}\sin(c + dx)}{d\sqrt{a + a\cos(c + dx)}}\end{aligned}$$

Mathematica [A] time = 0.0633067, size = 39, normalized size = 1.08

$$\frac{2\tan\left(\frac{1}{2}(c + dx)\right)\sqrt{\sec(c + dx)}\sqrt{a(\cos(c + dx) + 1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]*Tan[(c + d*x)/2])/d
```

Maple [A] time = 0.455, size = 50, normalized size = 1.4

$$-2\,{\frac { \left(-1+\cos \left(dx+c \right) \right) \cos \left(dx+c \right) \left(\left(\cos \left(dx+c \right) \right) ^{-1} \right) ^{3/2}\sqrt {a \left(1+\cos \left(dx+c \right) \right) }}{d\sin \left(dx+c \right) }}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/d*(-1+cos(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/
2)/sin(d*x+c)
```

Maxima [B] time = 1.61124, size = 132, normalized size = 3.67

$$\frac{2\left(\frac{\sqrt{2}\sqrt{a}\sin(dx+c)}{\cos(dx+c)+1} - \frac{\sqrt{2}\sqrt{a}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(2)*sqrt(a)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*sqrt(a)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/
2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(3/2))
```

Fricas [A] time = 1.56346, size = 112, normalized size = 3.11

$$\frac{2\,\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x
+ c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.343 $\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.108421, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4222, 2774, 216}

$$\frac{2\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx\\ &= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{d}\\ &= \frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A] time = 0.0870714, size = 70, normalized size = 1.23

$$\frac{\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Co
s[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/d
```

Maple [B] time = 0.514, size = 100, normalized size = 1.8

$$-2\,\frac{\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left((\cos(dx+c))^2-1\right)}{d(\sin(dx+c))^2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] -2/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*arctan(sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/s
in(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 1.79158, size = 197, normalized size = 3.46

$$\sqrt{a}\arctan\left(\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\,\arctan(\sin(2\,dx+2\,c),\cos(2\,dx+2\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + si
n(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(d*
x + c))/d
```

Fricas [A] time = 1.72048, size = 325, normalized size = 5.7

$$\left[\frac{\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{a\cos(dx+c)+a}\sqrt{-a}\sqrt{\cos(dx+c)}\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{d},-\frac{2\,\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(a*cos(d*x + c) + a)*sqrt(-a)*sqr
t(cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))/d, -
2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c)))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.344 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.163116, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4222, 2770, 2774, 216}

$$\frac{\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sqrt{\sec(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}dx$$
$$= \frac{a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos(c+dx}}{\sqrt{\cos(c+dx)}}$$
$$= \frac{a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}dx, x, -\right.}{d}$$
$$= \frac{\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.122019, size = 97, normalized size = 1.05

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{c}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*Si
n[(c + d*x)/2]))/(2*d)
```

Maple [A] time = 0.528, size = 132, normalized size = 1.4

$$\frac{\cos(dx+c)(-1+\cos(dx+c))^2}{d(\sin(dx+c))^4}\sqrt{a(1+\cos(dx+c))}\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos}{1+c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)/cos(d*x+c)))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/s
in(d*x+c)^4
```

Maxima [B] time = 1.95485, size = 1068, normalized size = 11.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c)
```

```
 - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))/d
```

Fricas [A] time = 1.70676, size = 251, normalized size = 2.73

$$-\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
 c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a(\cos(c+dx)+1)}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.345 $\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=136

$$\frac{a\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+\ldots}}$$

```
[Out] (3*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos
[c + d*x]]*Sec[c + d*x]^(3/2)) + (3*a*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.226686, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4222, 2770, 2774, 216}

$$\frac{a\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{3\sqrt{a}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{3a\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (3*Sqrt[a]*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*Sin[c + d*x])/(2*d*Sqrt[a + a*Cos
[c + d*x]]*Sec[c + d*x]^(3/2)) + (3*a*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}\,dx$$

$$= \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a$$

$$= \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{8}\left(3\sqrt{\cos(c+$$

$$= \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{3a\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(3\sqrt{\cos(c+d}{}$$

$$= \frac{3\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx}$$

Mathematica [A] time = 0.256725, size = 111, normalized size = 0.82

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(2\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c
+ d*x]]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*
(2*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2])))/(8*d)
```

Maple [A] time = 0.536, size = 169, normalized size = 1.2

$$-\frac{\cos(dx+c)(-1+\cos(dx+c))^3}{4d(\sin(dx+c))^6}\sqrt{a(1+\cos(dx+c))}\left(2\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)+3\sin(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(2*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c)))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^
6
```

Maxima [B] time = 2.03035, size = 1430, normalized size = 10.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*
c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
) - 1)))/d
```

Fricas [A] time = 1.70987, size = 308, normalized size = 2.26

$$-\frac{3\,\sqrt{a}(\cos\left(dx+c\right)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}\left(2\,\cos(dx+c)^2+3\,\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos
(d*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*(2*cos(d*x +
c)^2 + 3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(cos(c + d*x) + 1))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a\cos\left(dx+c\right)+a}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.346 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{26a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{208a^2\sin(c+dx)\sqrt{\sec}}{105d\sqrt{a\cos(c+dx}}$$

```
[Out] (208*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]])
+ (104*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]
) + (26*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]
) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.308021, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2762, 21, 2772, 2771}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{26a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d\sqrt{a\cos(c+dx)+a}}+\frac{104a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+\frac{208a^2\sin(c+dx)\sqrt{\sec}}{105d\sqrt{a\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (208*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]])
+ (104*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*x]]
) + (26*a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d*Sqrt[a + a*Cos[c + d*x]]
) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} - \frac{1}{7}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{1}{}}{\cos^{\frac{7}{2}}(c}$$

$$= \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{7}\left(13a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+}}{\mathrm{co}}$$

$$= \frac{26a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{1}{35}\left(52a\sqrt{c}\right.$$

$$= \frac{104a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sec^{\frac{7}{2}}}{7d\sqrt{a}}$$

$$= \frac{208a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{104a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{26a^2\mathrm{se}}{35d}$$

Mathematica [A] time = 0.276753, size = 72, normalized size = 0.45

$$\frac{2a(117\cos(c+dx)+26\cos(2(c+dx))+26\cos(3(c+dx))+41)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(41 + 117*Cos[c + d*x] + 26*Cos[2*(c + d*x)
] + 26*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(105*d)
```

Maple [A] time = 0.428, size = 83, normalized size = 0.5

$$-\frac{2a\left(104(\cos(dx+c))^4-52(\cos(dx+c))^3-13(\cos(dx+c))^2-24\cos(dx+c)-15\right)\cos(dx+c)}{105d\sin(dx+c)}\left((\cos(dx+c))\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d*a*(104*cos(d*x+c)^4-52*cos(d*x+c)^3-13*cos(d*x+c)^2-24*cos(d*x+c)-
15)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 1.62388, size = 355, normalized size = 2.2

$$\frac{4\left(\frac{105\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{245\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{273\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{171\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{38\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^3}{105\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 4/105*(105*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 245*sqrt(2)*a^
(3/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 273*sqrt(2)*a^(3/2)*sin(d*x + c
)^5/(cos(d*x + c) + 1)^5 - 171*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 38*sqrt(2)*a^(3/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
^(9/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 1))
```

Fricas [A] time = 1.65068, size = 238, normalized size = 1.48

$$\frac{2\left(104\,a\cos\left(dx+c\right)^3+52\,a\cos\left(dx+c\right)^2+39\,a\cos\left(dx+c\right)+15\,a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(104*a*cos(d*x + c)^3 + 52*a*cos(d*x + c)^2 + 39*a*cos(d*x + c) + 15*
a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*x + c
)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

3.347 $\int (a + a\cos(c+dx))^{3/2} \sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=121

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{6a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{12a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (12*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (
6*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.23856, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2762, 21, 2772, 2771}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{6a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}+\frac{12a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{5d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (12*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (
6*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]]) + (2*
a^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{1}{5}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-}{\cos^{\frac{5}{2}}(c}$$

$$= \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\left(9a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a}}{\cos}$$

$$= \frac{6a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{1}{5}\left(6a\sqrt{\cos(}\right.$$

$$= \frac{12a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{6a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sec^{\frac{5}{2}}(c}{5d\sqrt{a+}}$$

Mathematica [A] time = 0.186384, size = 62, normalized size = 0.51

$$\frac{2a(3\cos(c+dx)+3\cos(2(c+dx))+4)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*S
ec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(5*d)
```

Maple [A] time = 0.382, size = 73, normalized size = 0.6

$$-\frac{2\,a\left(6\,(\cos(dx+c))^3-3\,(\cos(dx+c))^2-2\,\cos(dx+c)-1\right)\cos(dx+c)}{5\,d\sin(dx+c)}\left((\cos(dx+c))^{-1}\right)^{\frac{7}{2}}\sqrt{a\,(1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d*a*(6*cos(d*x+c)^3-3*cos(d*x+c)^2-2*cos(d*x+c)-1)*cos(d*x+c)*(1/cos(d
*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [B] time = 1.62323, size = 293, normalized size = 2.42

$$\frac{4\left(\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)^{2}}{5d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 4/5*(5*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sqrt(2)*a^(3/2)
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 2*sqrt(2)*a^(3/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)*
(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

Fricas [A] time = 1.62508, size = 197, normalized size = 1.63

$$\frac{2\left(6\,a\cos\left(dx+c\right)^{2}+3\,a\cos\left(dx+c\right)+a\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{5\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(6*a*cos(d*x + c)^2 + 3*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*si
n(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)
```

3.348 $\int (a + a\cos(c + dx))^{3/2} \sec^{\frac{5}{2}}(c + dx)\, dx$

Optimal. Leaf size=81

$$\frac{2a^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d\sqrt{a\cos(c + dx) + a}} + \frac{10a^2 \sin(c + dx)\sqrt{\sec(c + dx)}}{3d\sqrt{a\cos(c + dx) + a}}$$

```
[Out] (10*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (
2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.174701, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4222, 2762, 21, 2771}

$$\frac{2a^2 \sin(c + dx) \sec^{\frac{3}{2}}(c + dx)}{3d\sqrt{a\cos(c + dx) + a}} + \frac{10a^2 \sin(c + dx)\sqrt{\sec(c + dx)}}{3d\sqrt{a\cos(c + dx) + a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (10*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (
2*a^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{1}{3}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{5a}{2}\ \cdots}{\cos^{\frac{3}{2}}(c+\cdots}$$
$$= \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{1}{3}\left(5a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos\cdots}}{\cos^{\frac{3}{2}}(c\cdots}$$
$$= \frac{10a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.133855, size = 52, normalized size = 0.64

$$\frac{2a(5\cos(c+dx)+1)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cos[c + d*x])]*(1 + 5*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Tan
[(c + d*x)/2])/(3*d)
```

Maple [A] time = 0.387, size = 63, normalized size = 0.8

$$-\frac{2a\left(5(\cos(dx+c))^2-4\cos(dx+c)-1\right)\cos(dx+c)}{3d\sin(dx+c)}\left((\cos(dx+c))^{-1}\right)^{\frac{5}{2}}\sqrt{a(1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*a*(5*cos(d*x+c)^2-4*cos(d*x+c)-1)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a
*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 1.58027, size = 169, normalized size = 2.09

$$\frac{4\left(\frac{3\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{3d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*(3*sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sqrt(2)*a^(3/2)*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*sqrt(2)*a^(3/2)*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/2)*(-sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)^(5/2))
```

Fricas [A] time = 1.6024, size = 166, normalized size = 2.05

$$\frac{2\,(5\,a\cos\left(dx+c\right)+a)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(5*a*cos(d*x + c) + a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*
x + c)^2 + d*cos(d*x + c))*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

3.349 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=96

$$\frac{2a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.186378, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2762, 21, 2774, 216}

$$\frac{2a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{a}{2} \cdots}{\sqrt{\cos(c+\cdots}}$$

$$= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\cos \cdots}}{\sqrt{\cos(c \cdots}}$$

$$= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\operatorname{Subst}\left(\int \cdots\right.}{d}$$

$$= \frac{2a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\sin \cdots}{d\sqrt{a+a\cos(c \cdots}}$$

Mathematica [A] time = 0.179012, size = 85, normalized size = 0.89

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(2\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(Sqrt[2]*
ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*Sin[(c + d*x)/2]))/
d
```

Maple [B] time = 0.4, size = 168, normalized size = 1.8

$$2\,\frac{\cos(dx+c)\,a\left((\cos(dx+c))^{-1}\right)^{3/2}\sqrt{a(1+\cos(dx+c))}}{d(1+\cos(dx+c))}\left(\cos(dx+c)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{\frac{\cdots}{1+\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(3/2),x)
```

```
[Out] 2/d*a*(cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d
*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+sin(d*x+c))
*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [B] time = 2.03895, size = 1346, normalized size = 14.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*d)
```

Fricas [A] time = 1.71392, size = 259, normalized size = 2.7

$$-\frac{2\left((a\cos(dx+c)+a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{a\cos(dx+c)+a}a\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d
*x + c))/(sqrt(a)*sin(d*x + c))) - sqrt(a*cos(d*x + c) + a)*a*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.350 $\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{3a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.177563, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2763, 21, 2774, 216}

$$\frac{3a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}+\frac{a^2\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (3*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
```

```
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\ldots}{\sqrt{\cos(\ldots}}$$

$$= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \ldots$$

$$= \frac{a^2\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(3a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\ldots}{d}$$

$$= \frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{a^2\sin(c\ldots}{d\sqrt{a+a\cos(c+d\ldots}}$$

Mathematica [A] time = 0.150304, size = 99, normalized size = 1.04

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(3\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\ldots\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(3*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]
]*Sin[(c + d*x)/2]))/(2*d)
```

Maple [A] time = 0.463, size = 130, normalized size = 1.4

$$-\frac{a\left((\cos(dx+c))^2-1\right)}{d(\sin(dx+c))^2}\left(\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\right)\sqrt{(\cos(dx+c\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*a*(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3*arctan(sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1/cos(d*x+c))^(1/2)*(a*(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^
2-1)
```

Maxima [B] time = 1.97764, size = 1084, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.71337, size = 258, normalized size = 2.72

$$\frac{\sqrt{a\cos\left(dx+c\right)+a}a\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a*cos(d*x + c) + a)*a*sqrt(cos(d*x + c))*sin(d*x + c) - 3*(a*cos(d*x
+ c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(
a)*sin(d*x + c))))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.351 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{a^2\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+}}$$

```
[Out] (7*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*Sin[c + d*x])/(2*d*Sqrt[a + a*C
os[c + d*x]]*Sec[c + d*x]^(3/2)) + (7*a^2*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.238039, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2763, 21, 2770, 2774, 216}

$$\frac{a^2\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{7a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{7a^2\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (7*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*Sin[c + d*x])/(2*d*Sqrt[a + a*C
os[c + d*x]]*Sec[c + d*x]^(3/2)) + (7*a^2*Sin[c + d*x])/(4*d*Sqrt[a + a*Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\,(}{\sqrt{a+}}$$

$$= \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{4}\left(7a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx}$$

$$= \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{1}{8}\left(7a\sqrt{\cos\right.$$

$$= \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{7a^2\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(7a\sqrt{\cos(c\right.}{}$$

$$= \frac{7a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{a^2\sin(c+dx)}{2d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+}$$

Mathematica [A] time = 0.253478, size = 111, normalized size = 0.79

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(-5\sin\left(\frac{1}{2}(c+dx)\right)+6\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)+7\sqrt{2}\,\mathrm{si}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(7*Sqrt[2
]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] - 5*Sin[(c + d*x)/2]
+ 6*Sin[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(8*d)
```

Maple [A] time = 0.463, size = 170, normalized size = 1.2

$$\frac{a\left(-1+\cos\left(dx+c\right)\right)^{2}\cos\left(dx+c\right)}{4\,d\left(\sin\left(dx+c\right)\right)^{4}}\left(2\,\sin\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\cos\left(dx+c\right)+7\,\sin\left(dx+c\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}+7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*a*(-1+cos(d*x+c))^2*(2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)+7*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*arctan(sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c
)))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)
^4
```

Maxima [B] time = 2.03693, size = 1458, normalized size = 10.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
2*c) + a*sin(2*d*x + 2*c) - (a*cos(2*d*x + 2*c) - 6*a)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) + (a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a*cos(2*d*x + 2*c) + (a*cos(2*d*x + 2*c) - 6*a)*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*a)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 7*(a*arctan2((cos(2*d*x + 2*c)^
2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*si
n(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - a*arctan2((cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - a*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1) + a*arctan2((cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
```

```
(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.73359, size = 316, normalized size = 2.26

$$-\frac{7\left(a\cos\left(dx+c\right)+a\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2\,a\cos(dx+c)^2+7\,a\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(7*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*a*cos(d*x + c)^2 + 7*a*cos(d*x +
c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.352 $\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{11a^2\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)}{3d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{a\,\mathrm{co}}}\right.}{8d}$$

```
[Out] (11*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*
Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (11*a^2*Sin[c + d*x])/(12*d*Sqrt[a + a*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (11*a^2*Sin[c + d*x])/(8*d*Sqrt[a + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.305351, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2763, 21, 2770, 2774, 216}

$$\frac{11a^2\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)}{3d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{11a^{3/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\,\mathrm{s}}{\sqrt{a\,\mathrm{co}}}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (11*a^(3/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*Sin[c + d*x])/(3*d*Sqrt[a + a*
Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (11*a^2*Sin[c + d*x])/(12*d*Sqrt[a + a*
Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (11*a^2*Sin[c + d*x])/(8*d*Sqrt[a + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}\,dx$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\right.}{\sqrt{a+}}$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{6}\left(11a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{8}\left(11a\sqrt{\mathrm{co}}\right.$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{8d\sqrt{a+a}}$$

$$= \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{11a^2\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{8d\sqrt{a+a}}$$

$$= \frac{11a^{3/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{a^2\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c}$$

Mathematica [A] time = 0.540369, size = 126, normalized size = 0.7

$$\frac{a\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(33\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\left(26\sin\left(\frac{1}{2}(c+\right.\right.\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sqrt[Sec[
c + d*x]]*(33*Sqrt[2]*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x
]]*(26*Sin[(c + d*x)/2] + 9*Sin[(3*(c + d*x))/2] + 2*Sin[(5*(c + d*x))/2]))
)/(48*d)
```

Maple [A] time = 0.474, size = 205, normalized size = 1.1

$$-\frac{a(-1+\cos(dx+c))^3\cos(dx+c)}{24d(\sin(dx+c))^6}\left(8\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+22\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d*a*(-1+cos(d*x+c))^3*(8*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)^2+22*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+33
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+33*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/
(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [B] time = 2.4454, size = 2622, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*x
+ 3*c) - (a*cos(3*d*x + 3*c) - a)*sin(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((3*a*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1
1*a*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (3*a*cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))) + 5*a*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))) - 8*a)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a)
 + 33*(a*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
 sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arcta
n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(
```

```
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3
*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - a
*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
 cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
+ 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - a*arctan2
((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)) + 1) + a*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^
(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))),
 cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1))*sqrt(a))/
d
```

Fricas [A] time = 1.74842, size = 350, normalized size = 1.94

$$-\frac{33\,(a\cos\left(dx+c\right)+a)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,a\cos(dx+c)^3+22\,a\cos(dx+c)^2+33\,a\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{24\,(d\cos\left(dx+c\right)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(33*(a*cos(d*x + c) + a)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt
(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a*cos(d*x + c)^3 + 22*a*cos(d*x
 + c)^2 + 33*a*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.353 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=201

$$\frac{2a^2\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}+\frac{38a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{146a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+$$

```
[Out] (1168*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]])
 + (584*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]
]) + (146*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*
x]]) + (38*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*
d)
```

Rubi [A] time = 0.413759, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2762, 2980, 2772, 2771}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{9d}+\frac{38a^3\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d\sqrt{a\cos(c+dx)+a}}+\frac{146a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d\sqrt{a\cos(c+dx)+a}}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```


```
[Out] (1168*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]])
 + (584*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(315*d*Sqrt[a + a*Cos[c + d*x]
]) + (146*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105*d*Sqrt[a + a*Cos[c + d*
x]]) + (38*a^3*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Cos[c + d*
x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(9*
d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
```

```
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} - \frac{1}{9}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{38a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+a}{9d}$$

$$= \frac{146a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{38a^3\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a}}{}$$

$$= \frac{584a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\cos(c+dx)}} + \frac{38a^3\sec^{\frac{7}{2}}}{63d\sqrt{a}}$$

$$= \frac{1168a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{584a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\cos(c+dx)}} + \frac{146a^3\text{se}}{105d}$$

Mathematica [A] time = 5.37714, size = 84, normalized size = 0.42

$$\frac{a^2(698\cos(c+dx)+803\cos(2(c+dx))+146\cos(3(c+dx))+146\cos(4(c+dx))+727)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{9}{2}}(c+dx)\sqrt{}}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(727 + 698*Cos[c + d*x] + 803*Cos[2*(c + d*
x)] + 146*Cos[3*(c + d*x)] + 146*Cos[4*(c + d*x)])*Sec[c + d*x]^(9/2)*Tan[(
c + d*x)/2])/(315*d)
```

Maple [A] time = 0.405, size = 95, normalized size = 0.5

$$-\frac{2 a^{2} \left(584 \left(\cos \left(dx+c\right)\right)^{5}-292 \left(\cos \left(dx+c\right)\right)^{4}-73 \left(\cos \left(dx+c\right)\right)^{3}-89 \left(\cos \left(dx+c\right)\right)^{2}-95 \cos \left(dx+c\right)-35\right) \cos (}{315 d \sin \left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(11/2),x)
```

```
[Out] -2/315/d*a^2*(584*cos(d*x+c)^5-292*cos(d*x+c)^4-73*cos(d*x+c)^3-89*cos(d*x+
c)^2-95*cos(d*x+c)-35)*cos(d*x+c)*(1/cos(d*x+c))^(11/2)*(a*(1+cos(d*x+c)))^
(1/2)/sin(d*x+c)
```

Maxima [A] time = 1.63878, size = 390, normalized size = 1.94

$$\frac{8 \left(\frac{315 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)}{\cos(dx+c)+1}-\frac{945 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{1449 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1287 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{572 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{104 \sqrt{2} a^{\frac{5}{2}} \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right.}{315 d \left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}} \left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{11}{2}} \left(\frac{3 \sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3 \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] 8/315*(315*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 945*sqrt(2)*a^
(5/2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1449*sqrt(2)*a^(5/2)*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 - 1287*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 + 572*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 104*s
qrt(2)*a^(5/2)*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)*(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)^3/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(-si
n(d*x + c)/(cos(d*x + c) + 1) + 1)^(11/2)*(3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 1))
```

Fricas [A] time = 1.66153, size = 285, normalized size = 1.42

$$\frac{2 \left(584 a^{2} \cos \left(dx+c\right)^{4}+292 a^{2} \cos \left(dx+c\right)^{3}+219 a^{2} \cos \left(dx+c\right)^{2}+130 a^{2} \cos \left(dx+c\right)+35 a^{2}\right) \sqrt{a \cos \left(dx+c\right)+a} \, \mathrm{s}}{315 \left(d \cos \left(dx+c\right)^{5}+d \cos \left(dx+c\right)^{4}\right) \sqrt{\cos \left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(584*a^2*cos(d*x + c)^4 + 292*a^2*cos(d*x + c)^3 + 219*a^2*cos(d*x +
c)^2 + 130*a^2*cos(d*x + c) + 35*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)
/((d*cos(d*x + c)^5 + d*cos(d*x + c)^4)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.354 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=161

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{46a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d\sqrt{a\cos(c+dx)+a}}+\frac{92a}{}$$

```
[Out] (92*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) +
(46*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) +
(6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (2
*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.346785, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2762, 2980, 2772, 2771}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{7d}+\frac{6a^3\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\cos(c+dx)+a}}+\frac{46a^3\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d\sqrt{a\cos(c+dx)+a}}+\frac{92a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (92*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) +
(46*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d*Sqrt[a + a*Cos[c + d*x]]) +
(6*a^3*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sqrt[a + a*Cos[c + d*x]]) + (2
*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} - \frac{1}{7}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{6a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx}{7d}$$

$$= \frac{46a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{6a^3\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos}}{}$$

$$= \frac{92a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{46a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d\sqrt{a+a\cos(c+dx)}} + \frac{6a^3\sec^{\frac{5}{2}}(c+}{7d\sqrt{a+a}}$$

Mathematica [A] time = 5.34543, size = 74, normalized size = 0.46

$$\frac{a^2(93\cos(c+dx)+23\cos(2(c+dx))+23\cos(3(c+dx))+29)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(29 + 93*Cos[c + d*x] + 23*Cos[2*(c + d*x)]
 + 23*Cos[3*(c + d*x)])*Sec[c + d*x]^(7/2)*Tan[(c + d*x)/2])/(21*d)
```

Maple [A] time = 0.392, size = 85, normalized size = 0.5

$$-\frac{2a^2\left(46(\cos(dx+c))^4-23(\cos(dx+c))^3-11(\cos(dx+c))^2-9\cos(dx+c)-3\right)\cos(dx+c)}{21d\sin(dx+c)}\left((\cos(dx+c))^{-1}\right)^{\frac{9}{2}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d*a^2*(46*cos(d*x+c)^4-23*cos(d*x+c)^3-11*cos(d*x+c)^2-9*cos(d*x+c)-3
)*cos(d*x+c)*(1/cos(d*x+c))^(9/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 1.57842, size = 328, normalized size = 2.04

$$\frac{8\left(\frac{21\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{56\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{36\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^2}{21\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{9}{2}}\left(\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 8/21*(21*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 56*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 36*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 8*sqrt(2)*a^(5/2)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)*(sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 1)^2/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)
*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(9/2)*(2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1))
```

Fricas [A] time = 1.64463, size = 244, normalized size = 1.52

$$\frac{2\left(46\,a^2\cos(dx+c)^3+23\,a^2\cos(dx+c)^2+12\,a^2\cos(dx+c)+3\,a^2\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{21\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*(46*a^2*cos(d*x + c)^3 + 23*a^2*cos(d*x + c)^2 + 12*a^2*cos(d*x + c) +
 3*a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/((d*cos(d*x + c)^4 + d*cos(d*
x + c)^3)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.355 $\int (a + a\cos(c + dx))^{5/2} \sec^{\frac{7}{2}}(c + dx)\, dx$

Optimal. Leaf size=121

$$\frac{22a^3 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2 \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d} + \frac{86a^3 \sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (86*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) +
(22*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.285106, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4222, 2762, 2980, 2771}

$$\frac{22a^3 \sin(c+dx) \sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}} + \frac{2a^2 \sin(c+dx) \sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{5d} + \frac{86a^3 \sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (86*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) +
(22*a^3*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) +
(2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2771

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Simp[(-2*b^2*Cos[e + f*x])/(f*(b*c + a*d)*S
qrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{22a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+d}{5d}$$

$$= \frac{86a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{22a^3\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\,c}}{}$$

Mathematica [A] time = 0.275582, size = 64, normalized size = 0.53

$$\frac{a^2(28\cos(c+dx)+43\cos(2(c+dx))+49)\tan\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sqrt{a(\cos(c+dx)+1)}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (a^2*Sqrt[a*(1 + Cos[c + d*x])]*(49 + 28*Cos[c + d*x] + 43*Cos[2*(c + d*x)]
)*Sec[c + d*x]^(5/2)*Tan[(c + d*x)/2])/(15*d)
```

Maple [A] time = 0.375, size = 75, normalized size = 0.6

$$-\frac{2a^2\left(43(\cos(dx+c))^3-29(\cos(dx+c))^2-11\cos(dx+c)-3\right)\cos(dx+c)}{15d\sin(dx+c)}\left((\cos(dx+c))^{-1}\right)^{\frac{7}{2}}\sqrt{a(1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*a^2*(43*cos(d*x+c)^3-29*cos(d*x+c)^2-11*cos(d*x+c)-3)*cos(d*x+c)*(1
/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)
```

Maxima [A] time = 1.56611, size = 204, normalized size = 1.69

$$\frac{8\left(\frac{15\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{28\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8\sqrt{2}a^{\frac{5}{2}}\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{15d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*(15*sqrt(2)*a^(5/2)*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sqrt(2)*a^(5/
2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 28*sqrt(2)*a^(5/2)*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 8*sqrt(2)*a^(5/2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(7/2)*(-sin(d*x + c)/(cos(d*x +
 c) + 1) + 1)^(7/2))
```

Fricas [A] time = 1.63454, size = 212, normalized size = 1.75

$$\frac{2\left(43\,a^{2}\cos\left(dx+c\right)^{2}+14\,a^{2}\cos\left(dx+c\right)+3\,a^{2}\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(43*a^2*cos(d*x + c)^2 + 14*a^2*cos(d*x + c) + 3*a^2)*sqrt(a*cos(d*x +
 c) + a)*sin(d*x + c)/((d*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sqrt(cos(d*x +
 c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.356 $\int (a + a\cos(c+dx))^{5/2} \sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d} + \frac{2a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} + \frac{14a^3\sin(c+d\ldots}{3d\sqrt{a\cos(\ldots}}$$

```
[Out] (2*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (14*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.287646, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2762, 2980, 2774, 216}

$$\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}{3d} + \frac{2a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d} + \frac{14a^3\sin(c+d\ldots}{3d\sqrt{a\cos(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (14*a^3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
```

```
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(\right.$$

$$= \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c-}{3d}$$

$$= \frac{14a^3\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c-}{3d}$$

$$= \frac{2a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{14a^3\sqrt{\sec(c+dx)}\,\mathrm{s}}{3d\sqrt{a+a\cos(}}$$

Mathematica [C] time = 6.29505, size = 404, normalized size = 2.93

$$\frac{\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)(a(\cos(c+dx)+1))^{5/2}\left(256\sin^6\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(5/2)*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^5*Sqr
t[(1 - 2*Sin[c/2 + (d*x)/2]^2)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(256*
Cos[(c + d*x)/2]^4*HypergeometricPFQ[{3/2, 2, 7/2}, {1, 9/2}, 2*Sin[c/2 + (
d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6 + 512*Hypergeometric2F1[3/2, 7/2, 9/2, 2*Si
n[c/2 + (d*x)/2]^2]*Sin[c/2 + (d*x)/2]^6*(2 - 3*Sin[c/2 + (d*x)/2]^2 + Sin[
c/2 + (d*x)/2]^4) + (21*Sqrt[2]*ArcSin[Sqrt[2]*Sqrt[Sin[c/2 + (d*x)/2]^2]]*
(15 - 10*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (d*x)/2]^4))/Sqrt[Sin[c/2 + (d*
x)/2]^2] - 14*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(45 + 30*Sin[c/2 + (d*x)/2]^
2 - 31*Sin[c/2 + (d*x)/2]^4 + 12*Sin[c/2 + (d*x)/2]^6)))/(672*d)
```

Maple [B] time = 0.405, size = 268, normalized size = 1.9

$$-\frac{2 a^{2} \cos \left(d x+c\right) \left(\sin \left(d x+c\right)\right)^{2}}{3 d \left(-1+\cos \left(d x+c\right)\right) \left(1+\cos \left(d x+c\right)\right)^{2}} \left(3 \arctan \left(\frac{\sin \left(d x+c\right)}{\cos \left(d x+c\right)} \sqrt{\frac{\cos \left(d x+c\right)}{1+\cos \left(d x+c\right)}}\right) \left(\cos \left(d x+c\right)\right)^{2} \left(\frac{\cos \left(d x+c\right)}{1+\cos \left(d x+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*a^2*(3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c
))*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+6*arctan(sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)+3*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+8*cos(d*x+c)*sin(d*x+c)+sin(d*x+c))*cos
(d*x+c)*sin(d*x+c)^2*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(
d*x+c))/(1+cos(d*x+c))^2
```

Maxima [B] time = 2.11449, size = 1883, normalized size = 13.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
```

```
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A] time = 1.71203, size = 344, normalized size = 2.49

$$-\frac{2\left(3\left(a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8\,a^{2}\cos(dx+c)+a^{2}\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*(a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqrt(a)*arctan(sqrt(a*cos(d
*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a^2*cos(d*x +
c) + a^2)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.357 $\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=134

$$\frac{5a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (5*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.282289, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2762, 2981, 2774, 216}

$$\frac{5a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{d}-\frac{a^3\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (5*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d - (a^3*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c +
 d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/d
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,s}{d}$$

$$= -\frac{a^3\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{2a^2\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}\,s}{d}$$

$$= \frac{5a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{a^3\sin(c}{d\sqrt{a+a\cos(c+dx}}$$

Mathematica [C] time = 3.07871, size = 202, normalized size = 1.51

$$\frac{\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(a(\cos(c+dx)+1))^{5/2}\left(6\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\,{}_3F\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Se
c[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F
1[1/2, 3/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 24*(3 + Cos[c + d*x])*Hypergeometr
ic2F1[3/2, 5/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 6*Csc[(c + d*x)
/2]^2*HypergeometricPFQ[{3/2, 2, 5/2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[
c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

Maple [A] time = 0.414, size = 186, normalized size = 1.4

$$\frac{a^2\cos(dx+c)}{d(1+\cos(dx+c))}\left(5\cos(dx+c)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\cos(dx+c)\sin(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(3/2),x)
```

```
[Out] 1/d*a^2*(5*cos(d*x+c)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/c
os(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+cos(d*x+c)*sin(d*x+c)+5*arctan
(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+2*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*
x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [B] time = 2.05394, size = 1314, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*
x + c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)*sqrt(a) + 5*(a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1))) + 1) - a^2*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + 1) + a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a)
 + 8*(a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
+ c) - (a^2*cos(d*x + c) - a^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)))*sqrt(a))/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*d)
```

Fricas [A] time = 1.75789, size = 298, normalized size = 2.22

$$-\frac{5\left(a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(a^{2}\cos(dx+c)+2\,a^{2}\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -(5*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) - (a^2*cos(d*x + c) + 2*a^2)*sqrt(a*co
s(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.358 $\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{19a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\cos(c+}}{2d\sqrt{\sec(c+dx)}}$$

```
[Out] (19*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (9*a^3*Sin[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.292256, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2763, 2981, 2774, 216}

$$\frac{19a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{4d}+\frac{9a^3\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\cos(c+}}{2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (19*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (9*a^3*Sin[c + d*x])/(4*d*Sqrt[a +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c +
 d*x])/(2*d*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+}}{}$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{8}$$

$$= \frac{9a^3\sin(c+dx)}{4d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} - $$

$$= \frac{19a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d} + \frac{9a^3\sin}{4d\sqrt{a+a\cos(c+}}$$

Mathematica [C] time = 3.03465, size = 202, normalized size = 1.44

$$\frac{\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(a(\cos(c+dx)+1))^{5/2}\left(2\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right){}_3F\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Se
c[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F
1[1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] + 8*(3 + Cos[c + d*x])*Hypergeometri
c2F1[3/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 + 2*Csc[(c + d*x)/
2]^2*HypergeometricPFQ[{3/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[c
 + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

Maple [A] time = 0.489, size = 166, normalized size = 1.2

$$-\frac{a^2\left((\cos(dx+c))^2-1\right)}{4d(\sin(dx+c))^2}\left(2\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)+11\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+19\,\mathrm{arct}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*a^2*(2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+11*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+19*arctan(sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] time = 2.08593, size = 1493, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*((a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x
+ 2*c) + a^2*sin(2*d*x + 2*c) - (a^2*cos(2*d*x + 2*c) - 10*a^2)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (a^2*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) - a^2*cos(2*d*x + 2*c) + 10*a^2 + (a^2*cos(2*d*x
 + 2*c) - 10*a^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 19*(a^2*ar
ctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))
 + 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) - 1) - a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
+ a^2*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.73859, size = 329, normalized size = 2.35

$$-\frac{19\left(a^{2}\cos\left(dx+c\right)+a^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(2\,a^{2}\cos(dx+c)^{2}+11\,a^{2}\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(19*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*s
qrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (2*a^2*cos(d*x + c)^2 + 11*a^2*
cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*
cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.359 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=180

$$\frac{13a^3\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\sec^{\frac{3}{2}}(c+dx)}+\frac{25a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a\cos}}\right.}{8d}$$

```
[Out] (25*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (13*a^3*Sin[c + d*x])/(12*d*Sqrt[a
+ a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c
 + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (25*a^3*Sin[c + d*x])/(8*d*Sqrt[a + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.360539, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2763, 2981, 2770, 2774, 216}

$$\frac{13a^3\sin(c+dx)}{12d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{3d\sec^{\frac{3}{2}}(c+dx)}+\frac{25a^{5/2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{a\cos}}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (25*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (13*a^3*Sin[c + d*x])/(12*d*Sqrt[a
+ a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin[c
 + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (25*a^3*Sin[c + d*x])/(8*d*Sqrt[a + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
```

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a}$$

$$= \frac{13a^3\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{8}\left(25a^2\sqrt{\text{co}}\right.$$

$$= \frac{13a^3\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{25}{8d\sqrt{a+a\,\text{co}}}$$

$$= \frac{13a^3\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{25}{8d\sqrt{a+a\,\text{co}}}$$

$$= \frac{25a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{8d} + \frac{13a^3\sin(c+dx)}{12d\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c}$$

Mathematica [C] time = 3.12753, size = 202, normalized size = 1.12

$$\frac{\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(a(\cos(c+dx)+1))^{5/2}\left(-2\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\,3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Se
c[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F
1[-1/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 8*(3 + Cos[c + d*x])*Hypergeometr
ic2F1[1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 2*Csc[(c + d*x)
/2]^2*HypergeometricPFQ[{1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*Sin[
c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

Maple [A] time = 0.47, size = 207, normalized size = 1.2

$$\frac{a^2 (-1+\cos (dx+c))^2 \cos (dx+c)}{24\, d\, (\sin (dx+c))^4}\left(8\,\sin (dx+c)\sqrt{\frac{\cos (dx+c)}{1+\cos (dx+c)}}\,(\cos (dx+c))^2+34\,\sin (dx+c)\sqrt{\frac{\cos (dx+c)}{1+\cos (dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/24/d*a^2*(-1+cos(d*x+c))^2*(8*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*cos(d*x+c)^2+34*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+7
5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+75*arctan(sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)
/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)^4
```

Maxima [B] time = 2.43227, size = 2651, normalized size = 14.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(4*(a^2*cos(3/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(3*d*
x + 3*c) - (a^2*cos(3*d*x + 3*c) - a^2)*sin(3/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1)))*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*sqrt(a) + 30*(cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))) + 1)^(1/4)*((a^2*sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 5*a^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - (a^2*cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c))) + 3*a^2*cos(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) - 4*a^2)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
1)))*sqrt(a) + 75*(a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arc
tan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(
```

```
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*
arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1))) + 1) - a^2*arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(s
in(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) - cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d
*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)
^(1/4)*(cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))
- 1) - a^2*arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2
+ sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan
2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) + 1) + a^2*arctan2((cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c),
 cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)) - 1))*sqrt(a))/d
```

Fricas [A] time = 1.75807, size = 363, normalized size = 2.02

$$-\frac{75\left(a^2\cos(dx+c)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(8a^2\cos(dx+c)^3+34a^2\cos(dx+c)^2+75a^2\cos(dx+c)\right)\sqrt{a\cos(dx+c)+a}\,s}{\sqrt{\cos(dx+c)}}}{24\left(d\cos(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(75*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*
sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (8*a^2*cos(d*x + c)^3 + 34*a^2
*cos(d*x + c)^2 + 75*a^2*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.360 $\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{163a^3\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{17a^3\sin(c+dx)}{24d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{4d\sec^{\frac{5}{2}}(c+dx)}+\frac{163}{}$$

```
[Out] (163*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (17*a^3*Sin[c + d*x])/(24*d*Sqrt[
a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (163*a^3*Sin[c + d*x])/(96*d*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (163*a^3*Sin[c + d*x])/(64*d*Sqrt[a
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.418572, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2763, 2981, 2770, 2774, 216}

$$\frac{163a^3\sin(c+dx)}{96d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{17a^3\sin(c+dx)}{24d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\cos(c+dx)+a}}+\frac{a^2\sin(c+dx)\sqrt{a\cos(c+dx)+a}}{4d\sec^{\frac{5}{2}}(c+dx)}+\frac{163}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (163*a^(5/2)*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (17*a^3*Sin[c + d*x])/(24*d*Sqrt[
a + a*Cos[c + d*x]]*Sec[c + d*x]^(5/2)) + (a^2*Sqrt[a + a*Cos[c + d*x]]*Sin
[c + d*x])/(4*d*Sec[c + d*x]^(5/2)) + (163*a^3*Sin[c + d*x])/(96*d*Sqrt[a +
 a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)) + (163*a^3*Sin[c + d*x])/(64*d*Sqrt[a
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
```

```
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(a+a\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}\,dx$$

$$= \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a$$

$$= \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{48}\left(163a^2\sqrt{\cos$$

$$= \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{163a^3}{96d\sqrt{a+a\cos$$

$$= \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{163a^3}{96d\sqrt{a+a\cos$$

$$= \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)} + \frac{a^2\sqrt{a+a\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{163a^3}{96d\sqrt{a+a\cos$$

$$= \frac{163a^{5/2}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64d} + \frac{17a^3\sin(c+dx)}{24d\sqrt{a+a\cos(c+dx)}\sec^{\frac{5}{2}}(c+$$

Mathematica [C] time = 3.11125, size = 202, normalized size = 0.92

$$\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}(a(\cos(c+dx)+1))^{5/2}\left(-6\sin^4(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right){}_3F_2$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(a*(1 + Cos[c + d*x]))^(5/2)*Sec[(c + d*x)/2]^4*Sqrt[Se
c[c + d*x]]*(7*(89 + 28*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Hypergeometric2F
1[-3/2, 1/2, 7/2, 2*Sin[(c + d*x)/2]^2] - 24*(3 + Cos[c + d*x])*Hypergeomet
ric2F1[-1/2, 3/2, 9/2, 2*Sin[(c + d*x)/2]^2]*Sin[c + d*x]^2 - 6*Csc[(c + d*
x)/2]^2*HypergeometricPFQ[{-1/2, 3/2, 2}, {1, 9/2}, 2*Sin[(c + d*x)/2]^2]*S
in[c + d*x]^4)*Tan[(c + d*x)/2])/(420*d)
```

Maple [A] time = 0.493, size = 242, normalized size = 1.1

$$-\frac{a^2(-1+\cos(dx+c))^3\cos(dx+c)}{192\,d(\sin(dx+c))^6}\left(48\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^3\sin(dx+c)+184\sin(dx+c)\sqrt{\frac{\cos(dx}{1+\cos(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d*a^2*(-1+cos(d*x+c))^3*(48*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)^3*sin(d*x+c)+184*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)^2+326*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+489*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+489*arctan(sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)/cos(d*x+c)))*cos(d*x+c)*(a*(1+cos(d*x+c)))^(1/2)/(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)/(1/cos(d*x+c))^(3/2)/sin(d*x+c)^6
```

Maxima [B] time = 3.38942, size = 10058, normalized size = 45.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/768*(10*(cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))) + 1)^(3/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d
*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^3 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*co
s(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c))*cos(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 12*(a^2*sin(4*d*x + 4*c)^3
 + (a^2*cos(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(2*a^2*cos(1/2
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*
d*x + 4*c) - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c
), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + 12*(a^2*sin(4*d*x + 4*c)^3 + (a^2*cos(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4
*c))*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))
 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*x + 4*c)^2 - 3*a^2*cos(4*d*x +
 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d
*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3
2*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c)
 + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(16*a^2*
```

```
cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^2 - 19*a^2*cos(4*d*x + 4*c) +
3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 2*(64*a^2*cos
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 19*a^2
*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*si
n(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 12*(4*a^2*cos(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c)^2 + a^2*sin(4*d*x
 + 4*c)^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(3/2*ar
ctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 - 8
*a^2*cos(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 14*a^2*cos(4*d*x +
4*c)^2 + 19*a^2*cos(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d
*x + 4*c)^2 - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
 + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x
 + 4*c)^3 - 2*a^2*cos(4*d*x + 4*c)^2 - 13*a^2*cos(4*d*x + 4*c) + (3*a^2*cos
(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2 - 8*a^2)*sin(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + (8*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*sin(4*d*
x + 4*c)^2 - 3*a^2*cos(4*d*x + 4*c) + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(
4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))^2 + 32*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 2*(16*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*sin(4*d*x + 4*c)^
2 - 19*a^2*cos(4*d*x + 4*c) + 3*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) - 2*(64*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*
c)))*sin(4*d*x + 4*c) + 19*a^2*sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))))*cos(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 4*(3*a^2*cos(4*d*x + 4*c)^3 - 11*a^2*cos(4*d*x + 4*c)^2 + 8*a^2*cos
(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*c)^2)*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 3*(2*a^2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)
 - 2*(a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d
*x + 4*c))))*sin(3/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*(3
*a^2*cos(4*d*x + 4*c) - 8*a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c) - 8*a^2)*sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(3/2*arctan2(s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c))) + 1)))*sqrt(a) - 6*(cos(1/2*arctan2(sin(4*d*
x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*
x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^
(1/4)*((3*a^2*cos(4*d*x + 4*c)^2*sin(4*d*x + 4*c) + 3*a^2*sin(4*d*x + 4*c)^
3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x +
4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d
*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 + 4
*(3*a^2*sin(4*d*x + 4*c)^3 + 3*(a^2*cos(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x +
4*c) + a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x
+ 4*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(3*a^2*sin(4*d*x + 4*c)^3 + 160*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c)))*sin(4*d*x + 4*c) + (3*a^2*cos(4*d*x + 4*c)^2 + 6*a^2*cos(
4*d*x + 4*c) + 43*a^2)*sin(4*d*x + 4*c) - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*sin(1/4*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c)))^2 + 2*(6*a^2*sin(4*d*x + 4*c)^3 + 3*a^2*cos(1/4*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 6*(a^2*cos(4*d*x + 4*c)^2 - a
^2*cos(4*d*x + 4*c))*sin(4*d*x + 4*c) - (320*a^2*cos(4*d*x + 4*c)^2 + 320*a
^2*sin(4*d*x + 4*c)^2 - 317*a^2*cos(4*d*x + 4*c) - 3*a^2)*sin(1/4*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) - 2*(20*a^2*cos(4*d*x + 4*c)^2 + 26*a^2*sin(4*d*x + 4*c)^2 -
 317*a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x
```

```
 + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 8*(
10*a^2*cos(4*d*x + 4*c)^2 + 13*a^2*sin(4*d*x + 4*c)^2 - 160*a^2*sin(4*d*x +
 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 10*a^2*cos(4*d
*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 3*(a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))
*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - (160*a^2*cos(4*d*x
+ 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*sin(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)) - (3*a^2*cos(4*d*x + 4*c)^3 + 120*a^2*cos(4*d*x + 4*c
)^2 - 160*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*
x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^3 - 3*
a^2*sin(4*d*x + 4*c)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 4*(3*a^2*cos(4*d*x + 4*c)^3 + 74*a^2*cos(4*d*x + 4*c)^2 - 197*a^2*cos(4*d*
x + 4*c) + (3*a^2*cos(4*d*x + 4*c) + 80*a^2)*sin(4*d*x + 4*c)^2 + 120*a^2 -
 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d*x + 4*
c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 3*(a^2*cos(4*d*x + 4*c) + 40*
a^2)*sin(4*d*x + 4*c)^2 + 4*(3*a^2*cos(4*d*x + 4*c)^3 + 126*a^2*cos(4*d*x +
 4*c)^2 + 243*a^2*cos(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(
4*d*x + 4*c)^2 + 120*a^2 - 40*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c
)^2 + 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4
*d*x + 4*c))) - 80*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2
*cos(4*d*x + 4*c) + a^2)*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*(6*a^2*cos(4
*d*x + 4*c)^3 + 214*a^2*cos(4*d*x + 4*c)^2 - 3*a^2*sin(4*d*x + 4*c)*sin(1/4
*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 240*a^2*cos(4*d*x + 4*c) +
2*(3*a^2*cos(4*d*x + 4*c) + 110*a^2)*sin(4*d*x + 4*c)^2 - (160*a^2*cos(4*d*
x + 4*c)^2 + 160*a^2*sin(4*d*x + 4*c)^2 - 157*a^2*cos(4*d*x + 4*c) - 3*a^2)
*cos(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*cos(1/2*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c))) - (80*a^2*cos(4*d*x + 4*c)^2 + 80*a^2*sin(
4*d*x + 4*c)^2 + 3*a^2*cos(4*d*x + 4*c))*cos(1/4*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))) + 2*(320*a^2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c)))^2*sin(4*d*x + 4*c) + 157*a^2*cos(1/4*arctan2(sin(4*d*x + 4*c), co
s(4*d*x + 4*c)))*sin(4*d*x + 4*c) + 8*(80*a^2*cos(1/4*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) - (3*a^2*cos(4*d*x + 4*c) + 110*a^
2)*sin(4*d*x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) -
 6*(a^2*cos(4*d*x + 4*c) + 40*a^2)*sin(4*d*x + 4*c) + 3*(a^2*cos(4*d*x + 4*
c) + a^2)*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*sin(1/2*arctan2(sin(1/2*arctan2(
sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))) + 1)))*sqrt(a) + 489*((a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d
*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*co
s(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^
2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x +
4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2
*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/
2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c)
)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
 + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))
*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
 + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + si
n(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(si
```

```
n(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(
sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) + 1) - (a^2*cos(4*d*x + 4*c)^2 + a^2
*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 -
2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x +
 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(
4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2
+ 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c)
)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*a
rctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*
x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2(-(c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin
(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), c
os(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)
)) + 1))*sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - cos(1/4*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*arctan2(sin(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1))), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))
)^2 + sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*ar
ctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*(cos(1/4*arctan2(sin(
4*d*x + 4*c), cos(4*d*x + 4*c)))*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x
+ 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c))) + 1)) + sin(1/4*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(1/2*
arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arcta
n2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1))) - 1) - (a^2*cos(4*d*x + 4*c)
^2 + a^2*sin(4*d*x + 4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4
*c)^2 - 2*a^2*cos(4*d*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos
(4*d*x + 4*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*
a^2*cos(4*d*x + 4*c) + a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4
*c)))^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*
x + 4*c))*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*c
os(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*
sin(4*d*x + 4*c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arc
tan2((cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d
*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))) + 1)), (cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + s
in(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arct
an2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))) + 1)) + 1) + (a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x +
4*c)^2 + 4*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - 2*a^2*cos(4*d
*x + 4*c) + a^2)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4
*(a^2*cos(4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 + 2*a^2*cos(4*d*x + 4*c)
+ a^2)*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*(a^2*cos(
4*d*x + 4*c)^2 + a^2*sin(4*d*x + 4*c)^2 - a^2*cos(4*d*x + 4*c))*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) - 4*(4*a^2*cos(1/2*arctan2(sin(4*
d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + a^2*sin(4*d*x + 4*c))*sin
(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*arctan2((cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(1/4)*sin(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x
+ 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) + 1)), (cos(
1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*
```

```
d*x + 4*c), cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(
4*d*x + 4*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c))), cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))) +
 1)) - 1))*sqrt(a))/((4*(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - 2*cos(4*
d*x + 4*c) + 1)*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + 4*
(cos(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 + 2*cos(4*d*x + 4*c) + 1)*sin(1/2*
arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + cos(4*d*x + 4*c)^2 + 4*(co
s(4*d*x + 4*c)^2 + sin(4*d*x + 4*c)^2 - cos(4*d*x + 4*c))*cos(1/2*arctan2(s
in(4*d*x + 4*c), cos(4*d*x + 4*c))) + sin(4*d*x + 4*c)^2 - 4*(4*cos(1/2*arc
tan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))*sin(4*d*x + 4*c) + sin(4*d*x + 4*
c))*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*d)
```

Fricas [A] time = 1.84049, size = 404, normalized size = 1.84

$$-\frac{489\left(a^2\cos\left(dx+c\right)+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\left(48\,a^2\cos(dx+c)^4+184\,a^2\cos(dx+c)^3+326\,a^2\cos(dx+c)^2+489\,a^2\cos(\right.}{\sqrt{\cos(dx+c)}}}{192\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/192*(489*(a^2*cos(d*x + c) + a^2)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a
)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - (48*a^2*cos(d*x + c)^4 + 184
*a^2*cos(d*x + c)^3 + 326*a^2*cos(d*x + c)^2 + 489*a^2*cos(d*x + c))*sqrt(a
*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.361 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=154

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{\cos(c+dx)+1}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26\sin(c+\ldots}{15d\sqrt{\ldots}}$$

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[1 + Cos
[c + d*x]]) - (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[1 + Cos[c + d*
x]]) + (2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[1 + Cos[c + d*x]])
```

Rubi [A] time = 0.284152, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4222, 2779, 2984, 12, 2781, 216}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{\cos(c+dx)+1}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}+\frac{26\sin(c+\ldots}{15d\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[1 + Cos[c + d*x]],x]
```


```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[1 + Cos
[c + d*x]]) - (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[1 + Cos[c + d*
x]]) + (2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[1 + Cos[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
```

```
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}} - \frac{1}{5}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1-4\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+a}}$$

$$= -\frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}} - \frac{1}{15}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}} - \frac{1}{15}$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}} - \left(\sqrt{\right.$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{1+\cos(c+dx)}} + \frac{\left(\sqrt{\right.}{}$$

$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{1+\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(}{15d}$$

Mathematica [C] time = 7.76938, size = 1540, normalized size = 10.

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(
-1))^(7/2)*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105
*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x
)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hyperg
eometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
```

```
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1
770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hy
pergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeome
tric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*S
in[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
 - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*
Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2,
 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Si
n[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*d*Sqrt[1 + Cos[c +
 d*x]]*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

Maple [B] time = 0.356, size = 294, normalized size = 1.9

$$\frac{\sqrt{2}\cos(dx+c)(\sin(dx+c))^4}{30\,d(-1+\cos(dx+c))^2(1+\cos(dx+c))^3}\left(15\,(\cos(dx+c))^3\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}\sqrt{2}+45\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/30/d*2^(1/2)*(15*cos(d*x+c)^3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+45*cos(d*x+c)^2*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+45*cos(d*x+c)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+15*arcs
in((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)+26
*cos(d*x+c)^2*sin(d*x+c)-2*cos(d*x+c)*sin(d*x+c)+6*sin(d*x+c))*cos(d*x+c)*(
1/cos(d*x+c))^(7/2)*sin(d*x+c)^4*(2+2*cos(d*x+c))^(1/2)/(-1+cos(d*x+c))^2/(
1+cos(d*x+c))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: RuntimeError

Fricas [A] time = 1.81763, size = 362, normalized size = 2.35

$$\frac{15\left(\sqrt{2}\cos\left(dx+c\right)^{3}+\sqrt{2}\cos\left(dx+c\right)^{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\left(13\cos(dx+c)^{2}-\cos(dx+c)+3\right)\sqrt{\cos(dx+c)+1}\sin(dx}{\sqrt{\cos(dx+c)}}}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*(sqrt(2)*cos(d*x + c)^3 + sqrt(2)*cos(d*x + c)^2)*arctan(sqrt(2)*s
qrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(13*cos(d*x + c)
^2 - cos(d*x + c) + 3)*sqrt(cos(d*x + c) + 1)*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{7}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(cos(d*x + c) + 1), x)
```

3.362 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{\cos(c+dx)+1}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\cos(c+dx)+1}}$$

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/d - (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[1 + Cos[c +
d*x]]) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[1 + Cos[c + d*x]])
```

Rubi [A] time = 0.201423, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4222, 2779, 2984, 12, 2781, 216}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{\cos(c+dx)+1}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/d - (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[1 + Cos[c +
d*x]]) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[1 + Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} - \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1-2\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+a}$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} - \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} - \frac{\left(\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{1+\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c-}{3d\sqrt{1-}}$$

Mathematica [C] time = 6.63411, size = 473, normalized size = 4.01

$$-\frac{2\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\,{}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)+1\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(
-1))^(7/2)*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 +
12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (
d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*
x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]
```

```
*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[1 + Cos[c + d*x]]
)
```

Maple [B] time = 0.332, size = 228, normalized size = 1.9

$$\frac{\sqrt{2}\cos(dx+c)(\sin(dx+c))^2}{6d(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}(\cos(dx+c))^2\sqrt{2}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+6\left(\frac{c}{1+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*2^(1/2)*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*2^(1/2)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))+6*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x
+c)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+3*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)*2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+2*cos(d*x+c)*sin(d*x+c)-
2*sin(d*x+c))*cos(d*x+c)*(2+2*cos(d*x+c))^(1/2)*sin(d*x+c)^2*(1/cos(d*x+c))
^(5/2)/(-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.84904, size = 328, normalized size = 2.78

$$-\frac{3\left(\sqrt{2}\cos(dx+c)^2+\sqrt{2}\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\sqrt{\cos(dx+c)+1}(\cos(dx+c)-1)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*cos(d*x + c))*arctan(sqrt(2)*sqrt
(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*sqrt(cos(d*x + c) +
 1)*(cos(d*x + c) - 1)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(cos(d*x + c) + 1), x)
```

3.363 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Cos[c +
 d*x]])
```

Rubi [A] time = 0.118816, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4222, 2779, 2781, 216}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{\cos(c+dx)+1}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[1 + Cos[c +
 d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{1+\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} - \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx}}$$

$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}} + \frac{\left(\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+}{1+\cos(}\right.}{d}$$

$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}}$$

Mathematica [C] time = 1.85072, size = 178, normalized size = 2.17

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}\cos(c+dx)(\cos(c+dx)+2)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-\cos(c+dx)+\cos(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 +
 Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c
+ d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (H
ypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d
*x]*Tan[c + d*x])/10))/(d*Sqrt[1 + Cos[c + d*x]])
```

Maple [A] time = 0.318, size = 144, normalized size = 1.8

$$\frac{\sqrt{2}\cos(dx+c)}{2d(1+\cos(dx+c))}\left(\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)+\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)*(2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)+2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsi
n((-1+cos(d*x+c))/sin(d*x+c))+2*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)
*(2+2*cos(d*x+c))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.79921, size = 252, normalized size = 3.07

$$\frac{\left(\sqrt{2}\cos\left(dx+c\right)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\sqrt{\cos(dx+c)+1}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d\cos\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] ((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqr
t(cos(d*x + c))/sin(d*x + c)) + 2*sqrt(cos(d*x + c) + 1)*sin(d*x + c)/sqrt(
cos(d*x + c)))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{\cos\left(dx+c\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(cos(d*x + c) + 1), x)
```

3.364 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/d
```

Rubi [A] time = 0.0787229, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4222, 2781, 216}

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/d
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx\\
&= -\frac{\left(\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)}{d}\\
&= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}
\end{aligned}$$

Mathematica [A] time = 0.107214, size = 68, normalized size = 1.45

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[1 + Cos[c + d*x]],x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])/d
```

Maple [A] time = 0.374, size = 82, normalized size = 1.7

$$\frac{(\cos(dx+c))^2-1}{d(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{2+2\cos(dx+c)}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(1/cos(d*x+c))^(1/2)*(2+2*cos(d*x+c))^(1/2)*arcsin((-1+cos(d*x+c))/sin(
d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.78513, size = 112, normalized size = 2.38

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x +
 c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{\cos(c+dx)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(cos(c + d*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{\cos(dx+c)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(cos(d*x + c) + 1), x)
```

3.365 $\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=94

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (2*ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.153836, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4222, 2777, 2774, 216, 2781}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/d) + (2*ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sqrt[Sec[c + d*x]])/d
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
```

```
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{1+\cos(c+dx)}}\,dx$$
$$= -\left(\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{1+\cos(c+dx)}}\,dx\right) + \left(\sqrt{\cos\ldots}\right.$$
$$= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)}{d} + \frac{\left(\sqrt{2}\sqrt{\ldots}\right.}{\ldots}$$
$$= -\frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} + \frac{2\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)\sqrt{\ldots}}{\ldots}$$

Mathematica [C] time = 0.56827, size = 171, normalized size = 1.82

$$\frac{i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)\left(-\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+\ldots}\right.\right.}{d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c +
d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)
*(c + d*x))]])*Cos[(c + d*x)/2])/(d*E^((I/2)*(c + d*x))*Sqrt[1 + Cos[c + d*
x]])
```

Maple [A] time = 0.36, size = 134, normalized size = 1.4

$$\frac{\sqrt{2}\cos(dx+c)\left(-1+\cos(dx+c)\right)^2}{2d\left(\sin(dx+c)\right)^4}\sqrt{2+2\cos(dx+c)}\left(\sqrt{2}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\ldots}{1\ldots}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)*(2+2*cos(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(2^(1/2)*
arcsin((-1+cos(d*x+c))/sin(d*x+c))+2*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)/cos(d*x+c)))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 1.81508, size = 204, normalized size = 2.17

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right) - 2\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x +
 c)) - 2*arctan(sqrt(cos(d*x + c) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(c+dx)+1}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(cos(c + d*x) + 1)*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sqrt(sec(d*x + c))), x)
```

3.366 $\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}+\frac{\sin(c}{d\sqrt{\cos(c+dx)+}}$$

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/d - (ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/d + Sin[c + d*x]/(d*Sqrt[1 + Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])
```

Rubi [A] time = 0.226502, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4222, 2778, 2982, 2781, 216, 2774}

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)}{d}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{\cos(c+dx)+1}}\right)}{d}+\frac{\sin(c}{d\sqrt{\cos(c+dx)+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[2]*ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]]*Sqrt[Se
c[c + d*x]])/d - (ArcSin[Sin[c + d*x]/Sqrt[1 + Cos[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/d + Sin[c + d*x]/(d*Sqrt[1 + Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2781

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> -Dist[Sqrt[2]/(Sqrt[a]*f), Subst[Int[1/Sqrt[1 - x
^2], x], x, (b*Cos[e + f*x])/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, d, e
, f}, x] && EqQ[a^2 - b^2, 0] && EqQ[d, a/b] && GtQ[a, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{1+\cos(c+dx)}}\,dx$$
$$= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-1+\ldots}{\sqrt{\cos(c+dx}\ldots}$$
$$= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{1+\cos(c\ldots}}{\sqrt{\cos(c+\ldots}}$$
$$= \frac{\sin(c+dx)}{d\sqrt{1+\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\ldots\right.}{d}$$
$$= \frac{\sqrt{2}\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d} - \frac{\sin^{-1}\left(\frac{\sin(c+dx)}{\sqrt{1+\cos(c+dx)}}\right)\sqrt{\cos(c+\ldots}}{d}$$

Mathematica [C] time = 0.826744, size = 257, normalized size = 2.06

$$\frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(-e^{i(c+dx)}+e^{2i(c+dx)}-e^{3i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+2\sqrt{2}e^{i(c+dx)}\sqrt{1\ldots}\right.}{4d\sqrt{\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(1 - E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) - E
^((3*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[
E^(I*(c + d*x))] + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - E^
(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c +
d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[1 + Cos[c + d*x]])
```

Maple [A] time = 0.386, size = 159, normalized size = 1.3

$$\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^3}{2d(\sin(dx+c))^6}\sqrt{2+2\cos(dx+c)}\left(-\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\sqrt{2}\arcsin\left(\frac{-1+\cos(dx+\ldots}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)*(2+2*cos(d*x+c))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+2^(1/2)*arcsin((-1+cos(d*x+c))/sin(d*
x+c))+arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)))/(1/c
os(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

Fricas [A] time = 1.78545, size = 365, normalized size = 2.92

$$-\frac{\left(\sqrt{2}\cos(dx+c)+\sqrt{2}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\left(\cos(dx+c)+1\right)\arctan\left(\frac{\sqrt{\cos(dx+c)+1}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\sqrt{\cos}}{d\cos(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -((sqrt(2)*cos(d*x + c) + sqrt(2))*arctan(sqrt(2)*sqrt(cos(d*x + c) + 1)*sq
rt(cos(d*x + c))/sin(d*x + c)) - (cos(d*x + c) + 1)*arctan(sqrt(cos(d*x + c
) + 1)*sqrt(cos(d*x + c))/sin(d*x + c)) - sqrt(cos(d*x + c) + 1)*sqrt(cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(1+cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(dx+c)+1}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(1+cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(cos(d*x + c) + 1)*sec(d*x + c)^(3/2)), x)
```

3.367 $\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=189

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
(26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (2*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.4211, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2779, 2984, 12, 2782, 205}

$$\frac{2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d\sqrt{a\cos(c+dx)+a}}+\frac{26\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(7/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
(26*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) - (2*S
ec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sec[c
+ d*x]^(5/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
```

```
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{5a}$$

$$= -\frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{15a^2}$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - \frac{(4\sqrt{}}{}$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} - (\sqrt{c}$$

$$= \frac{26\sqrt{\sec(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} - \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\cos(c+dx)}} + \frac{(2a}{}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{26\sqrt{\sec(c+dx)}\sin(c+}{15d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 7.76042, size = 1542, normalized size = 8.16

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(7/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^6*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(
-1))^(7/2)*(4725*Sin[c/2 + (d*x)/2]^2 - 48825*Sin[c/2 + (d*x)/2]^4 + 210105
```

```
*Sin[c/2 + (d*x)/2]^6 - 486630*Sin[c/2 + (d*x)/2]^8 + 655812*Sin[c/2 + (d*x
)/2]^10 - 710*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 40*Cos[(c + d*x)/2]^6*Hyperg
eometricPFQ[{2, 2, 2, 9/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^10 - 518760*Sin[c/2 + (d*x)/2]^12 + 1
770*Hypergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 + 226656*Sin[c/2 + (d*x)/2]^14 - 1500*Hy
pergeometric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/
2]^2)]*Sin[c/2 + (d*x)/2]^14 - 42048*Sin[c/2 + (d*x)/2]^16 + 440*Hypergeome
tric2F1[2, 9/2, 11/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*S
in[c/2 + (d*x)/2]^16 + 4725*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]
 - 56700*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*
Sin[c/2 + (d*x)/2]^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)] + 291060*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]]*Sin[c/2 + (d*x)/2]^4*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2
]^2)] - 833760*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*Sin[c/2 + (d*x)/2]^6*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)] + 1458000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]]*Sin[c/2 + (d*x)/2]^8*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)] - 1598400*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
(d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^10*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)] + 1080000*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^12*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)] - 414720*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^14*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
+ 2*Sin[c/2 + (d*x)/2]^2)] + 69120*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
2*Sin[c/2 + (d*x)/2]^2)]]*Sin[c/2 + (d*x)/2]^16*Sqrt[Sin[c/2 + (d*x)/2]^2/(
-1 + 2*Sin[c/2 + (d*x)/2]^2)] + 60*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2,
 2, 9/2}, {1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Si
n[c/2 + (d*x)/2]^10*(-5 + 4*Sin[c/2 + (d*x)/2]^2)))/(675*d*Sqrt[a*(1 + Cos[
c + d*x])]*(-1 + 2*Sin[c/2 + (d*x)/2]^2))
```

Maple [A] time = 0.436, size = 294, normalized size = 1.6

$$\frac{\sqrt{2}\cos(dx+c)(\sin(dx+c))^4}{15\,da(-1+\cos(dx+c))^2(1+\cos(dx+c))^3}\left(15\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{5/2}+45$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/15/d*2^(1/2)/a*(15*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*(cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+13*2^(1/2)*cos(d*x+c)^2*sin(d*x+
c)-cos(d*x+c)*sin(d*x+c)*2^(1/2)+3*2^(1/2)*sin(d*x+c))*cos(d*x+c)*sin(d*x+c
)^4*(1/cos(d*x+c))^(7/2)*(a*(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))^2/(1+cos(
d*x+c))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88342, size = 392, normalized size = 2.07

$$\frac{\frac{15\sqrt{2}\left(a\cos(dx+c)^3+a\cos(dx+c)^2\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}\left(13\cos(dx+c)^2-\cos(dx+c)+3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*sqrt(2)*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2)*arctan(sqrt(2)*sqrt(
a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*
sqrt(a*cos(d*x + c) + a)*(13*cos(d*x + c)^2 - cos(d*x + c) + 3)*sin(d*x + c
)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{7}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(7/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.368 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=151

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)}}\right.}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.291383, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2779, 2984, 12, 2782, 205}

$$\frac{2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d\sqrt{a\cos(c+dx)+a}}-\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)}}\right.}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (2*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]]) + (2*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(3*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
```

```
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{3a}$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{3a^2}$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int$$

$$= -\frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}} - \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{ad}} - \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 6.56287, size = 475, normalized size = 3.15

$$-\frac{2\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\cot\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\sin^8\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{7}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)+12\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (-2*Cot[c/2 + (d*x)/2]*Csc[c/2 + (d*x)/2]^4*((1 - 2*Sin[c/2 + (d*x)/2]^2)^(
-1))^(7/2)*(12*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2, 7/2}, {1, 9/2},
Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8 +
12*Hypergeometric2F1[2, 7/2, 9/2, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
```

```
*x)/2]^2)]*Sin[c/2 + (d*x)/2]^8*(4 - 7*Sin[c/2 + (d*x)/2]^2 + 3*Sin[c/2 + (
d*x)/2]^4) + 7*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1
 + 2*Sin[c/2 + (d*x)/2]^2)]*(15 - 20*Sin[c/2 + (d*x)/2]^2 + 8*Sin[c/2 + (d*
x)/2]^4)*(ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]
*(3 - 6*Sin[c/2 + (d*x)/2]^2) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]*(-3 + 7*Sin[c/2 + (d*x)/2]^2))))/(63*d*Sqrt[a*(1 + Cos[c + d*
x])])
```

Maple [A] time = 0.44, size = 227, normalized size = 1.5

$$\frac{\sqrt{2}\cos(dx+c)(\sin(dx+c))^2}{3\,da(-1+\cos(dx+c))(1+\cos(dx+c))^2}\left(3\,(\cos(dx+c))^2\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+6\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/3/d*2^(1/2)/a*(3*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-
1+cos(d*x+c))/sin(d*x+c))+6*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ar
csin((-1+cos(d*x+c))/sin(d*x+c))+3*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arcsin
((-1+cos(d*x+c))/sin(d*x+c))+cos(d*x+c)*sin(d*x+c)*2^(1/2)-2^(1/2)*sin(d*x+
c))*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(
-1+cos(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85062, size = 358, normalized size = 2.37

$$-\frac{\frac{3\sqrt{2}\left(a\cos(dx+c)^2+a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}(\cos(dx+c)-1)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(2)*(a*cos(d*x + c)^2 + a*cos(d*x + c))*arctan(sqrt(2)*sqrt(a*c
os(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqr
t(a*cos(d*x + c) + a)*(cos(d*x + c) - 1)*sin(d*x + c)/sqrt(cos(d*x + c)))/(
a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.369 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.179632, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2779, 12, 2782, 205}

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a
 + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) +
(2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx}{a}$$

$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} - \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx}}$$

$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}} + \frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{}{\sqrt{\cos(c+}}\right.}{d}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+d}{d\sqrt{a+a\cos(c+dx)}}$$

Mathematica [C] time = 1.82312, size = 180, normalized size = 1.59

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}\cos(c+dx)(\cos(c+dx)+2)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-\cos(c+dx)+\cos(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]*((Cos[c + d*x]*(2 +
 Cos[c + d*x])*Csc[(c + d*x)/2]^4*(1 - Cos[c + d*x] + ArcTanh[Sqrt[-(Sec[c
+ d*x]*Sin[(c + d*x)/2]^2)]]*Cos[c + d*x]*Sqrt[2 - 2*Sec[c + d*x]]))/2 - (H
ypergeometric2F1[2, 5/2, 7/2, -(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]*Sin[c + d
*x]*Tan[c + d*x])/10))/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.434, size = 142, normalized size = 1.3

$$\frac{\sqrt{2}\cos(dx+c)}{da(1+\cos(dx+c))}\left(\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+\sqrt{2}\sin(dx+c)+\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/a*(cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))+2^(1/2)*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ar
csin((-1+cos(d*x+c))/sin(d*x+c)))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos
```

```
(d*x+c)))^(1/2)/(1+cos(d*x+c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.79043, size = 282, normalized size = 2.5

$$\frac{\frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\frac{2\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a) + 2*sqrt(a*cos(d*x + c) + a)*
sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(a*cos(d*x + c) + a), x)
```

3.370 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.115159, antiderivative size = 76, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4222, 2782, 205}

$$\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a +
 a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx &= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx\\
&= -\frac{\left(2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a^2+ax^2}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)}{d}\\
&= \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] time = 0.0757927, size = 71, normalized size = 1.27

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*ArcTan[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]]*Cos[(c + d*x)/2]*Sqrt[Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a*(1 + Cos[c + d*x])])
```

Maple [A] time = 0.44, size = 88, normalized size = 1.6

$$\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{da(\sin(dx+c))^2}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/a*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x
+c)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81414, size = 404, normalized size = 7.21

$$\left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2d}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right.}{\sqrt{a}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(-1/a)
*sqrt(cos(d*x + c))*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(
```

```
cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/d, -sqrt(2)*arctan(sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/(sqrt(a)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a*(cos(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{a\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(a*cos(d*x + c) + a), x)
```

3.371 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=105

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]
]])/(Sqrt[a]*d) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])])/(Sqrt[a]*d)
```

Rubi [A] time = 0.251864, antiderivative size = 135, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2777, 2774, 216, 2782, 205}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}-\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*
Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2777

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(a_) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c
 + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b, Int[1/(Sqrt[a + b*Sin[e +
f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= -\left(\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\,dx\right) + \frac{\left(\sqrt{\mathrm{co}\right.}{}$$

$$= -\frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -\frac{a\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)}{ad} + \frac{\left(2a\sqrt{\mathrm{c}\right.}{}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+d}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a}}\right.}{}$$

Mathematica [C] time = 0.238786, size = 173, normalized size = 1.65

$$\frac{i\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos\left(\frac{1}{2}(c+dx)\right)\left(-\sinh^{-1}\left(e^{i(c+dx)}\right)+\sqrt{2}\tanh^{-1}\left(\frac{-1+e^{i(c+dx)}}{\sqrt{2}\sqrt{1+e^{2i(c+dx)}}}\right)+\tanh^{-1}\left(\sqrt{1+e^{2i}}\right.\right.}{d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (I*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*(-ArcSinh[E^(I*(c + d*x))] + Sqrt[2]*ArcTanh[(-1 + E^(I*(c +
d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + ArcTanh[Sqrt[1 + E^((2*I)
*(c + d*x))]])*Cos[(c + d*x)/2])/(d*E^((I/2)*(c + d*x))*Sqrt[a*(1 + Cos[c +
 d*x])])
```

Maple [A] time = 0.422, size = 134, normalized size = 1.3

$$\frac{\sqrt{2}\cos(dx+c)\left(-1+\cos(dx+c)\right)^2}{da\left(\sin(dx+c)\right)^4}\sqrt{a\left(1+\cos(dx+c)\right)}\left(\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)+\arcsin\left(\frac{-1+\mathrm{c}}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)/a*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(2^(1/2
)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+arcsin((-
1+cos(d*x+c))/sin(d*x+c)))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))
^(3/2)/sin(d*x+c)^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04716, size = 263, normalized size = 2.5

$$\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a}\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))
/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a*(cos(c + d*x) + 1))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.372 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=168

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{t}}{\sqrt{}}$$

```
[Out] -((ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x]
)/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(d*Sqrt[a + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.385706, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2778, 2982, 2782, 205, 2774, 216}

$$-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}+\frac{\sin(c+dx)}{d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\mathrm{t}}{\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)) + (Sqrt[2]*ArcTan[(Sqrt[a]*Sin[c + d*x]
)/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + Sin[c + d*x]/(d*Sqrt[a + a*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2778

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-2*d*Cos[e + f*x]*(c + d*Sin[e + f*x])
^(n - 1))/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)*Simp[a*c*d - b*(2*d^2*(n - 1) + c^2*(2*
n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
```

```
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$$

$$= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-a+\cdots}{\sqrt{\cos(c+d\cdots}}}{2a}$$

$$= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cdots}{\sqrt{\cos(c\cdots}}$$

$$= \frac{\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int \cdots\right.}{ad}$$

$$= -\frac{\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}d} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{a}\sin(\cdots}{\sqrt{2}\sqrt{\cos(c+dx)}\cdots}\right.\cdots}{\cdots}$$

Mathematica [C] time = 0.422286, size = 259, normalized size = 1.54

$$\frac{ie^{-2i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{\sec(c+dx)}\left(-e^{i(c+dx)}+e^{2i(c+dx)}-e^{3i(c+dx)}+e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+2\sqrt{2}e^{i(c+dx}\cdots\right.}{4d\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((I/4)*(1 + E^(I*(c + d*x)))*(1 - E^(I*(c + d*x)) + E^((2*I)*(c + d*x)) - E
^((3*I)*(c + d*x)) + E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[
E^(I*(c + d*x))] + 2*Sqrt[2]*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] - E^
(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c +
d*x))]])*Sqrt[Sec[c + d*x]])/(d*E^((2*I)*(c + d*x))*Sqrt[a*(1 + Cos[c + d*x
])])
```

Maple [A] time = 0.449, size = 167, normalized size = 1.

$$\frac{\sqrt{2}\cos(dx+c)(-1+\cos(dx+c))^{3}}{2\,da(\sin(dx+c))^{6}}\sqrt{a(1+\cos(dx+c))}\left(-\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\sqrt{2}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 1/2/d*2^(1/2)/a*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(-2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2^(1/2)*arctan(sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))+2*arcsin((-1+cos(d*x+c))/sin
(d*x+c)))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)
^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Fricas [A] time = 2.02265, size = 420, normalized size = 2.5

$$\frac{\sqrt{a}(\cos(dx+c)+1)\arctan\left(\frac{\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}(a\cos(dx+c)+a)\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}+\sqrt{a\cos(dx+c)+a}}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a)*(cos(d*x + c) + 1)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x +
c))/(sqrt(a)*sin(d*x + c))) - sqrt(2)*(a*cos(d*x + c) + a)*arctan(sqrt(2)*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c)))/sqrt(a)
+ sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x +
c) + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.373 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-$$

```
[Out] (11*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]]) - (
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (7*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.510614, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2766, 2984, 12, 2782, 205}

$$\frac{11\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{7\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6ad\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\cos(c+dx)+a)^{3/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (11*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- (19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]]) - (
Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (7*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
```

```
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{7a}{2}-2a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{3a}$$

$$= -\frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{7\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6ad\sqrt{a+a\cos(c+dx)}}$$

$$= \frac{11\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{19\sqrt{\sec(c+dx)}\sin(c}{6ad\sqrt{a+a\cos(c+}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

[Out] $Aborted

Maple [A] time = 0.441, size = 258, normalized size = 1.3

$$-\frac{\cos(dx+c)\sin(dx+c)\sqrt{2}}{12\,a^2d\left(-1+\cos(dx+c)\right)\left(1+\cos(dx+c)\right)^2}\left(-33\left(\cos(dx+c)\right)^2\sin(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/12/d*2^(1/2)/a^2*(-33*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-66*cos(d*x+c)*sin(d*x+c)*arcsin(
(-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-33*sin(d*x+c)
*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+19*co
s(d*x+c)^3*2^(1/2)-7*cos(d*x+c)^2*2^(1/2)-16*cos(d*x+c)*2^(1/2)+4*2^(1/2))*
cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/(-1+cos
(d*x+c))/(1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.92235, size = 451, normalized size = 2.29

$$-\frac{33\sqrt{2}\left(\cos(dx+c)^3+2\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\sqrt{a\cos(dx+c)+a}\left(19\cos(dx+c)^2\right.}{\sqrt{\cos(dx}}}{12\left(a^2d\cos(dx+c)^3+2\,a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*(33*sqrt(2)*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a
)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(19*cos(d*x + c)^2 + 12*cos(d*x + c)
 - 4)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(
d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.374 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (-7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (5*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.352862, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2766, 2984, 12, 2782, 205}

$$-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{5\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\cos(c+dx)+a}}-\frac{\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d)
- (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(3/2)) + (5*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
```

```
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{5a}{2}-a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\cos(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} - \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}} + \frac{5\sqrt{\sec(c+dx)}\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}} + \frac{\left(7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.}{}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+}{2d(a+a\cos(c+dx)}$$

Mathematica [C] time = 6.48889, size = 458, normalized size = 2.92

$$2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{4\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^4\left(\frac{1}{2}(c+dx)\right){}_3F_2\left(2,2,\frac{5}{2};1,\frac{9}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{70\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-35} - \frac{1}{6}\left(1\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^3*Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c
/2 + (d*x)/2]^2)^(-1))^(3/2)*((4*Cos[(c + d*x)/2]^4*HypergeometricPFQ[{2, 2
```

```
, 5/2}, {1, 9/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*Sin[c
/2 + (d*x)/2]^2)/(-35 + 70*Sin[c/2 + (d*x)/2]^2) - (Csc[c/2 + (d*x)/2]^6*(1
 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*(-3*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]
^2)]]*(-25 + 91*Sin[c/2 + (d*x)/2]^2 - 100*Sin[c/2 + (d*x)/2]^4 + 34*Sin[c/
2 + (d*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*
(-75 + 298*Sin[c/2 + (d*x)/2]^2 - 350*Sin[c/2 + (d*x)/2]^4 + 124*Sin[c/2 +
(d*x)/2]^6)))/6))/(d*(a*(1 + Cos[c + d*x]))^(3/2))
```

Maple [A] time = 0.398, size = 183, normalized size = 1.2

$$\frac{\cos(dx+c)\sqrt{2}}{4a^2d\sin(dx+c)(1+\cos(dx+c))}\left(7\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\cos(dx+c)\sin(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+7\arcsin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] 1/4/d*2^(1/2)/a^2*(7*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+7*arcsin((-1+cos(d*x+c))/sin(d*x+c))*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*cos(d*x+c)^2*2^(1/2)+cos(d*x+
c)*2^(1/2)+4*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1
/2)/sin(d*x+c)/(1+cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.90468, size = 381, normalized size = 2.43

$$\frac{7\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\sqrt{a\cos(dx+c)+a}(5\cos(dx+c)+4)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(7*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*sq
rt(a*cos(d*x + c) + a)*(5*cos(d*x + c) + 4)*sin(d*x + c)/sqrt(cos(d*x + c))
)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(3/2), x)
```

3.375 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=117

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) -
 Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.21931, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2766, 12, 2782, 205}

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) -
 Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{3a}{2\sqrt{\cos(c+dx)}\sqrt{a+a}}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+}}}{4a}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{2a^2+a}\right.}{2d}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} - \frac{\sin(c+d}{2d(a+a\cos(c+dx))^{3/}}$$

Mathematica [A] time = 0.476787, size = 99, normalized size = 0.85

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(3\cot^2\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx))}\right)+2\right)}{4ad\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((2 + 3*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]
^2*Sqrt[2 - 2*Sec[c + d*x]])*Tan[(c + d*x)/2])/(4*a*d*Sqrt[a*(1 + Cos[c + d
*x])]*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.408, size = 151, normalized size = 1.3

$$-\frac{\sqrt{2}\left((\cos(dx+c))^2-1\right)}{4a^2d(\sin(dx+c))^3}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(\sqrt{2}\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-3\arcsin\left(\frac{-1}{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(3/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*(2^(1/2)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*sin(d*x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 1.87113, size = 354, normalized size = 3.03

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+2\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(3*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*s
qrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a(\cos(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a*(cos(c + d*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(3/2), x)
```

3.376 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

```
[Out] (ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + S
in[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.215751, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2764, 12, 2782, 205}

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sin(c+dx)}{2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c
 + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + S
in[c + d*x]/(2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{2\sqrt{\cos(}}}{2a^2}$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c}}}{4a}$$
$$= \frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\Big(}{}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d} + \frac{\text{s}}{2d(a+a\cos(c}$$

Mathematica [A] time = 0.425541, size = 140, normalized size = 1.2

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\left(\sqrt{\cos(c+dx)+1}\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\right)}{2ad\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]]*
(ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[1 + Cos[c + d*x]] +
 2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)/2]))/(2*a*d*Sqrt[a*(
1 + Cos[c + d*x])])
```

Maple [A] time = 0.39, size = 156, normalized size = 1.3

$$-\frac{\cos(dx+c)\sqrt{2}(-1+\cos(dx+c))^2}{4a^2d(\sin(dx+c))^5}\sqrt{a(1+\cos(dx+c))}\left(\sqrt{2}\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+\arcsin\left(\frac{-1+\cos(dx+}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^2*(2
^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+arcsin((-1+cos(d*x+c))/
sin(d*x+c))*sin(d*x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*
```

```
x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Fricas [A] time = 1.89708, size = 351, normalized size = 3.

$$-\frac{\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*
sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqr
t(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.377 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=174

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{}{2d\sqrt{}}$$

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x]
)^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.396089, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2765, 2982, 2782, 205, 2774, 216}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{}{2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[S
ec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x]
)^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
```

```
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{}}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\mathrm{Su}}{}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} - \frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{\cos(c+dx)}}\right)}{}$$

Mathematica [C] time = 6.52325, size = 316, normalized size = 1.82

$$\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{2\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}-\frac{2\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}+\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{\tan\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{(a(\cos(c+dx)+1))^{3/2}} - \frac{i\sqrt{2}e^{-\frac{1}{2}i(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] ((-I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*(2*ArcSinh[E^(I*(c + d*x))] + (5*ArcTanh[(1 - E^(I*(c + d*
x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 2*ArcTanh[Sqrt[1 +
 E^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1
+ Cos[c + d*x]))^(3/2)) + (Cos[c/2 + (d*x)/2]^3*Sqrt[Sec[c + d*x]]*((-2*Cos
[(d*x)/2]*Sin[c/2])/d - (2*Cos[c/2]*Sin[(d*x)/2])/d + (Sec[c/2]*Sec[c/2 + (
d*x)/2]^2*Sin[(d*x)/2])/d + (Sec[c/2 + (d*x)/2]*Tan[c/2])/d))/(a*(1 + Cos[c
 + d*x]))^(3/2)
```

Maple [A] time = 0.393, size = 203, normalized size = 1.2

$$-\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{3}\cos\left(dx+c\right)}{4\,a^{2}d\left(\sin\left(dx+c\right)\right)^{7}}\sqrt{a\left(1+\cos\left(dx+c\right)\right)}\left(4\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\right)\sqrt{2}\sin\left(dx+c\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^3*cos(d*x+c)*(4
*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*si
n(d*x+c)+2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5*arcsin((-1+
cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [A] time = 2.65621, size = 521, normalized size = 2.99

$$\frac{5\,\sqrt{2}\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-8\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\sqrt{a}}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)
*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 8*(c
os(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)
*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*sq
rt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c)
```

```
+ a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.378 $\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$-\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{}{2d}$$

```
[Out] (-3*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + (9*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x
])^(3/2)*Sec[c + d*x]^(3/2)) + (3*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.547534, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2765, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{3/2}d}+\frac{9\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (-3*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + (9*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) - Sin[c + d*x]/(2*d*(a + a*Cos[c + d*x
])^(3/2)*Sec[c + d*x]^(3/2)) + (3*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos}}{}}{2a^2}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= -\frac{\sin(c+dx)}{2d(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)} + \frac{3\sin(c+dx)}{2ad\sqrt{a+a\cos(c+dx)}\sqrt{\sec(c+dx)}}$$

$$= -\frac{3\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d} + \frac{9\tan^{-1}\left(\frac{\sqrt{a}\sin(c}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a}}\right.}{}$$

Mathematica [C] time = 6.55013, size = 316, normalized size = 1.48

$$\frac{\cos^3\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(\frac{2\sin\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{d}+\frac{2\cos\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{d}-\frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{\tan\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{(a(\cos(c+dx)+1))^{3/2}} + \frac{3i\sqrt{2}e^{-\frac{1}{2}i(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((3*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*(ArcSinh[E^(I*(c + d*x))] + (3*ArcTanh[(1 - E^(I*(c + d*x
)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - ArcTanh[Sqrt[1 + E^
((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^3)/(d*E^((I/2)*(c + d*x))*(a*(1 + C
os[c + d*x]))^(3/2)) + (Cos[c/2 + (d*x)/2]^3*Sqrt[Sec[c + d*x]]*((2*Cos[(3*
d*x)/2]*Sin[(3*c)/2])/d - (Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/d +
(2*Cos[(3*c)/2]*Sin[(3*d*x)/2])/d - (Sec[c/2 + (d*x)/2]*Tan[c/2])/d))/(a*(1
 + Cos[c + d*x]))^(3/2)
```

Maple [A] time = 0.42, size = 235, normalized size = 1.1

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^4\cos(dx+c)}{4\,a^2d\,(\sin(dx+c))^9}\sqrt{a\,(1+\cos(dx+c))}\left(2\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos(dx+c))^2+6\arctan\left(\frac{\sin(dx}{\cos(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(3/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/4/d*2^(1/2)/a^2*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*cos(d*x+c)*(2
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+6*arctan(sin(d*x+c)
```

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+2^(1/2)*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3*2^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)+9*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x+c)
)^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [A] time = 2.67085, size = 572, normalized size = 2.67

$$-\frac{9\sqrt{2}\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-12\left(\cos(dx+c)^2+2\cos(dx+c)+\right.}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4*(9*sqrt(2)*(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2
)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 12*
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) +
a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*
(2*cos(d*x + c)^2 + 3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d
*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.379 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$\frac{95\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(}\right.}{16\sqrt{2}a^{5/2}d}$$

```
[Out] (163*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - (299*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]
]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (
17*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (
95*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.640021, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2766, 2978, 2984, 12, 2782, 205}

$$\frac{95\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{48a^2d\sqrt{a\cos(c+dx)+a}}-\frac{299\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\cos(c+dx)+a}}+\frac{163\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(}\right.}{16\sqrt{2}a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (163*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - (299*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]
]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (
17*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (
95*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
```

```
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{11a}{2}-3a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{8}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{95\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}}$$

$$= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{299\sqrt{\sec(c+dx)}\sin(c+dx)}{48a^2d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{17\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{16ad(a+a\cos(c+dx))^{3/2}}$$

$$= \frac{163\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{299\sqrt{\sec(c+dx)}\sin}{48a^2d\sqrt{a+a\cos(}}$$

<u>Mathematica</u> [C] time = 7.98176, size = 641, normalized size = 2.7

$$-\frac{\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\cot^5\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(640\sin^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^8\left(\frac{1}{2}(c+dx)\right)\,{}_5F_4\left(2,2,2,2,\frac{7}{2};\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] -(Cot[c/2 + (d*x)/2]^5*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^4*((1 - 2*Sin[
c/2 + (d*x)/2]^2)^(-1))^(7/2)*(640*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2,
 2, 2, 2, 7/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*
x)/2]^2)]*Sin[c/2 + (d*x)/2]^12 - 1280*Cos[(c + d*x)/2]^6*HypergeometricPFQ
[{2, 2, 2, 7/2}, {1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)
/2]^2)]*Sin[c/2 + (d*x)/2]^12*(-6 + 5*Sin[c/2 + (d*x)/2]^2) + 33*(1 - 2*Sin
[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2
)]*(-105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]]*
Cos[(c + d*x)/2]^4*(-10935 + 72902*Sin[c/2 + (d*x)/2]^2 - 188110*Sin[c/2 +
(d*x)/2]^4 + 234156*Sin[c/2 + (d*x)/2]^6 - 140732*Sin[c/2 + (d*x)/2]^8 + 33
208*Sin[c/2 + (d*x)/2]^10) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d
*x)/2]^2)]*(-1148175 + 10333785*Sin[c/2 + (d*x)/2]^2 - 38990350*Sin[c/2 + (
d*x)/2]^4 + 79946462*Sin[c/2 + (d*x)/2]^6 - 96281836*Sin[c/2 + (d*x)/2]^8 +
 68243596*Sin[c/2 + (d*x)/2]^10 - 26448512*Sin[c/2 + (d*x)/2]^12 + 4344400*
Sin[c/2 + (d*x)/2]^14))))/(41580*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 0.446, size = 316, normalized size = 1.3

$$\frac{\cos(dx+c)\sqrt{2}}{96\,da^{3}\sin(dx+c)\,(1+\cos(dx+c))^{2}}\left(-489\,(\cos(dx+c))^{3}\sin(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/96/d*2^(1/2)/a^3*(-489*cos(d*x+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin
(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-1467*cos(d*x+c)^2*sin(d*x+c)*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-1467*cos(
d*x+c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)+299*2^(1/2)*cos(d*x+c)^4-489*sin(d*x+c)*arcsin((-1+cos(d*x+c))/
sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+204*cos(d*x+c)^3*2^(1/2)-343*
cos(d*x+c)^2*2^(1/2)-192*cos(d*x+c)*2^(1/2)+32*2^(1/2))*cos(d*x+c)*(1/cos(d
*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.90572, size = 544, normalized size = 2.3

$$-\frac{489\,\sqrt{2}\left(\cos(dx+c)^{4}+3\,\cos(dx+c)^{3}+3\,\cos(dx+c)^{2}+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\,(299\,\mathrm{c}}{}}{96\left(a^{3}d\cos(dx+c)^{4}+3\,a^{3}d\cos(dx+c)^{3}+3\,a^{3}d\cos(dx+c)^{2}+a^{3}d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/96*(489*sqrt(2)*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 +
cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x
+ c))/(sqrt(a)*sin(d*x + c))) + 2*(299*cos(d*x + c)^3 + 503*cos(d*x + c)^2
+ 160*cos(d*x + c) - 32)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x
 + c)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x +
c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{(a\cos (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.380 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=197

$$\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)\sqrt{\sec(c+d}}{16ad(a\cos(c+dx)+a)^{3}}$$

```
[Out] (-75*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (4
9*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.494683, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2766, 2978, 2984, 12, 2782, 205}

$$\frac{49\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\cos(c+dx)+a}}-\frac{75\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{13\sin(c+dx)\sqrt{\sec(c+d}}{16ad(a\cos(c+dx)+a)^{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-75*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*C
os[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d
) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Cos[c + d*x])^(5/2)) - (1
3*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)) + (4
9*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{9a}{2}-2a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{8a^4}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} -$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}} - \frac{13\sqrt{\sec(c+dx)}\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}} + \frac{49\sqrt{\sec(c+dx)}\sin(c+dx)}{16a^2d\sqrt{a+a\cos(c+dx)}} +$$

$$= -\frac{75\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+d}{4d(a+a\cos(c+dx))^{5/}}$$

Mathematica [C] time = 6.75247, size = 508, normalized size = 2.58

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}\cos^5\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\frac{8\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^6\left(\frac{1}{2}(c+dx)\right)\,{}_4F_3\left(2,2,2,\frac{5}{2};1,1,\frac{11}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{315\left(2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}+\frac{1}{120}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^5*Sec[(c + d*x)/2]^4*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c
/2 + (d*x)/2]^2)^(-1))^(3/2)*((8*Cos[(c + d*x)/2]^6*HypergeometricPFQ[{2, 2
, 2, 5/2}, {1, 1, 11/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]*Sin[c/2 + (d*x)/2]^2)/(315*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) + (Csc[c/2 + (d
*x)/2]^8*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*S
in[c/2 + (d*x)/2]^2)]*(-15*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/
2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^4*(-343 + 1465*Sin[c/2 + (d*x)/2]^2 - 20
21*Sin[c/2 + (d*x)/2]^4 + 824*Sin[c/2 + (d*x)/2]^6) + Sqrt[Sin[c/2 + (d*x)/
2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-5145 + 33980*Sin[c/2 + (d*x)/2]^2 - 8
7764*Sin[c/2 + (d*x)/2]^4 + 109737*Sin[c/2 + (d*x)/2]^6 - 66122*Sin[c/2 + (
d*x)/2]^8 + 15344*Sin[c/2 + (d*x)/2]^10)))/120))/(d*(a*(1 + Cos[c + d*x]))^
(5/2))
```

Maple [A] time = 0.428, size = 258, normalized size = 1.3

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))\cos(dx+c)}{32\,da^3(\sin(dx+c))^3(1+\cos(dx+c))}\left(-75\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2\sin(dx+c)\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(-1+cos(d*x+c))*(-75*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))+49*cos(d*x+c)^3*2
^(1/2)-150*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)+36*cos(d*x+c)^2*2^(1/2)-75*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-53*cos(d*x+c)*2^
(1/2)-32*2^(1/2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/
sin(d*x+c)^3/(1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.9474, size = 473, normalized size = 2.4

$$\frac{75\sqrt{2}\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\sqrt{a\cos(dx+c)+a}(49\,c}{}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(75*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(49*cos(d*x + c)^2 + 85*cos(d*x
 + c) + 32)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(5/2), x)
```

3.381 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{19\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d\sqrt{\sec(c}}$$

```
[Out] (19*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (9*Si
n[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.351771, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2766, 2978, 12, 2782, 205}

$$\frac{19\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{9\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d\sqrt{\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```


```
[Out] (19*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
 - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (9*Si
n[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{7a}{2}-a\cos(c+dx}{\sqrt{\cos(c+dx)}(a+a\cos(c}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos\right.}}{}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(19\sqrt{c\right.}}{}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{9\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(19\sqrt{c\right.}}{}$$

$$= \frac{19\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{}}$$

Mathematica [A] time = 0.942985, size = 131, normalized size = 0.83

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\left(76\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx))}\right)-\cos(c+dx)(9\cos(c+dx)+13)\sec^4\left(\frac{1}{2}(c+d\right.\right.}{64\sqrt{2}a^2d\sqrt{1-\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((76*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]] - Cos[c + d*x]*(13 +
 9*Cos[c + d*x])*Sec[(c + d*x)/2]^4*Sqrt[2 - 2*Sec[c + d*x]])*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(64*Sqrt[2]*a^2*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[1 - S
ec[c + d*x]])
```

Maple [A] time = 0.444, size = 222, normalized size = 1.4

$$\frac{\cos(dx+c)\sqrt{2}\,(-1+\cos(dx+c))^2}{32\,da^3(\sin(dx+c))^5}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(9\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
*(-1+cos(d*x+c))^2*(9*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^
2+4*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-19*arcsin((-1+cos(
d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-13*2^(1/2)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)-19*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/sin(d*x+c)^5/
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

Fricas [A] time = 1.88155, size = 466, normalized size = 2.97

$$-\frac{19\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+\frac{2\sqrt{a\cos(dx+c)+a}(9\cos}{}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(19*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*
sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)
*sin(d*x + c))) + 2*sqrt(a*cos(d*x + c) + a)*(9*cos(d*x + c)^2 + 13*cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos
(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(5/2), x)
```

3.382 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=157

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{}{4d\sqrt{\sec(c}}$$

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
+ Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + Sin[c
+ d*x]/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.350278, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2764, 2978, 12, 2782, 205}

$$\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{}{4d\sqrt{\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
+ Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + Sin[c
+ d*x]/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c}}}{4a^2}$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= \frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+d}}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} + \frac{}{4d(a+a\cos}$$

Mathematica [A] time = 0.666025, size = 122, normalized size = 0.78

$$\frac{-2\tan^3\left(\frac{1}{2}(c+dx)\right)+48\sin^4\left(\frac{1}{2}(c+dx)\right)\csc^3(c+dx)-5\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right.}\right.}{32a^2d\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-5*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]*Sqrt
[2 - 2*Sec[c + d*x]] + 48*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 2*Tan[(c + d*
x)/2]^3)/(32*a^2*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.423, size = 221, normalized size = 1.4

$$\frac{\cos(dx+c)\sqrt{2}\,(-1+\cos(dx+c))^3}{32\,da^3(\sin(dx+c))^7}\sqrt{a(1+\cos(dx+c))}\left(\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+5\arcsin\left(\frac{-1+\cos}{\sin(d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^3*(2
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+5*arcsin((-1+cos(d*x+
c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+4*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+5*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-5*2^(1/2)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos
(d*x+c)))^(3/2)/sin(d*x+c)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Fricas [A] time = 1.89319, size = 460, normalized size = 2.93

$$-\frac{5\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\sqrt{a\cos(dx+c)+a}(\cos(}{\sqrt{}}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(5*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(cos(d*x + c)^2 + 5*cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
 + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.383 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{7\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d\sqrt{\sec(c+}}$$

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
- Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (7*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.35199, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2765, 2978, 12, 2782, 205}

$$\frac{3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{7\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{}{4d\sqrt{\sec(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (3*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d)
- Sin[c + d*x]/(4*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (7*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos}}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c+}}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d} - \frac{}{4d(a+a\cos}$$

Mathematica [A] time = 0.71445, size = 164, normalized size = 1.04

$$\frac{\sqrt{\cos(c+dx)}(\cos(c+dx)+1)^{3/2}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(6\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)+1}\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)\right.}{32d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])^(3/2)*Sec[(c + d*x)/2]*Sqrt[Sec[c +
d*x]]*(6*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Cos[(c + d*x)/2]
^2*Sqrt[1 + Cos[c + d*x]] - Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(Sin[(c +
```

```
d*x)/2] - 7*Sin[(3*(c + d*x))/2])))/(32*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 0.415, size = 222, normalized size = 1.4

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^4\cos(dx+c)}{32\,da^3(\sin(dx+c))^9}\sqrt{a\,(1+\cos(dx+c))}\left(7\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+3\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^4*cos(d*x+c)*(
7*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+3*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-4*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)+3*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-3*2^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)/sin(d*x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [A] time = 1.91081, size = 463, normalized size = 2.95

$$-\frac{3\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-\frac{2\sqrt{a\cos(dx+c)+a}(7\cos}{}}{32\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(3*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(7*cos(d*x + c)^2 + 3*cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.384 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-$$

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - (43*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*
x])^(5/2)*Sec[c + d*x]^(3/2)) - (11*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*
x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.528317, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2765, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}-\frac{43\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - (43*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[
Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c + d*
x])^(5/2)*Sec[c + d*x]^(3/2)) - (11*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d*
x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c}}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{11\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sqrt{\sec(c}}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} - \frac{43\tan^{-1}\left(\frac{\sqrt{a}\,\mathrm{si}}{\sqrt{2}\sqrt{\cos(c+dx}}\right.}{}$$

Mathematica [C] time = 2.24476, size = 373, normalized size = 1.74

$$\frac{e^{-\frac{1}{2}i(c+dx)}\left(\frac{1}{16}ie^{-2i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\cos\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{2}\left(-7e^{i(c+dx)}-8e^{2i(c+dx)}+8e^{3i(c+dx)}+7e^{4i(c+dx)}+15e^{5i(c+dx)}+16\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (((I/16)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-43*(1 + E^(I*(c
+ d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqr
t[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(-15 - 7*E^(I*(c + d*x)) - 8
*E^((2*I)*(c + d*x)) + 8*E^((3*I)*(c + d*x)) + 7*E^((4*I)*(c + d*x)) + 15*E
^((5*I)*(c + d*x)) + 16*(1 + E^(I*(c + d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x)
)]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2])/E^((2*I)*(c +
 d*x)) - (16*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqr
t[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2]^5)/(4*
d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 0.424, size = 320, normalized size = 1.5

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^5\cos(dx+c)}{32\,da^3(\sin(dx+c))^{11}}\sqrt{a(1+\cos(dx+c))}\left(15\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^2+32\cos(dx+c)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(
15*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+32*cos(d*x+c)*2^(
1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*
x+c)+43*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-4*2^(1/2)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+32*arctan(sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+43*arcsin((-1+cos(d
*x+c))/sin(d*x+c))*sin(d*x+c)-11*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))
/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^11
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [A] time = 3.75379, size = 662, normalized size = 3.09

$$\frac{43\sqrt{2}\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-64\left(\cos(dx+c)^3+3\right.}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/32*(43*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)*s
qrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*
sin(d*x + c))) - 64*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1
)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d
*x + c))) - 2*sqrt(a*cos(d*x + c) + a)*(15*cos(d*x + c)^2 + 11*cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

3.385 $\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=254

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{115\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{}$$

```
[Out] (-5*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + (115*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c +
d*x])^(5/2)*Sec[c + d*x]^(5/2)) - (15*Sin[c + d*x])/(16*a*d*(a + a*Cos[c +
d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (35*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.666436, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4222, 2765, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{5/2}d}+\frac{35\sin(c+dx)}{16a^2d\sqrt{\sec(c+dx)}\sqrt{a\cos(c+dx)+a}}+\frac{115\sqrt{\cos(c+dx)}\sqrt{\sec(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)),x]
```


```
[Out] (-5*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + (115*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) - Sin[c + d*x]/(4*d*(a + a*Cos[c +
d*x])^(5/2)*Sec[c + d*x]^(5/2)) - (15*Sin[c + d*x])/(16*a*d*(a + a*Cos[c +
d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (35*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}}{}}{4a^2}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{\sin(c+dx)}{4d(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{15\sin(c+dx)}{16ad(a+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{5\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d} + \frac{115\tan^{-1}\left(\frac{\sqrt{a}\sin}{\sqrt{2}\sqrt{\cos(c+dx)}}\right.}{}$$

Mathematica [C] time = 3.17602, size = 412, normalized size = 1.62

$$\frac{e^{-\frac{1}{2}i(c+dx)}\left(40i\sqrt{2}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^5\left(\frac{1}{2}(c+dx)\right)\sinh^{-1}\left(e^{i(c+dx)}\right)+115i\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^5\left(\frac{1}{2}(c+dx\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((40*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2]^5 + (115*I)*Sq
rt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]
*ArcTanh[(1 - E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])]*Cos
[(c + d*x)/2]^5 - ((I/16)*(-8 - 47*E^(I*(c + d*x)) - 39*E^((2*I)*(c + d*x))
 - 16*E^((3*I)*(c + d*x)) + 16*E^((4*I)*(c + d*x)) + 39*E^((5*I)*(c + d*x))
 + 47*E^((6*I)*(c + d*x)) + 8*E^((7*I)*(c + d*x)) + 40*E^(I*(c + d*x))*(1 +
 E^(I*(c + d*x)))^4*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[Sqrt[1 + E^((2*I)
*(c + d*x))]])*Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]])/E^((3*I)*(c + d*x)))/(4
*d*E^((I/2)*(c + d*x))*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 0.463, size = 352, normalized size = 1.4

$$-\frac{\sqrt{2}(-1+\cos(dx+c))^6\cos(dx+c)}{32\,da^3(\sin(dx+c))^{13}}\sqrt{a(1+\cos(dx+c))}\left(16\,(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\sqrt{2}\sqrt{\frac{\cos(dx}{1+\cos(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(5/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/32/d*2^(1/2)/a^3*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^6*cos(d*x+c)*(
16*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+80*cos(d*x+c)*2^(1/2)*arctan(sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+115*arcsin((
-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-20*2^(1/2)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+80*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+115*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*sin(d*x+c)-35*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos(d*x+c)
)^(7/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^13
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2)), x)
```

Fricas [A] time = 3.76728, size = 693, normalized size = 2.73

$$-\frac{115\sqrt{2}\left(\cos(dx+c)^{3}+3\cos(dx+c)^{2}+3\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-160\left(\cos(dx+c\right.}{32\left(a^{3}d\cos(dx+c)^{3}+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/32*(115*sqrt(2)*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c) + 1)
*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a
)*sin(d*x + c))) - 160*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x + c)
+ 1)*sqrt(a)*arctan(sqrt(a*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*si
n(d*x + c))) - 2*(16*cos(d*x + c)^3 + 55*cos(d*x + c)^2 + 35*cos(d*x + c))*
sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x +
c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(5/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(5/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2)), x)
```

3.386 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=277

$$\frac{193\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^3d\sqrt{a\cos(c+dx)+a}}-\frac{109\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{629\sin(c+dx)\sqrt{\sec(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}+\frac{1015\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

```
[Out] (1015*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*
d) - (629*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x
]]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) -
(23*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) -
(109*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2))
 + (193*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]]
)
```

Rubi [A] time = 0.780325, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2766, 2978, 2984, 12, 2782, 205}

$$\frac{193\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^3d\sqrt{a\cos(c+dx)+a}}-\frac{109\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{629\sin(c+dx)\sqrt{\sec(c+dx)}}{64a^3d\sqrt{a\cos(c+dx)+a}}+\frac{1015\sqrt{\cos(c+dx)}\sqrt{\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```


```
[Out] (1015*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*
d) - (629*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x
]]) - (Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) -
(23*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) -
(109*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c + d*x])^(3/2))
 + (193*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*a^3*d*Sqrt[a + a*Cos[c + d*x]]
)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{15a}{2}-4a\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.}{2}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{64a^2d(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{109\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{64a^2d(a+a\cos(c+dx))^{3/2}}$$

$$= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= -\frac{629\sqrt{\sec(c+dx)}\sin(c+dx)}{64a^3d\sqrt{a+a\cos(c+dx)}} - \frac{\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{23\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{48ad(a+a\cos(c+dx))^{5/2}}$$

$$= \frac{1015\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{629\sqrt{\sec(c+dx)}\,\mathrm{si}}{64a^3d\sqrt{a+a\cos}}$$

Mathematica [C] time = 8.37468, size = 696, normalized size = 2.51

$$\frac{\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{7/2}\cot^7\left(\frac{c}{2}+\frac{dx}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-7680\sin^{14}\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^{10}\left(\frac{1}{2}(c+dx)\right){}_6F_5\left(2,2,2,2,\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Cot[c/2 + (d*x)/2]^7*Csc[c/2 + (d*x)/2]^4*Sec[(c + d*x)/2]^6*((1 - 2*Sin[c
/2 + (d*x)/2]^2)^(-1))^(7/2)*(-7680*Cos[(c + d*x)/2]^10*HypergeometricPFQ[{
2, 2, 2, 2, 2, 7/2}, {1, 1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c
/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14 + 19200*Cos[(c + d*x)/2]^8*Hypergeo
metricPFQ[{2, 2, 2, 2, 7/2}, {1, 1, 1, 15/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*
Sin[c/2 + (d*x)/2]^2)]*Sin[c/2 + (d*x)/2]^14*(-7 + 6*Sin[c/2 + (d*x)/2]^2)
+ 143*(1 - 2*Sin[c/2 + (d*x)/2]^2)^3*Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[
c/2 + (d*x)/2]^2)]*(315*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 +
 (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(351384 - 2928877*Sin[c/2 + (d*x)/2]^2 +
9953934*Sin[c/2 + (d*x)/2]^4 - 17629526*Sin[c/2 + (d*x)/2]^6 + 17139064*Sin
[c/2 + (d*x)/2]^8 - 8670660*Sin[c/2 + (d*x)/2]^10 + 1793816*Sin[c/2 + (d*x)
/2]^12) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(-110685
960 + 1291549455*Sin[c/2 + (d*x)/2]^2 - 6601900452*Sin[c/2 + (d*x)/2]^4 + 1
9406027859*Sin[c/2 + (d*x)/2]^6 - 36160322412*Sin[c/2 + (d*x)/2]^8 + 443132
22590*Sin[c/2 + (d*x)/2]^10 - 35736693140*Sin[c/2 + (d*x)/2]^12 + 183052542
```

```
12*Sin[c/2 + (d*x)/2]^14 - 5410719584*Sin[c/2 + (d*x)/2]^16 + 704274992*Sin
[c/2 + (d*x)/2]^18))))/(3243240*d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [A] time = 0.457, size = 390, normalized size = 1.4

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))\cos(dx+c)}{384\,da^4(\sin(dx+c))^3(1+\cos(dx+c))^2}\left(-3045\,(\cos(dx+c))^4\sin(dx+c)\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\arcsin\left(\frac{-1+\cos(dx}{\sin(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(-1+cos(d*x+c))*(-3045*cos(d*x+c)^4*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(3/2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))-12180*cos(d*x
+c)^3*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)-18270*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))
*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+1887*cos(d*x+c)^5*2^(1/2)-12180*cos(d*x+
c)*sin(d*x+c)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c))
)^(3/2)+3195*2^(1/2)*cos(d*x+c)^4-3045*sin(d*x+c)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-831*cos(d*x+c)^3*2^(1/2)-3355*c
os(d*x+c)^2*2^(1/2)-1024*cos(d*x+c)*2^(1/2)+128*2^(1/2))*cos(d*x+c)*(1/cos(
d*x+c))^(5/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/(1+cos(d*x+c))^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 2.01543, size = 640, normalized size = 2.31

$$-\frac{3045\,\sqrt{2}\left(\cos(dx+c)^5+4\,\cos(dx+c)^4+6\,\cos(dx+c)^3+4\,\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}}{\sqrt{a}\sin(dx+}\right.}{384\left(a^4d\cos(dx+c)^5+4\,a^4d\cos(dx+c)^4+6\,a^4d\cos(dx+c)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(3045*sqrt(2)*(cos(d*x + c)^5 + 4*cos(d*x + c)^4 + 6*cos(d*x + c)^3
 + 4*cos(d*x + c)^2 + cos(d*x + c))*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x +
c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(1887*cos(d*x + c)^4
 + 5082*cos(d*x + c)^3 + 4251*cos(d*x + c)^2 + 896*cos(d*x + c) - 128)*sqrt
(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^5
 + 4*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2
 + a^4*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.387 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=237

$$\frac{691\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{199\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{363\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}\right.}{64\sqrt{2}a^{7/2}d}$$

```
[Out] (-363*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*
d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - (
19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - (
199*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
 + (691*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]
])
```

Rubi [A] time = 0.630605, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2766, 2978, 2984, 12, 2782, 205}

$$\frac{691\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^3d\sqrt{a\cos(c+dx)+a}}-\frac{199\sin(c+dx)\sqrt{\sec(c+dx)}}{192a^2d(a\cos(c+dx)+a)^{3/2}}-\frac{363\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+}\right.}{64\sqrt{2}a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (-363*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*
d) - (Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(a + a*Cos[c + d*x])^(7/2)) - (
19*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)) - (
199*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2))
 + (691*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*a^3*d*Sqrt[a + a*Cos[c + d*x]
])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{13a}{2}-3a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^{5/2}}\,dx}{6a^2}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{24a}$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{199\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{199\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{199\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{\sqrt{\sec(c+dx)}\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}} - \frac{19\sqrt{\sec(c+dx)}\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}} - \frac{199\sqrt{\sec(c+dx)}\sin(c+dx)}{192a^2d(a+a\cos(c+dx))^{3/2}} +$$

$$= -\frac{363\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{\sqrt{\sec(c+dx)}\sin(c+}{6d(a+a\cos(c+dx))}$$

Mathematica [C] time = 6.879, size = 561, normalized size = 2.37

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)^{3/2}\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{16\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\cos^8\left(\frac{1}{2}(c+dx)\right){}_5F_4\left(2,2,2,2,\frac{5}{2};1,1,1,\frac{13}{2};\frac{\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1}\right)}{3465\left(2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)} - \right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^7*Sec[(c + d*x)/2]^6*Sin[c/2 + (d*x)/2]*((1 - 2*Sin[c
/2 + (d*x)/2]^2)^(-1))^(3/2)*((16*Cos[(c + d*x)/2]^8*HypergeometricPFQ[{2,
2, 2, 2, 5/2}, {1, 1, 1, 13/2}, Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x
)/2]^2)]*Sin[c/2 + (d*x)/2]^2)/(3465*(-1 + 2*Sin[c/2 + (d*x)/2]^2)) - (Csc[
c/2 + (d*x)/2]^10*(1 - 2*Sin[c/2 + (d*x)/2]^2)^2*Sqrt[Sin[c/2 + (d*x)/2]^2/
(-1 + 2*Sin[c/2 + (d*x)/2]^2)]*(105*ArcTanh[Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 +
 2*Sin[c/2 + (d*x)/2]^2)]]*Cos[(c + d*x)/2]^6*(2187 - 12908*Sin[c/2 + (d*x)
/2]^2 + 27986*Sin[c/2 + (d*x)/2]^4 - 26380*Sin[c/2 + (d*x)/2]^6 + 8752*Sin[
c/2 + (d*x)/2]^8) + Sqrt[Sin[c/2 + (d*x)/2]^2/(-1 + 2*Sin[c/2 + (d*x)/2]^2)
]*(-229635 + 2120790*Sin[c/2 + (d*x)/2]^2 - 8267707*Sin[c/2 + (d*x)/2]^4 +
17646926*Sin[c/2 + (d*x)/2]^6 - 22251094*Sin[c/2 + (d*x)/2]^8 + 16548816*Si
n[c/2 + (d*x)/2]^10 - 6712984*Sin[c/2 + (d*x)/2]^12 + 1144608*Sin[c/2 + (d*
x)/2]^14)))/1680))/(d*(a*(1 + Cos[c + d*x]))^(7/2))
```

Maple [A] time = 0.435, size = 326, normalized size = 1.4

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))^2\cos(dx+c)}{384\,da^4\,(\sin(dx+c))^5\,(1+\cos(dx+c))}\left(1089\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\arcsin\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*(-1+cos(d*x+c))^2*(1089*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)+3267*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*arcsin((-1+cos(d*x+c))/si
n(d*x+c))-691*2^(1/2)*cos(d*x+c)^4+3267*arcsin((-1+cos(d*x+c))/sin(d*x+c))*
cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-1183*cos(d*x+c)^3*2
^(1/2)+1089*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)+275*cos(d*x+c)^2*2^(1/2)+1215*cos(d*x+c)*2^(1/2)+384*2^(1/
2))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/(
1+cos(d*x+c))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.9625, size = 571, normalized size = 2.41

$$\frac{1089\,\sqrt{2}\left(\cos(dx+c)^4+4\,\cos(dx+c)^3+6\,\cos(dx+c)^2+4\,\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{384\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(1089*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(691*cos(d*x + c)^3 + 1874*cos(d*x
 + c)^2 + 1599*cos(d*x + c) + 384)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sq
rt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*
cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(a*cos(d*x + c) + a)^(7/2), x)
```

3.388 $\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{103\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{63\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{\cdots}{16ad\cdots}$$

```
[Out] (63*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
 - Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) - (5*Si
n[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (103*S
in[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.480572, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2766, 2978, 12, 2782, 205}

$$-\frac{103\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{63\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{\cdots}{16ad\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (63*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
 - Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) - (5*Si
n[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (103*S
in[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{11a}{2}-2a\cos(c+d}{\sqrt{\cos(c+dx)}(a+a\cos(c}}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos\right.}{}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{}{192a^2}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{}{192a^2}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{5\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}} - \frac{}{192a^2}$$

$$= \frac{63\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}}$$

Mathematica [A] time = 3.69234, size = 153, normalized size = 0.78

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(6048\cos^6\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\tanh^{-1}\left(\sqrt{\sin^2\left(\frac{1}{2}(c+dx)\right)(-\sec(c+dx))}\right)-2(532}{3072\sqrt{2}a^3d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\cos(c+dx)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + a*Cos[c + d*x])^(7/2),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(-2*(493 + 532*Cos[c + d*x] + 103*Cos[2*(c + d*x)])*Sqr
t[2 - 2*Sec[c + d*x]] + 6048*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2
)]]*Cos[(c + d*x)/2]^6*Sec[c + d*x])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3*d*
Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])
```

Maple [A] time = 0.461, size = 288, normalized size = 1.5

$$-\frac{\cos(dx+c)\sqrt{2}(-1+\cos(dx+c))^3}{384\,da^4(\sin(dx+c))^7}\sqrt{(\cos(dx+c))^{-1}}\sqrt{a(1+\cos(dx+c))}\left(103\,(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+cos(d*x+c)*a)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)*(-1+cos(d*x+c))^3*(103*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)-189*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+163*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-378*arcsin((-1+cos(d*x
+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-71*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)-189*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-195*2
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/sin(d*x+c)^7/(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(a\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

Fricas [A] time = 1.92119, size = 560, normalized size = 2.84

$$-\frac{189\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)}{384\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/384*(189*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(c
os(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(103*cos(d*x + c)^3 + 266*cos(d*x
+ c)^2 + 195*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d
*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x
```

```
+ c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+a*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec (dx + c)}}{(a\cos (dx + c) + a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+a*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(a*cos(d*x + c) + a)^(7/2), x)
```

3.389 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=197

$$-\frac{5\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{\ }{16ad\sqrt{\ }}$$

```
[Out] (13*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
 + Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + Sin[c
 + d*x]/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (5*Sin[c +
 d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.481076, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2764, 2978, 12, 2782, 205}

$$-\frac{5\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{13\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{\ }{16ad\sqrt{\ }}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (13*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
 + Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + Sin[c
 + d*x]/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) - (5*Sin[c +
 d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2764

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a*d*n - b*c*
(m + 1) - b*d*(m + n + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && L
tQ[m, -1] && LtQ[0, n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c,
 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
```

```
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cdots}{\sqrt{\cos(c\cdots}}}{6a^2}$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d\cdots}}$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d\cdots}}$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d\cdots}}$$

$$= \frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+d\cdots}}$$

$$= \frac{13\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} + \frac{\cdots}{6d(a+a\,\mathrm{co}\cdots}$$

Mathematica [A] time = 0.843406, size = 125, normalized size = 0.63

$$\frac{\sin(c+dx)(4\cos(c+dx)-5\cos(2(c+dx))+73)\sec^6\left(\frac{1}{2}(c+dx)\right)-312\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{2-2\sec(c+dx)}\tanh^{-1}\left(\sqrt{\mathrm{s}\cdots}\right.}{3072a^3d\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-312*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*Cot[(c + d*x)/2]*Sq
rt[2 - 2*Sec[c + d*x]] + (73 + 4*Cos[c + d*x] - 5*Cos[2*(c + d*x)])*Sec[(c
+ d*x)/2]^6*Sin[c + d*x])/(3072*a^3*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[c
 + d*x]])
```

Maple [A] time = 0.427, size = 288, normalized size = 1.5

$$-\frac{\cos(dx+c)\sqrt{2}(-1+\cos(dx+c))^4}{384\,da^4(\sin(dx+c))^9}\sqrt{a(1+\cos(dx+c))}\left(-5(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+39\arcsin\left(\frac{-1}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*(-1+cos(d*x+c))^4*
(-5*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*arcsin((-1+co
s(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+7*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2+78*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c
)*sin(d*x+c)+37*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+39*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-39*2^(1/2)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2))/(1/cos(d*x+c))^(1/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/sin(d*
x+c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c))), x)
```

Fricas [A] time = 1.97095, size = 552, normalized size = 2.8

$$-\frac{39\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)+}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/384*(39*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) + 2*(5*cos(d*x + c)^3 - 2*cos(d*x + c)^
2 - 39*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c
)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2
 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sqrt(sec(d*x + c))), x)
```

3.390 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{17\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{}{16ad\sqrt{\mathrm{se}}}$$

```
[Out] (7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
- Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + (3*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (17*Sin
[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.496976, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2765, 2978, 12, 2782, 205}

$$\frac{17\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}+\frac{}{16ad\sqrt{\mathrm{se}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (7*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
- Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + (3*Sin
[c + d*x])/(16*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (17*Sin
[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos}}}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+dx)}} + \frac{3\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(c+}}$$

$$= \frac{7\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{}{6d(a+a\cos}$$

Mathematica [A] time = 3.64394, size = 153, normalized size = 0.78

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(2(140\cos(c+dx)+17\cos(2(c+dx))+59)\sqrt{2-2\sec(c+dx)}+672\cos^6\left(\frac{1}{2}(c+dx)\right)\right.}{3072\sqrt{2}a^3d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(2*(59 + 140*Cos[c + d*x] + 17*Cos[2*(c + d*x)])*Sqrt[2
 - 2*Sec[c + d*x]] + 672*ArcTanh[Sqrt[-(Sec[c + d*x]*Sin[(c + d*x)/2]^2)]]*
Cos[(c + d*x)/2]^6*Sec[c + d*x])*Tan[(c + d*x)/2])/(3072*Sqrt[2]*a^3*d*Sqrt
[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])])
```

Maple [A] time = 0.424, size = 288, normalized size = 1.5

$$\frac{\sqrt{2}\,(-1+\cos(dx+c))^5\cos(dx+c)}{384\,da^4(\sin(dx+c))^{11}}\sqrt{a(1+\cos(dx+c))}\left(17\,(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}+53\sqrt{2}\sqrt{\frac{\cos(d}{1+\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^5*cos(d*x+c)*(
17*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+53*2^(1/2)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+21*arcsin((-1+cos(d*x+c))/sin(d*x+
c))*cos(d*x+c)^2*sin(d*x+c)-49*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)+42*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-21*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*arcsin((-1+cos(d*x+c))/sin(d*x+c)
)*sin(d*x+c))/(1/cos(d*x+c))^(3/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/sin(d*
x+c)^11
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [A] time = 2.16779, size = 555, normalized size = 2.82

$$-\frac{21\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/384*(21*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(17*cos(d*x + c)^3 + 70*cos(d*x + c
)^2 + 21*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(3/2)), x)
```

3.391 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{67\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{}{6d\sec^{\frac{3}{2}}(}$$

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
- Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)) - (13*Si
n[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (67*Si
n[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.48544, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2765, 2977, 2978, 12, 2782, 205}

$$\frac{67\sin(c+dx)}{192a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{5\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a\cos(c+dx)+a}}\right)}{64\sqrt{2}a^{7/2}d}-\frac{}{6d\sec^{\frac{3}{2}}(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (5*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos
[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d)
- Sin[c + d*x]/(6*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)) - (13*Si
n[c + d*x])/(48*a*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (67*Si
n[c + d*x])/(192*a^2*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
```

```
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(}}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{13\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sqrt{\sec(}}$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{64\sqrt{2}a^{7/2}d} - \frac{}{6d(a+a}$$

Mathematica [A] time = 4.55703, size = 196, normalized size = 0.99

$$\frac{\cos^7\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(15\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\sin^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}}\right)+\sqrt{2}\sin\right.}{24a^4d\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(\cos(c+dx)+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^7*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[Sec[
c + d*x]]*(15*ArcSin[Sin[(c + d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*Sqrt[Cos[(c
 + d*x)/2]^2] + Sqrt[2]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sin[(c + d*x)
/2]*(33 - 26*Tan[(c + d*x)/2]^2 + 8*Tan[(c + d*x)/2]^4)))/(24*a^4*d*Sqrt[Co
s[(c + d*x)/2]^2]*(1 + Cos[c + d*x])^4)
```

Maple [A] time = 0.426, size = 288, normalized size = 1.5

$$-\frac{\sqrt{2}(-1+\cos(dx+c))^6\cos(dx+c)}{384\,da^4(\sin(dx+c))^{13}}\sqrt{a(1+\cos(dx+c))}\left(67(\cos(dx+c))^3\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}-17\sqrt{2}\sqrt{\frac{\cos(}{1+\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^6*cos(d*x+c)*
(67*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-17*2^(1/2)*(cos(
```

```
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+15*arcsin((-1+cos(d*x+c))/sin(d*x
+c))*cos(d*x+c)^2*sin(d*x+c)-35*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+30*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-15*2^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+15*arcsin((-1+cos(d*x+c))/sin(d*x+c
))*sin(d*x+c))/(1/cos(d*x+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d
*x+c)^13
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [A] time = 1.89747, size = 555, normalized size = 2.82

$$-\frac{15\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-2(}{384\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(15*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(67*cos(d*x + c)^3 + 50*cos(d*x + c
)^2 + 15*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x +
 c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)
^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(5/2)), x)
```

3.392 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=254

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{49\sin(c+dx)}{64a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{177\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{}$$

```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) - (177*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*(a + a*Cos[c + d
*x])^(7/2)*Sec[c + d*x]^(5/2)) - (17*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d
*x])^(5/2)*Sec[c + d*x]^(3/2)) - (49*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c +
 d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.668365, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2765, 2977, 2982, 2782, 205, 2774, 216}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}-\frac{49\sin(c+dx)}{64a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{177\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```


```
[Out] (2*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*x
]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) - (177*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sq
rt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt
[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*(a + a*Cos[c + d
*x])^(7/2)*Sec[c + d*x]^(5/2)) - (17*Sin[c + d*x])/(48*a*d*(a + a*Cos[c + d
*x])^(5/2)*Sec[c + d*x]^(3/2)) - (49*Sin[c + d*x])/(64*a^2*d*(a + a*Cos[c +
 d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}}{}}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{17\sin(c+dx)}{48ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} - \frac{177\tan^{-1}\left(\frac{\sqrt{a}\sin(}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{}}\right.}{}$$

Mathematica [C] time = 6.79598, size = 454, normalized size = 1.79

$$\frac{\cos^7\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\left(-\frac{247\sin\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{12d} - \frac{247\cos\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{12d} + \frac{\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3d} - \frac{41\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{12d} + \frac{379}{}\right.}{(a(\cos(c+dx)+1))^{7/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] ((-I/4)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(
c + d*x))]*(64*ArcSinh[E^(I*(c + d*x))] + (177*ArcTanh[(1 - E^(I*(c + d*x))
)/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])])/Sqrt[2] - 64*ArcTanh[Sqrt[1 + E
^((2*I)*(c + d*x))]])*Cos[c/2 + (d*x)/2]^7)/(Sqrt[2]*d*E^((I/2)*(c + d*x))*
(a*(1 + Cos[c + d*x]))^(7/2)) + (Cos[c/2 + (d*x)/2]^7*Sqrt[Sec[c + d*x]]*((
-247*Cos[(d*x)/2]*Sin[c/2])/(12*d) - (247*Cos[c/2]*Sin[(d*x)/2])/(12*d) + (
379*Sec[c/2]*Sec[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(24*d) - (41*Sec[c/2]*Sec[c
/2 + (d*x)/2]^4*Sin[(d*x)/2])/(12*d) + (Sec[c/2]*Sec[c/2 + (d*x)/2]^6*Sin[(
d*x)/2])/(3*d) + (379*Sec[c/2 + (d*x)/2]*Tan[c/2])/(24*d) - (41*Sec[c/2 + (
d*x)/2]^3*Tan[c/2])/(12*d) + (Sec[c/2 + (d*x)/2]^5*Tan[c/2])/(3*d)))/(a*(1
+ Cos[c + d*x]))^(7/2)
```

Maple [B] time = 0.434, size = 440, normalized size = 1.7

$$-\frac{\sqrt{2}(-1+\cos(dx+c))^7\cos(dx+c)}{384\,da^4(\sin(dx+c))^{15}}\sqrt{a(1+\cos(dx+c))}\left(384\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\right)(\cos(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*cos(d*x+c)*
(384*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*
x+c)^2*sin(d*x+c)*2^(1/2)+247*cos(d*x+c)^3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)+768*cos(d*x+c)*2^(1/2)*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)/cos(d*x+c))*sin(d*x+c)+531*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos
(d*x+c)^2*sin(d*x+c)+115*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+
c)^2+384*arctan(sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*2^
(1/2)*sin(d*x+c)+1062*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x
+c)-215*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+531*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*sin(d*x+c)-147*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))/(1/cos(d*x+c))^(7/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^1
5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7/2)), x)
```

Fricas [A] time = 4.38393, size = 782, normalized size = 3.08

$$\frac{531\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-}{384\left(a^4d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/384*(531*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2 +
4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(co
s(d*x + c))/(sqrt(a)*sin(d*x + c))) - 768*(cos(d*x + c)^4 + 4*cos(d*x + c)^
3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x +
c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(247*cos(d*x + c)^3
+ 362*cos(d*x + c)^2 + 147*cos(d*x + c))*sqrt(a*cos(d*x + c) + a)*sin(d*x +
 c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*
a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\cos\left(dx+c\right)+a\right)^{\frac{7}{2}}\sec\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(7/2)), x)
```

3.393 $\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$-\frac{259\sin(c+dx)}{192a^2d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{189\sin(}{64a^3d\sqrt{\sec(c+dx)}}$$

```
[Out] (-7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) + (637*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*(a + a*Cos[c +
d*x])^(7/2)*Sec[c + d*x]^(7/2)) - (7*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d
*x])^(5/2)*Sec[c + d*x]^(5/2)) - (259*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
 + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (189*Sin[c + d*x])/(64*a^3*d*Sqrt[a +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.816935, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4222, 2765, 2977, 2983, 2982, 2782, 205, 2774, 216}

$$-\frac{259\sin(c+dx)}{192a^2d\sec^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^{3/2}}-\frac{7\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a\cos(c+dx)+a}}\right)}{a^{7/2}d}+\frac{189\sin(}{64a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(9/2)),x]
```


```
[Out] (-7*ArcSin[(Sqrt[a]*Sin[c + d*x])/Sqrt[a + a*Cos[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sqrt[Sec[c + d*x]])/(a^(7/2)*d) + (637*ArcTan[(Sqrt[a]*Sin[c + d*x])/(S
qrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Cos[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqr
t[Sec[c + d*x]])/(64*Sqrt[2]*a^(7/2)*d) - Sin[c + d*x]/(6*d*(a + a*Cos[c +
d*x])^(7/2)*Sec[c + d*x]^(7/2)) - (7*Sin[c + d*x])/(16*a*d*(a + a*Cos[c + d
*x])^(5/2)*Sec[c + d*x]^(5/2)) - (259*Sin[c + d*x])/(192*a^2*d*(a + a*Cos[c
 + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + (189*Sin[c + d*x])/(64*a^3*d*Sqrt[a +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
 + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2982

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2774

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[-2/f, Subst[Int[1/Sqrt[1 - x^2/a], x], x, (b*Cos
[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && Eq
Q[a^2 - b^2, 0] && EqQ[d, a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{7/2}\sec^{\frac{9}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+a\cos(c+dx))^{7/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{6a^2}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{\sin(c+dx)}{6d(a+a\cos(c+dx))^{7/2}\sec^{\frac{7}{2}}(c+dx)} - \frac{7\sin(c+dx)}{16ad(a+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c}$$

$$= -\frac{7\sin^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{7/2}d} + \frac{637\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{2}\sqrt{\cos(c}}\right.}{}$$

Mathematica [C] time = 3.515, size = 460, normalized size = 1.56

$$\frac{e^{-\frac{1}{2}i(c+dx)}\sqrt{a(\cos(c+dx)+1)}\left(672i\sqrt{2}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^7\left(\frac{1}{2}(c+dx)\right)\sinh^{-1}\left(e^{i(c+dx)}\right)-\frac{1}{64}ie^{-4i(c+dx)}\sqrt{\frac{e^{i(c+}}{1+e^{2i(c}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(9/2)),x]
```

```
[Out] ((((-I/64)*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*(-1911*E^(I*(c +
 d*x))*(1 + E^(I*(c + d*x)))^6*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcTanh[(1 - E
^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*(-96 - 1
003*E^(I*(c + d*x)) - 2169*E^((2*I)*(c + d*x)) - 2297*E^((3*I)*(c + d*x)) -
 779*E^((4*I)*(c + d*x)) + 779*E^((5*I)*(c + d*x)) + 2297*E^((6*I)*(c + d*x
)) + 2169*E^((7*I)*(c + d*x)) + 1003*E^((8*I)*(c + d*x)) + 96*E^((9*I)*(c +
 d*x)) + 672*E^(I*(c + d*x))*(1 + E^(I*(c + d*x)))^6*Sqrt[1 + E^((2*I)*(c +
 d*x))]*ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))*Cos[(c + d*x)/2])/E^((4*I)
*(c + d*x)) + (672*I)*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x))
)]*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))]*Cos[(c + d*x)/2]^
7)*Sqrt[a*(1 + Cos[c + d*x])])/(24*a^4*d*E^((I/2)*(c + d*x))*(1 + Cos[c + d
*x])^4)
```

Maple [A] time = 0.458, size = 472, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(7/2)/sec(d*x+c)^(9/2),x)
```

```
[Out] -1/384/d*2^(1/2)/a^4*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^8*cos(d*x+c)*
(192*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+907*cos(d*x+c)^
3*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1344*arctan(sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+1911*
arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c)+343*2^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+2688*cos(d*x+c)*2^(1/2)*arctan(s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/cos(d*x+c))*sin(d*x+c)+3822*arc
sin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-875*2^(1/2)*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1344*arctan(sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)/cos(d*x+c))*2^(1/2)*sin(d*x+c)+1911*arcsin((-1+cos(d*x+c)
)/sin(d*x+c))*sin(d*x+c)-567*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/
cos(d*x+c))^(9/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(11/2)/sin(d*x+c)^17
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(9/2)), x)
```

Fricas [A] time = 5.24897, size = 817, normalized size = 2.78

$$-\frac{1911\sqrt{2}\left(\cos(dx+c)^4+4\cos(dx+c)^3+6\cos(dx+c)^2+4\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{\sqrt{a}\sin(dx+c)}\right)-}{384\,(a^4d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/384*(1911*sqrt(2)*(cos(d*x + c)^4 + 4*cos(d*x + c)^3 + 6*cos(d*x + c)^2
+ 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) + a)*sqrt(
cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2688*(cos(d*x + c)^4 + 4*cos(d*x +
c)^3 + 6*cos(d*x + c)^2 + 4*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(a*cos(d*x
 + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(192*cos(d*x + c)
^4 + 1099*cos(d*x + c)^3 + 1442*cos(d*x + c)^2 + 567*cos(d*x + c))*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(7/2)/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{7}{2}}\sec(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(7/2)/sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(7/2)*sec(d*x + c)^(9/2)), x)
```

3.394 $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{73\sin(c+dx)}{1024a^3d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{33\sin(c+dx)}{256a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}}+\frac{45\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,t}{1024}$$

```
[Out] (45*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1024*Sqrt[2]*a^(9/2)*
d) - Sin[c + d*x]/(8*d*(a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(3/2)) - (5*
Sin[c + d*x])/(32*a*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + (33*
Sin[c + d*x])/(256*a^2*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (
73*Sin[c + d*x])/(1024*a^3*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.631394, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2765, 2977, 2978, 12, 2782, 205}

$$\frac{73\sin(c+dx)}{1024a^3d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}+\frac{33\sin(c+dx)}{256a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}}+\frac{45\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,t}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (45*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1024*Sqrt[2]*a^(9/2)*
d) - Sin[c + d*x]/(8*d*(a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(3/2)) - (5*
Sin[c + d*x])/(32*a*d*(a + a*Cos[c + d*x])^(7/2)*Sqrt[Sec[c + d*x]]) + (33*
Sin[c + d*x])/(256*a^2*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) + (
73*Sin[c + d*x])/(1024*a^3*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
```

```
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos}}{}}{8a^2}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+}}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{3}{2}}(c+dx)} - \frac{5\sin(c+dx)}{32ad(a+a\cos(c+dx))^{7/2}\sqrt{\sec(c+}}$$

$$= \frac{45\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{1024\sqrt{2}a^{9/2}d} - \frac{}{8d(a+a\,\mathrm{co}}$$

Mathematica [A] time = 4.3907, size = 163, normalized size = 0.69

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2(999\cos(c+dx)+702\cos(2(c+dx))+73\cos(3(c+dx))+882)\sqrt{2-2\sec(c+dx)}+\right.}{65536\sqrt{2}a^4d\sqrt{-(\sec(c+dx)-1)\sec(c+dx)}\sqrt{a(\cos(}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^6*(2*(882 + 999*Cos[c + d*x] + 702*Cos[2*(c + d*x)] + 73*
Cos[3*(c + d*x)])*Sqrt[2 - 2*Sec[c + d*x]] + 5760*ArcTanh[Sqrt[-(Sec[c + d*
x]*Sin[(c + d*x)/2]^2)]]*Cos[(c + d*x)/2]^8*Sec[c + d*x])*Tan[(c + d*x)/2])
/(65536*Sqrt[2]*a^4*d*Sqrt[a*(1 + Cos[c + d*x])]*Sqrt[-((-1 + Sec[c + d*x])
*Sec[c + d*x])])
```

Maple [A] time = 0.434, size = 354, normalized size = 1.5

$$\frac{\sqrt{2}(-1+\cos(dx+c))^7\cos(dx+c)}{2048\,da^5(\sin(dx+c))^{15}}\sqrt{a(1+\cos(dx+c))}\left(73\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}(\cos(dx+c))^4+278(\cos(dx+c))^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(9/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] 1/2048/d*2^(1/2)/a^5*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^7*cos(d*x+c)*
(73*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+278*cos(d*x+c)^3
*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+45*arcsin((-1+cos(d*x+c))/sin(d*
x+c))*cos(d*x+c)^3*sin(d*x+c)-156*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)^2+135*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^2*sin(d*x+c
)-150*2^(1/2)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+135*arcsin((-1+c
os(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-45*2^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)+45*arcsin((-1+cos(d*x+c))/sin(d*x+c))*sin(d*x+c))/(1/cos(d*x
+c))^(5/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(7/2)/sin(d*x+c)^15
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{9}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [A] time = 1.99829, size = 651, normalized size = 2.75

$$-\frac{45\sqrt{2}\left(\cos(dx+c)^5+5\cos(dx+c)^4+10\cos(dx+c)^3+10\cos(dx+c)^2+5\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos}}{}\right.}{2048\left(a^5d\cos(dx+c)^5+5a^5d\cos(dx+c)^4+10a^5d\cos(dx+c)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2048*(45*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
+ 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*cos
(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(73*cos(d*x +
 c)^4 + 351*cos(d*x + c)^3 + 195*cos(d*x + c)^2 + 45*cos(d*x + c))*sqrt(a*c
os(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^5*d*cos(d*x + c)^5 + 5
*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^2 +
 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(9/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{9}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(5/2)), x)
```

3.395 $\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=237

$$\frac{853\sin(c+dx)}{3072a^3d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{187\sin(c+dx)}{768a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}}+\frac{35\sqrt{\cos(c+dx)}\sqrt{\sec(c+a}}{10}$$

```
[Out] (35*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1024*Sqrt[2]*a^(9/2)*
d) - Sin[c + d*x]/(8*d*(a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)) - (19
*Sin[c + d*x])/(96*a*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)) - (18
7*Sin[c + d*x])/(768*a^2*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) +
 (853*Sin[c + d*x])/(3072*a^3*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]])
```

Rubi [A] time = 0.636186, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2765, 2977, 2978, 12, 2782, 205}

$$\frac{853\sin(c+dx)}{3072a^3d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{3/2}}-\frac{187\sin(c+dx)}{768a^2d\sqrt{\sec(c+dx)}(a\cos(c+dx)+a)^{5/2}}+\frac{35\sqrt{\cos(c+dx)}\sqrt{\sec(c+a}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (35*ArcTan[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Co
s[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(1024*Sqrt[2]*a^(9/2)*
d) - Sin[c + d*x]/(8*d*(a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(5/2)) - (19
*Sin[c + d*x])/(96*a*d*(a + a*Cos[c + d*x])^(7/2)*Sec[c + d*x]^(3/2)) - (18
7*Sin[c + d*x])/(768*a^2*d*(a + a*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]) +
 (853*Sin[c + d*x])/(3072*a^3*d*(a + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2782

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[(-2*a)/f, Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*S
in[e + f*x]])], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+a\cos(c+dx))^{9/2}\sec^{\frac{7}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+a\cos(c+dx))^{9/2}}\,dx$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int -}{8a^2}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(}$$

$$= -\frac{\sin(c+dx)}{8d(a+a\cos(c+dx))^{9/2}\sec^{\frac{5}{2}}(c+dx)} - \frac{19\sin(c+dx)}{96ad(a+a\cos(c+dx))^{7/2}\sec^{\frac{3}{2}}(}$$

$$= \frac{35\tan^{-1}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{\cos(c+dx)}\sqrt{a+a\cos(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{1024\sqrt{2}a^{9/2}d} - \frac{}{8d(a+a}$$

Mathematica [A] time = 6.09238, size = 395, normalized size = 1.67

$$\frac{2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\frac{1}{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}}\sqrt{1-2\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)}\cos^9\left(\frac{c}{2}+\frac{dx}{2}\right)\left(1-\sin^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^{9/2}\left(\frac{1}{8}\left(\frac{}{1-\sin^2\left(\frac{c}{2}+\right.}\right.\right.}{d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Cos[c + d*x])^(9/2)*Sec[c + d*x]^(7/2)),x]
```

```
[Out] (2*Cos[c/2 + (d*x)/2]^9*Sin[c/2 + (d*x)/2]*Sqrt[(1 - 2*Sin[c/2 + (d*x)/2]^2
)^(-1)]*Sqrt[1 - 2*Sin[c/2 + (d*x)/2]^2]*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 +
(d*x)/2]^2)^(9/2)*((35*ArcSin[Sin[c/2 + (d*x)/2]/Sqrt[Cos[(c + d*x)/2]^2]]*
Sqrt[Cos[(c + d*x)/2]^2]*Csc[c/2 + (d*x)/2])/(128*(1 - Sec[(c + d*x)/2]^2*S
in[c/2 + (d*x)/2]^2)^(9/2)) + (35/(16*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*
x)/2]^2)^4) + 35/(24*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2)^3) + 7/(
6*(1 - Sec[(c + d*x)/2]^2*Sin[c/2 + (d*x)/2]^2)^2) + (1 - Sec[(c + d*x)/2]^
2*Sin[c/2 + (d*x)/2]^2)^(-1))/8))/(d*Sqrt[Cos[(c + d*x)/2]^2]*(a*(1 + Cos[c
 + d*x]))^(9/2))
```

Maple [A] time = 0.431, size = 354, normalized size = 1.5

$$-\frac{\sqrt{2}\,(-1+\cos(dx+c))^{8}\cos(dx+c)}{6144\,da^{5}\,(\sin(dx+c))^{17}}\sqrt{a\,(1+\cos(dx+c))}\left(853\,\sqrt{2}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos(dx+c))^{4}+105\,\arcsin\left(\frac{-1+}{\mathrm{si}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(d*x+c)*a)^(9/2)/sec(d*x+c)^(7/2),x)
```

```
[Out] -1/6144/d*2^(1/2)/a^5*(a*(1+cos(d*x+c)))^(1/2)*(-1+cos(d*x+c))^8*cos(d*x+c)
*(853*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4+105*arcsin((-1
+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)^3*sin(d*x+c)-34*cos(d*x+c)^3*2^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)+315*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(
d*x+c)^2*sin(d*x+c)-364*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)^2+315*arcsin((-1+cos(d*x+c))/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-350*2^(1/2
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+105*arcsin((-1+cos(d*x+c))/s
in(d*x+c))*sin(d*x+c)-105*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2))/(1/cos
(d*x+c))^(7/2)/(cos(d*x+c)/(1+cos(d*x+c)))^(9/2)/sin(d*x+c)^17
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a\cos(dx+c)+a)^{\frac{9}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(7/2)), x)
```

Fricas [A] time = 2.36078, size = 655, normalized size = 2.76

$$-\frac{105\,\sqrt{2}\left(\cos(dx+c)^{5}+5\,\cos(dx+c)^{4}+10\,\cos(dx+c)^{3}+10\,\cos(dx+c)^{2}+5\,\cos(dx+c)+1\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(}}{\sqrt{}}\right.}{6144\left(a^{5}d\cos(dx+c)^{5}+5\,a^{5}d\cos(dx+c)^{4}+10\,a^{5}d\cos(dx+c)^{3}+10\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/6144*(105*sqrt(2)*(cos(d*x + c)^5 + 5*cos(d*x + c)^4 + 10*cos(d*x + c)^3
 + 10*cos(d*x + c)^2 + 5*cos(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(a*co
s(d*x + c) + a)*sqrt(cos(d*x + c))/(sqrt(a)*sin(d*x + c))) - 2*(853*cos(d*x
 + c)^4 + 819*cos(d*x + c)^3 + 455*cos(d*x + c)^2 + 105*cos(d*x + c))*sqrt(
a*cos(d*x + c) + a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^5*d*cos(d*x + c)^5
+ 5*a^5*d*cos(d*x + c)^4 + 10*a^5*d*cos(d*x + c)^3 + 10*a^5*d*cos(d*x + c)^
2 + 5*a^5*d*cos(d*x + c) + a^5*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))**(9/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\cos(dx+c)+a)^{\frac{9}{2}}\sec(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*cos(d*x+c))^(9/2)/sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cos(d*x + c) + a)^(9/2)*sec(d*x + c)^(7/2)), x)
```

3.396 $\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{4a^2\sin(c+dx)\sqrt[4]{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (4*a^2*Sec[c + d*x]^(1/4)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.118211, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {4222, 2762, 8}

$$\frac{4a^2\sin(c+dx)\sqrt[4]{\sec(c+dx)}}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/4),x]
```

```
[Out] (4*a^2*Sec[c + d*x]^(1/4)*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2762

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*(b*c - a*d)*Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d
)), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 2)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c*(
m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (I
ntegerQ[m] && EqQ[c, 0]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\cos(c+dx))^{3/2}\sec^{\frac{5}{4}}(c+dx)\,dx &= \left(\sqrt[4]{\cos(c+dx)}\sqrt[4]{\sec(c+dx)}\right)\int \frac{(a+a\cos(c+dx))^{3/2}}{\cos^{\frac{5}{4}}(c+dx)}\,dx\\ &= \frac{4a^2\sqrt[4]{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}-\left(4a\sqrt[4]{\cos(c+dx)}\sqrt[4]{\sec(c+dx)}\right)\int 0\,dx\\ &= \frac{4a^2\sqrt[4]{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.114656, size = 51, normalized size = 1.34

$$\frac{2\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt[4]{\sec(c+dx)}(a(\cos(c+dx)+1))^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/4),x]
```

```
[Out] (2*(a*(1 + Cos[c + d*x]))^(3/2)*Sec[(c + d*x)/2]^2*Sec[c + d*x]^(1/4)*Tan[(
c + d*x)/2])/d
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int (a+\cos(dx+c)\,a)^{\frac{3}{2}}(\sec(dx+c))^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/4),x)
```

```
[Out] int((a+cos(d*x+c)*a)^(3/2)*sec(d*x+c)^(5/4),x)
```

Maxima [B] time = 1.49958, size = 163, normalized size = 4.29

$$\frac{4\left(\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sqrt{2}a^{\frac{3}{2}}\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)^{\frac{5}{4}}\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] 4*(sqrt(2)*a^(3/2)*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(2)*a^(3/2)*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/(d*(sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/
4)*(-sin(d*x + c)/(cos(d*x + c) + 1) + 1)^(5/4)*(sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 1)^(1/4))
```

Fricas [A] time = 2.09714, size = 115, normalized size = 3.03

$$\frac{4\sqrt{a\cos(dx+c)+a}\,a\sin(dx+c)}{(d\cos(dx+c)+d)\cos(dx+c)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4*sqrt(a*cos(d*x + c) + a)*a*sin(d*x + c)/((d*cos(d*x + c) + d)*cos(d*x + c
)^(1/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.397 $\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$

Optimal. Leaf size=302

$$-\frac{a^4\left(8m^2+40m+35\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}-\frac{4a^4(2m+5)\sin(c+dx)\cos^{m+2}(c}{d(m+2)(m}$$

```
[Out] (a^4*(55 + 29*m + 4*m^2)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 +
 m)*(4 + m)) + (Cos[c + d*x]^(1 + m)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x
])/(d*(4 + m)) + (2*(5 + m)*Cos[c + d*x]^(1 + m)*(a^4 + a^4*Cos[c + d*x])*S
in[c + d*x])/(d*(3 + m)*(4 + m)) - (a^4*(35 + 40*m + 8*m^2)*Cos[c + d*x]^(1
 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c +
d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2]) - (4*a^4*(5 + 2*m)*C
os[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.531488, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2763, 2976, 2968, 3023, 2748, 2643}

$$-\frac{a^4\left(8m^2+40m+35\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}-\frac{4a^4(2m+5)\sin(c+dx)\cos^{m+2}(c}{d(m+2)(m}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] (a^4*(55 + 29*m + 4*m^2)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 +
 m)*(4 + m)) + (Cos[c + d*x]^(1 + m)*(a^2 + a^2*Cos[c + d*x])^2*Sin[c + d*x
])/(d*(4 + m)) + (2*(5 + m)*Cos[c + d*x]^(1 + m)*(a^4 + a^4*Cos[c + d*x])*S
in[c + d*x])/(d*(3 + m)*(4 + m)) - (a^4*(35 + 40*m + 8*m^2)*Cos[c + d*x]^(1
 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c +
d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2]) - (4*a^4*(5 + 2*m)*C
os[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx = \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{d(4+m)} + \frac{\int \cos^m(c+dx)(a+a\cos(c}{}$$

$$= \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{d(4+m)} + \frac{2(5+m)\cos^{1+m}(c+dx)\left(a\right.}{d(3+}$$

$$= \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos(c+dx)\right)^2\sin(c+dx)}{d(4+m)} + \frac{2(5+m)\cos^{1+m}(c+dx)\left(a\right.}{d(3+}$$

$$= \frac{a^4\left(55+29m+4m^2\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)\left(12+7m+m^2\right)} + \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos\right.}{d(4+n}$$

$$= \frac{a^4\left(55+29m+4m^2\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)\left(12+7m+m^2\right)} + \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos\right.}{d(4+n}$$

$$= \frac{a^4\left(55+29m+4m^2\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)\left(12+7m+m^2\right)} + \frac{\cos^{1+m}(c+dx)\left(a^2+a^2\cos\right.}{d(4+n}$$

Mathematica [F] time = 3.23122, size = 0, normalized size = 0.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^4\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4,x]
```

```
[Out] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^4, x]
```

Maple [F] time = 2.855, size = 0, normalized size = 0.

$$\int \left(\cos\left(dx+c\right)\right)^{m}\left(a+\cos\left(dx+c\right)a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^4,x)
```

```
[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(dx+c\right)+a\right)^{4}\cos\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{4}\cos\left(dx+c\right)^{4}+4\,a^{4}\cos\left(dx+c\right)^{3}+6\,a^{4}\cos\left(dx+c\right)^{2}+4\,a^{4}\cos\left(dx+c\right)+a^{4}\right)\cos\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((a^4*cos(d*x + c)^4 + 4*a^4*cos(d*x + c)^3 + 6*a^4*cos(d*x + c)^2
+ 4*a^4*cos(d*x + c) + a^4)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

3.398 $\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx$

Optimal. Leaf size=232

$$-\frac{a^3(4m+5)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{a^3(4m+11)\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1}{d(m+2)(m+3)\sqrt{\sin^2}}$$

```
[Out] (a^3*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (Co
s[c + d*x]^(1 + m)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) - (a^
3*(5 + 4*m)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) -
 (a^3*(11 + 4*m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4
+ m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^
2])
```

Rubi [A] time = 0.307741, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2763, 2968, 3023, 2748, 2643}

$$-\frac{a^3(4m+5)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{a^3(4m+11)\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1}{d(m+2)(m+3)\sqrt{\sin^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3,x]
```



```
[Out] (a^3*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (Co
s[c + d*x]^(1 + m)*(a^3 + a^3*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) - (a^
3*(5 + 4*m)*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) -
 (a^3*(11 + 4*m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4
+ m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^
2])
```


Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx = \frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{d(3+m)}+\frac{\int \cos^m(c+dx)(a+a\cos(c+dx)}{}$$

$$=\frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\sin(c+dx)}{d(3+m)}+\frac{\int \cos^m(c+dx)\left(2a^3(2+m)\right.}{}$$

$$=\frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)}+\frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\mathrm{s}}{d(3+m)}$$

$$=\frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)}+\frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\mathrm{s}}{d(3+m)}$$

$$=\frac{a^3(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)}+\frac{\cos^{1+m}(c+dx)\left(a^3+a^3\cos(c+dx)\right)\mathrm{s}}{d(3+m)}$$

Mathematica [F] time = 1.35356, size = 0, normalized size = 0.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^3\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3,x]
```

```
[Out] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^3, x]
```

Maple [F] time = 2.316, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m\,(a+\cos(dx+c)\,a)^3\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^3,x)
```

```
[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^3\cos(dx+c)^3+3\,a^3\cos(dx+c)^2+3\,a^3\cos(dx+c)+a^3\right)\cos(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((a^3*cos(d*x + c)^3 + 3*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) +
a^3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^3\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

3.399 $\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx$

Optimal. Leaf size=173

$$-\frac{a^2(2m+3)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2a^2\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+}{2}\right.}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (a^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - (a^2*(3 + 2*m)*Cos[c
+ d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a^2*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.139103, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2763, 2748, 2643}

$$-\frac{a^2(2m+3)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2a^2\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+}{2}\right.}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - (a^2*(3 + 2*m)*Cos[c
+ d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a^2*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx = \frac{a^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} + \frac{\int \cos^m(c+dx)\left(a^2(3+2m)+2a^2(2+m)\cos\right.}{2+m}$$
$$= \frac{a^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} + \left(2a^2\right)\int \cos^{1+m}(c+dx)\,dx + \frac{\left(a^2(3+2m)\right)}{2}$$
$$= \frac{a^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} - \frac{a^2(3+2m)\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2}\right.}{d(1+m)(2+m)\sqrt{\sin^2(}}$$

Mathematica [F] time = 0.534615, size = 0, normalized size = 0.

$$\int \cos^m(c+dx)(a+a\cos(c+dx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2,x]
```

```
[Out] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x])^2, x]
```

Maple [F] time = 2.961, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m\,(a+\cos(dx+c)\,a)^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^2,x)
```

```
[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\cos(dx+c)^m\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2\right)\cos(dx+c)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

3.400 $\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx$

Optimal. Leaf size=131

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{a\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(c\right.}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (a*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0637415, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2748, 2643}

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{a\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(c\right.}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + a*Cos[c + d*x]),x]
```


```
[Out] -((a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (a*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```


Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^m(c+dx)(a+a\cos(c+dx))\,dx &= a\int \cos^m(c+dx)\,dx+a\int \cos^{1+m}(c+dx)\,dx\\ &=-\frac{a\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)\sqrt{\sin^2(c+dx)}}-\frac{a\cos^{2+m}(c+}{}\end{aligned}$$

Mathematica [C] time = 0.997566, size = 208, normalized size = 1.59

$$\frac{ia2^{-m-2}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^{m+1}(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left((m-1)m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{1-m}{2};-e^{2i(c+dx)}\right)+(m+1\right.}{d(m-1)m(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^m*(a + a*Cos[c + d*x]),x]
```

```
[Out] (I*2^(-2 - m)*a*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^(1 + m)*(1 + Co
s[c + d*x])*((-1 + m)*m*Hypergeometric2F1[1, (1 + m)/2, (1 - m)/2, -E^((2*I
)*(c + d*x))] + E^(I*(c + d*x))*(1 + m)*(2*(-1 + m)*Hypergeometric2F1[1, (2
 + m)/2, 1 - m/2, -E^((2*I)*(c + d*x))] + E^(I*(c + d*x))*m*Hypergeometric2
F1[1, (3 + m)/2, (3 - m)/2, -E^((2*I)*(c + d*x))]))*Sec[(c + d*x)/2]^2)/(d*
(-1 + m)*m*(1 + m))
```

Maple [F] time = 1.062, size = 0, normalized size = 0.

$$\int \left(\cos \left(dx+c\right)\right)^{m} \left(a+\cos \left(dx+c\right) a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+cos(d*x+c)*a),x)
```

```
[Out] int(cos(d*x+c)^m*(a+cos(d*x+c)*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \cos \left(dx+c\right)+a\right) \cos \left(dx+c\right)^{m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a \cos \left(dx+c\right)+a\right) \cos \left(dx+c\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a \left(\int \cos \left(c+dx\right) \cos^{m} \left(c+dx\right)\, dx+\int \cos^{m} \left(c+dx\right)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+a*cos(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*cos(c + d*x)**m, x) + Integral(cos(c + d*x)**m, x)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+a)\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.401 $\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\cos^2(c+dx)\right)}{ad\sqrt{\sin^2(c+dx)}}+\frac{m\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{ad(m+1)\sqrt{\sin^2(c+dx)}}\cdot$$

```
[Out] (Cos[c + d*x]^m*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - (Cos[c + d*x]^m*Hy
pergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*Sqr
t[Sin[c + d*x]^2]) + (m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*(1 + m)*Sqrt[Sin[c + d*x]
^2])
```

Rubi [A] time = 0.125815, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2769, 2748, 2643}

$$-\frac{\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\cos^2(c+dx)\right)}{ad\sqrt{\sin^2(c+dx)}}+\frac{m\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{ad(m+1)\sqrt{\sin^2(c+dx)}}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a + a*Cos[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^m*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])) - (Cos[c + d*x]^m*Hy
pergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*Sqr
t[Sin[c + d*x]^2]) + (m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)
/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*(1 + m)*Sqrt[Sin[c + d*x]
^2])
```

Rule 2769

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a
*f*(a + b*Sin[e + f*x])), x] + Dist[(d*n)/(a*b), Int[(c + d*Sin[e + f*x])^(
n - 1)*(a - b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && (IntegerQ[2*n
] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx = \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{m\int \cos^{-1+m}(c+dx)(a-a\cos(c+dx))\,dx}{a^2}$$
$$= \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{m\int \cos^{-1+m}(c+dx)\,dx}{a} - \frac{m\int \cos^m(c+dx)\,dx}{a}$$
$$= \frac{\cos^m(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{ad\sqrt{\sin^2(c+dx)}} + \frac{m\cos}{}$$

Mathematica [F] time = 0.815791, size = 0, normalized size = 0.

$$\int \frac{\cos^m(c+dx)}{a+a\cos(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x]),x]
```

```
[Out] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x]), x]
```

Maple [F] time = 0.845, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m}{a+\cos(dx+c)\,a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a+cos(d*x+c)*a),x)
```

```
[Out] int(cos(d*x+c)^m/(a+cos(d*x+c)*a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^m}{a\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^{m}(c+dx)}{\cos(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a+a*cos(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**m/(cos(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{m}}{a\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a), x)
```

3.402 $\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{(1-2m)m\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{3a^2d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{2(1-m)(m+1)\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2}{3a^2d(m+2)\sqrt{\sin^2(c}$$

```
[Out] (-2*(1 - m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x]))
 - (Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((1 -
 2*m)*m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(3*a^2*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) - (2*(1
 - m)*(1 + m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(3*a^2*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.304709, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2766, 2978, 2748, 2643}

$$\frac{(1-2m)m\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{3a^2d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{2(1-m)(m+1)\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2}{3a^2d(m+2)\sqrt{\sin^2(c}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2,x]
```

```
[Out] (-2*(1 - m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(3*a^2*d*(1 + Cos[c + d*x]))
 - (Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2) + ((1 -
 2*m)*m*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(3*a^2*d*(1 + m)*Sqrt[Sin[c + d*x]^2]) - (2*(1
 - m)*(1 + m)*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(3*a^2*d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
```

```
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx = -\frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^m(c+dx)(a(2-m)+am\cos(c+dx))}{a+a\cos(c+dx)}\,dx}{3a^2}$$

$$= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \cos^m(c+dx)\left(-a^2(\right.}{}$$

$$= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} - \frac{((1-2m)m)\int \cos^m(c}{3a^2}$$

$$= -\frac{2(1-m)\cos^{1+m}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{\cos^{1+m}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{(1-2m)m\cos^{1+m}(c-}{}$$

Mathematica [F] time = 1.05838, size = 0, normalized size = 0.

$$\int \frac{\cos^m(c+dx)}{(a+a\cos(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2,x]

[Out] Integrate[Cos[c + d*x]^m/(a + a*Cos[c + d*x])^2, x]
```

Maple [F] time = 0.492, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m}{(a+\cos(dx+c)a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a+cos(d*x+c)*a)^2,x)

[Out] int(cos(d*x+c)^m/(a+cos(d*x+c)*a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^m}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cos^m(c+dx)}{\cos^2(c+dx)+2\cos(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a+a*cos(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**m/(cos(c + d*x)**2 + 2*cos(c + d*x) + 1), x)/a**2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{(a\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(a*cos(d*x + c) + a)^2, x)
```

3.403 $\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=150

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7b\sin(c+dx)\cos^5(c+dx)}{48d}$$

```
[Out] (35*b*x)/128 + (a*Sin[c + d*x])/d + (35*b*Cos[c + d*x]*Sin[c + d*x])/(128*d
) + (35*b*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (7*b*Cos[c + d*x]^5*Sin[c
+ d*x])/(48*d) + (b*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)
/d + (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.101661, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2633, 2635, 8}

$$-\frac{a\sin^7(c+dx)}{7d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7b\sin(c+dx)\cos^5(c+dx)}{48d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Cos[c + d*x]),x]
```

```
[Out] (35*b*x)/128 + (a*Sin[c + d*x])/d + (35*b*Cos[c + d*x]*Sin[c + d*x])/(128*d
) + (35*b*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (7*b*Cos[c + d*x]^5*Sin[c
+ d*x])/(48*d) + (b*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)
/d + (3*a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^7(c+dx)\,dx + b\int \cos^8(c+dx)\,dx$$
$$= \frac{b\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}(7b)\int \cos^6(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int\left(1-3x^2+\right.\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{b\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a\,\mathrm{s}}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{35b\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{48d} +$$
$$= \frac{a\sin(c+dx)}{d} + \frac{35b\cos(c+dx)\sin(c+dx)}{128d} + \frac{35b\cos^3(c+dx)\sin(c+dx)}{192d} +$$
$$= \frac{35bx}{128} + \frac{a\sin(c+dx)}{d} + \frac{35b\cos(c+dx)\sin(c+dx)}{128d} + \frac{35b\cos^3(c+dx)\sin(c}{192d}$$

Mathematica [A] time = 0.212083, size = 135, normalized size = 0.9

$$-\frac{a\sin^7(c+dx)}{7d} + \frac{3a\sin^5(c+dx)}{5d} - \frac{a\sin^3(c+dx)}{d} + \frac{a\sin(c+dx)}{d} + \frac{35b(c+dx)}{128d} + \frac{7b\sin(2(c+dx))}{32d} + \frac{7b\sin(4(c+}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Cos[c + d*x]),x]
```

```
[Out] (35*b*(c + d*x))/(128*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*a
*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^7)/(7*d) + (7*b*Sin[2*(c + d*x)])/
(32*d) + (7*b*Sin[4*(c + d*x)])/(128*d) + (b*Sin[6*(c + d*x)])/(96*d) + (b*
Sin[8*(c + d*x)])/(1024*d)
```

Maple [A] time = 0.035, size = 100, normalized size = 0.7

$$\frac{1}{d}\left(b\left(\frac{\sin(dx+c)}{8}\left((\cos(dx+c))^7 + \frac{7(\cos(dx+c))^5}{6} + \frac{35(\cos(dx+c))^3}{24} + \frac{35\cos(dx+c)}{16}\right) + \frac{35dx}{128} + \frac{35c}{128}\right) + \frac{a\,\mathrm{si}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(b*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x
+c))*sin(d*x+c)+35/128*d*x+35/128*c)+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c
)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.984367, size = 142, normalized size = 0.95

$$-\frac{3072\left(5\sin(dx+c)^7 - 21\sin(dx+c)^5 + 35\sin(dx+c)^3 - 35\sin(dx+c)\right)a + 35\left(128\sin(2dx+2c)^3 - 840dx -\right.}{107520d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/107520*(3072*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 -
 35*sin(d*x + c))*a + 35*(128*sin(2*d*x + 2*c)^3 - 840*d*x - 840*c - 3*sin(
```

```
8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*b)/d
```

Fricas [A] time = 2.26971, size = 289, normalized size = 1.93

$$\frac{3675\,bdx + \left(1680\,b\cos\left(dx+c\right)^{7} + 1920\,a\cos\left(dx+c\right)^{6} + 1960\,b\cos\left(dx+c\right)^{5} + 2304\,a\cos\left(dx+c\right)^{4} + 2450\,b\cos\left(dx+c\right)^{4}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/13440*(3675*b*d*x + (1680*b*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 + 1960
*b*cos(d*x + c)^5 + 2304*a*cos(d*x + c)^4 + 2450*b*cos(d*x + c)^3 + 3072*a*
cos(d*x + c)^2 + 3675*b*cos(d*x + c) + 6144*a)*sin(d*x + c))/d
```

Sympy [A] time = 13.7775, size = 286, normalized size = 1.91

$$\begin{cases} \frac{16a\sin^{7}\left(c+dx\right)}{35d} + \frac{8a\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d} + \frac{2a\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d} + \frac{a\sin\left(c+dx\right)\cos^{6}\left(c+dx\right)}{d} + \frac{35bx\sin^{8}\left(c+dx\right)}{128} + \frac{35bx\sin^{6}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{32} \\ x\left(a+b\cos\left(c\right)\right)\cos^{7}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**7/(35*d) + 8*a*sin(c + d*x)**5*cos(c + d*x)**
2/(5*d) + 2*a*sin(c + d*x)**3*cos(c + d*x)**4/d + a*sin(c + d*x)*cos(c + d*
x)**6/d + 35*b*x*sin(c + d*x)**8/128 + 35*b*x*sin(c + d*x)**6*cos(c + d*x)*
*2/32 + 105*b*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*b*x*sin(c + d*x)**2
*cos(c + d*x)**6/32 + 35*b*x*cos(c + d*x)**8/128 + 35*b*sin(c + d*x)**7*cos
(c + d*x)/(128*d) + 385*b*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 511*b*s
in(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*b*sin(c + d*x)*cos(c + d*x)**7/
(128*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**7, True))
```

Giac [A] time = 1.36918, size = 165, normalized size = 1.1

$$\frac{35}{128}\,bx + \frac{b\sin\left(8\,dx+8\,c\right)}{1024\,d} + \frac{a\sin\left(7\,dx+7\,c\right)}{448\,d} + \frac{b\sin\left(6\,dx+6\,c\right)}{96\,d} + \frac{7\,a\sin\left(5\,dx+5\,c\right)}{320\,d} + \frac{7\,b\sin\left(4\,dx+4\,c\right)}{128\,d} + \frac{7\,a\sin\left(3\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 35/128*b*x + 1/1024*b*sin(8*d*x + 8*c)/d + 1/448*a*sin(7*d*x + 7*c)/d + 1/9
6*b*sin(6*d*x + 6*c)/d + 7/320*a*sin(5*d*x + 5*c)/d + 7/128*b*sin(4*d*x + 4
*c)/d + 7/64*a*sin(3*d*x + 3*c)/d + 7/32*b*sin(2*d*x + 2*c)/d + 35/64*a*sin
(d*x + c)/d
```

3.404 $\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=128

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^7(c+dx)}{7d}+\frac{3b\sin$$

```
[Out] (5*a*x)/16 + (b*Sin[c + d*x])/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (b*Sin[c + d*x]^3)/d + (3*b*Sin[c + d*x]^5)/(5*d) - (b*Sin[c + d*x]
^7)/(7*d)
```

Rubi [A] time = 0.0857035, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2635, 8, 2633}

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^7(c+dx)}{7d}+\frac{3b\sin$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Cos[c + d*x]),x]
```

```
[Out] (5*a*x)/16 + (b*Sin[c + d*x])/d + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (b*Sin[c + d*x]^3)/d + (3*b*Sin[c + d*x]^5)/(5*d) - (b*Sin[c + d*x]
^7)/(7*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^6(c+dx)\,dx + b\int \cos^7(c+dx)\,dx$$
$$= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int\left(1-3x^2+3x\right.\right.}{}$$
$$= \frac{b\sin(c+dx)}{d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{b\sin^3}{}$$
$$= \frac{b\sin(c+dx)}{d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos}{}$$
$$= \frac{5ax}{16} + \frac{b\sin(c+dx)}{d} + \frac{5a\cos(c+dx)\sin(c+dx)}{16d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} -$$

Mathematica [A] time = 0.181262, size = 89, normalized size = 0.7

$$\frac{35a(45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))+60c+60dx)-960b\sin^7(c+dx)+4032b\sin^5(c+dx)-6720}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Cos[c + d*x]),x]
```

```
[Out] (6720*b*Sin[c + d*x] - 6720*b*Sin[c + d*x]^3 + 4032*b*Sin[c + d*x]^5 - 960*
b*Sin[c + d*x]^7 + 35*a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c +
 d*x)] + Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A] time = 0.033, size = 90, normalized size = 0.7

$$\frac{1}{d}\left(\frac{b\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)+a\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/7*b*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x
+5/16*c))
```

Maxima [A] time = 0.965041, size = 127, normalized size = 0.99

$$-\frac{35\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)a+192\left(5\sin(dx+c)^7-21\sin(dx+c)^5\right.}{6720d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6720*(35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48
*sin(2*d*x + 2*c))*a + 192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d
*x + c)^3 - 35*sin(d*x + c))*b)/d
```

Fricas [A] time = 2.28918, size = 244, normalized size = 1.91

$$\frac{525\,adx + \left(240\,b\cos\left(dx+c\right)^{6} + 280\,a\cos\left(dx+c\right)^{5} + 288\,b\cos\left(dx+c\right)^{4} + 350\,a\cos\left(dx+c\right)^{3} + 384\,b\cos\left(dx+c\right)^{2} + \cdots}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(525*a*d*x + (240*b*cos(d*x + c)^6 + 280*a*cos(d*x + c)^5 + 288*b*co
s(d*x + c)^4 + 350*a*cos(d*x + c)^3 + 384*b*cos(d*x + c)^2 + 525*a*cos(d*x
+ c) + 768*b)*sin(d*x + c))/d
```

Sympy [A] time = 8.76348, size = 238, normalized size = 1.86

$$\begin{cases} \frac{5ax\sin^{6}(c+dx)}{16} + \frac{15ax\sin^{4}(c+dx)\cos^{2}(c+dx)}{16} + \frac{15ax\sin^{2}(c+dx)\cos^{4}(c+dx)}{16} + \frac{5ax\cos^{6}(c+dx)}{16} + \frac{5a\sin^{5}(c+dx)\cos(c+dx)}{16d} + \frac{5a\sin^{3}(c+dx)\cos^{3}\cdots}{6d} \\ x\left(a+b\cos\left(c\right)\right)\cos^{6}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 16*b*sin(c + d*x)*
*7/(35*d) + 8*b*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*b*sin(c + d*x)**3
*cos(c + d*x)**4/d + b*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(a + b
*cos(c))*cos(c)**6, True))
```

Giac [A] time = 1.39491, size = 144, normalized size = 1.12

$$\frac{5}{16}\,ax + \frac{b\sin\left(7\,dx+7\,c\right)}{448\,d} + \frac{a\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{7\,b\sin\left(5\,dx+5\,c\right)}{320\,d} + \frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{7\,b\sin\left(3\,dx+3\,c\right)}{64\,d} + \frac{15\,a\,\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x + 1/448*b*sin(7*d*x + 7*c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 7/320*
b*sin(5*d*x + 5*c)/d + 3/64*a*sin(4*d*x + 4*c)/d + 7/64*b*sin(3*d*x + 3*c)/
d + 15/64*a*sin(2*d*x + 2*c)/d + 35/64*b*sin(d*x + c)/d
```

3.405 $\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=114

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5b\sin(c+$$

```
[Out] (5*b*x)/16 + (a*Sin[c + d*x])/d + (5*b*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*b*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.081893, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2633, 2635, 8}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5b\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Cos[c + d*x]),x]
```

```
[Out] (5*b*x)/16 + (a*Sin[c + d*x])/d + (5*b*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
(5*b*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d) - (2*a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^5(c+dx)\,dx + b\int \cos^6(c+dx)\,dx$$
$$= \frac{b\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5b)\int \cos^4(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1-2x^2+\right.\right.}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{5b\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{b\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{2a}{}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{5b\cos(c+dx)\sin(c+dx)}{16d} + \frac{5b\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{b\,c}{}$$
$$= \frac{5bx}{16} + \frac{a\sin(c+dx)}{d} + \frac{5b\cos(c+dx)\sin(c+dx)}{16d} + \frac{5b\cos^3(c+dx)\sin(c+d}{24d}$$

Mathematica [A] time = 0.10248, size = 78, normalized size = 0.68

$$\frac{192a\sin^5(c+dx) - 640a\sin^3(c+dx) + 960a\sin(c+dx) + 5b(45\sin(2(c+dx)) + 9\sin(4(c+dx)) + \sin(6(c+dx)) +}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Cos[c + d*x]),x]
```

```
[Out] (960*a*Sin[c + d*x] - 640*a*Sin[c + d*x]^3 + 192*a*Sin[c + d*x]^5 + 5*b*(60
*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)]))
/(960*d)
```

Maple [A] time = 0.036, size = 80, normalized size = 0.7

$$\frac{1}{d}\left(b\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5 + \frac{5(\cos(dx+c))^3}{4} + \frac{15\cos(dx+c)}{8}\right) + \frac{5dx}{16} + \frac{5c}{16}\right) + \frac{a\sin(dx+c)}{5}\left(\frac{8}{3} + (\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(b*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)+1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.97169, size = 113, normalized size = 0.99

$$\frac{64\left(3\sin(dx+c)^5 - 10\sin(dx+c)^3 + 15\sin(dx+c)\right)a - 5\left(4\sin(2dx+2c)^3 - 60dx - 60c - 9\sin(4dx+4c) - 4\right.}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a - 5*(4
*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2
*c))*b)/d
```

Fricas [A] time = 2.2048, size = 204, normalized size = 1.79

$$\frac{75\,bdx + \left(40\,b\cos\left(dx+c\right)^{5} + 48\,a\cos\left(dx+c\right)^{4} + 50\,b\cos\left(dx+c\right)^{3} + 64\,a\cos\left(dx+c\right)^{2} + 75\,b\cos\left(dx+c\right) + 128\,a\right)\sin\left(\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(75*b*d*x + (40*b*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 + 50*b*cos(d*x
 + c)^3 + 64*a*cos(d*x + c)^2 + 75*b*cos(d*x + c) + 128*a)*sin(d*x + c))/d
```

Sympy [A] time = 4.37404, size = 216, normalized size = 1.89

$$\begin{cases} \frac{8a\sin^{5}(c+dx)}{15d} + \frac{4a\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^{4}(c+dx)}{d} + \frac{5bx\sin^{6}(c+dx)}{16} + \frac{15bx\sin^{4}(c+dx)\cos^{2}(c+dx)}{16} + \frac{15bx\sin^{2}(c+dx)\cos^{4}(c+dx}{16} \\ x\left(a+b\cos\left(c\right)\right)\cos^{5}\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d + 5*b*x*sin(c + d*x)**6/16 + 15*b
*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b*x*sin(c + d*x)**2*cos(c + d*x)
**4/16 + 5*b*x*cos(c + d*x)**6/16 + 5*b*sin(c + d*x)**5*cos(c + d*x)/(16*d)
 + 5*b*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b*sin(c + d*x)*cos(c + d*
x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**5, True))
```

Giac [A] time = 1.36121, size = 124, normalized size = 1.09

$$\frac{5}{16}\,bx + \frac{b\sin\left(6\,dx+6\,c\right)}{192\,d} + \frac{a\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{3\,b\sin\left(4\,dx+4\,c\right)}{64\,d} + \frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{15\,b\sin\left(2\,dx+2\,c\right)}{64\,d} + \frac{5\,a\sin}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*b*x + 1/192*b*sin(6*d*x + 6*c)/d + 1/80*a*sin(5*d*x + 5*c)/d + 3/64*b*
sin(4*d*x + 4*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 15/64*b*sin(2*d*x + 2*c)/d
 + 5/8*a*sin(d*x + c)/d
```

3.406 $\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=92

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] (3*a*x)/8 + (b*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*b*Sin[c + d*x]^3)/(3*d) + (b*Sin[c
 + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0639376, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2635, 8, 2633}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Cos[c + d*x]),x]
```

```
[Out] (3*a*x)/8 + (b*Sin[c + d*x])/d + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*b*Sin[c + d*x]^3)/(3*d) + (b*Sin[c
 + d*x]^5)/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + b\int \cos^5(c+dx)\,dx$$
$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int \left(1-2x^2+x^4\right.\right.}{d}$$
$$= \frac{b\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{2b\sin^3}{3}$$
$$= \frac{3ax}{8} + \frac{b\sin(c+dx)}{d} + \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} -$$

Mathematica [A] time = 0.130354, size = 89, normalized size = 0.97

$$\frac{3a(c+dx)}{8d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d} + \frac{b\sin^5(c+dx)}{5d} - \frac{2b\sin^3(c+dx)}{3d} + \frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Cos[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*Sin[c + d*x])/d - (2*b*Sin[c + d*x]^3)/(3*d) + (
b*Sin[c + d*x]^5)/(5*d) + (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)])
/(32*d)
```

Maple [A] time = 0.033, size = 70, normalized size = 0.8

$$\frac{1}{d}\left(\frac{b\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/5*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a*(1/4*(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 0.963489, size = 93, normalized size = 1.01

$$\frac{15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a+32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)b}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a + 32*(3
*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*b)/d
```

Fricas [A] time = 1.9223, size = 173, normalized size = 1.88

$$\frac{45adx+\left(24b\cos(dx+c)^4+30a\cos(dx+c)^3+32b\cos(dx+c)^2+45a\cos(dx+c)+64b\right)\sin(dx+c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(45*a*d*x + (24*b*cos(d*x + c)^4 + 30*a*cos(d*x + c)^3 + 32*b*cos(d*x
 + c)^2 + 45*a*cos(d*x + c) + 64*b)*sin(d*x + c))/d
```

Sympy [A] time = 2.32742, size = 168, normalized size = 1.83

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{8b\sin^5(c+dx)}{15d} + \frac{4b\sin \\ x(a+b\cos(c))\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*b*sin(c + d*x)**5/(15*d) + 4*b*sin(c
 + d*x)**3*cos(c + d*x)**2/(3*d) + b*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d,
0)), (x*(a + b*cos(c))*cos(c)**4, True))
```

Giac [A] time = 1.36078, size = 104, normalized size = 1.13

$$\frac{3}{8}ax + \frac{b\sin(5dx+5c)}{80d} + \frac{a\sin(4dx+4c)}{32d} + \frac{5b\sin(3dx+3c)}{48d} + \frac{a\sin(2dx+2c)}{4d} + \frac{5b\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x + 1/80*b*sin(5*d*x + 5*c)/d + 1/32*a*sin(4*d*x + 4*c)/d + 5/48*b*si
n(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 5/8*b*sin(d*x + c)/d
```

3.407 $\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=76

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] (3*b*x)/8 + (a*Sin[c + d*x])/d + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0586476, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2633, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x]),x]
```

```
[Out] (3*b*x)/8 + (a*Sin[c + d*x])/d + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b
*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^3(c+dx)\,dx + b\int \cos^4(c+dx)\,dx$$
$$= \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3b)\int \cos^2(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1-x^2\right) d\right.}{d}$$
$$= \frac{a\sin(c+dx)}{d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a\,\text{si}}{}$$
$$= \frac{3bx}{8} + \frac{a\sin(c+dx)}{d} + \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+dx}{4d}$$

Mathematica [A] time = 0.0966939, size = 73, normalized size = 0.96

$$-\frac{a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} + \frac{3b(c+dx)}{8d} + \frac{b\sin(2(c+dx))}{4d} + \frac{b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x]),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d) + (b*
Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.033, size = 60, normalized size = 0.8

$$\frac{1}{d}\left(b\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{3dx}{8} + \frac{3c}{8}\right) + \frac{a\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.954641, size = 77, normalized size = 1.01

$$-\frac{32\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)a - 3\left(12dx + 12c + \sin(4dx+4c) + 8\sin(2dx+2c)\right)b}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) + 8*sin(2*d*x + 2*c))*b)/d
```

Fricas [A] time = 1.8701, size = 136, normalized size = 1.79

$$\frac{9bdx + \left(6b\cos(dx+c)^3 + 8a\cos(dx+c)^2 + 9b\cos(dx+c) + 16a\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(9*b*d*x + (6*b*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 + 9*b*cos(d*x + c)
 + 16*a)*sin(d*x + c))/d
```

Sympy [A] time = 1.18761, size = 144, normalized size = 1.89

$$\begin{cases} \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3bx\sin^4(c+dx)}{8} + \frac{3bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3bx\cos^4(c+dx)}{8} + \frac{3b\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5b\sin(c+dx)}{} \\ x(a+b\cos(c))\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d + 3
*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x*co
s(c + d*x)**4/8 + 3*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b*sin(c + d*x)
*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**3, True))
```

Giac [A] time = 1.32354, size = 84, normalized size = 1.11

$$\frac{3}{8}bx + \frac{b\sin(4dx+4c)}{32d} + \frac{a\sin(3dx+3c)}{12d} + \frac{b\sin(2dx+2c)}{4d} + \frac{3a\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/32*b*sin(4*d*x + 4*c)/d + 1/12*a*sin(3*d*x + 3*c)/d + 1/4*b*sin
(2*d*x + 2*c)/d + 3/4*a*sin(d*x + c)/d
```

3.408 $\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] (a*x)/2 + (b*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (b*Sin
[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.047293, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 2635, 8, 2633}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*x)/2 + (b*Sin[c + d*x])/d + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (b*Sin
[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+b\cos(c+dx))\,dx &= a\int \cos^2(c+dx)\,dx+b\int \cos^3(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx-\frac{b\,\mathrm{Subst}\left(\int\left(1-x^2\right)dx,x,-\sin(c+d\right.}{d}\\ &= \frac{ax}{2}+\frac{b\sin(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}-\frac{b\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.064984, size = 57, normalized size = 1.06

$$\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (b*Sin[c + d*x])/d - (b*Sin[c + d*x]^3)/(3*d) + (a*Si
n[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.033, size = 49, normalized size = 0.9

$$\frac{1}{d}\left(\frac{b\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(1/3*b*(2+cos(d*x+c)^2)*sin(d*x+c)+a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c))
```

Maxima [A] time = 0.945717, size = 62, normalized size = 1.15

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a-4\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)b}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a - 4*(sin(d*x + c)^3 - 3*sin(d*x
+ c))*b)/d
```

Fricas [A] time = 1.90571, size = 105, normalized size = 1.94

$$\frac{3\,adx+\left(2\,b\cos(dx+c)^2+3\,a\cos(dx+c)+4\,b\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*d*x + (2*b*cos(d*x + c)^2 + 3*a*cos(d*x + c) + 4*b)*sin(d*x + c))/
d
```

Sympy [A] time = 0.601227, size = 92, normalized size = 1.7

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} + \frac{a\sin(c+dx)\cos(c+dx)}{2d} + \frac{2b\sin^3(c+dx)}{3d} + \frac{b\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ x(a+b\cos(c))\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) + 2*b*sin(c + d*x)**3/(3*d) + b*sin(c + d*x)*cos(c + d*x)
**2/d, Ne(d, 0)), (x*(a + b*cos(c))*cos(c)**2, True))
```

Giac [A] time = 1.30865, size = 63, normalized size = 1.17

$$\frac{1}{2}ax + \frac{b\sin(3dx+3c)}{12d} + \frac{a\sin(2dx+2c)}{4d} + \frac{3b\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x + 1/12*b*sin(3*d*x + 3*c)/d + 1/4*a*sin(2*d*x + 2*c)/d + 3/4*b*sin(
d*x + c)/d
```

3.409 $\int \cos(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=38

$$\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

```
[Out] (b*x)/2 + (a*Sin[c + d*x])/d + (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0153514, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2734}

$$\frac{a\sin(c+dx)}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x]),x]
```

```
[Out] (b*x)/2 + (a*Sin[c + d*x])/d + (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))\,dx=\frac{bx}{2}+\frac{a\sin(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0630121, size = 35, normalized size = 0.92

$$\frac{4a\sin(c+dx)+b(2(c+dx)+\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x]),x]
```

```
[Out] (4*a*Sin[c + d*x] + b*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.036, size = 38, normalized size = 1.

$$\frac{1}{d}\left(b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c)),x)
```

```
[Out] 1/d*(b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*sin(d*x+c))
```

Maxima [A] time = 0.954672, size = 46, normalized size = 1.21

$$\frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))b + 4\,a\sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*b + 4*a*sin(d*x + c))/d
```

Fricas [A] time = 1.79919, size = 72, normalized size = 1.89

$$\frac{bdx + (b\cos(dx + c) + 2\,a)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + (b*cos(d*x + c) + 2*a)*sin(d*x + c))/d
```

Sympy [A] time = 0.236868, size = 66, normalized size = 1.74

$$\begin{cases} \frac{a\sin(c+dx)}{d} + \frac{bx\sin^2(c+dx)}{2} + \frac{bx\cos^2(c+dx)}{2} + \frac{b\sin(c+dx)\cos(c+dx)}{2d} & \text{for}\ d \neq 0 \\ x(a + b\cos(c))\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((a*sin(c + d*x)/d + b*x*sin(c + d*x)**2/2 + b*x*cos(c + d*x)**2/2
 + b*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(a + b*cos(c))*cos(c),
True))
```

Giac [A] time = 1.40161, size = 42, normalized size = 1.11

$$\frac{1}{2}\,bx + \frac{b\sin(2\,dx + 2\,c)}{4\,d} + \frac{a\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*b*x + 1/4*b*sin(2*d*x + 2*c)/d + a*sin(d*x + c)/d
```

3.410 $\int (a+b\cos(c+dx))\,dx$

Optimal. Leaf size=15

$$ax+\frac{b\sin(c+dx)}{d}$$

```
[Out] a*x + (b*Sin[c + d*x])/d
```

Rubi [A] time = 0.0085139, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2637}

$$ax+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Cos[c + d*x],x]
```

```
[Out] a*x + (b*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\,dx &= ax+b\int \cos(c+dx)\,dx\\ &= ax+\frac{b\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0063226, size = 26, normalized size = 1.73

$$ax+\frac{b\sin(c)\cos(dx)}{d}+\frac{b\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Cos[c + d*x],x]
```

```
[Out] a*x + (b*Cos[d*x]*Sin[c])/d + (b*Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.023, size = 16, normalized size = 1.1

$$ax+\frac{b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*cos(d*x+c),x)
```

[Out] a*x+b*sin(d*x+c)/d

Maxima [A] time = 0.947299, size = 20, normalized size = 1.33

$$ax + \frac{b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(d*x+c),x, algorithm="maxima")

[Out] a*x + b*sin(d*x + c)/d

Fricas [A] time = 1.86152, size = 38, normalized size = 2.53

$$\frac{adx + b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(d*x+c),x, algorithm="fricas")

[Out] (a*d*x + b*sin(d*x + c))/d

Sympy [A] time = 0.125019, size = 17, normalized size = 1.13

$$ax + b\left(\begin{cases} \frac{\sin(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\cos(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(d*x+c),x)

[Out] a*x + b*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True))

Giac [A] time = 1.36589, size = 20, normalized size = 1.33

$$ax + \frac{b\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*cos(d*x+c),x, algorithm="giac")

[Out] a*x + b*sin(d*x + c)/d

3.411 $\int (a+b\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=16

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+bx$$

```
[Out] b*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rubi [A] time = 0.0232678, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2735, 3770}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] b*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\sec(c+dx)\,dx &= bx+a\int \sec(c+dx)\,dx\\ &= bx+\frac{a\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0066724, size = 16, normalized size = 1.

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] b*x + (a*ArcTanh[Sin[c + d*x]])/d
```

Maple [A] time = 0.046, size = 30, normalized size = 1.9

$$bx + \frac{a \ln(\sec(dx+c) + \tan(dx+c))}{d} + \frac{cb}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] b*x+1/d*a*ln(sec(d*x+c)+tan(d*x+c))+b*c/d
```

Maxima [A] time = 0.976505, size = 38, normalized size = 2.38

$$\frac{(dx+c)b + a\log(\sec(dx+c) + \tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*b + a*log(sec(d*x + c) + tan(d*x + c)))/d
```

Fricas [B] time = 1.871, size = 95, normalized size = 5.94

$$\frac{2\,bdx + a\log(\sin(dx+c) + 1) - a\log(-\sin(dx+c) + 1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1))/d
```

Sympy [B] time = 4.84301, size = 49, normalized size = 3.06

$$a\left(\begin{cases} \frac{x\tan(c)\sec(c)}{\tan(c)+\sec(c)} + \frac{x\sec^2(c)}{\tan(c)+\sec(c)} & \text{for } d = 0 \\ \frac{\log(\tan(c+dx)+\sec(c+dx))}{d} & \text{otherwise} \end{cases}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] a*Piecewise((x*tan(c)*sec(c)/(tan(c) + sec(c)) + x*sec(c)**2/(tan(c) + sec(
c)), Eq(d, 0)), (log(tan(c + d*x) + sec(c + d*x))/d, True)) + b*x
```

Giac [B] time = 1.38204, size = 58, normalized size = 3.62

$$\frac{(dx+c)b + a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] ((d*x + c)*b + a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)))/d
```

3.412 $\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=24

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0369524, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3767, 8, 3770}

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\sec^2(c+dx)\,dx &= a\int \sec^2(c+dx)\,dx+b\int \sec(c+dx)\,dx\\ &= \frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\,\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{d}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))}{d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0070144, size = 24, normalized size = 1.

$$\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.05, size = 32, normalized size = 1.3

$$\frac{a\tan(dx+c)}{d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] a*tan(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.969617, size = 51, normalized size = 2.12

$$\frac{b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,a\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*a*tan(d*x + c))/
d
```

Fricas [B] time = 1.88524, size = 162, normalized size = 6.75

$$\frac{b\cos(dx+c)\log(\sin(dx+c)+1)-b\cos(dx+c)\log(-\sin(dx+c)+1)+2\,a\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*cos(d*x + c)*log(sin(d*x + c) + 1) - b*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*a*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\cos(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.34074, size = 85, normalized size = 3.54

$$\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.413 $\int (a+b\cos (c+dx))\sec ^3(c+dx)\,dx$

Optimal. Leaf size=47

$$\frac{a\tanh ^{-1}(\sin (c+dx))}{2d}+\frac{a\tan (c+dx)\sec (c+dx)}{2d}+\frac{b\tan (c+dx)}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rubi [A] time = 0.0489279, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2748, 3768, 3770, 3767, 8}

$$\frac{a\tanh ^{-1}(\sin (c+dx))}{2d}+\frac{a\tan (c+dx)\sec (c+dx)}{2d}+\frac{b\tan (c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^3(c+dx)\,dx = a\int \sec^3(c+dx)\,dx + b\int \sec^2(c+dx)\,dx$$
$$= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx - \frac{b\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c}{d}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\tan(c+dx)}{d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0145189, size = 47, normalized size = 1.

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Tan[c + d*x])/d + (a*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Maple [A] time = 0.052, size = 51, normalized size = 1.1

$$\frac{a\sec(dx+c)\tan(dx+c)}{2d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{b\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+b*tan(d*x+c
)/d
```

Maxima [A] time = 0.968342, size = 78, normalized size = 1.66

$$-\frac{a\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 4b\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) - 4*b*tan(d*x + c))/d
```

Fricas [A] time = 1.89198, size = 198, normalized size = 4.21

$$\frac{a\cos(dx+c)^2\log(\sin(dx+c)+1) - a\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2(2b\cos(dx+c)+a)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - a*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) + 2*(2*b*cos(d*x + c) + a)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\cos(c + dx))\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] time = 1.45567, size = 142, normalized size = 3.02

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/
2*d*x + 1/2*c) + 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/
d
```

3.414 $\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=63

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0528197, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3767, 3768, 3770}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (a*Tan[c + d*x])/d + (b*Sec[c + d*x]*Tan[
c + d*x])/(2*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))\sec^4(c+dx)\,dx &= a\int \sec^4(c+dx)\,dx+b\int \sec^3(c+dx)\,dx\\ &= \frac{b\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}b\int \sec(c+dx)\,dx-\frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\right.}{d}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec(c+dx)\tan(c+dx)}{2d}+\frac{a\tan^3(}{3}\end{aligned}$$

Mathematica [A] time = 0.1598, size = 60, normalized size = 0.95

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*
(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.056, size = 72, normalized size = 1.1

$$\frac{2a\tan(dx+c)}{3d}+\frac{a\tan(dx+c)(\sec(dx+c))^2}{3d}+\frac{b\sec(dx+c)\tan(dx+c)}{2d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a*tan(d*x+c)/d+1/3/d*a*tan(d*x+c)*sec(d*x+c)^2+1/2*b*sec(d*x+c)*tan(d*x
+c)/d+1/2/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01816, size = 95, normalized size = 1.51

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a-3b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a - 3*b*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.9198, size = 236, normalized size = 3.75

$$\frac{3b\cos(dx+c)^3\log(\sin(dx+c)+1)-3b\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(4a\cos(dx+c)^2+3b\cos(dx+c)+\right.}{12d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*b*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^2 + 3*b*cos(d*x + c) + 2*a)*sin(d*x +
 c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44481, size = 165, normalized size = 2.62

$$\frac{3\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(6*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 - 4*
a*tan(1/2*d*x + 1/2*c)^3 + 6*a*tan(1/2*d*x + 1/2*c) + 3*b*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.415 $\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*
x]^3)/(3*d)
```

Rubi [A] time = 0.065147, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3768, 3770, 3767}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^3(c+dx)}{3d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Tan[c + d*x])/d + (3*a*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*
x]^3)/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^5(c+dx)\,dx = a\int \sec^5(c+dx)\,dx + b\int \sec^4(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx - \frac{b\,\mathrm{Subst}\left(\int \left(1+x^2\right) \right.}{d}$$
$$= \frac{b\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{b\,\mathrm{t}}{}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\tan(c+dx)}{d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec}{}$$

Mathematica [A] time = 0.226101, size = 76, normalized size = 0.89

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx)) + \tan(c+dx)\sec(c+dx)\right)}{8d} + \frac{b\left(\frac{1}{3}\tan^3(c+dx) + \tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (b*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.056, size = 92, normalized size = 1.1

$$\frac{a\tan(dx+c)(\sec(dx+c))^3}{4d} + \frac{3a\sec(dx+c)\tan(dx+c)}{8d} + \frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{2b\tan(dx+c)}{3d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*a*tan(d*x+c)*sec(d*x+c)^3+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+2/3*b*tan(d*x+c)/d+1/3/d*b*tan(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.98934, size = 128, normalized size = 1.51

$$\frac{16\left(\tan(dx+c)^3 + 3\tan(dx+c)\right)b - 3a\left(\frac{2\left(3\sin(dx+c)^3 - 5\sin(dx+c)\right)}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*b - 3*a*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
 + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.91005, size = 266, normalized size = 3.13

$$\frac{9\,a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-9\,a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,b\cos\left(dx+c\right)^{3}+9\,a\cos\left(dx+c\right)^{2}\right.}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(9*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 9*a*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 2*(16*b*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 + 8*b*cos(d*x
 + c) + 6*a)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44514, size = 221, normalized size = 2.6

$$\frac{9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+40\,b\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(9*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + 2*(15*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2*d*x + 1/2*c)^7 +
 9*a*tan(1/2*d*x + 1/2*c)^5 + 40*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*tan(1/2*d*x
 + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c) + 24*
b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.416 $\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c$$

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*b*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*a*Tan[c +
d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0701793, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2748, 3767, 3768, 3770}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Tan[c + d*x])/d + (3*b*Sec[c + d*x]*
Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (2*a*Tan[c +
d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^6(c+dx)\,dx = a\int \sec^6(c+dx)\,dx + b\int \sec^5(c+dx)\,dx$$
$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3b)\int \sec^3(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \left(1+2x^2+x^4\right.\right.}{d}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{2a\,\text{ta}}{}$$
$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\tan(c+dx)}{d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c}{}$$

Mathematica [A] time = 0.324824, size = 88, normalized size = 0.87

$$\frac{a\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{b\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3b\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*b*(ArcTanh[Sin[c + d*x]] + Sec[c
 + d*x]*Tan[c + d*x]))/(8*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Ta
n[c + d*x]^5/5))/d
```

Maple [A] time = 0.053, size = 112, normalized size = 1.1

$$\frac{8a\tan(dx+c)}{15d} + \frac{a\tan(dx+c)(\sec(dx+c))^4}{5d} + \frac{4a\tan(dx+c)(\sec(dx+c))^2}{15d} + \frac{b(\sec(dx+c))^3\tan(dx+c)}{4d} + \frac{3b\,\text{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 8/15*a*tan(d*x+c)/d+1/5/d*a*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a*tan(d*x+c)*sec
(d*x+c)^2+1/4*b*sec(d*x+c)^3*tan(d*x+c)/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+3/8
/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.987564, size = 144, normalized size = 1.43

$$\frac{16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a - 15b\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1} - 3\log(\sin(dx+c)+1)+3\,\text{l}\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 15*b
*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
+ 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 1.98316, size = 304, normalized size = 3.01

$$\frac{45\,b\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\,b\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(64\,a\cos\left(dx+c\right)^4+45\,b\cos\left(\right.\right.}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(45*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*b*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 2*(64*a*cos(d*x + c)^4 + 45*b*cos(d*x + c)^3 + 32*a*co
s(d*x + c)^2 + 30*b*cos(d*x + c) + 24*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43671, size = 240, normalized size = 2.38

$$\frac{45\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-160\,a\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(45*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(120*a*tan(1/2*d*x + 1/2*c)^9 - 75*b*tan(1/2*d*x + 1/2*c)
^9 - 160*a*tan(1/2*d*x + 1/2*c)^7 + 30*b*tan(1/2*d*x + 1/2*c)^7 + 464*a*tan
(1/2*d*x + 1/2*c)^5 - 160*a*tan(1/2*d*x + 1/2*c)^3 - 30*b*tan(1/2*d*x + 1/2
*c)^3 + 120*a*tan(1/2*d*x + 1/2*c) + 75*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 - 1)^5)/d
```

3.417 $\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=150

$$\frac{(6a^2+5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+5b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a^2+5b^2)+\frac{2ab\sin^5(c+dx)}{5d}-\frac{4ab\,\mathrm{s}}{}$$

```
[Out] ((6*a^2 + 5*b^2)*x)/16 + (2*a*b*Sin[c + d*x])/d + ((6*a^2 + 5*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(16*d) + ((6*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(
24*d) + (b^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (4*a*b*Sin[c + d*x]^3)/(3
*d) + (2*a*b*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.106189, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2633, 3014, 2635, 8}

$$\frac{(6a^2+5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{(6a^2+5b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a^2+5b^2)+\frac{2ab\sin^5(c+dx)}{5d}-\frac{4ab\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + 5*b^2)*x)/16 + (2*a*b*Sin[c + d*x])/d + ((6*a^2 + 5*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(16*d) + ((6*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(
24*d) + (b^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (4*a*b*Sin[c + d*x]^3)/(3
*d) + (2*a*b*Sin[c + d*x]^5)/(5*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^5(c+dx)\,dx + \int \cos^4(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$
$$= \frac{b^2\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}\left(6a^2+5b^2\right)\int \cos^4(c+dx)\,dx - \frac{(2ab)\,\text{Subs}}{}$$
$$= \frac{2ab\sin(c+dx)}{d} + \frac{\left(6a^2+5b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{b^2\cos^5(c+dx)\sin}{6d}$$
$$= \frac{2ab\sin(c+dx)}{d} + \frac{\left(6a^2+5b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{\left(6a^2+5b^2\right)\cos^3(c}{24d}$$
$$= \frac{1}{16}\left(6a^2+5b^2\right)x + \frac{2ab\sin(c+dx)}{d} + \frac{\left(6a^2+5b^2\right)\cos(c+dx)\sin(c+dx)}{16d} +$$

Mathematica [A] time = 0.303745, size = 123, normalized size = 0.82

$$\frac{5\left(\left(48a^2+45b^2\right)\sin(2(c+dx))+\left(6a^2+9b^2\right)\sin(4(c+dx))+72a^2c+72a^2dx+b^2\sin(6(c+dx))+60b^2c+60b^2dx\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (1920*a*b*Sin[c + d*x] - 1280*a*b*Sin[c + d*x]^3 + 384*a*b*Sin[c + d*x]^5 +
 5*(72*a^2*c + 60*b^2*c + 72*a^2*d*x + 60*b^2*d*x + (48*a^2 + 45*b^2)*Sin[2
*(c + d*x)] + (6*a^2 + 9*b^2)*Sin[4*(c + d*x)] + b^2*Sin[6*(c + d*x)]))/(96
0*d)
```

Maple [A] time = 0.034, size = 120, normalized size = 0.8

$$\frac{1}{d}\left(b^2\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{2ab\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+2/5*a*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*(
1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 0.99005, size = 162, normalized size = 1.08

$$\frac{30(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^2+128\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)ab}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/960*(30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2 + 128
*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a*b - 5*(4*sin(2*
d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^
2)/d
```

Fricas [A] time = 1.98242, size = 273, normalized size = 1.82

$$\frac{15\left(6a^2+5b^2\right)dx+\left(40b^2\cos\left(dx+c\right)^5+96ab\cos\left(dx+c\right)^4+128ab\cos\left(dx+c\right)^2+10\left(6a^2+5b^2\right)\cos\left(dx+c\right)^3+256\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(6*a^2 + 5*b^2)*d*x + (40*b^2*cos(d*x + c)^5 + 96*a*b*cos(d*x + c
)^4 + 128*a*b*cos(d*x + c)^2 + 10*(6*a^2 + 5*b^2)*cos(d*x + c)^3 + 256*a*b
+ 15*(6*a^2 + 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.49266, size = 343, normalized size = 2.29

$$\begin{cases}\frac{3a^2x\sin^4(c+dx)}{8}+\frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^2x\cos^4(c+dx)}{8}+\frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{16ab\sin^5(c+dx)}{15d}+\frac{8}{} \\ x\left(a+b\cos(c)\right)^2\cos^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 16*a*b*sin(c + d*x)**5/
(15*d) + 8*a*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 2*a*b*sin(c + d*x)*c
os(c + d*x)**4/d + 5*b**2*x*sin(c + d*x)**6/16 + 15*b**2*x*sin(c + d*x)**4*
cos(c + d*x)**2/16 + 15*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**2*
x*cos(c + d*x)**6/16 + 5*b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*b**2*
sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b**2*sin(c + d*x)*cos(c + d*x)**
5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**4, True))
```

Giac [A] time = 1.37907, size = 171, normalized size = 1.14

$$\frac{1}{16}\left(6a^2+5b^2\right)x+\frac{b^2\sin\left(6dx+6c\right)}{192d}+\frac{ab\sin\left(5dx+5c\right)}{40d}+\frac{5ab\sin\left(3dx+3c\right)}{24d}+\frac{5ab\sin\left(dx+c\right)}{4d}+\frac{\left(2a^2+3b^2\right)\sin\left(\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(6*a^2 + 5*b^2)*x + 1/192*b^2*sin(6*d*x + 6*c)/d + 1/40*a*b*sin(5*d*x
+ 5*c)/d + 5/24*a*b*sin(3*d*x + 3*c)/d + 5/4*a*b*sin(d*x + c)/d + 1/64*(2*a
^2 + 3*b^2)*sin(4*d*x + 4*c)/d + 1/64*(16*a^2 + 15*b^2)*sin(2*d*x + 2*c)/d
```

3.418 $\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=111

$$-\frac{(a^2+2b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{3abx}{4}$$

```
[Out] (3*a*b*x)/4 + ((a^2 + b^2)*Sin[c + d*x])/d + (3*a*b*Cos[c + d*x]*Sin[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - ((a^2 + 2*b^2)*Sin[c
+ d*x]^3)/(3*d) + (b^2*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.108267, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2635, 8, 3013, 373}

$$-\frac{(a^2+2b^2)\sin^3(c+dx)}{3d}+\frac{(a^2+b^2)\sin(c+dx)}{d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{3abx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/4 + ((a^2 + b^2)*Sin[c + d*x])/d + (3*a*b*Cos[c + d*x]*Sin[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(2*d) - ((a^2 + 2*b^2)*Sin[c
+ d*x]^3)/(3*d) + (b^2*Sin[c + d*x]^5)/(5*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^4(c+dx)\,dx + \int \cos^3(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$
$$= \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}(3ab)\int \cos^2(c+dx)\,dx - \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a^2\right.\right.}{}$$
$$= \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{4}(3ab)\int 1\,dx - \frac{\text{Su}}{}$$
$$= \frac{3abx}{4} + \frac{\left(a^2+b^2\right)\sin(c+dx)}{d} + \frac{3ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{ab\cos^3(c+dx)\text{si}}{2d}$$

Mathematica [A] time = 0.134224, size = 85, normalized size = 0.77

$$\frac{-80\left(a^2+2b^2\right)\sin^3(c+dx)+240\left(a^2+b^2\right)\sin(c+dx)+15ab(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))+48b^2\sin}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (240*(a^2 + b^2)*Sin[c + d*x] - 80*(a^2 + 2*b^2)*Sin[c + d*x]^3 + 48*b^2*Si
n[c + d*x]^5 + 15*a*b*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/(240*d)
```

Maple [A] time = 0.034, size = 95, normalized size = 0.9

$$\frac{1}{d}\left(\frac{b^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+2ab\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c)+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(1/5*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*(2+cos(d*x+c)^2
)*sin(d*x+c))
```

Maxima [A] time = 0.995854, size = 127, normalized size = 1.14

$$-\frac{80\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2-15(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))ab-16\left(3\sin(dx+c)^5-1\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(80*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^2 - 15*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b - 16*(3*sin(d*x + c)^5 - 10*sin(d*x
+ c)^3 + 15*sin(d*x + c))*b^2)/d
```

Fricas [A] time = 1.90697, size = 215, normalized size = 1.94

$$\frac{45\,abdx + \left(12\,b^2\cos\left(dx+c\right)^4 + 30\,ab\cos\left(dx+c\right)^3 + 45\,ab\cos\left(dx+c\right) + 4\left(5\,a^2+4\,b^2\right)\cos\left(dx+c\right)^2 + 40\,a^2 + 32\,b^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(45*a*b*d*x + (12*b^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 + 45*a*b*
cos(d*x + c) + 4*(5*a^2 + 4*b^2)*cos(d*x + c)^2 + 40*a^2 + 32*b^2)*sin(d*x
+ c))/d
```

Sympy [A] time = 2.35245, size = 221, normalized size = 1.99

$$\begin{cases} \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^2(c+dx)}{d} + \frac{3abx\sin^4(c+dx)}{4} + \frac{3abx\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{3abx\cos^4(c+dx)}{4} + \frac{3ab\sin^3(c+dx)\cos(c+dx)}{4d} + \\ x\left(a+b\cos\left(c\right)\right)^2\cos^3\left(c\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2
/d + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2/2
+ 3*a*b*x*cos(c + d*x)**4/4 + 3*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 5*
a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) + 8*b**2*sin(c + d*x)**5/(15*d) + 4*
b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + b**2*sin(c + d*x)*cos(c + d*x)
**4/d, Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c)**3, True))
```

Giac [A] time = 1.34549, size = 138, normalized size = 1.24

$$\frac{3}{4}\,abx + \frac{b^2\sin\left(5\,dx+5\,c\right)}{80\,d} + \frac{ab\sin\left(4\,dx+4\,c\right)}{16\,d} + \frac{ab\sin\left(2\,dx+2\,c\right)}{2\,d} + \frac{\left(4\,a^2+5\,b^2\right)\sin\left(3\,dx+3\,c\right)}{48\,d} + \frac{\left(6\,a^2+5\,b^2\right)\sin}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/4*a*b*x + 1/80*b^2*sin(5*d*x + 5*c)/d + 1/16*a*b*sin(4*d*x + 4*c)/d + 1/2
*a*b*sin(2*d*x + 2*c)/d + 1/48*(4*a^2 + 5*b^2)*sin(3*d*x + 3*c)/d + 1/8*(6*
a^2 + 5*b^2)*sin(d*x + c)/d
```

3.419 $\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{(4a^2+3b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2+3b^2\right)-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

```
[Out] ((4*a^2 + 3*b^2)*x)/8 + (2*a*b*Sin[c + d*x])/d + ((4*a^2 + 3*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*b*
Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0920934, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 2633, 3014, 2635, 8}

$$\frac{(4a^2+3b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(4a^2+3b^2\right)-\frac{2ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a^2 + 3*b^2)*x)/8 + (2*a*b*Sin[c + d*x])/d + ((4*a^2 + 3*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (2*a*b*
Sin[c + d*x]^3)/(3*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^3(c+dx)\,dx + \int \cos^2(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$
$$= \frac{b^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\left(4a^2+3b^2\right)\int \cos^2(c+dx)\,dx - \frac{(2ab)\,\text{Subs}}{}$$
$$= \frac{2ab\sin(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{b^2\cos^3(c+dx)\sin(}{4d}$$
$$= \frac{1}{8}\left(4a^2+3b^2\right)x + \frac{2ab\sin(c+dx)}{d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8d} + \frac{b}{}$$

Mathematica [A] time = 0.156146, size = 86, normalized size = 0.85

$$\frac{24\left(a^2+b^2\right)\sin(2(c+dx)) + 48a^2c + 48a^2dx - 64ab\sin^3(c+dx) + 192ab\sin(c+dx) + 3b^2\sin(4(c+dx)) + 36b^2c + }{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (48*a^2*c + 36*b^2*c + 48*a^2*d*x + 36*b^2*d*x + 192*a*b*Sin[c + d*x] - 64*
a*b*Sin[c + d*x]^3 + 24*(a^2 + b^2)*Sin[2*(c + d*x)] + 3*b^2*Sin[4*(c + d*x
)])/(96*d)
```

Maple [A] time = 0.034, size = 89, normalized size = 0.9

$$\frac{1}{d}\left(b^2\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{2ab\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+a^2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*a
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A] time = 0.984981, size = 111, normalized size = 1.1

$$\frac{24(2dx+2c+\sin(2dx+2c))a^2 - 64\left(\sin(dx+c)^3 - 3\sin(dx+c)\right)ab + 3(12dx+12c+\sin(4dx+4c)+8\sin(2}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 64*(sin(d*x + c)^3 - 3*sin(
d*x + c))*a*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*b
^2)/d
```

Fricas [A] time = 1.85724, size = 184, normalized size = 1.82

$$\frac{3\left(4a^2+3b^2\right)dx+\left(6b^2\cos(dx+c)^3+16ab\cos(dx+c)^2+32ab+3\left(4a^2+3b^2\right)\cos(dx+c)\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(4*a^2 + 3*b^2)*d*x + (6*b^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2
 + 32*a*b + 3*(4*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.24016, size = 211, normalized size = 2.09

$$\begin{cases}\frac{a^2x\sin^2(c+dx)}{2}+\frac{a^2x\cos^2(c+dx)}{2}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{4ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3b^2x\sin^4(c+dx)}{8}+\frac{3b^2x\sin^2(c+dx)\text{co}}{4}\\ x\left(a+b\cos(c)\right)^2\cos^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) + 4*a*b*sin(c + d*x)**3/(3*d) + 2*a*b*sin(c + d*
x)*cos(c + d*x)**2/d + 3*b**2*x*sin(c + d*x)**4/8 + 3*b**2*x*sin(c + d*x)**
2*cos(c + d*x)**2/4 + 3*b**2*x*cos(c + d*x)**4/8 + 3*b**2*sin(c + d*x)**3*c
os(c + d*x)/(8*d) + 5*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (
x*(a + b*cos(c))**2*cos(c)**2, True))
```

Giac [A] time = 1.74242, size = 111, normalized size = 1.1

$$\frac{1}{8}\left(4a^2+3b^2\right)x+\frac{b^2\sin(4dx+4c)}{32d}+\frac{ab\sin(3dx+3c)}{6d}+\frac{3ab\sin(dx+c)}{2d}+\frac{\left(a^2+b^2\right)\sin(2dx+2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2 + 3*b^2)*x + 1/32*b^2*sin(4*d*x + 4*c)/d + 1/6*a*b*sin(3*d*x + 3
*c)/d + 3/2*a*b*sin(d*x + c)/d + 1/4*(a^2 + b^2)*sin(2*d*x + 2*c)/d
```

3.420 $\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=71

$$\frac{2(a^2+b^2)\sin(c+dx)}{3d}+\frac{\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+abx$$

```
[Out] a*b*x + (2*(a^2 + b^2)*Sin[c + d*x])/(3*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]
)/(3*d) + ((a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0498053, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$\frac{2(a^2+b^2)\sin(c+dx)}{3d}+\frac{\sin(c+dx)(a+b\cos(c+dx))^2}{3d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] a*b*x + (2*(a^2 + b^2)*Sin[c + d*x])/(3*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]
)/(3*d) + ((a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\cos(c+dx))^2\,dx &= \frac{(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (2b+2a\cos(c+dx))(a+b\cos(c+dx\\ &= abx+\frac{2(a^2+b^2)\sin(c+dx)}{3d}+\frac{ab\cos(c+dx)\sin(c+dx)}{3d}+\frac{(a+b\cos(c+d}{3c}\end{aligned}$$

Mathematica [A] time = 0.153382, size = 59, normalized size = 0.83

$$\frac{3(4a^2+3b^2)\sin(c+dx)+b(12a(c+dx)+6a\sin(2(c+dx))+b\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 + 3*b^2)*Sin[c + d*x] + b*(12*a*(c + d*x) + 6*a*Sin[2*(c + d*x)]
+ b*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A] time = 0.033, size = 63, normalized size = 0.9

$$\frac{1}{d}\left(\frac{b^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+2\,ab\,(1/2\ \cos(dx+c)\sin(dx+c)+1/2\,dx+c/2)+a^2\sin(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1
/2*d*x+1/2*c)+a^2*sin(d*x+c))
```

Maxima [A] time = 0.966371, size = 81, normalized size = 1.14

$$\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))ab-2\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)b^2+6\,a^2\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b - 2*(sin(d*x + c)^3 - 3*sin(d*x
 + c))*b^2 + 6*a^2*sin(d*x + c))/d
```

Fricas [A] time = 1.8673, size = 124, normalized size = 1.75

$$\frac{3\,abdx+\left(b^2\cos(dx+c)^2+3\,ab\cos(dx+c)+3\,a^2+2\,b^2\right)\sin(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*d*x + (b^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) + 3*a^2 + 2*b^2)*
sin(d*x + c))/d
```

Sympy [A] time = 0.611332, size = 107, normalized size = 1.51

$$\begin{cases}\frac{a^2\sin(c+dx)}{d}+abx\sin^2(c+dx)+abx\cos^2(c+dx)+\frac{ab\sin(c+dx)\cos(c+dx)}{d}+\frac{2b^2\sin^3(c+dx)}{3d}+\frac{b^2\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d\neq 0\\ x\,(a+b\cos(c))^2\cos(c) & \text{otherwis}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)/d + a*b*x*sin(c + d*x)**2 + a*b*x*cos(c + d*x)
**2 + a*b*sin(c + d*x)*cos(c + d*x)/d + 2*b**2*sin(c + d*x)**3/(3*d) + b**2
*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(a + b*cos(c))**2*cos(c), Tr
ue))
```

Giac [A] time = 1.39079, size = 81, normalized size = 1.14

$$abx + \frac{b^2 \sin(3\,dx + 3\,c)}{12\,d} + \frac{ab \sin(2\,dx + 2\,c)}{2\,d} + \frac{(4\,a^2 + 3\,b^2) \sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a*b*x + 1/12*b^2*sin(3*d*x + 3*c)/d + 1/2*a*b*sin(2*d*x + 2*c)/d + 1/4*(4*a
^2 + 3*b^2)*sin(d*x + c)/d
```

3.421 $\int (a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] ((2*a^2 + b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*
x])/(2*d)
```

Rubi [A] time = 0.0149788, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*a^2 + b^2)*x)/2 + (2*a*b*Sin[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*
x])/(2*d)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\,dx=\frac{1}{2}\left(2a^2+b^2\right)x+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0716666, size = 46, normalized size = 0.92

$$\frac{2\left(2a^2+b^2\right)(c+dx)+8ab\sin(c+dx)+b^2\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*x)])/(4*
d)
```

Maple [A] time = 0.031, size = 51, normalized size = 1.

$$\frac{1}{d}\left(b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\,ab\sin(dx+c)+a^2(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*sin(d*x+c)+a^2*(d*
x+c))
```

Maxima [A] time = 0.954685, size = 59, normalized size = 1.18

$$a^2x + \frac{(2\,dx + 2\,c + \sin(2\,dx + 2\,c))b^2}{4\,d} + \frac{2\,ab\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*b^2/d + 2*a*b*sin(d*x + c)/d
```

Fricas [A] time = 1.80501, size = 93, normalized size = 1.86

$$\frac{\left(2\,a^2 + b^2\right)dx + \left(b^2\cos(dx + c) + 4\,ab\right)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a^2 + b^2)*d*x + (b^2*cos(d*x + c) + 4*a*b)*sin(d*x + c))/d
```

Sympy [A] time = 0.281447, size = 78, normalized size = 1.56

$$\begin{cases} a^2x + \frac{2ab\sin(c+dx)}{d} + \frac{b^2x\sin^2(c+dx)}{2} + \frac{b^2x\cos^2(c+dx)}{2} + \frac{b^2\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x\,(a + b\cos(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sin(c + d*x)/d + b**2*x*sin(c + d*x)**2/2 + b**2*
x*cos(c + d*x)**2/2 + b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d, 0)), (x*(
a + b*cos(c))**2, True))
```

Giac [A] time = 1.52084, size = 58, normalized size = 1.16

$$\frac{1}{2}\left(2\,a^2 + b^2\right)x + \frac{b^2\sin(2\,dx + 2\,c)}{4\,d} + \frac{2\,ab\sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 + b^2)*x + 1/4*b^2*sin(2*d*x + 2*c)/d + 2*a*b*sin(d*x + c)/d
```

3.422 $\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx$

Optimal. Leaf size=33

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2abx+\frac{b^2\sin(c+dx)}{d}$$

```
[Out] 2*a*b*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (b^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.0624424, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2746, 2735, 3770}

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2abx+\frac{b^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x],x]
```

```
[Out] 2*a*b*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (b^2*Sin[c + d*x])/d
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec(c+dx)\,dx &= \frac{b^2\sin(c+dx)}{d}+\int \left(a^2+2ab\cos(c+dx)\right)\sec(c+dx)\,dx\\ &= 2abx+\frac{b^2\sin(c+dx)}{d}+a^2\int \sec(c+dx)\,dx\\ &= 2abx+\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0133475, size = 46, normalized size = 1.39

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+2abx+\frac{b^2\sin(c)\cos(dx)}{d}+\frac{b^2\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x],x]
```

```
[Out] 2*a*b*x + (a^2*ArcTanh[Sin[c + d*x]])/d + (b^2*Cos[d*x]*Sin[c])/d + (b^2*Co
s[c]*Sin[d*x])/d
```

Maple [A] time = 0.049, size = 49, normalized size = 1.5

$$2\,abx+{\frac{{b}^{2}\sin \left(dx+c \right) }{d}}+{\frac{{a}^{2}\ln \left(\sec \left(dx+c \right) +\tan \left(dx+c \right) \right) }{d}}+2\,{\frac{abc}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c),x)
```

```
[Out] 2*a*b*x+b^2*sin(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b*c
```

Maxima [A] time = 0.98336, size = 57, normalized size = 1.73

$$\frac{2\,(dx+c)ab+a^{2}\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+b^{2}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(d*x + c)*a*b + a^2*log(sec(d*x + c) + tan(d*x + c)) + b^2*sin(d*x + c))
/d
```

Fricas [A] time = 2.00354, size = 131, normalized size = 3.97

$$\frac{4\,abdx+a^{2}\log\left(\sin\left(dx+c\right)+1\right)-a^{2}\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^{2}\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*d*x + a^2*log(sin(d*x + c) + 1) - a^2*log(-sin(d*x + c) + 1) + 2
*b^2*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\cos\left(c+dx\right)\right)^{2}\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] time = 1.37555, size = 105, normalized size = 3.18

$$\frac{2\,(dx+c)ab + a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c),x, algorithm="giac")
```

```
[Out] (2*(d*x + c)*a*b + a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^
2 + 1))/d
```

3.423 $\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx$

Optimal. Leaf size=33

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+b^2x$$

```
[Out] b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.0656476, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2789, 3770, 3012, 8}

$$\frac{a^2\tan(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d + (a^2*Tan[c + d*x])/d
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^2(c+dx)\,dx &= (2ab)\int \sec(c+dx)\,dx+\int \left(a^2+b^2\cos^2(c+dx)\right)\sec^2(c+dx)\,dx\\ &= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}+b^2\int 1\,dx\\ &= b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0845777, size = 32, normalized size = 0.97

$$\frac{a^2\tan(c+dx)+2ab\tanh^{-1}(\sin(c+dx))+b^2dx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*d*x + 2*a*b*ArcTanh[Sin[c + d*x]] + a^2*Tan[c + d*x])/d
```

Maple [A] time = 0.053, size = 49, normalized size = 1.5

$$b^2x+2\,\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\tan(dx+c)}{d}+\frac{b^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x)
```

```
[Out] b^2*x+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+a^2*tan(d*x+c)/d+1/d*b^2*c
```

Maxima [A] time = 0.975303, size = 65, normalized size = 1.97

$$\frac{(dx+c)b^2+ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)*b^2 + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + a^2*
tan(d*x + c))/d
```

Fricas [B] time = 1.96573, size = 193, normalized size = 5.85

$$\frac{b^2dx\cos(dx+c)+ab\cos(dx+c)\log(\sin(dx+c)+1)-ab\cos(dx+c)\log(-\sin(dx+c)+1)+a^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x*cos(d*x + c) + a*b*cos(d*x + c)*log(sin(d*x + c) + 1) - a*b*cos(d*
x + c)*log(-sin(d*x + c) + 1) + a^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\cos(c+dx))^2\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [B] time = 1.57318, size = 104, normalized size = 3.15

$$\frac{(dx+c)b^2 + 2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*b^2 + 2*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*b*log(abs(t
an(1/2*d*x + 1/2*c) - 1)) - 2*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c
)^2 - 1))/d
```

3.424 $\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx$

Optimal. Leaf size=59

$$\frac{(a^2+2b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}$$

```
[Out] ((a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (a^2
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0784044, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3767, 8, 3012, 3770}

$$\frac{(a^2+2b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*Tan[c + d*x])/d + (a^2
*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^3(c+dx)\,dx = (2ab)\int \sec^2(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\left(a^2+2b^2\right)\int \sec(c+dx)\,dx - \frac{(2ab)\,\mathrm{Subst}(\int 1\,d$$
$$= \frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0132483, size = 67, normalized size = 1.14

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2\tan(c+dx)\sec(c+dx)}{2d} + \frac{2ab\tan(c+dx)}{d} + \frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^3,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) + (b^2*ArcTanh[Sin[c + d*x]])/d + (2*a*b*
Tan[c + d*x])/d + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.059, size = 78, normalized size = 1.3

$$\frac{a^2\sec(dx+c)\tan(dx+c)}{2d} + \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{2d} + 2\frac{ab\tan(dx+c)}{d} + \frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^2*sec(d*x+c)*tan(d*x+c)+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b*t
an(d*x+c)/d+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.958304, size = 117, normalized size = 1.98

$$-\frac{a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) - 2b^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) - 8a}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 8*a*b*tan(d*x + c))/d
```

Fricas [A] time = 1.99757, size = 236, normalized size = 4.

$$\frac{\left(a^2+2b^2\right)\cos(dx+c)^2\log(\sin(dx+c)+1) - \left(a^2+2b^2\right)\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\left(4ab\cos(dx+c) + a\right.}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 + 2*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (a^2 + 2*b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(4*a*b*cos(d*x + c) + a^2)*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4239, size = 171, normalized size = 2.9

$$\frac{\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\cdots\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (a^2 + 2*b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^3 + a^2*tan(1/2*d*x + 1/2*c) + 4*a*b*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.425 $\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{(2a^2+3b^2)\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}$$

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + ((2*a^2 + 3*b^2)*Tan[c + d*x])/(3*d) + (a*b
*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.0890891, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2789, 3768, 3770, 3012, 3767, 8}

$$\frac{(2a^2+3b^2)\tan(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(c+dx)}{3d}+\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + ((2*a^2 + 3*b^2)*Tan[c + d*x])/(3*d) + (a*b
*Sec[c + d*x]*Tan[c + d*x])/d + (a^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^4(c+dx)\,dx = (2ab)\int \sec^3(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\sec^2(c+dx)\tan(c+dx)}{3d} + (ab)\int \sec(c+d$$
$$= \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{ab\sec(c+dx)\tan(c+dx)}{d} + \frac{a^2\sec^2(c+dx)\tan(c}{3d}$$
$$= \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+}{d}$$

Mathematica [A] time = 0.217512, size = 71, normalized size = 0.89

$$\frac{a^2\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{ab\tan(c+dx)\sec(c+dx)}{d} + \frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^4,x]
```

```
[Out] (a*b*ArcTanh[Sin[c + d*x]])/d + (b^2*Tan[c + d*x])/d + (a*b*Sec[c + d*x]*Ta
n[c + d*x])/d + (a^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.06, size = 89, normalized size = 1.1

$$\frac{2a^2\tan(dx+c)}{3d} + \frac{a^2(\sec(dx+c))^2\tan(dx+c)}{3d} + \frac{ab\sec(dx+c)\tan(dx+c)}{d} + \frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a^2*tan(d*x+c)/d+1/3*a^2*sec(d*x+c)^2*tan(d*x+c)/d+a*b*sec(d*x+c)*tan(d
*x+c)/d+1/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^2*tan(d*x+c)
```

Maxima [A] time = 0.985262, size = 113, normalized size = 1.41

$$\frac{2\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2 - 3ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + 6b^2\tan(dx}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 - 3*a*b*(2*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*b^2*tan
```

```
(d*x + c))/d
```

Fricas [A] time = 2.04616, size = 259, normalized size = 3.24

$$\frac{3\,ab\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,ab\cos\left(dx+c\right)+\left(2\,a^{2}+3\,b^{2}\right)\text{c}\right.}{6\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*b*cos(d*x + c)^3*log(
-sin(d*x + c) + 1) + 2*(3*a*b*cos(d*x + c) + (2*a^2 + 3*b^2)*cos(d*x + c)^2
 + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.48634, size = 240, normalized size = 3.

$$\frac{3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*tan(1/2*d*x + 1/2*c
)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a^2*tan(1/2*d*x + 1/2*c)^3 - 6*b^2*t
an(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/
2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.426 $\int (a+b\cos(c+dx))^2 \sec^5(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{(3a^2+4b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2+4b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{2ab\tan^3(c+$$

```
[Out] ((3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*Tan[c + d*x])/d + ((
3*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (2*a*b*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0950722, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3767, 3012, 3768, 3770}

$$\frac{(3a^2+4b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3a^2+4b^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{2ab\tan^3(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```


```
[Out] ((3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (2*a*b*Tan[c + d*x])/d + ((
3*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (2*a*b*Tan[c + d*x]^3)/(3*d)
```


Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^5(c+dx)\,dx = (2ab)\int \sec^4(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\left(3a^2+4b^2\right)\int \sec^3(c+dx)\,dx - \frac{(2ab)\,\text{Subst}\,(\int$$
$$= \frac{2ab\tan(c+dx)}{d} + \frac{\left(3a^2+4b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^2\sec^3(c+dx)\tan(c+}{4d}$$
$$= \frac{\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{2ab\tan(c+dx)}{d} + \frac{\left(3a^2+4b^2\right)\sec(c+dx)\,\text{t}}{8d}$$

Mathematica [A] time = 0.275158, size = 82, normalized size = 0.75

$$\frac{3\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(3\left(3a^2+4b^2\right)\sec(c+dx)+6a^2\sec^3(c+dx)+16ab\left(\tan^2(c+dx)+3\right)\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(3*a^2 + 4*b^2)*
Sec[c + d*x] + 6*a^2*Sec[c + d*x]^3 + 16*a*b*(3 + Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.062, size = 142, normalized size = 1.3

$$\frac{a^2(\sec(dx+c))^3\tan(dx+c)}{4d}+\frac{3a^2\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{4ab\tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+3/8/d*a^2*sec(d*x+c)*tan(d*x+c)+3/8/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+4/3*a*b*tan(d*x+c)/d+2/3/d*a*b*tan(d*x+c)*sec(d*
x+c)^2+1/2/d*b^2*tan(d*x+c)*sec(d*x+c)+1/2/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.984156, size = 194, normalized size = 1.76

$$\frac{32\left(\tan(dx+c)^3+3\tan(dx+c)\right)ab-3a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b - 3*a^2*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*b^2*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 2.10362, size = 332, normalized size = 3.02

$$\frac{3\left(3a^2+4b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3a^2+4b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(32ab\cos\left(\right.\right.}{48d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(3*a^2 + 4*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(3*a^2 + 4
*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(32*a*b*cos(d*x + c)^3 + 16
*a*b*cos(d*x + c) + 3*(3*a^2 + 4*b^2)*cos(d*x + c)^2 + 6*a^2)*sin(d*x + c))
/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43151, size = 348, normalized size = 3.16

$$3\left(3a^2+4b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(3a^2+4b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-48ab\tan\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^2 + 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^2 + 4*b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^2*tan(1/2*d*x + 1/2*c)^7 -
 48*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 9*a^2*tan(
1/2*d*x + 1/2*c)^5 + 80*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*b^2*tan(1/2*d*x + 1
/2*c)^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 80*a*b*tan(1/2*d*x + 1/2*c)^3 - 12
*b^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c) + 48*a*b*tan(1/2*
d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)
/d
```

3.427 $\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{(4a^2+5b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2+5b^2)\tan(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan(}{}$$

```
[Out] (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) + ((4*a^2 + 5*b^2)*Tan[c + d*x])/(5*d)
+ (3*a*b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a*b*Sec[c + d*x]^3*Tan[c + d*x
])/(2*d) + (a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*a^2 + 5*b^2)*Tan[c
 + d*x]^3)/(15*d)
```

Rubi [A] time = 0.108473, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2789, 3768, 3770, 3012, 3767}

$$\frac{(4a^2+5b^2)\tan^3(c+dx)}{15d}+\frac{(4a^2+5b^2)\tan(c+dx)}{5d}+\frac{a^2\tan(c+dx)\sec^4(c+dx)}{5d}+\frac{3ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{ab\tan(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^6,x]
```


```
[Out] (3*a*b*ArcTanh[Sin[c + d*x]])/(4*d) + ((4*a^2 + 5*b^2)*Tan[c + d*x])/(5*d)
+ (3*a*b*Sec[c + d*x]*Tan[c + d*x])/(4*d) + (a*b*Sec[c + d*x]^3*Tan[c + d*x
])/(2*d) + (a^2*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*a^2 + 5*b^2)*Tan[c
 + d*x]^3)/(15*d)
```


Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^6(c+dx)\,dx = (2ab)\int \sec^5(c+dx)\,dx + \int \left(a^2+b^2\cos^2(c+dx)\right)\sec^6(c+dx)\,dx$$
$$= \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^2\sec^4(c+dx)\tan(c+dx)}{5d} + \frac{1}{2}(3ab)\int \sec^3(c$$
$$= \frac{3ab\sec(c+dx)\tan(c+dx)}{4d} + \frac{ab\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{a^2\sec^4(c+dx)}{5d}$$
$$= \frac{3ab\tanh^{-1}(\sin(c+dx))}{4d} + \frac{\left(4a^2+5b^2\right)\tan(c+dx)}{5d} + \frac{3ab\sec(c+dx)\tan(c}{4d}$$

Mathematica [A] time = 0.561164, size = 118, normalized size = 0.87

$$\frac{a^2\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d} + \frac{ab\tan(c+dx)\sec^3(c+dx)}{2d} + \frac{3ab\left(\tanh^{-1}(\sin(c+dx))+\tan(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^6,x]
```

```
[Out] (a*b*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) + (3*a*b*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x]))/(4*d) + (b^2*(Tan[c + d*x] + Tan[c + d*x]^3/3))/
d + (a^2*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d
```

Maple [A] time = 0.061, size = 157, normalized size = 1.2

$$\frac{8a^2\tan(dx+c)}{15d} + \frac{a^2(\sec(dx+c))^4\tan(dx+c)}{5d} + \frac{4a^2(\sec(dx+c))^2\tan(dx+c)}{15d} + \frac{ab(\sec(dx+c))^3\tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x)
```

```
[Out] 8/15*a^2*tan(d*x+c)/d+1/5*a^2*sec(d*x+c)^4*tan(d*x+c)/d+4/15*a^2*sec(d*x+c)
^2*tan(d*x+c)/d+1/2*a*b*sec(d*x+c)^3*tan(d*x+c)/d+3/4*a*b*sec(d*x+c)*tan(d*
x+c)/d+3/4/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*b^2*tan(d*x+c)+1/3/d*b^2*t
an(d*x+c)*sec(d*x+c)^2
```

Maxima [A] time = 0.970859, size = 178, normalized size = 1.32

$$\frac{8\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^2+40\left(\tan(dx+c)^3+3\tan(dx+c)\right)b^2-15ab\left(\frac{2\left(3\sin(dx+c\right.}{\sin(dx+c)^4-}\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/120*(8*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + 40*
(tan(d*x + c)^3 + 3*tan(d*x + c))*b^2 - 15*a*b*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 2.09217, size = 352, normalized size = 2.61

$$\frac{45\,ab\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-45\,ab\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(45\,ab\cos\left(dx+c\right)^{3}+8\left(4\,a^{2}+\ldots\right.\right.}{120\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/120*(45*a*b*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*a*b*cos(d*x + c)^5*
log(-sin(d*x + c) + 1) + 2*(45*a*b*cos(d*x + c)^3 + 8*(4*a^2 + 5*b^2)*cos(d
*x + c)^4 + 30*a*b*cos(d*x + c) + 4*(4*a^2 + 5*b^2)*cos(d*x + c)^2 + 12*a^2
)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39617, size = 367, normalized size = 2.72

$$45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+60\,b^{2}\tan\left(\frac{1}{2}\,dx+\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/60*(45*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*a*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) - 2*(60*a^2*tan(1/2*d*x + 1/2*c)^9 - 75*a*b*tan(1/2*d*x +
1/2*c)^9 + 60*b^2*tan(1/2*d*x + 1/2*c)^9 - 80*a^2*tan(1/2*d*x + 1/2*c)^7 +
30*a*b*tan(1/2*d*x + 1/2*c)^7 - 160*b^2*tan(1/2*d*x + 1/2*c)^7 + 232*a^2*ta
n(1/2*d*x + 1/2*c)^5 + 200*b^2*tan(1/2*d*x + 1/2*c)^5 - 80*a^2*tan(1/2*d*x
+ 1/2*c)^3 - 30*a*b*tan(1/2*d*x + 1/2*c)^3 - 160*b^2*tan(1/2*d*x + 1/2*c)^3
 + 60*a^2*tan(1/2*d*x + 1/2*c) + 75*a*b*tan(1/2*d*x + 1/2*c) + 60*b^2*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.428 $\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=170

$$-\frac{a\left(a^2+6b^2\right)\sin^3(c+dx)}{3d}+\frac{a\left(a^2+3b^2\right)\sin(c+dx)}{d}+\frac{b\left(18a^2+5b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\left(18a^2+5b^2\right)\sin(c}{16}$$

```
[Out] (9*a^2*b*x)/8 + (5*b^3*x)/16 + (a*(a^2 + 3*b^2)*Sin[c + d*x])/d + (b*(18*a^
2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (b*(18*a^2 + 5*b^2)*Cos[c +
d*x]^3*Sin[c + d*x])/(24*d) + (b^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (a*
(a^2 + 6*b^2)*Sin[c + d*x]^3)/(3*d) + (3*a*b^2*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.214977, antiderivative size = 193, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3023, 2748, 2633, 2635, 8}

$$-\frac{a\left(5a^2+12b^2\right)\sin^3(c+dx)}{15d}+\frac{a\left(5a^2+12b^2\right)\sin(c+dx)}{5d}+\frac{b\left(18a^2+5b^2\right)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\left(18a^2+5b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (b*(18*a^2 + 5*b^2)*x)/16 + (a*(5*a^2 + 12*b^2)*Sin[c + d*x])/(5*d) + (b*(1
8*a^2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (b*(18*a^2 + 5*b^2)*Cos[
c + d*x]^3*Sin[c + d*x])/(24*d) + (13*a*b^2*Cos[c + d*x]^4*Sin[c + d*x])/(3
0*d) + (b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])*Sin[c + d*x])/(6*d) - (a*(5
*a^2 + 12*b^2)*Sin[c + d*x]^3)/(15*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{6d} + \frac{1}{6}\int \cos^3(c+dx)\left(2a\left(3a^2+2b\right.\right.$$
$$= \frac{13ab^2\cos^4(c+dx)\sin(c+dx)}{30d} + \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{6d}$$
$$= \frac{13ab^2\cos^4(c+dx)\sin(c+dx)}{30d} + \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{6d}$$
$$= \frac{b\left(18a^2+5b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{13ab^2\cos^4(c+dx)\sin(c+dx)}{30d} + \frac{b^2\,\mathrm{c}}{}$$
$$= \frac{a\left(5a^2+12b^2\right)\sin(c+dx)}{5d} + \frac{b\left(18a^2+5b^2\right)\cos(c+dx)\sin(c+dx)}{16d} + \frac{b\left(18a^2+\right.}{}$$
$$= \frac{1}{16}b\left(18a^2+5b^2\right)x + \frac{a\left(5a^2+12b^2\right)\sin(c+dx)}{5d} + \frac{b\left(18a^2+5b^2\right)\cos(c+dx)\sin}{16d}$$

Mathematica [A] time = 0.328951, size = 159, normalized size = 0.94

$$\frac{360a\left(2a^2+5b^2\right)\sin(c+dx)+45\left(16a^2b+5b^3\right)\sin(2(c+dx))+90a^2b\sin(4(c+dx))+1080a^2bc+1080a^2bdx+80a^3\sin}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (1080*a^2*b*c + 300*b^3*c + 1080*a^2*b*d*x + 300*b^3*d*x + 360*a*(2*a^2 + 5
*b^2)*Sin[c + d*x] + 45*(16*a^2*b + 5*b^3)*Sin[2*(c + d*x)] + 80*a^3*Sin[3*
(c + d*x)] + 300*a*b^2*Sin[3*(c + d*x)] + 90*a^2*b*Sin[4*(c + d*x)] + 45*b^
3*Sin[4*(c + d*x)] + 36*a*b^2*Sin[5*(c + d*x)] + 5*b^3*Sin[6*(c + d*x)])/(9
60*d)
```

Maple [A] time = 0.038, size = 145, normalized size = 0.9

$$\frac{1}{d}\left(b^3\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{3ab^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+3/5*a*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a
^2*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^3*(
2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.99068, size = 196, normalized size = 1.15

$$-\frac{320\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)a^{3}-90\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)a^{2}b-192\left(3\sin\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(320*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3 - 90*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 192*(3*sin(d*x + c)^5 - 10*sin(
d*x + c)^3 + 15*sin(d*x + c))*a*b^2 + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60
*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^3)/d
```

Fricas [A] time = 2.0297, size = 324, normalized size = 1.91

$$\frac{15\left(18\,a^{2}b+5\,b^{3}\right)dx+\left(40\,b^{3}\cos\left(dx+c\right)^{5}+144\,ab^{2}\cos\left(dx+c\right)^{4}+10\left(18\,a^{2}b+5\,b^{3}\right)\cos\left(dx+c\right)^{3}+160\,a^{3}+384\,ab}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(18*a^2*b + 5*b^3)*d*x + (40*b^3*cos(d*x + c)^5 + 144*a*b^2*cos(d
*x + c)^4 + 10*(18*a^2*b + 5*b^3)*cos(d*x + c)^3 + 160*a^3 + 384*a*b^2 + 16
*(5*a^3 + 12*a*b^2)*cos(d*x + c)^2 + 15*(18*a^2*b + 5*b^3)*cos(d*x + c))*si
n(d*x + c))/d
```

Sympy [A] time = 4.89508, size = 393, normalized size = 2.31

$$\begin{cases}\frac{2a^{3}\sin^{3}(c+dx)}{3d}+\frac{a^{3}\sin(c+dx)\cos^{2}(c+dx)}{d}+\frac{9a^{2}bx\sin^{4}(c+dx)}{8}+\frac{9a^{2}bx\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{9a^{2}bx\cos^{4}(c+dx)}{8}+\frac{9a^{2}b\sin^{3}(c+dx)\cos(c+dx}{8d} \\ x\left(a+b\cos\left(c\right)\right)^{3}\cos^{3}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d + 9*a**2*b*x*sin(c + d*x)**4/8 + 9*a**2*b*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 9*a**2*b*x*cos(c + d*x)**4/8 + 9*a**2*b*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 15*a**2*b*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 8*a*b**2*sin(c + d
*x)**5/(5*d) + 4*a*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 3*a*b**2*sin(c
+ d*x)*cos(c + d*x)**4/d + 5*b**3*x*sin(c + d*x)**6/16 + 15*b**3*x*sin(c +
d*x)**4*cos(c + d*x)**2/16 + 15*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
```

```
 5*b**3*x*cos(c + d*x)**6/16 + 5*b**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) +
 5*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*b**3*sin(c + d*x)*cos(c
+ d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*cos(c))**3*cos(c)**3, True))
```

Giac [A] time = 1.3278, size = 203, normalized size = 1.19

$$\frac{b^3\sin(6\,dx+6\,c)}{192\,d}+\frac{3\,ab^2\sin(5\,dx+5\,c)}{80\,d}+\frac{1}{16}\left(18\,a^2b+5\,b^3\right)x+\frac{3\left(2\,a^2b+b^3\right)\sin(4\,dx+4\,c)}{64\,d}+\frac{\left(4\,a^3+15\,ab^2\right)\sin(3\,c}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*b^3*sin(6*d*x + 6*c)/d + 3/80*a*b^2*sin(5*d*x + 5*c)/d + 1/16*(18*a^2
*b + 5*b^3)*x + 3/64*(2*a^2*b + b^3)*sin(4*d*x + 4*c)/d + 1/48*(4*a^3 + 15*
a*b^2)*sin(3*d*x + 3*c)/d + 3/64*(16*a^2*b + 5*b^3)*sin(2*d*x + 2*c)/d + 3/
8*(2*a^3 + 5*a*b^2)*sin(d*x + c)/d
```

3.429 $\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=180

$$-\frac{(-52a^2b^2+3a^4-16b^4)\sin(c+dx)}{30bd}-\frac{(3a^2-16b^2)\sin(c+dx)(a+b\cos(c+dx))^2}{60bd}-\frac{a(6a^2-71b^2)\sin(c+dx)\cos(}{120d}$$

```
[Out] (a*(4*a^2 + 9*b^2)*x)/8 - ((3*a^4 - 52*a^2*b^2 - 16*b^4)*Sin[c + d*x])/(30*
b*d) - (a*(6*a^2 - 71*b^2)*Cos[c + d*x]*Sin[c + d*x])/(120*d) - ((3*a^2 - 1
6*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*b*d) - (a*(a + b*Cos[c + d*
x])^3*Sin[c + d*x])/(20*b*d) + ((a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d
)
```

Rubi [A] time = 0.220759, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2791, 2753, 2734}

$$-\frac{(-52a^2b^2+3a^4-16b^4)\sin(c+dx)}{30bd}-\frac{(3a^2-16b^2)\sin(c+dx)(a+b\cos(c+dx))^2}{60bd}-\frac{a(6a^2-71b^2)\sin(c+dx)\cos(}{120d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (a*(4*a^2 + 9*b^2)*x)/8 - ((3*a^4 - 52*a^2*b^2 - 16*b^4)*Sin[c + d*x])/(30*
b*d) - (a*(6*a^2 - 71*b^2)*Cos[c + d*x]*Sin[c + d*x])/(120*d) - ((3*a^2 - 1
6*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(60*b*d) - (a*(a + b*Cos[c + d*
x])^3*Sin[c + d*x])/(20*b*d) + ((a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*b*d
)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5bd} + \frac{\int(4b-a\cos(c+dx))(a+b\cos(c+dx))^3\,dx}{5b}$$
$$= -\frac{a(a+b\cos(c+dx))^3\sin(c+dx)}{20bd} + \frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5bd} + \frac{\int(a+b$$
$$= -\frac{\left(3a^2-16b^2\right)(a+b\cos(c+dx))^2\sin(c+dx)}{60bd} - \frac{a(a+b\cos(c+dx))^3\sin(c+d$$
$$= \frac{1}{8}a\left(4a^2+9b^2\right)x - \frac{\left(3a^4-52a^2b^2-16b^4\right)\sin(c+dx)}{30bd} - \frac{a\left(6a^2-71b^2\right)\cos(c+a$$

Mathematica [A] time = 0.305776, size = 130, normalized size = 0.72

$$\frac{60b\left(18a^2+5b^2\right)\sin(c+dx)+120\left(a^3+3ab^2\right)\sin(2(c+dx))+120a^2b\sin(3(c+dx))+240a^3c+240a^3dx+45ab^2\sin(4}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (240*a^3*c + 540*a*b^2*c + 240*a^3*d*x + 540*a*b^2*d*x + 60*b*(18*a^2 + 5*b
^2)*Sin[c + d*x] + 120*(a^3 + 3*a*b^2)*Sin[2*(c + d*x)] + 120*a^2*b*Sin[3*(
c + d*x)] + 50*b^3*Sin[3*(c + d*x)] + 45*a*b^2*Sin[4*(c + d*x)] + 6*b^3*Sin
[5*(c + d*x)])/(480*d)
```

Maple [A] time = 0.034, size = 123, normalized size = 0.7

$$\frac{1}{d}\left(\frac{b^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+3ab^2\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(1/5*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+3*a*b^2*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^2*b*(2+cos(d*x+c)^2
)*sin(d*x+c)+a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 0.952877, size = 161, normalized size = 0.89

$$\frac{120(2dx+2c+\sin(2dx+2c))a^3-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2b+45(12dx+12c+\sin(4dx+4c)+8\sin}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/480*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 480*(sin(d*x + c)^3 - 3*s
in(d*x + c))*a^2*b + 45*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2
*c))*a*b^2 + 32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*b^
3)/d
```

Fricas [A] time = 1.89415, size = 265, normalized size = 1.47

$$\frac{15\left(4a^3+9ab^2\right)dx+\left(24b^3\cos(dx+c)^4+90ab^2\cos(dx+c)^3+240a^2b+64b^3+8\left(15a^2b+4b^3\right)\cos(dx+c)^2+1\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/120*(15*(4*a^3 + 9*a*b^2)*d*x + (24*b^3*cos(d*x + c)^4 + 90*a*b^2*cos(d*x
 + c)^3 + 240*a^2*b + 64*b^3 + 8*(15*a^2*b + 4*b^3)*cos(d*x + c)^2 + 15*(4*
a^3 + 9*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 2.59505, size = 284, normalized size = 1.58

$$\begin{cases}\frac{a^3x\sin^2(c+dx)}{2}+\frac{a^3x\cos^2(c+dx)}{2}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2b\sin^3(c+dx)}{d}+\frac{3a^2b\sin(c+dx)\cos^2(c+dx)}{d}+\frac{9ab^2x\sin^4(c+dx)}{8}+\frac{9ab^2x\sin^2(\\ x\left(a+b\cos(c)\right)^3\cos^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**2/2 + a**3*x*cos(c + d*x)**2/2 + a**3*sin(c
 + d*x)*cos(c + d*x)/(2*d) + 2*a**2*b*sin(c + d*x)**3/d + 3*a**2*b*sin(c +
d*x)*cos(c + d*x)**2/d + 9*a*b**2*x*sin(c + d*x)**4/8 + 9*a*b**2*x*sin(c +
d*x)**2*cos(c + d*x)**2/4 + 9*a*b**2*x*cos(c + d*x)**4/8 + 9*a*b**2*sin(c +
 d*x)**3*cos(c + d*x)/(8*d) + 15*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
+ 8*b**3*sin(c + d*x)**5/(15*d) + 4*b**3*sin(c + d*x)**3*cos(c + d*x)**2/(3
*d) + b**3*sin(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(a + b*cos(c))**3*
cos(c)**2, True))
```

Giac [A] time = 1.86832, size = 167, normalized size = 0.93

$$\frac{b^3\sin(5dx+5c)}{80d}+\frac{3ab^2\sin(4dx+4c)}{32d}+\frac{1}{8}\left(4a^3+9ab^2\right)x+\frac{\left(12a^2b+5b^3\right)\sin(3dx+3c)}{48d}+\frac{\left(a^3+3ab^2\right)\sin(2d}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/80*b^3*sin(5*d*x + 5*c)/d + 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(4*a^3 +
9*a*b^2)*x + 1/48*(12*a^2*b + 5*b^3)*sin(3*d*x + 3*c)/d + 1/4*(a^3 + 3*a*b^
2)*sin(2*d*x + 2*c)/d + 1/8*(18*a^2*b + 5*b^3)*sin(d*x + c)/d
```

3.430 $\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=121

$$\frac{a\left(a^2+4b^2\right)\sin(c+dx)}{2d}+\frac{b\left(2a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}bx\left(4a^2+b^2\right)+\frac{\sin(c+dx)(a+b\cos(c+dx))^3}{4d}+\frac{a\,\text{s}}{}$$

```
[Out] (3*b*(4*a^2 + b^2)*x)/8 + (a*(a^2 + 4*b^2)*Sin[c + d*x])/(2*d) + (b*(2*a^2
+ 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(a + b*Cos[c + d*x])^2*Sin[c
 + d*x])/(4*d) + ((a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.116425, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$\frac{a\left(a^2+4b^2\right)\sin(c+dx)}{2d}+\frac{b\left(2a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}bx\left(4a^2+b^2\right)+\frac{\sin(c+dx)(a+b\cos(c+dx))^3}{4d}+\frac{a\,\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (3*b*(4*a^2 + b^2)*x)/8 + (a*(a^2 + 4*b^2)*Sin[c + d*x])/(2*d) + (b*(2*a^2
+ 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*(a + b*Cos[c + d*x])^2*Sin[c
 + d*x])/(4*d) + ((a + b*Cos[c + d*x])^3*Sin[c + d*x])/(4*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\cos(c+dx))^3\,dx &= \frac{(a+b\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{4}\int(3b+3a\cos(c+dx))(a+b\cos(c+dx))^2\,d\\ &= \frac{a(a+b\cos(c+dx))^2\sin(c+dx)}{4d}+\frac{(a+b\cos(c+dx))^3\sin(c+dx)}{4d}+\frac{1}{12}\int(a+\\ &= \frac{3}{8}b\left(4a^2+b^2\right)x+\frac{a\left(a^2+4b^2\right)\sin(c+dx)}{2d}+\frac{b\left(2a^2+3b^2\right)\cos(c+dx)\sin(c+dx}{8d}\end{aligned}$$

Mathematica [A] time = 0.263678, size = 100, normalized size = 0.83

$$\frac{8a\left(4a^2+9b^2\right)\sin(c+dx)+b\left(8\left(3a^2+b^2\right)\sin(2(c+dx))+48a^2c+48a^2dx+8ab\sin(3(c+dx))+b^2\sin(4(c+dx))+1\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (8*a*(4*a^2 + 9*b^2)*Sin[c + d*x] + b*(48*a^2*c + 12*b^2*c + 48*a^2*d*x + 1
2*b^2*d*x + 8*(3*a^2 + b^2)*Sin[2*(c + d*x)] + 8*a*b*Sin[3*(c + d*x)] + b^2
*Sin[4*(c + d*x)]))/(32*d)
```

Maple [A] time = 0.033, size = 102, normalized size = 0.8

$$\frac{1}{d}\left(b^3\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+ab^2\left(2+(\cos(dx+c))^2\right)\sin(dx+c)+3a^2b(1/$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a*b^2
*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+a^3*sin(d*x+c))
```

Maxima [A] time = 0.965021, size = 128, normalized size = 1.06

$$\frac{24(2dx+2c+\sin(2dx+2c))a^2b-32\left(\sin(dx+c)^3-3\sin(dx+c)\right)ab^2+(12dx+12c+\sin(4dx+4c)+8\sin(}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/32*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b - 32*(sin(d*x + c)^3 - 3*si
n(d*x + c))*a*b^2 + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))
*b^3 + 32*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.89638, size = 197, normalized size = 1.63

$$\frac{3\left(4a^2b+b^3\right)dx+\left(2b^3\cos(dx+c)^3+8ab^2\cos(dx+c)^2+8a^3+16ab^2+3\left(4a^2b+b^3\right)\cos(dx+c)\right)\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(4*a^2*b + b^3)*d*x + (2*b^3*cos(d*x + c)^3 + 8*a*b^2*cos(d*x + c)^2
 + 8*a^3 + 16*a*b^2 + 3*(4*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.35326, size = 233, normalized size = 1.93

$$\begin{cases} \frac{a^3\sin(c+dx)}{d}+\frac{3a^2bx\sin^2(c+dx)}{2}+\frac{3a^2bx\cos^2(c+dx)}{2}+\frac{3a^2b\sin(c+dx)\cos(c+dx)}{2d}+\frac{2ab^2\sin^3(c+dx)}{d}+\frac{3ab^2\sin(c+dx)\cos^2(c+dx)}{d}+\frac{3b^3x\sin^4}{8} \\ x(a+b\cos(c))^3\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)/d + 3*a**2*b*x*sin(c + d*x)**2/2 + 3*a**2*b*x*
cos(c + d*x)**2/2 + 3*a**2*b*sin(c + d*x)*cos(c + d*x)/(2*d) + 2*a*b**2*sin
(c + d*x)**3/d + 3*a*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*b**3*x*sin(c +
 d*x)**4/8 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b**3*x*cos(c +
d*x)**4/8 + 3*b**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*b**3*sin(c + d*x)
*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**3*cos(c), True))
```

Giac [A] time = 2.18567, size = 130, normalized size = 1.07

$$\frac{b^3 \sin(4\,dx+4\,c)}{32\,d}+\frac{ab^2 \sin(3\,dx+3\,c)}{4\,d}+\frac{3}{8}\left(4\,a^2b+b^3\right)x+\frac{\left(3\,a^2b+b^3\right)\sin(2\,dx+2\,c)}{4\,d}+\frac{\left(4\,a^3+9\,ab^2\right)\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/32*b^3*sin(4*d*x + 4*c)/d + 1/4*a*b^2*sin(3*d*x + 3*c)/d + 3/8*(4*a^2*b +
 b^3)*x + 1/4*(3*a^2*b + b^3)*sin(2*d*x + 2*c)/d + 1/4*(4*a^3 + 9*a*b^2)*si
n(d*x + c)/d
```

3.431 $\int (a+b\cos (c+dx))^3 \, dx$

Optimal. Leaf size=76

$$\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}+a^3x+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3\sin^3(c+dx)}{3d}$$

```
[Out] a^3*x + (3*a*b^2*x)/2 + (b*(3*a^2 + b^2)*Sin[c + d*x])/d + (3*a*b^2*Cos[c +
 d*x]*Sin[c + d*x])/(2*d) - (b^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0691565, antiderivative size = 90, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2656, 2734}

$$\frac{2b\left(4a^2+b^2\right)\sin(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2+3b^2\right)+\frac{5ab^2\sin(c+dx)\cos(c+dx)}{6d}+\frac{b\sin(c+dx)(a+b\cos(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/2 + (2*b*(4*a^2 + b^2)*Sin[c + d*x])/(3*d) + (5*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (b*(a + b*Cos[c + d*x])^2*Sin[c + d*x])
/(3*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\,dx=\frac{b(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+b\cos(c+dx))\left(3a^2+2b^2+5ab\cos(c+dx)\right)$$

$$=\frac{1}{2}a\left(2a^2+3b^2\right)x+\frac{2b\left(4a^2+b^2\right)\sin(c+dx)}{3d}+\frac{5ab^2\cos(c+dx)\sin(c+dx)}{6d}+\frac{b(a+b\cos}{}$$

Mathematica [A] time = 0.12404, size = 80, normalized size = 1.05

$$\frac{9b\left(4a^2+b^2\right)\sin(c+dx)+12a^3c+12a^3dx+9ab^2\sin(2(c+dx))+18ab^2c+18ab^2dx+b^3\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sin[
c + d*x] + 9*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.032, size = 76, normalized size = 1.

$$\frac{1}{d}\left(\frac{b^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+3\,ab^2\left(1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2\right)+3\,a^2b\sin(dx+c)+a^3(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/d*(1/3*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+3*a^2*b*sin(d*x+c)+a^3*(d*x+c))
```

Maxima [A] time = 0.944881, size = 97, normalized size = 1.28

$$a^3x+\frac{3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))ab^2}{4\,d}-\frac{\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)b^3}{3\,d}+\frac{3\,a^2b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 3/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b^2/d - 1/3*(sin(d*x + c)^3
- 3*sin(d*x + c))*b^3/d + 3*a^2*b*sin(d*x + c)/d
```

Fricas [A] time = 1.95134, size = 153, normalized size = 2.01

$$\frac{3\left(2\,a^3+3\,ab^2\right)dx+\left(2\,b^3\cos(dx+c)^2+9\,ab^2\cos(dx+c)+18\,a^2b+4\,b^3\right)\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*d*x + (2*b^3*cos(d*x + c)^2 + 9*a*b^2*cos(d*x + c)
 + 18*a^2*b + 4*b^3)*sin(d*x + c))/d
```

Sympy [A] time = 0.666712, size = 128, normalized size = 1.68

$$\begin{cases} a^3x+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2x\sin^2(c+dx)}{2}+\frac{3ab^2x\cos^2(c+dx)}{2}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2b^3\sin^3(c+dx)}{3d}+\frac{b^3\sin(c+dx)\cos^2(c+dx)}{d} & \text{for } d \\ x\,(a+b\cos(c))^3 & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sin(c + d*x)/d + 3*a*b**2*x*sin(c + d*x)**2/2
+ 3*a*b**2*x*cos(c + d*x)**2/2 + 3*a*b**2*sin(c + d*x)*cos(c + d*x)/(2*d) +
 2*b**3*sin(c + d*x)**3/(3*d) + b**3*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d,
0)), (x*(a + b*cos(c))**3, True))
```

Giac [A] time = 1.34079, size = 97, normalized size = 1.28

$$\frac{b^3 \sin(3\,dx + 3\,c)}{12\,d} + \frac{3\,ab^2 \sin(2\,dx + 2\,c)}{4\,d} + \frac{1}{2}\left(2\,a^3 + 3\,ab^2\right)x + \frac{3\left(4\,a^2b + b^3\right)\sin(dx + c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*b^3*sin(3*d*x + 3*c)/d + 3/4*a*b^2*sin(2*d*x + 2*c)/d + 1/2*(2*a^3 + 3
*a*b^2)*x + 3/4*(4*a^2*b + b^3)*sin(d*x + c)/d
```

3.432 $\int (a+b\cos(c+dx))^3 \sec(c+dx)\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}bx\left(6a^2+b^2\right)+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

```
[Out] (b*(6*a^2 + b^2)*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (5*a*b^2*Sin[c + d*
x])/(2*d) + (b^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.11458, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2793, 3023, 2735, 3770}

$$\frac{1}{2}bx\left(6a^2+b^2\right)+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2\sin(c+dx)(a+b\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x],x]
```

```
[Out] (b*(6*a^2 + b^2)*x)/2 + (a^3*ArcTanh[Sin[c + d*x]])/d + (5*a*b^2*Sin[c + d*
x])/(2*d) + (b^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec(c+dx)\,dx &= \frac{b^2(a+b\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int \left(2a^3+b\left(6a^2+b^2\right)\cos(c+dx)+5ab\right.\\ &= \frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2(a+b\cos(c+dx))\sin(c+dx)}{2d}+\frac{1}{2}\int \left(2a^3+b\left(6a^2+b\right.\right.\\ &= \frac{1}{2}b\left(6a^2+b^2\right)x+\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2(a+b\cos(c+dx))\sin(c+dx)}{2d}+a^3\int\\ &= \frac{1}{2}b\left(6a^2+b^2\right)x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{5ab^2\sin(c+dx)}{2d}+\frac{b^2(a+b\cos(c\right.}{}\end{aligned}$$

Mathematica [A] time = 0.139916, size = 105, normalized size = 1.44

$$\frac{2b\left(6a^2+b^2\right)(c+dx)-4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+12}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x],x]
```

```
[Out] (2*b*(6*a^2 + b^2)*(c + d*x) - 4*a^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] + 4*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 12*a*b^2*Sin[c + d*x]
 + b^3*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.055, size = 90, normalized size = 1.2

$$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+3a^2bx+3\frac{a^2bc}{d}+3\frac{ab^2\sin(dx+c)}{d}+\frac{b^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{b^3x}{2}+\frac{b^3c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c),x)
```

```
[Out] 1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b*x+3/d*a^2*b*c+3*a*b^2*sin(d*x+c)/
d+1/2/d*b^3*cos(d*x+c)*sin(d*x+c)+1/2*b^3*x+1/2/d*b^3*c
```

Maxima [A] time = 0.965056, size = 93, normalized size = 1.27

$$\frac{12(dx+c)a^2b+(2dx+2c+\sin(2dx+2c))b^3+4a^3\log(\sec(dx+c)+\tan(dx+c))+12ab^2\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*(12*(d*x + c)*a^2*b + (2*d*x + 2*c + sin(2*d*x + 2*c))*b^3 + 4*a^3*log(
sec(d*x + c) + tan(d*x + c)) + 12*a*b^2*sin(d*x + c))/d
```

Fricas [A] time = 1.98894, size = 176, normalized size = 2.41

$$\frac{a^3 \log (\sin (dx + c) + 1) - a^3 \log (-\sin (dx + c) + 1) + \left(6 a^2 b + b^3\right) dx + \left(b^3 \cos (dx + c) + 6 ab^2\right) \sin (dx + c)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*log(sin(d*x + c) + 1) - a^3*log(-sin(d*x + c) + 1) + (6*a^2*b + b^
3)*d*x + (b^3*cos(d*x + c) + 6*a*b^2)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \cos (c + dx))^3 \sec (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] time = 1.43241, size = 185, normalized size = 2.53

$$\frac{2 a^3 \log \left(\left|\tan \left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) - 2 a^3 \log \left(\left|\tan \left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + \left(6 a^2 b + b^3\right)(dx + c) + \frac{2 \left(6 ab^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - b^3 \tan \left(\frac{1}{2} dx \cdots\right.\right.}{\left(\tan \left(\cdots\right.\right.}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a^3*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (6*a^2*b + b^3)*(d*x + c) + 2*(6*a*b^2*tan(1/2*d*x + 1/2*c)
^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c) + b^3*tan(1/
2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.433 $\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx$

Optimal. Leaf size=68

$$-\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}+3ab^2x$$

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b*(a^2 - b^2)*Sin[c + d*x]
)/d + (a^2*(a + b*Cos[c + d*x])*Tan[c + d*x])/d
```

Rubi [A] time = 0.122414, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2792, 3023, 2735, 3770}

$$-\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\tan(c+dx)(a+b\cos(c+dx))}{d}+3ab^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (b*(a^2 - b^2)*Sin[c + d*x]
)/d + (a^2*(a + b*Cos[c + d*x])*Tan[c + d*x])/d
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec^2(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))\tan(c+dx)}{d}+\int \left(3a^2b+3ab^2\cos(c+dx)-b\left(a^2-b^2\right)\text{co}\right.\\ &= -\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{a^2(a+b\cos(c+dx))\tan(c+dx)}{d}+\int \left(3a^2b+3ab^2\right.\\ &= 3ab^2x-\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{a^2(a+b\cos(c+dx))\tan(c+dx)}{d}+\left(3a^2b\right)\int\\ &= 3ab^2x+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\left(a^2-b^2\right)\sin(c+dx)}{d}+\frac{a^2(a+b\cos(c+}{d}\end{aligned}$$

Mathematica [A] time = 0.333023, size = 88, normalized size = 1.29

$$\frac{a^3\tan(c+dx)+3ab\left(-a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+bc+bdx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^2,x]
```

```
[Out] (3*a*b*(b*c + b*d*x - a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + a*Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b^3*Sin[c + d*x] + a^3*Tan[c + d*x])/
d
```

Maple [A] time = 0.059, size = 68, normalized size = 1.

$$3\,ab^2x+\frac{b^3\sin(dx+c)}{d}+\frac{a^3\tan(dx+c)}{d}+3\,\frac{a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\,\frac{ab^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x)
```

```
[Out] 3*a*b^2*x+1/d*b^3*sin(d*x+c)+a^3*tan(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c)+tan(d
*x+c))+3/d*a*b^2*c
```

Maxima [A] time = 0.983287, size = 89, normalized size = 1.31

$$\frac{6\,(dx+c)ab^2+3\,a^2b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,b^3\sin(dx+c)+2\,a^3\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*(d*x + c)*a*b^2 + 3*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 2*b^3*sin(d*x + c) + 2*a^3*tan(d*x + c))/d
```

Fricas [A] time = 2.0692, size = 246, normalized size = 3.62

$$\frac{6\,ab^2dx\cos(dx+c)+3\,a^2b\cos(dx+c)\log(\sin(dx+c)+1)-3\,a^2b\cos(dx+c)\log(-\sin(dx+c)+1)+2\left(b^3\cos(\right.}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*d*x*cos(d*x + c) + 3*a^2*b*cos(d*x + c)*log(sin(d*x + c) + 1)
- 3*a^2*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(b^3*cos(d*x + c) + a^3)*
sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42718, size = 174, normalized size = 2.56

$$\frac{3\,(dx+c)ab^2+3\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{\tan\left(\frac{1}{2}\,dx\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (3*(d*x + c)*a*b^2 + 3*a^2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^2*b*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - b^3*tan
(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/
(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.434 $\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx$

Optimal. Leaf size=79

$$\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}+b^3x$$

```
[Out] b^3*x + (a*(a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^2*b*Tan[c + d*
x])/(2*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.134308, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2792, 3021, 2735, 3770}

$$\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos(c+dx))}{2d}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```


```
[Out] b^3*x + (a*(a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^2*b*Tan[c + d*
x])/(2*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^3(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(5a^2b+a\left(a^2+6b^2\right)\cos\right.$$
$$= \frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2(a+b\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \left(a\left(\right.\right.$$
$$= b^3x+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2(a+b\cos(c+dx))\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}$$
$$= b^3x+\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^2b\tan(c+dx)}{2d}+\frac{a^2(a+b\cos(c}{}$$

Mathematica [A] time = 0.175937, size = 55, normalized size = 0.7

$$\frac{a\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))+a^2\tan(c+dx)(a\sec(c+dx)+6b)+2b^3dx}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^3*d*x + a*(a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]] + a^2*(6*b + a*Sec[c + d
*x])*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.06, size = 95, normalized size = 1.2

$$\frac{a^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+3\frac{a^2b\tan(dx+c)}{d}+3\frac{ab^2\ln(\sec(dx+c)+\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^3*sec(d*x+c)*tan(d*x+c)+1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b
*tan(d*x+c)/d+3/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c))+b^3*x+1/d*b^3*c
```

Maxima [A] time = 0.965618, size = 136, normalized size = 1.72

$$\frac{4(dx+c)b^3-a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6ab^2(\log(\sin(dx+c)+1)-\log(\sin(}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*b^3 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*a*b^2*(log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 12*a^2*b*tan(d*x + c))/d
```

Fricas [A] time = 1.99616, size = 281, normalized size = 3.56

$$\frac{4\,b^3 dx\cos\left(dx+c\right)^2+\left(a^3+6\,ab^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(a^3+6\,ab^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^3*d*x*cos(d*x + c)^2 + (a^3 + 6*a*b^2)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - (a^3 + 6*a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(6*a^
2*b*cos(d*x + c) + a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.80078, size = 193, normalized size = 2.44

$$\frac{2\,(dx+c)b^3+\left(a^3+6\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(a^3+6\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^2b}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*b^3 + (a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 (a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x +
1/2*c)^3 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + 6*a^
2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.435 $\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx$

Optimal. Leaf size=109

$$\frac{a\left(2a^2+9b^2\right)\tan(c+dx)}{3d}+\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{6d}+\frac{a^2\tan(c+dx)\sec^2(}{}$$

```
[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*a^2 + 9*b^2)*Tan[c
+ d*x])/(3*d) + (7*a^2*b*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a^2*(a + b*Cos
[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.182106, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3021, 2748, 3767, 8, 3770}

$$\frac{a\left(2a^2+9b^2\right)\tan(c+dx)}{3d}+\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{7a^2b\tan(c+dx)\sec(c+dx)}{6d}+\frac{a^2\tan(c+dx)\sec^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

```
[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(2*a^2 + 9*b^2)*Tan[c
+ d*x])/(3*d) + (7*a^2*b*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (a^2*(a + b*Cos
[c + d*x])*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^4(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \left(7a^2b+a\left(2a^2+9b^2\right)\cos\right.$$
$$=\frac{7a^2b\sec(c+dx)\tan(c+dx)}{6d}+\frac{a^2(a+b\cos(c+dx))\sec^2(c+dx)\tan(c+dx)}{3d}+$$
$$=\frac{7a^2b\sec(c+dx)\tan(c+dx)}{6d}+\frac{a^2(a+b\cos(c+dx))\sec^2(c+dx)\tan(c+dx)}{3d}+$$
$$=\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{7a^2b\sec(c+dx)\tan(c+dx)}{6d}+\frac{a^2(a+b\,\mathrm{co}}{}$$
$$=\frac{b\left(3a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\left(2a^2+9b^2\right)\tan(c+dx)}{3d}+\frac{7a^2b\sec(c+d}{6}$$

Mathematica [A] time = 0.268197, size = 70, normalized size = 0.64

$$\frac{\left(9a^2b+6b^3\right)\tanh^{-1}(\sin(c+dx))+a\tan(c+dx)\left(2a^2\tan^2(c+dx)+6a^2+9ab\sec(c+dx)+18b^2\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^4,x]
```

```
[Out] ((9*a^2*b + 6*b^3)*ArcTanh[Sin[c + d*x]] + a*Tan[c + d*x]*(6*a^2 + 18*b^2 +
 9*a*b*Sec[c + d*x] + 2*a^2*Tan[c + d*x]^2))/(6*d)
```

Maple [A] time = 0.065, size = 118, normalized size = 1.1

$$\frac{2a^3\tan(dx+c)}{3d}+\frac{a^3\tan(dx+c)(\sec(dx+c))^2}{3d}+\frac{3a^2b\sec(dx+c)\tan(dx+c)}{2d}+\frac{3a^2b\ln(\sec(dx+c)+\tan(dx+c}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a^3*tan(d*x+c)/d+1/3/d*a^3*tan(d*x+c)*sec(d*x+c)^2+3/2*a^2*b*sec(d*x+c)
*tan(d*x+c)/d+3/2/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+3/d*a*b^2*tan(d*x+c)+1/
d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.971871, size = 153, normalized size = 1.4

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{3}-9\,a^{2}b\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6\,b^{3}(\log(\mathrm{si}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 - 9*a^2*b*(2*sin(d*x + c)/(si
n(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*b^3*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*a*b^2*tan(d*x + c))/d
```

Fricas [A] time = 1.95837, size = 309, normalized size = 2.83

$$\frac{3\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,a^{2}b+2\,b^{3}\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(9\,a^{2}b\,\mathrm{co}\right.}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(3*a^2*b + 2*b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*a^2*b
 + 2*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(9*a^2*b*cos(d*x + c) +
 2*a^3 + 2*(2*a^3 + 9*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^
3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.57902, size = 277, normalized size = 2.54

$$\frac{3\left(3\,a^{2}b+2\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,a^{2}b+2\,b^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,a^{2}b\tan\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(3*a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*a^2*b +
2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*a^3*tan(1/2*d*x + 1/2*c)^5
 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 4*a^3
*tan(1/2*d*x + 1/2*c)^3 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2*d
*x + 1/2*c) + 9*a^2*b*tan(1/2*d*x + 1/2*c) + 18*a*b^2*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.436 $\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx$

Optimal. Leaf size=133

$$\frac{b\left(2a^2+b^2\right)\tan(c+dx)}{d}+\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\left(a^2+4b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{3a^2b\tan(c+$$

```
[Out] (3*a*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (b*(2*a^2 + b^2)*Tan[c +
d*x])/d + (3*a*(a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^2*b*Se
c[c + d*x]^2*Tan[c + d*x])/(4*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.203476, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{b\left(2a^2+b^2\right)\tan(c+dx)}{d}+\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a\left(a^2+4b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{3a^2b\tan(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```


```
[Out] (3*a*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (b*(2*a^2 + b^2)*Tan[c +
d*x])/d + (3*a*(a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (3*a^2*b*Se
c[c + d*x]^2*Tan[c + d*x])/(4*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec^5(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \left(9a^2b+3a\left(a^2+4b^2\right)\cos\right.\\
&= \frac{3a^2b\sec^2(c+dx)\tan(c+dx)}{4d}+\frac{a^2(a+b\cos(c+dx))\sec^3(c+dx)\tan(c+dx)}{4d}\\
&= \frac{3a^2b\sec^2(c+dx)\tan(c+dx)}{4d}+\frac{a^2(a+b\cos(c+dx))\sec^3(c+dx)\tan(c+dx)}{4d}\\
&= \frac{3a\left(a^2+4b^2\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{3a^2b\sec^2(c+dx)\tan(c+dx)}{4d}+\frac{a^2(a+}{}\\
&= \frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(2a^2+b^2\right)\tan(c+dx)}{d}+\frac{3a\left(a^2+4b^2\right)\mathrm{se}}{}\end{aligned}$$

Mathematica [A] time = 0.414429, size = 90, normalized size = 0.68

$$\frac{3a\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8b\left(a^2\tan^2(c+dx)+3a^2+b^2\right)+3a\left(a^2+4b^2\right)\sec(c+dx)+2a^3\sec^3(\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^5,x]
```

```
[Out] (3*a*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*a*(a^2 + 4*b^2)*
Sec[c + d*x] + 2*a^3*Sec[c + d*x]^3 + 8*b*(3*a^2 + b^2 + a^2*Tan[c + d*x]^2
)))/(8*d)
```

Maple [A] time = 0.066, size = 160, normalized size = 1.2

$$\frac{a^3\tan(dx+c)(\sec(dx+c))^3}{4\,d}+\frac{3\,a^3\sec(dx+c)\tan(dx+c)}{8\,d}+\frac{3\,a^3\ln(\sec(dx+c)+\tan(dx+c))}{8\,d}+2\,\frac{a^2b\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x)
```

```
[Out] 1/4/d*a^3*tan(d*x+c)*sec(d*x+c)^3+3/8/d*a^3*sec(d*x+c)*tan(d*x+c)+3/8/d*a^3
*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*b*tan(d*x+c)/d+a^2*b*sec(d*x+c)^2*tan(d*x+
c)/d+3/2/d*a*b^2*tan(d*x+c)*sec(d*x+c)+3/2/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c)
)+1/d*b^3*tan(d*x+c)
```

Maxima [A] time = 0.966777, size = 213, normalized size = 1.6

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{2}b-a^{3}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b - a^3*(2*(3*sin(d*x + c)^3
 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 12*a*b^2*(2*sin(d*x + c)/(sin(d*x +
c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 16*b^3*tan(d*x
 + c))/d
```

Fricas [A] time = 2.03084, size = 348, normalized size = 2.62

$$\frac{3\left(a^{3}+4\,ab^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^{3}+4\,ab^{2}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,a^{2}b\cos\left(\right.\right.}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(a^3 + 4*a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 + 4*a
*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*a^2*b*cos(d*x + c) + 8*(
2*a^2*b + b^3)*cos(d*x + c)^3 + 2*a^3 + 3*(a^3 + 4*a*b^2)*cos(d*x + c)^2)*s
in(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.69653, size = 446, normalized size = 3.35

$$\frac{3\left(a^{3}+4\,ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(a^{3}+4\,ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,a^{2}b\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 + 4*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(a^3 + 4*a*b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 2
4*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 8*b^3*ta
n(1/2*d*x + 1/2*c)^7 + 3*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 24*b^3*tan(1/2*d*x + 1/2*c)^
5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b
^2*tan(1/2*d*x + 1/2*c)^3 - 24*b^3*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*tan(1/2*d
*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c)
 + 8*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.437 $\int (a+b\cos(c+dx))^3 \sec^6(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{a\left(4a^2+15b^2\right)\tan^3(c+dx)}{15d}+\frac{a\left(4a^2+15b^2\right)\tan(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(9a^2+4b^2\right)\tan(c}{8d}$$

```
[Out] (b*(9*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(4*a^2 + 15*b^2)*Tan[c
 + d*x])/(5*d) + (b*(9*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (11*
a^2*b*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c
 + d*x]^4*Tan[c + d*x])/(5*d) + (a*(4*a^2 + 15*b^2)*Tan[c + d*x]^3)/(15*d)
```

Rubi [A] time = 0.216605, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3021, 2748, 3767, 3768, 3770}

$$\frac{a\left(4a^2+15b^2\right)\tan^3(c+dx)}{15d}+\frac{a\left(4a^2+15b^2\right)\tan(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\left(9a^2+4b^2\right)\tan(c}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```


```
[Out] (b*(9*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(4*a^2 + 15*b^2)*Tan[c
 + d*x])/(5*d) + (b*(9*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (11*
a^2*b*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (a^2*(a + b*Cos[c + d*x])*Sec[c
 + d*x]^4*Tan[c + d*x])/(5*d) + (a*(4*a^2 + 15*b^2)*Tan[c + d*x]^3)/(15*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^3\sec^6(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \left(11a^2b+a\left(4a^2+15b^2\right)\text{co}\right.\\ &= \frac{11a^2b\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{a^2(a+b\cos(c+dx))\sec^4(c+dx)\tan(c+dx)}{5d}\\ &= \frac{11a^2b\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{a^2(a+b\cos(c+dx))\sec^4(c+dx)\tan(c+dx)}{5d}\\ &= \frac{b\left(9a^2+4b^2\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{11a^2b\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{a^2(a+}{}\\ &= \frac{b\left(9a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\left(4a^2+15b^2\right)\tan(c+dx)}{5d}+\frac{b\left(9a^2+4b^2\right)}{}\end{aligned}$$

Mathematica [A] time = 0.839601, size = 120, normalized size = 0.71

$$\frac{15b\left(9a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(8a\left(5\left(2a^2+3b^2\right)\tan^2(c+dx)+15\left(a^2+3b^2\right)+3a^2\tan^4(c+dx)\right)+\right.}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^6,x]
```

```
[Out] (15*b*(9*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*b*(9*a^2 + 4
*b^2)*Sec[c + d*x] + 90*a^2*b*Sec[c + d*x]^3 + 8*a*(15*(a^2 + 3*b^2) + 5*(2
*a^2 + 3*b^2)*Tan[c + d*x]^2 + 3*a^2*Tan[c + d*x]^4)))/(120*d)
```

Maple [A] time = 0.064, size = 206, normalized size = 1.2

$$\frac{8a^3\tan(dx+c)}{15d}+\frac{a^3\tan(dx+c)(\sec(dx+c))^4}{5d}+\frac{4a^3\tan(dx+c)(\sec(dx+c))^2}{15d}+\frac{3a^2b(\sec(dx+c))^3\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x)
```

```
[Out] 8/15*a^3*tan(d*x+c)/d+1/5/d*a^3*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a^3*tan(d*x+
c)*sec(d*x+c)^2+3/4*a^2*b*sec(d*x+c)^3*tan(d*x+c)/d+9/8*a^2*b*sec(d*x+c)*ta
n(d*x+c)/d+9/8/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b^2*tan(d*x+c)+1/d*a
*b^2*tan(d*x+c)*sec(d*x+c)^2+1/2/d*b^3*tan(d*x+c)*sec(d*x+c)+1/2/d*b^3*ln(s
ec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.976783, size = 244, normalized size = 1.44

$$\frac{16\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)a^{3}+240\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)ab^{2}-45\,a^{2}b\left(\frac{2\left(3\sin\cdots\right.}{\sin(dx\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/240*(16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
0*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^2 - 45*a^2*b*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 2.00369, size = 421, normalized size = 2.49

$$\frac{15\left(9\,a^{2}b+4\,b^{3}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(9\,a^{2}b+4\,b^{3}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(4\cdots\right.\right.}{24\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(9*a^2*b + 4*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(9*a^
2*b + 4*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(4*a^3 + 15*a*b^
2)*cos(d*x + c)^4 + 90*a^2*b*cos(d*x + c) + 15*(9*a^2*b + 4*b^3)*cos(d*x +
c)^3 + 24*a^3 + 8*(4*a^3 + 15*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38761, size = 495, normalized size = 2.93

$$15\left(9a^2b+4b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(9a^2b+4b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(120a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-225a^2b\,\mathrm{t}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/120*(15*(9*a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(9*a^2*
b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*a^3*tan(1/2*d*x + 1/
2*c)^9 - 225*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^
9 - 60*b^3*tan(1/2*d*x + 1/2*c)^9 - 160*a^3*tan(1/2*d*x + 1/2*c)^7 + 90*a^2
*b*tan(1/2*d*x + 1/2*c)^7 - 960*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 120*b^3*tan(
1/2*d*x + 1/2*c)^7 + 464*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*a*b^2*tan(1/2*d*
x + 1/2*c)^5 - 160*a^3*tan(1/2*d*x + 1/2*c)^3 - 90*a^2*b*tan(1/2*d*x + 1/2*
c)^3 - 960*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*b^3*tan(1/2*d*x + 1/2*c)^3 +
120*a^3*tan(1/2*d*x + 1/2*c) + 225*a^2*b*tan(1/2*d*x + 1/2*c) + 360*a*b^2*t
an(1/2*d*x + 1/2*c) + 60*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

3.438 $\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=247

$$-\frac{(168a^2b^2+35a^4+24b^4)\sin^3(c+dx)}{105d}+\frac{(168a^2b^2+35a^4+24b^4)\sin(c+dx)}{35d}+\frac{b^2(37a^2+6b^2)\sin(c+dx)\cos^4(c+$$

```
[Out] (a*b*(6*a^2 + 5*b^2)*x)/4 + ((35*a^4 + 168*a^2*b^2 + 24*b^4)*Sin[c + d*x])/
(35*d) + (a*b*(6*a^2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a*b*(6*a^
2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (b^2*(37*a^2 + 6*b^2)*Cos[c
 + d*x]^4*Sin[c + d*x])/(35*d) + (8*a*b^3*Cos[c + d*x]^5*Sin[c + d*x])/(21*
d) + (b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) - ((35*
a^4 + 168*a^2*b^2 + 24*b^4)*Sin[c + d*x]^3)/(105*d)
```

Rubi [A] time = 0.400815, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2793, 3033, 3023, 2748, 2633, 2635, 8}

$$-\frac{(168a^2b^2+35a^4+24b^4)\sin^3(c+dx)}{105d}+\frac{(168a^2b^2+35a^4+24b^4)\sin(c+dx)}{35d}+\frac{b^2(37a^2+6b^2)\sin(c+dx)\cos^4(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (a*b*(6*a^2 + 5*b^2)*x)/4 + ((35*a^4 + 168*a^2*b^2 + 24*b^4)*Sin[c + d*x])/
(35*d) + (a*b*(6*a^2 + 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a*b*(6*a^
2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(6*d) + (b^2*(37*a^2 + 6*b^2)*Cos[c
 + d*x]^4*Sin[c + d*x])/(35*d) + (8*a*b^3*Cos[c + d*x]^5*Sin[c + d*x])/(21*
d) + (b^2*Cos[c + d*x]^4*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(7*d) - ((35*
a^4 + 168*a^2*b^2 + 24*b^4)*Sin[c + d*x]^3)/(105*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\cos(c+dx))^4\,dx &= \frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{7d}+\frac{1}{7}\int \cos^3(c+dx)(a+b\cos(c\\
&= \frac{8ab^3\cos^5(c+dx)\sin(c+dx)}{21d}+\frac{b^2\cos^4(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{7d}\\
&= \frac{b^2\left(37a^2+6b^2\right)\cos^4(c+dx)\sin(c+dx)}{35d}+\frac{8ab^3\cos^5(c+dx)\sin(c+dx)}{21d}+\frac{b^2 c}{}\\
&= \frac{b^2\left(37a^2+6b^2\right)\cos^4(c+dx)\sin(c+dx)}{35d}+\frac{8ab^3\cos^5(c+dx)\sin(c+dx)}{21d}+\frac{b^2 c}{}\\
&= \frac{ab\left(6a^2+5b^2\right)\cos^3(c+dx)\sin(c+dx)}{6d}+\frac{b^2\left(37a^2+6b^2\right)\cos^4(c+dx)\sin(c+d}{35d}\\
&= \frac{\left(35a^4+168a^2b^2+24b^4\right)\sin(c+dx)}{35d}+\frac{ab\left(6a^2+5b^2\right)\cos(c+dx)\sin(c+dx)}{4d}+\\
&= \frac{1}{4}ab\left(6a^2+5b^2\right)x+\frac{\left(35a^4+168a^2b^2+24b^4\right)\sin(c+dx)}{35d}+\frac{ab\left(6a^2+5b^2\right)\cos(}{4d}\end{aligned}$$

Mathematica [A] time = 0.408403, size = 181, normalized size = 0.73

$$\frac{1680ab\left(6a^2+5b^2\right)(c+dx)+21b^2\left(24a^2+7b^2\right)\sin(5(c+dx))+420ab\left(16a^2+15b^2\right)\sin(2(c+dx))+420ab\left(2a^2+3b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (1680*a*b*(6*a^2 + 5*b^2)*(c + d*x) + 105*(48*a^4 + 240*a^2*b^2 + 35*b^4)*S
in[c + d*x] + 420*a*b*(16*a^2 + 15*b^2)*Sin[2*(c + d*x)] + 35*(16*a^4 + 120
*a^2*b^2 + 21*b^4)*Sin[3*(c + d*x)] + 420*a*b*(2*a^2 + 3*b^2)*Sin[4*(c + d*
x)] + 21*b^2*(24*a^2 + 7*b^2)*Sin[5*(c + d*x)] + 140*a*b^3*Sin[6*(c + d*x)]
 + 15*b^4*Sin[7*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.052, size = 190, normalized size = 0.8

$$\frac{1}{d}\left(\frac{b^4\sin(dx+c)}{7}\left(\frac{16}{5}+(\cos(dx+c))^6+\frac{6(\cos(dx+c))^4}{5}+\frac{8(\cos(dx+c))^2}{5}\right)+4ab^3\left(1/6\left((\cos(dx+c))^5+5/4(c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(1/7*b^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+
c)+4*a*b^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+6/5*a^2*b^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
+4*a^3*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a
^4*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 0.978239, size = 259, normalized size = 1.05

$$-\frac{560\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^4-210(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^3b-672\left(3\sin(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/1680*(560*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^4 - 210*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3*b - 672*(3*sin(d*x + c)^5 - 10*si
n(d*x + c)^3 + 15*sin(d*x + c))*a^2*b^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x
 - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a*b^3 + 48*(5*sin(d*x +
 c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*b^4)/d
```

Fricas [A] time = 1.97599, size = 416, normalized size = 1.68

$$\frac{105\left(6a^3b+5ab^3\right)dx+\left(60b^4\cos(dx+c)^6+280ab^3\cos(dx+c)^5+72\left(7a^2b^2+b^4\right)\cos(dx+c)^4+280a^4+1344a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/420*(105*(6*a^3*b + 5*a*b^3)*d*x + (60*b^4*cos(d*x + c)^6 + 280*a*b^3*cos
(d*x + c)^5 + 72*(7*a^2*b^2 + b^4)*cos(d*x + c)^4 + 280*a^4 + 1344*a^2*b^2
+ 192*b^4 + 70*(6*a^3*b + 5*a*b^3)*cos(d*x + c)^3 + 4*(35*a^4 + 168*a^2*b^2
 + 24*b^4)*cos(d*x + c)^2 + 105*(6*a^3*b + 5*a*b^3)*cos(d*x + c))*sin(d*x +
```

```
c))/d
```

Sympy [A] time = 8.5724, size = 495, normalized size = 2.

$$\begin{cases} \frac{2a^4 \sin^3(c+dx)}{3d} + \frac{a^4 \sin(c+dx)\cos^2(c+dx)}{d} + \frac{3a^3bx\sin^4(c+dx)}{2} + 3a^3bx\sin^2(c+dx)\cos^2(c+dx) + \frac{3a^3bx\cos^4(c+dx)}{2} + \frac{3a^3b\sin^3(c+dx)\cos \\ x(a+b\cos(c))^4\cos^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((2*a**4*sin(c + d*x)**3/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**2
/d + 3*a**3*b*x*sin(c + d*x)**4/2 + 3*a**3*b*x*sin(c + d*x)**2*cos(c + d*x)
**2 + 3*a**3*b*x*cos(c + d*x)**4/2 + 3*a**3*b*sin(c + d*x)**3*cos(c + d*x)/
(2*d) + 5*a**3*b*sin(c + d*x)*cos(c + d*x)**3/(2*d) + 16*a**2*b**2*sin(c +
d*x)**5/(5*d) + 8*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**2/d + 6*a**2*b**2
*sin(c + d*x)*cos(c + d*x)**4/d + 5*a*b**3*x*sin(c + d*x)**6/4 + 15*a*b**3*
x*sin(c + d*x)**4*cos(c + d*x)**2/4 + 15*a*b**3*x*sin(c + d*x)**2*cos(c + d
*x)**4/4 + 5*a*b**3*x*cos(c + d*x)**6/4 + 5*a*b**3*sin(c + d*x)**5*cos(c +
d*x)/(4*d) + 10*a*b**3*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) + 11*a*b**3*si
n(c + d*x)*cos(c + d*x)**5/(4*d) + 16*b**4*sin(c + d*x)**7/(35*d) + 8*b**4*
sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*b**4*sin(c + d*x)**3*cos(c + d*x)
**4/d + b**4*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(a + b*cos(c))**
4*cos(c)**3, True))
```

Giac [A] time = 1.36568, size = 266, normalized size = 1.08

$$\frac{b^4\sin(7dx+7c)}{448d} + \frac{ab^3\sin(6dx+6c)}{48d} + \frac{1}{4}\left(6a^3b+5ab^3\right)x + \frac{\left(24a^2b^2+7b^4\right)\sin(5dx+5c)}{320d} + \frac{\left(2a^3b+3ab^3\right)\sin(4d}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/448*b^4*sin(7*d*x + 7*c)/d + 1/48*a*b^3*sin(6*d*x + 6*c)/d + 1/4*(6*a^3*b
 + 5*a*b^3)*x + 1/320*(24*a^2*b^2 + 7*b^4)*sin(5*d*x + 5*c)/d + 1/16*(2*a^3
*b + 3*a*b^3)*sin(4*d*x + 4*c)/d + 1/192*(16*a^4 + 120*a^2*b^2 + 21*b^4)*si
n(3*d*x + 3*c)/d + 1/16*(16*a^3*b + 15*a*b^3)*sin(2*d*x + 2*c)/d + 1/64*(48
*a^4 + 240*a^2*b^2 + 35*b^4)*sin(d*x + c)/d
```

3.439 $\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=235

$$-\frac{a\left(-121a^2b^2+4a^4-128b^4\right)\sin(c+dx)}{60bd}-\frac{\left(4a^2-25b^2\right)\sin(c+dx)(a+b\cos(c+dx))^3}{120bd}-\frac{a\left(4a^2-53b^2\right)\sin(c+dx)(}{120bd}$$

```
[Out] ((8*a^4 + 36*a^2*b^2 + 5*b^4)*x)/16 - (a*(4*a^4 - 121*a^2*b^2 - 128*b^4)*Si
n[c + d*x])/(60*b*d) - ((8*a^4 - 178*a^2*b^2 - 75*b^4)*Cos[c + d*x]*Sin[c +
 d*x])/(240*d) - (a*(4*a^2 - 53*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
120*b*d) - ((4*a^2 - 25*b^2)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(120*b*d)
 - (a*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d) + ((a + b*Cos[c + d*x])
^5*Sin[c + d*x])/(6*b*d)
```

Rubi [A] time = 0.320216, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2791, 2753, 2734}

$$-\frac{a\left(-121a^2b^2+4a^4-128b^4\right)\sin(c+dx)}{60bd}-\frac{\left(4a^2-25b^2\right)\sin(c+dx)(a+b\cos(c+dx))^3}{120bd}-\frac{a\left(4a^2-53b^2\right)\sin(c+dx)(}{120bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 36*a^2*b^2 + 5*b^4)*x)/16 - (a*(4*a^4 - 121*a^2*b^2 - 128*b^4)*Si
n[c + d*x])/(60*b*d) - ((8*a^4 - 178*a^2*b^2 - 75*b^4)*Cos[c + d*x]*Sin[c +
 d*x])/(240*d) - (a*(4*a^2 - 53*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(
120*b*d) - ((4*a^2 - 25*b^2)*(a + b*Cos[c + d*x])^3*Sin[c + d*x])/(120*b*d)
 - (a*(a + b*Cos[c + d*x])^4*Sin[c + d*x])/(30*b*d) + ((a + b*Cos[c + d*x])
^5*Sin[c + d*x])/(6*b*d)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^4\,dx = \frac{(a+b\cos(c+dx))^5\sin(c+dx)}{6bd} + \frac{\int(5b-a\cos(c+dx))(a+b\cos(c+dx))^4\,dx}{6b}$$

$$= -\frac{a(a+b\cos(c+dx))^4\sin(c+dx)}{30bd} + \frac{(a+b\cos(c+dx))^5\sin(c+dx)}{6bd} + \frac{\int(a+b$$

$$= -\frac{\left(4a^2-25b^2\right)(a+b\cos(c+dx))^3\sin(c+dx)}{120bd} - \frac{a(a+b\cos(c+dx))^4\sin(c+d}{30bd}$$

$$= -\frac{a\left(4a^2-53b^2\right)(a+b\cos(c+dx))^2\sin(c+dx)}{120bd} - \frac{\left(4a^2-25b^2\right)(a+b\cos(c+d}{120bd}$$

$$= \frac{1}{16}\left(8a^4+36a^2b^2+5b^4\right)x - \frac{a\left(4a^4-121a^2b^2-128b^4\right)\sin(c+dx)}{60bd} - \frac{\left(8a^4-178\right.}{}$$

Mathematica [A] time = 0.446306, size = 156, normalized size = 0.66

$$\frac{60\left(36a^2b^2+8a^4+5b^4\right)(c+dx)+45b^2\left(4a^2+b^2\right)\sin(4(c+dx))+480ab\left(6a^2+5b^2\right)\sin(c+dx)+80ab\left(4a^2+5b^2\right)\sin}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (60*(8*a^4 + 36*a^2*b^2 + 5*b^4)*(c + d*x) + 480*a*b*(6*a^2 + 5*b^2)*Sin[c
+ d*x] + 15*(16*a^4 + 96*a^2*b^2 + 15*b^4)*Sin[2*(c + d*x)] + 80*a*b*(4*a^2
 + 5*b^2)*Sin[3*(c + d*x)] + 45*b^2*(4*a^2 + b^2)*Sin[4*(c + d*x)] + 48*a*b
^3*Sin[5*(c + d*x)] + 5*b^4*Sin[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.036, size = 174, normalized size = 0.7

$$\frac{1}{d}\left(b^4\left(\frac{\sin(dx+c)}{6}\left((\cos(dx+c))^5+\frac{5(\cos(dx+c))^3}{4}+\frac{15\cos(dx+c)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{4ab^3\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
16*d*x+5/16*c)+4/5*a*b^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a
^2*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a^3
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c
))
```

Maxima [A] time = 0.979925, size = 230, normalized size = 0.98

$$\frac{240(2dx+2c+\sin(2dx+2c))a^4-1280\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3b+180(12dx+12c+\sin(4dx+4c)+8\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/960*(240*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 - 1280*(sin(d*x + c)^3 - 3*
sin(d*x + c))*a^3*b + 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x +
 2*c))*a^2*b^2 + 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c
))*a*b^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 4
8*sin(2*d*x + 2*c))*b^4)/d
```

Fricas [A] time = 1.98184, size = 358, normalized size = 1.52

$$\frac{15\left(8a^4+36a^2b^2+5b^4\right)dx+\left(40b^4\cos\left(dx+c\right)^5+192ab^3\cos\left(dx+c\right)^4+640a^3b+512ab^3+10\left(36a^2b^2+5b^4\right)\cos\ldots\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(8*a^4 + 36*a^2*b^2 + 5*b^4)*d*x + (40*b^4*cos(d*x + c)^5 + 192*a
*b^3*cos(d*x + c)^4 + 640*a^3*b + 512*a*b^3 + 10*(36*a^2*b^2 + 5*b^4)*cos(d
*x + c)^3 + 64*(5*a^3*b + 4*a*b^3)*cos(d*x + c)^2 + 15*(8*a^4 + 36*a^2*b^2
+ 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 5.04267, size = 459, normalized size = 1.95

$$\begin{cases}\frac{a^4x\sin^2\left(c+dx\right)}{2}+\frac{a^4x\cos^2\left(c+dx\right)}{2}+\frac{a^4\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{8a^3b\sin^3\left(c+dx\right)}{3d}+\frac{4a^3b\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}+\frac{9a^2b^2x\sin^4\left(c+dx\right)}{4}+\frac{9a^2b^2x\sin\ldots}{} \\ x\left(a+b\cos\left(c\right)\right)^4\cos^2\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x*sin(c + d*x)**2/2 + a**4*x*cos(c + d*x)**2/2 + a**4*sin(c
 + d*x)*cos(c + d*x)/(2*d) + 8*a**3*b*sin(c + d*x)**3/(3*d) + 4*a**3*b*sin(
c + d*x)*cos(c + d*x)**2/d + 9*a**2*b**2*x*sin(c + d*x)**4/4 + 9*a**2*b**2*
x*sin(c + d*x)**2*cos(c + d*x)**2/2 + 9*a**2*b**2*x*cos(c + d*x)**4/4 + 9*a
**2*b**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 15*a**2*b**2*sin(c + d*x)*cos
(c + d*x)**3/(4*d) + 32*a*b**3*sin(c + d*x)**5/(15*d) + 16*a*b**3*sin(c + d
*x)**3*cos(c + d*x)**2/(3*d) + 4*a*b**3*sin(c + d*x)*cos(c + d*x)**4/d + 5*
b**4*x*sin(c + d*x)**6/16 + 15*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 +
15*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**4*x*cos(c + d*x)**6/16
+ 5*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*b**4*sin(c + d*x)**3*cos(c
 + d*x)**3/(6*d) + 11*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)),
(x*(a + b*cos(c))**4*cos(c)**2, True))
```

Giac [A] time = 1.30023, size = 227, normalized size = 0.97

$$\frac{b^4\sin\left(6dx+6c\right)}{192d}+\frac{ab^3\sin\left(5dx+5c\right)}{20d}+\frac{1}{16}\left(8a^4+36a^2b^2+5b^4\right)x+\frac{3\left(4a^2b^2+b^4\right)\sin\left(4dx+4c\right)}{64d}+\frac{\left(4a^3b+5a\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/192*b^4*sin(6*d*x + 6*c)/d + 1/20*a*b^3*sin(5*d*x + 5*c)/d + 1/16*(8*a^4
+ 36*a^2*b^2 + 5*b^4)*x + 3/64*(4*a^2*b^2 + b^4)*sin(4*d*x + 4*c)/d + 1/12*
(4*a^3*b + 5*a*b^3)*sin(3*d*x + 3*c)/d + 1/64*(16*a^4 + 96*a^2*b^2 + 15*b^4
)*sin(2*d*x + 2*c)/d + 1/2*(6*a^3*b + 5*a*b^3)*sin(d*x + c)/d
```

3.440 $\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=170

$$\frac{2\left(28a^2b^2+3a^4+4b^4\right)\sin(c+dx)}{15d}+\frac{\left(3a^2+4b^2\right)\sin(c+dx)(a+b\cos(c+dx))^2}{15d}+\frac{ab\left(6a^2+29b^2\right)\sin(c+dx)\cos(c$$

```
[Out] (a*b*(4*a^2 + 3*b^2)*x)/2 + (2*(3*a^4 + 28*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(
15*d) + (a*b*(6*a^2 + 29*b^2)*Cos[c + d*x]*Sin[c + d*x])/(30*d) + ((3*a^2 +
 4*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(15*d) + (a*(a + b*Cos[c + d*x
])^3*Sin[c + d*x])/(5*d) + ((a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.203751, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2753, 2734}

$$\frac{2\left(28a^2b^2+3a^4+4b^4\right)\sin(c+dx)}{15d}+\frac{\left(3a^2+4b^2\right)\sin(c+dx)(a+b\cos(c+dx))^2}{15d}+\frac{ab\left(6a^2+29b^2\right)\sin(c+dx)\cos(c$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (a*b*(4*a^2 + 3*b^2)*x)/2 + (2*(3*a^4 + 28*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(
15*d) + (a*b*(6*a^2 + 29*b^2)*Cos[c + d*x]*Sin[c + d*x])/(30*d) + ((3*a^2 +
 4*b^2)*(a + b*Cos[c + d*x])^2*Sin[c + d*x])/(15*d) + (a*(a + b*Cos[c + d*x
])^3*Sin[c + d*x])/(5*d) + ((a + b*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\cos(c+dx))^4\,dx &= \frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{5}\int(4b+4a\cos(c+dx))(a+b\cos(c+dx)\\ &= \frac{a(a+b\cos(c+dx))^3\sin(c+dx)}{5d}+\frac{(a+b\cos(c+dx))^4\sin(c+dx)}{5d}+\frac{1}{20}\int(a\\ &= \frac{\left(3a^2+4b^2\right)(a+b\cos(c+dx))^2\sin(c+dx)}{15d}+\frac{a(a+b\cos(c+dx))^3\sin(c+dx}{5d}\\ &= \frac{1}{2}ab\left(4a^2+3b^2\right)x+\frac{2\left(3a^4+28a^2b^2+4b^4\right)\sin(c+dx)}{15d}+\frac{ab\left(6a^2+29b^2\right)\cos}{3}\end{aligned}$$

Mathematica [A] time = 0.489407, size = 133, normalized size = 0.78

$$\frac{30\left(36a^2b^2+8a^4+5b^4\right)\sin(c+dx)+b\left(240a\left(a^2+b^2\right)\sin(2(c+dx))+5\left(24a^2b+5b^3\right)\sin(3(c+dx))+480a^3c+480a\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (30*(8*a^4 + 36*a^2*b^2 + 5*b^4)*Sin[c + d*x] + b*(480*a^3*c + 360*a*b^2*c
+ 480*a^3*d*x + 360*a*b^2*d*x + 240*a*(a^2 + b^2)*Sin[2*(c + d*x)] + 5*(24*
a^2*b + 5*b^3)*Sin[3*(c + d*x)] + 30*a*b^2*Sin[4*(c + d*x)] + 3*b^3*Sin[5*(
c + d*x)]))/(240*d)
```

Maple [A] time = 0.034, size = 138, normalized size = 0.8

$$\frac{1}{d}\left(\frac{b^4\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4(\cos(dx+c))^2}{3}\right)+4ab^3\left(1/4\left((\cos(dx+c))^3+3/2\cos(dx+c)\right)\sin(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*b^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*a*b^3*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*a^2*b^2*(2+cos(d*x+
c)^2)*sin(d*x+c)+4*a^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*sin(
d*x+c))
```

Maxima [A] time = 1.00177, size = 180, normalized size = 1.06

$$\frac{120(2dx+2c+\sin(2dx+2c))a^3b-240\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^2b^2+15(12dx+12c+\sin(4dx+4c)+8\sin\right.}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/120*(120*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3*b - 240*(sin(d*x + c)^3 - 3
*sin(d*x + c))*a^2*b^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a*b^3 + 8*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))
*b^4 + 120*a^4*sin(d*x + c))/d
```

Fricas [A] time = 2.01594, size = 285, normalized size = 1.68

$$\frac{15\left(4a^3b+3ab^3\right)dx+\left(6b^4\cos(dx+c)^4+30ab^3\cos(dx+c)^3+30a^4+120a^2b^2+16b^4+4\left(15a^2b^2+2b^4\right)\cos(dx+\right.}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(15*(4*a^3*b + 3*a*b^3)*d*x + (6*b^4*cos(d*x + c)^4 + 30*a*b^3*cos(d*x
 + c)^3 + 30*a^4 + 120*a^2*b^2 + 16*b^4 + 4*(15*a^2*b^2 + 2*b^4)*cos(d*x +
c)^2 + 15*(4*a^3*b + 3*a*b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 2.54356, size = 301, normalized size = 1.77

$$\begin{cases} \frac{a^4\sin(c+dx)}{d} + 2a^3bx\sin^2(c+dx) + 2a^3bx\cos^2(c+dx) + \frac{2a^3b\sin(c+dx)\cos(c+dx)}{d} + \frac{4a^2b^2\sin^3(c+dx)}{d} + \frac{6a^2b^2\sin(c+dx)\cos^2(c+d}{d} \\ x(a+b\cos(c))^4\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)/d + 2*a**3*b*x*sin(c + d*x)**2 + 2*a**3*b*x*co
s(c + d*x)**2 + 2*a**3*b*sin(c + d*x)*cos(c + d*x)/d + 4*a**2*b**2*sin(c +
d*x)**3/d + 6*a**2*b**2*sin(c + d*x)*cos(c + d*x)**2/d + 3*a*b**3*x*sin(c +
 d*x)**4/2 + 3*a*b**3*x*sin(c + d*x)**2*cos(c + d*x)**2 + 3*a*b**3*x*cos(c
+ d*x)**4/2 + 3*a*b**3*sin(c + d*x)**3*cos(c + d*x)/(2*d) + 5*a*b**3*sin(c
+ d*x)*cos(c + d*x)**3/(2*d) + 8*b**4*sin(c + d*x)**5/(15*d) + 4*b**4*sin(c
 + d*x)**3*cos(c + d*x)**2/(3*d) + b**4*sin(c + d*x)*cos(c + d*x)**4/d, Ne(
d, 0)), (x*(a + b*cos(c))**4*cos(c), True))
```

Giac [A] time = 1.37816, size = 181, normalized size = 1.06

$$\frac{b^4\sin(5\,dx+5\,c)}{80\,d} + \frac{ab^3\sin(4\,dx+4\,c)}{8\,d} + \frac{1}{2}\left(4\,a^3b+3\,ab^3\right)x + \frac{\left(24\,a^2b^2+5\,b^4\right)\sin(3\,dx+3\,c)}{48\,d} + \frac{\left(a^3b+ab^3\right)\sin(2\,d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/80*b^4*sin(5*d*x + 5*c)/d + 1/8*a*b^3*sin(4*d*x + 4*c)/d + 1/2*(4*a^3*b +
 3*a*b^3)*x + 1/48*(24*a^2*b^2 + 5*b^4)*sin(3*d*x + 3*c)/d + (a^3*b + a*b^3
)*sin(2*d*x + 2*c)/d + 1/8*(8*a^4 + 36*a^2*b^2 + 5*b^4)*sin(d*x + c)/d
```

3.441 $\int (a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{ab\left(19a^2+16b^2\right)\sin(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)+\frac{b\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 + 16*b^2)*Sin[c + d*x])/(
6*d) + (b^2*(26*a^2 + 9*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (7*a*b*(a
+ b*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + (b*(a + b*Cos[c + d*x])^3*Sin[c
+ d*x])/(4*d)
```

Rubi [A] time = 0.146885, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2656, 2753, 2734}

$$\frac{ab\left(19a^2+16b^2\right)\sin(c+dx)}{6d}+\frac{b^2\left(26a^2+9b^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{1}{8}x\left(24a^2b^2+8a^4+3b^4\right)+\frac{b\sin(c+dx)(a+b\cos(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 + 16*b^2)*Sin[c + d*x])/(
6*d) + (b^2*(26*a^2 + 9*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d) + (7*a*b*(a
+ b*Cos[c + d*x])^2*Sin[c + d*x])/(12*d) + (b*(a + b*Cos[c + d*x])^3*Sin[c
+ d*x])/(4*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\,dx = \frac{b(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{4}\int (a+b\cos(c+dx))^2\left(4a^2+3b^2+7ab\cos(c+dx)\right.$$
$$= \frac{7ab(a+b\cos(c+dx))^2\sin(c+dx)}{12d} + \frac{b(a+b\cos(c+dx))^3\sin(c+dx)}{4d} + \frac{1}{12}\int (a+b\cos$$
$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x + \frac{ab\left(19a^2+16b^2\right)\sin(c+dx)}{6d} + \frac{b^2\left(26a^2+9b^2\right)\cos(c+dx)\sin}{24d}$$

Mathematica [A] time = 0.209031, size = 104, normalized size = 0.76

$$\frac{12\left(24a^2b^2+8a^4+3b^4\right)(c+dx)+24b^2\left(6a^2+b^2\right)\sin(2(c+dx))+96ab\left(4a^2+3b^2\right)\sin(c+dx)+32ab^3\sin(3(c+dx}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sin[c +
 d*x] + 24*b^2*(6*a^2 + b^2)*Sin[2*(c + d*x)] + 32*a*b^3*Sin[3*(c + d*x)] +
 3*b^4*Sin[4*(c + d*x)])/(96*d)
```

Maple [A] time = 0.033, size = 116, normalized size = 0.9

$$\frac{1}{d}\left(b^4\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{4ab^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}+6a^2b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4/3*a
*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^2*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d
*x+1/2*c)+4*a^3*b*sin(d*x+c)+a^4*(d*x+c))
```

Maxima [A] time = 0.993018, size = 150, normalized size = 1.09

$$a^4x + \frac{3(2dx+2c+\sin(2dx+2c))a^2b^2}{2d} - \frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)ab^3}{3d} + \frac{(12dx+12c+\sin(4dx+4c)+8}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 3/2*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b^2/d - 4/3*(sin(d*x + c)^
3 - 3*sin(d*x + c))*a*b^3/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*si
n(2*d*x + 2*c))*b^4/d + 4*a^3*b*sin(d*x + c)/d
```

Fricas [A] time = 1.96055, size = 224, normalized size = 1.64

$$\frac{3\left(8a^4+24a^2b^2+3b^4\right)dx+\left(6b^4\cos(dx+c)^3+32ab^3\cos(dx+c)^2+96a^3b+64ab^3+9\left(8a^2b^2+b^4\right)\cos(dx+c)\right.}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*d*x + (6*b^4*cos(d*x + c)^3 + 32*a*b^3
*cos(d*x + c)^2 + 96*a^3*b + 64*a*b^3 + 9*(8*a^2*b^2 + b^4)*cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [A] time = 1.28512, size = 240, normalized size = 1.75

$$\begin{cases} a^4x + \frac{4a^3b\sin(c+dx)}{d} + 3a^2b^2x\sin^2(c+dx) + 3a^2b^2x\cos^2(c+dx) + \frac{3a^2b^2\sin(c+dx)\cos(c+dx)}{d} + \frac{8ab^3\sin^3(c+dx)}{3d} + \frac{4ab^3\sin(c+dx)\cos\ldots}{d} \\ x(a+b\cos(c))^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*sin(c + d*x)/d + 3*a**2*b**2*x*sin(c + d*x)**2
 + 3*a**2*b**2*x*cos(c + d*x)**2 + 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)/d
+ 8*a*b**3*sin(c + d*x)**3/(3*d) + 4*a*b**3*sin(c + d*x)*cos(c + d*x)**2/d
+ 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
 3*b**4*x*cos(c + d*x)**4/8 + 3*b**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5
*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*cos(c))**4,
True))
```

Giac [A] time = 1.32937, size = 144, normalized size = 1.05

$$\frac{b^4\sin(4\,dx+4\,c)}{32\,d} + \frac{ab^3\sin(3\,dx+3\,c)}{3\,d} + \frac{1}{8}\left(8\,a^4 + 24\,a^2b^2 + 3\,b^4\right)x + \frac{\left(6\,a^2b^2 + b^4\right)\sin(2\,dx+2\,c)}{4\,d} + \frac{\left(4\,a^3b + 3\,ab^3\right)\sin\ldots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/32*b^4*sin(4*d*x + 4*c)/d + 1/3*a*b^3*sin(3*d*x + 3*c)/d + 1/8*(8*a^4 + 2
4*a^2*b^2 + 3*b^4)*x + 1/4*(6*a^2*b^2 + b^4)*sin(2*d*x + 2*c)/d + (4*a^3*b
+ 3*a*b^3)*sin(d*x + c)/d
```

3.442 $\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx$

Optimal. Leaf size=107

$$\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+2abx\left(2a^2+b^2\right)+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{3d}+\frac{b^2\sin(c+d}{}$$

```
[Out] 2*a*b*(2*a^2 + b^2)*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Sin[c + d*x])/(3*d) + (4*a*b^3*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (b^2*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.22764, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2793, 3033, 3023, 2735, 3770}

$$\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+2abx\left(2a^2+b^2\right)+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\sin(c+dx)\cos(c+dx)}{3d}+\frac{b^2\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x],x]
```

```
[Out] 2*a*b*(2*a^2 + b^2)*x + (a^4*ArcTanh[Sin[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Sin[c + d*x])/(3*d) + (4*a*b^3*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (b^2*(
a + b*Cos[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec(c+dx)\,dx = \frac{b^2(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (a+b\cos(c+dx))\left(3a^3+b\left(9a^2+2b^2\right)\right.$$

$$=\frac{4ab^3\cos(c+dx)\sin(c+dx)}{3d}+\frac{b^2(a+b\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{6}\int\left(6a^4+1\right.$$

$$=\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+\frac{4ab^3\cos(c+dx)\sin(c+dx)}{3d}+\frac{b^2(a+b\cos(c+dx}{3d}$$

$$=2ab\left(2a^2+b^2\right)x+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+\frac{4ab^3\cos(c+dx)\sin(c+dx)}{3d}+$$

$$=2ab\left(2a^2+b^2\right)x+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\left(17a^2+2b^2\right)\sin(c+dx)}{3d}+\frac{4ab^3}{}$$

Mathematica [A] time = 0.159085, size = 128, normalized size = 1.2

$$\frac{24ab\left(2a^2+b^2\right)(c+dx)+9b^2\left(8a^2+b^2\right)\sin(c+dx)-12a^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^4\log\left(\sin\left(\frac{1}{2}(c\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x],x]
```

```
[Out] (24*a*b*(2*a^2 + b^2)*(c + d*x) - 12*a^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*a^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*b^2*(8*a^2 + b
^2)*Sin[c + d*x] + 12*a*b^3*Sin[2*(c + d*x)] + b^4*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.058, size = 131, normalized size = 1.2

$$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+4a^3bx+4\frac{a^3bc}{d}+6\frac{a^2b^2\sin(dx+c)}{d}+2\frac{ab^3\cos(dx+c)\sin(dx+c)}{d}+2ab^3x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c),x)
```

```
[Out] 1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b*x+4/d*a^3*b*c+6/d*a^2*b^2*sin(d*x
+c)+2*a*b^3*cos(d*x+c)*sin(d*x+c)/d+2*a*b^3*x+2/d*a*b^3*c+1/3/d*sin(d*x+c)*
cos(d*x+c)^2*b^4+2/3/d*b^4*sin(d*x+c)
```

Maxima [A] time = 0.961537, size = 128, normalized size = 1.2

$$\frac{12\,(dx+c)a^{3}b+3\,(2\,dx+2\,c+\sin(2\,dx+2\,c))ab^{3}-\left(\sin(dx+c)^{3}-3\,\sin(dx+c)\right)b^{4}+3\,a^{4}\log(\sec(dx+c)+\tan(}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/3*(12*(d*x + c)*a^3*b + 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b^3 - (sin(d
*x + c)^3 - 3*sin(d*x + c))*b^4 + 3*a^4*log(sec(d*x + c) + tan(d*x + c)) +
18*a^2*b^2*sin(d*x + c))/d
```

Fricas [A] time = 2.05108, size = 239, normalized size = 2.23

$$\frac{3\,a^{4}\log(\sin(dx+c)+1)-3\,a^{4}\log(-\sin(dx+c)+1)+12\left(2\,a^{3}b+ab^{3}\right)dx+2\left(b^{4}\cos(dx+c)^{2}+6\,ab^{3}\cos(dx+c)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^4*log(sin(d*x + c) + 1) - 3*a^4*log(-sin(d*x + c) + 1) + 12*(2*a^3
*b + a*b^3)*d*x + 2*(b^4*cos(d*x + c)^2 + 6*a*b^3*cos(d*x + c) + 18*a^2*b^2
 + 2*b^4)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36577, size = 286, normalized size = 2.67

$$\frac{3\,a^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+6\left(2\,a^{3}b+ab^{3}\right)(dx+c)+\frac{2\left(18\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,a\right.}{}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a^4*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 6*(2*a^3*b + a*b^3)*(d*x + c) + 2*(18*a^2*b^2*tan(1/2*d*x +
 1/2*c)^5 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*x + 1/2*c)^5 +
```

```
 36*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^4*tan(1/2*d*x + 1/2*c)^3 + 18*a^2*
b^2*tan(1/2*d*x + 1/2*c) + 6*a*b^3*tan(1/2*d*x + 1/2*c) + 3*b^4*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.443 $\int (a+b\cos(c+dx))^4 \sec^2(c+dx)\,dx$

Optimal. Leaf size=114

$$-\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}b^2x\left(12a^2+b^2\right)+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+$$

```
[Out] (b^2*(12*a^2 + b^2)*x)/2 + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*(a^2
- 2*b^2)*Sin[c + d*x])/d - (b^2*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(2
*d) + (a^2*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.234006, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3033, 3023, 2735, 3770}

$$-\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{b^2\left(2a^2-b^2\right)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}b^2x\left(12a^2+b^2\right)+\frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*(12*a^2 + b^2)*x)/2 + (4*a^3*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*(a^2
- 2*b^2)*Sin[c + d*x])/d - (b^2*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(2
*d) + (a^2*(a + b*Cos[c + d*x])^2*Tan[c + d*x])/d
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
```

```
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec^2(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))^2\tan(c+dx)}{d} + \int (a+b\cos(c+dx))\left(4a^2b+3ab^2\cos(c+\right.$$

$$= -\frac{b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^2(a+b\cos(c+dx))^2\tan(c+dx)}{d} + \frac{1}{2}$$

$$= -\frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d} - \frac{b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^2(a+b\,c}{}$$

$$= \frac{1}{2}b^2\left(12a^2+b^2\right)x - \frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d} - \frac{b^2\left(2a^2-b^2\right)\cos(c+dx)\sin(c}{2d}$$

$$= \frac{1}{2}b^2\left(12a^2+b^2\right)x + \frac{4a^3b\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ab\left(a^2-2b^2\right)\sin(c+dx)}{d} - \frac{b^2}{}$$

Mathematica [A] time = 0.633494, size = 119, normalized size = 1.04

$$\frac{2b\left(b\left(12a^2+b^2\right)(c+dx)-8a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+8a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(b*(12*a^2 + b^2)*(c + d*x) - 8*a^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 8*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 16*a*b^3*Sin[c +
 d*x] + b^4*Sin[2*(c + d*x)] + 4*a^4*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.064, size = 109, normalized size = 1.

$$\frac{a^4\tan(dx+c)}{d} + 4\,\frac{a^3b\ln(\sec(dx+c)+\tan(dx+c))}{d} + 6\,a^2b^2x + 6\,\frac{a^2b^2c}{d} + 4\,\frac{ab^3\sin(dx+c)}{d} + \frac{b^4\cos(dx+c)\sin(dx}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x)
```

```
[Out] a^4*tan(d*x+c)/d+4/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6*a^2*b^2*x+6/d*a^2*b^
2*c+4/d*a*b^3*sin(d*x+c)+1/2/d*b^4*cos(d*x+c)*sin(d*x+c)+1/2*b^4*x+1/2/d*b^
4*c
```

Maxima [A] time = 0.985013, size = 122, normalized size = 1.07

$$\frac{24\,(dx+c)a^2b^2+(2\,dx+2\,c+\sin(2\,dx+2\,c))b^4+8\,a^3b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+16\,ab^3\sin(d}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(24*(d*x + c)*a^2*b^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*b^4 + 8*a^3*b*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 16*a*b^3*sin(d*x + c) + 4
*a^4*tan(d*x + c))/d
```

Fricas [A] time = 1.96007, size = 294, normalized size = 2.58

$$\frac{4\,a^3b\cos(dx+c)\log(\sin(dx+c)+1)-4\,a^3b\cos(dx+c)\log(-\sin(dx+c)+1)+\left(12\,a^2b^2+b^4\right)dx\cos(dx+c)+\left(\right.}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^3*b*cos(d*x + c)*log(sin(d*x + c) + 1) - 4*a^3*b*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (12*a^2*b^2 + b^4)*d*x*cos(d*x + c) + (b^4*cos(d*x + c
)^2 + 8*a*b^3*cos(d*x + c) + 2*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39385, size = 230, normalized size = 2.02

$$\frac{8\,a^3b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-8\,a^3b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\left(12\,a^2b^2+b^4\right)(dx+c)+\frac{2\left(8\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(8*a^3*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 8*a^3*b*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) - 4*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
 + (12*a^2*b^2 + b^4)*(d*x + c) + 2*(8*a*b^3*tan(1/2*d*x + 1/2*c)^3 - b^4*t
an(1/2*d*x + 1/2*c)^3 + 8*a*b^3*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.444 $\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx$

Optimal. Leaf size=108

$$-\frac{b^2(a^2-2b^2)\sin(c+dx)}{2d}+\frac{a^2(a^2+12b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3b\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos}{2d}$$

```
[Out] 4*a*b^3*x + (a^2*(a^2 + 12*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*(a^2 -
2*b^2)*Sin[c + d*x])/(2*d) + (3*a^3*b*Tan[c + d*x])/d + (a^2*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.252111, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3031, 3023, 2735, 3770}

$$-\frac{b^2(a^2-2b^2)\sin(c+dx)}{2d}+\frac{a^2(a^2+12b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^3b\tan(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)(a+b\cos}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```


```
[Out] 4*a*b^3*x + (a^2*(a^2 + 12*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*(a^2 -
2*b^2)*Sin[c + d*x])/(2*d) + (3*a^3*b*Tan[c + d*x])/d + (a^2*(a + b*Cos[c +
 d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^3(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int (a+b\cos(c+dx))\left(6a^2\right.\\ &= \frac{3a^3b\tan(c+dx)}{d}+\frac{a^2(a+b\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}\int \left(-\right.\\ &= -\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{2d}+\frac{3a^3b\tan(c+dx)}{d}+\frac{a^2(a+b\cos(c+dx))^2\sec(c}{2d}\\ &= 4ab^3x-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{2d}+\frac{3a^3b\tan(c+dx)}{d}+\frac{a^2(a+b\cos(c+dx)}{}\\ &= 4ab^3x+\frac{a^2\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\left(a^2-2b^2\right)\sin(c+dx)}{2d}+\frac{3a}{}\end{aligned}$$

Mathematica [A] time = 2.4354, size = 174, normalized size = 1.61

$$\frac{a\left(-2a\left(a^2+12b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2a\left(a^2+12b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^3,x]
```

```
[Out] (a*(16*b^3*c + 16*b^3*d*x - 2*a*(a^2 + 12*b^2)*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 2*a*(a^2 + 12*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ a^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^3/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2) + 4*b^4*Sin[c + d*x] + 16*a^3*b*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.067, size = 114, normalized size = 1.1

$$\frac{a^4\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{2d}+4\frac{a^3b\tan(dx+c)}{d}+6\frac{a^2b^2\ln(\sec(dx+c)+\tan(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x)
```

```
[Out] 1/2*a^4*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b
*tan(d*x+c)/d+6/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+4*a*b^3*x+4/d*a*b^3*c+1
/d*b^4*sin(d*x+c)
```

Maxima [A] time = 0.975915, size = 155, normalized size = 1.44

$$\frac{16\,(dx+c)ab^3 - a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 12\,a^2b^2(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(16*(d*x + c)*a*b^3 - a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^2*b^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) + 4*b^4*sin(d*x + c) + 16*a^3*b*tan(d*x + c))/d
```

Fricas [A] time = 2.02003, size = 324, normalized size = 3.

$$\frac{16\,ab^3dx\cos\left(dx+c\right)^2 + \left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - \left(a^4+12\,a^2b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a*b^3*d*x*cos(d*x + c)^2 + (a^4 + 12*a^2*b^2)*cos(d*x + c)^2*log(si
n(d*x + c) + 1) - (a^4 + 12*a^2*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
+ 2*(2*b^4*cos(d*x + c)^2 + 8*a^3*b*cos(d*x + c) + a^4)*sin(d*x + c))/(d*co
s(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47515, size = 239, normalized size = 2.21

$$\frac{8\,(dx+c)ab^3 + \frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + \left(a^4+12\,a^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - \left(a^4+12\,a^2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(8*(d*x + c)*a*b^3 + 4*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2
 + 1) + (a^4 + 12*a^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (a^4 + 12*a
^2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^4*tan(1/2*d*x + 1/2*c)^3
- 8*a^3*b*tan(1/2*d*x + 1/2*c)^3 + a^4*tan(1/2*d*x + 1/2*c) + 8*a^3*b*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.445 $\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{2ab\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^3b\tan(c+dx)\sec(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(}{}$$

```
[Out] b^4*x + (2*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^
2)*Tan[c + d*x])/(3*d) + (4*a^3*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (a^2*(
a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.251554, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3031, 3021, 2735, 3770}

$$\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{2ab\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^3b\tan(c+dx)\sec(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

```
[Out] b^4*x + (2*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/d + (a^2*(2*a^2 + 17*b^
2)*Tan[c + d*x])/(3*d) + (4*a^3*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + (a^2*(
a + b*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
```

```
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec^4(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\cos(c+dx))\left(8a\right.$$
$$= \frac{4a^3b\sec(c+dx)\tan(c+dx)}{3d}+\frac{a^2(a+b\cos(c+dx))^2\sec^2(c+dx)\tan(c+d}{3d}$$
$$= \frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{4a^3b\sec(c+dx)\tan(c+dx)}{3d}+\frac{a^2(a+b\cos(c}{}$$
$$= b^4x+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{4a^3b\sec(c+dx)\tan(c+dx)}{3d}+\frac{a^2(a+b}{}$$
$$= b^4x+\frac{2ab\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\left(2a^2+17b^2\right)\tan(c+dx)}{3d}+\frac{4}{}$$

Mathematica [A] time = 0.387917, size = 77, normalized size = 0.67

$$\frac{6ab\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))+3a^2\tan(c+dx)\left(a^2+2ab\sec(c+dx)+6b^2\right)+a^4\tan^3(c+dx)+3b^4dx}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^4,x]
```

```
[Out] (3*b^4*d*x + 6*a*b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]] + 3*a^2*(a^2 + 6*b^2
 + 2*a*b*Sec[c + d*x])*Tan[c + d*x] + a^4*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.069, size = 135, normalized size = 1.2

$$\frac{2a^4\tan(dx+c)}{3d}+\frac{a^4\tan(dx+c)(\sec(dx+c))^2}{3d}+2\frac{a^3b\sec(dx+c)\tan(dx+c)}{d}+2\frac{a^3b\ln(\sec(dx+c)+\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x)
```

```
[Out] 2/3*a^4*tan(d*x+c)/d+1/3/d*a^4*tan(d*x+c)*sec(d*x+c)^2+2*a^3*b*sec(d*x+c)*t
an(d*x+c)/d+2/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+6/d*a^2*b^2*tan(d*x+c)+4/d*
```

```
a*b^3*ln(sec(d*x+c)+tan(d*x+c))+b^4*x+1/d*b^4*c
```

Maxima [A] time = 0.976064, size = 169, normalized size = 1.47

$$\frac{\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{4}+3\,(dx+c)b^{4}-3\,a^{3}b\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+6}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 3*(d*x + c)*b^4 - 3*a^3*b*(2*s
in(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
 - 1)) + 6*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 18*a^2*b
^2*tan(d*x + c))/d
```

Fricas [A] time = 2.0155, size = 339, normalized size = 2.95

$$\frac{3\,b^{4}dx\cos\left(dx+c\right)^{3}+3\left(a^{3}b+2\,ab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^{3}b+2\,ab^{3}\right)\cos\left(dx+c\right)^{3}\log\left(-\sin\left(dx+c\right.\right.}{3\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*d*x*cos(d*x + c)^3 + 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3*log(sin(
d*x + c) + 1) - 3*(a^3*b + 2*a*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 (6*a^3*b*cos(d*x + c) + a^4 + 2*(a^4 + 9*a^2*b^2)*cos(d*x + c)^2)*sin(d*x
+ c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34265, size = 298, normalized size = 2.59

$$3\,(dx+c)b^{4}+6\left(a^{3}b+2\,ab^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\left(a^{3}b+2\,ab^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*b^4 + 6*(a^3*b + 2*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 6*(a^3*b + 2*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*a^4*tan(
1/2*d*x + 1/2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*
x + 1/2*c)^5 - 2*a^4*tan(1/2*d*x + 1/2*c)^3 - 36*a^2*b^2*tan(1/2*d*x + 1/2*
c)^3 + 3*a^4*tan(1/2*d*x + 1/2*c) + 6*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b
^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.446 $\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx$

Optimal. Leaf size=154

$$\frac{4ab\left(2a^2+3b^2\right)\tan(c+dx)}{3d}+\frac{\left(24a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\left(3a^2+22b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{5a}{}$$

```
[Out] ((3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a*b*(2*a^2
+ 3*b^2)*Tan[c + d*x])/(3*d) + (a^2*(3*a^2 + 22*b^2)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (5*a^3*b*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a^2*(a + b*Cos[
c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.335893, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3031, 3021, 2748, 3767, 8, 3770}

$$\frac{4ab\left(2a^2+3b^2\right)\tan(c+dx)}{3d}+\frac{\left(24a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\left(3a^2+22b^2\right)\tan(c+dx)\sec(c+dx)}{8d}+\frac{5a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

```
[Out] ((3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a*b*(2*a^2
+ 3*b^2)*Tan[c + d*x])/(3*d) + (a^2*(3*a^2 + 22*b^2)*Sec[c + d*x]*Tan[c + d
*x])/(8*d) + (5*a^3*b*Sec[c + d*x]^2*Tan[c + d*x])/(6*d) + (a^2*(a + b*Cos[
c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^5(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (a+b\cos(c+dx))\left(10\right.\\ &= \frac{5a^3b\sec^2(c+dx)\tan(c+dx)}{6d}+\frac{a^2(a+b\cos(c+dx))^2\sec^3(c+dx)\tan(c+\\ &= \frac{a^2\left(3a^2+22b^2\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{5a^3b\sec^2(c+dx)\tan(c+dx)}{6d}+\\ &= \frac{a^2\left(3a^2+22b^2\right)\sec(c+dx)\tan(c+dx)}{8d}+\frac{5a^3b\sec^2(c+dx)\tan(c+dx)}{6d}+\\ &= \frac{\left(3a^4+24a^2b^2+8b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\left(3a^2+22b^2\right)\sec(c+dx)\tan}{8d}\\ &= \frac{\left(3a^4+24a^2b^2+8b^4\right)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4ab\left(2a^2+3b^2\right)\tan(c+dx)}{3d}+\end{aligned}$$

Mathematica [A] time = 0.498054, size = 101, normalized size = 0.66

$$\frac{3\left(24a^2b^2+3a^4+8b^4\right)\tanh^{-1}(\sin(c+dx))+a\tan(c+dx)\left(32b\left(3\left(a^2+b^2\right)+a^2\tan^2(c+dx)\right)+9a\left(a^2+8b^2\right)\sec(c\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^5,x]
```

```
[Out] (3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]] + a*Tan[c + d*x]*(9*a
*(a^2 + 8*b^2)*Sec[c + d*x] + 6*a^3*Sec[c + d*x]^3 + 32*b*(3*(a^2 + b^2) +
```

```
a^2*Tan[c + d*x]^2)))/(24*d)
```

Maple [A] time = 0.074, size = 188, normalized size = 1.2

$$\frac{a^4 (\sec (dx+c))^3 \tan (dx+c)}{4d} + \frac{3a^4 \sec (dx+c) \tan (dx+c)}{8d} + \frac{3a^4 \ln (\sec (dx+c) + \tan (dx+c))}{8d} + \frac{8a^3 b \tan (dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x)
```

```
[Out] 1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^4*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^4
*ln(sec(d*x+c)+tan(d*x+c))+8/3*a^3*b*tan(d*x+c)/d+4/3*a^3*b*sec(d*x+c)^2*ta
n(d*x+c)/d+3/d*a^2*b^2*tan(d*x+c)*sec(d*x+c)+3/d*a^2*b^2*ln(sec(d*x+c)+tan(
d*x+c))+4/d*a*b^3*tan(d*x+c)+1/d*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.989196, size = 252, normalized size = 1.64

$$64 \left(\tan (dx+c)^3 + 3 \tan (dx+c)\right) a^3 b - 3a^4 \left(\frac{2 \left(3 \sin(dx+c)^3 - 5 \sin(dx+c)\right)}{\sin(dx+c)^4 - 2 \sin(dx+c)^2 + 1} - 3 \log (\sin (dx+c) + 1) + 3 \log (\sin (dx+c) - 1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(64*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3*b - 3*a^4*(2*(3*sin(d*x + c)
^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 72*a^2*b^2*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*b^4*(lo
g(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 192*a*b^3*tan(d*x + c))/d
```

Fricas [A] time = 1.89799, size = 394, normalized size = 2.56

$$\frac{3 \left(3a^4 + 24a^2b^2 + 8b^4\right) \cos (dx+c)^4 \log (\sin (dx+c) + 1) - 3 \left(3a^4 + 24a^2b^2 + 8b^4\right) \cos (dx+c)^4 \log (-\sin (dx+c) + 1}{48 d \cos (dx +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(
32*a^3*b*cos(d*x + c) + 6*a^4 + 32*(2*a^3*b + 3*a*b^3)*cos(d*x + c)^3 + 9*(
a^4 + 8*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43225, size = 486, normalized size = 3.16

$$\frac{3\left(3a^4+24a^2b^2+8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(3a^4+24a^2b^2+8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(15a^4\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 + 24*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(3*a^4 + 24*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(15*a^
4*tan(1/2*d*x + 1/2*c)^7 - 96*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 72*a^2*b^2*tan
(1/2*d*x + 1/2*c)^7 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 9*a^4*tan(1/2*d*x +
 1/2*c)^5 + 160*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 72*a^2*b^2*tan(1/2*d*x + 1/2
*c)^5 + 288*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*a^4*tan(1/2*d*x + 1/2*c)^3 - 1
60*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*a
*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*a^4*tan(1/2*d*x + 1/2*c) + 96*a^3*b*tan(1/
2*d*x + 1/2*c) + 72*a^2*b^2*tan(1/2*d*x + 1/2*c) + 96*a*b^3*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.447 $\int (a+b\cos(c+dx))^4 \sec^6(c+dx)\,dx$

Optimal. Leaf size=188

$$\frac{(60a^2b^2+8a^4+15b^4)\tan(c+dx)}{15d}+\frac{ab(3a^2+4b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(4a^2+27b^2)\tan(c+dx)\sec^2(c+dx)}{15d}+\cdots$$

```
[Out] (a*b*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + ((8*a^4 + 60*a^2*b^2 +
15*b^4)*Tan[c + d*x])/(15*d) + (a*b*(3*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*
x])/(2*d) + (a^2*(4*a^2 + 27*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (3*
a^3*b*Sec[c + d*x]^3*Tan[c + d*x])/(5*d) + (a^2*(a + b*Cos[c + d*x])^2*Sec[
c + d*x]^4*Tan[c + d*x])/(5*d)
```

Rubi [A] time = 0.36073, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2792, 3031, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(60a^2b^2+8a^4+15b^4)\tan(c+dx)}{15d}+\frac{ab(3a^2+4b^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(4a^2+27b^2)\tan(c+dx)\sec^2(c+dx)}{15d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```


```
[Out] (a*b*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]])/(2*d) + ((8*a^4 + 60*a^2*b^2 +
15*b^4)*Tan[c + d*x])/(15*d) + (a*b*(3*a^2 + 4*b^2)*Sec[c + d*x]*Tan[c + d*
x])/(2*d) + (a^2*(4*a^2 + 27*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d) + (3*
a^3*b*Sec[c + d*x]^3*Tan[c + d*x])/(5*d) + (a^2*(a + b*Cos[c + d*x])^2*Sec[
c + d*x]^4*Tan[c + d*x])/(5*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^4\sec^6(c+dx)\,dx &= \frac{a^2(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int (a+b\cos(c+dx))\left(12\right.\\
&= \frac{3a^3b\sec^3(c+dx)\tan(c+dx)}{5d}+\frac{a^2(a+b\cos(c+dx))^2\sec^4(c+dx)\tan(c+\right.}{5d}\\
&= \frac{a^2\left(4a^2+27b^2\right)\sec^2(c+dx)\tan(c+dx)}{15d}+\frac{3a^3b\sec^3(c+dx)\tan(c+dx)}{5d}+\\
&= \frac{a^2\left(4a^2+27b^2\right)\sec^2(c+dx)\tan(c+dx)}{15d}+\frac{3a^3b\sec^3(c+dx)\tan(c+dx)}{5d}+\\
&= \frac{ab\left(3a^2+4b^2\right)\sec(c+dx)\tan(c+dx)}{2d}+\frac{a^2\left(4a^2+27b^2\right)\sec^2(c+dx)\tan(c}{15d}\\
&= \frac{ab\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\left(8a^4+60a^2b^2+15b^4\right)\tan(c+dx)}{15d}+
\end{aligned}$$

Mathematica [A] time = 0.732868, size = 125, normalized size = 0.66

$$\frac{15ab\left(3a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(20a^2\left(a^2+3b^2\right)\tan^2(c+dx)+15ab\left(3a^2+4b^2\right)\sec(c+dx)+3\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^6,x]
```

```
[Out] (15*a*b*(3*a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(30*(a^4 + 6*a
^2*b^2 + b^4) + 15*a*b*(3*a^2 + 4*b^2)*Sec[c + d*x] + 30*a^3*b*Sec[c + d*x]
^3 + 20*a^2*(a^2 + 3*b^2)*Tan[c + d*x]^2 + 6*a^4*Tan[c + d*x]^4))/(30*d)
```

Maple [A] time = 0.068, size = 225, normalized size = 1.2

$$\frac{8\,a^4\tan(dx+c)}{15\,d}+\frac{a^4\tan(dx+c)\left(\sec(dx+c)\right)^4}{5\,d}+\frac{4\,a^4\tan(dx+c)\left(\sec(dx+c)\right)^2}{15\,d}+\frac{a^3b\left(\sec(dx+c)\right)^3\tan(dx+c)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x)
```

```
[Out] 8/15*a^4*tan(d*x+c)/d+1/5/d*a^4*tan(d*x+c)*sec(d*x+c)^4+4/15/d*a^4*tan(d*x+
c)*sec(d*x+c)^2+a^3*b*sec(d*x+c)^3*tan(d*x+c)/d+3/2*a^3*b*sec(d*x+c)*tan(d*
x+c)/d+3/2/d*a^3*b*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^2*b^2*tan(d*x+c)+2/d*a^2
*b^2*tan(d*x+c)*sec(d*x+c)^2+2/d*a*b^3*tan(d*x+c)*sec(d*x+c)+2/d*a*b^3*ln(s
ec(d*x+c)+tan(d*x+c))+1/d*b^4*tan(d*x+c)
```

Maxima [A] time = 1.00131, size = 263, normalized size = 1.4

$$4\left(3\,\tan(dx+c)^5+10\,\tan(dx+c)^3+15\,\tan(dx+c)\right)a^4+120\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)a^2b^2-15\,a^3b\left(\frac{2\left(3\,\sin(dx-\right.}{\sin(dx+c)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/60*(4*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 120*
(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2*b^2 - 15*a^3*b*(2*(3*sin(d*x + c)^3 -
 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 60*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)
^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 60*b^4*tan(d*x +
 c))/d
```

Fricas [A] time = 1.9966, size = 443, normalized size = 2.36

$$15\left(3\,a^3b+4\,ab^3\right)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\left(3\,a^3b+4\,ab^3\right)\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(30\,a^3b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/60*(15*(3*a^3*b + 4*a*b^3)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(3*a
^3*b + 4*a*b^3)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(30*a^3*b*cos(d*x
```

```
 + c) + 2*(8*a^4 + 60*a^2*b^2 + 15*b^4)*cos(d*x + c)^4 + 6*a^4 + 15*(3*a^3*
b + 4*a*b^3)*cos(d*x + c)^3 + 4*(2*a^4 + 15*a^2*b^2)*cos(d*x + c)^2)*sin(d*
x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**6,x)

[Out] Timed out
```

Giac [B] time = 1.43402, size = 622, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^6,x, algorithm="giac")

[Out] 1/30*(15*(3*a^3*b + 4*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(3*a^3
*b + 4*a*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*a^4*tan(1/2*d*x +
1/2*c)^9 - 75*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 180*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 60*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 30*b^4*tan(1/2*d*x + 1/2*c)^9 - 40
*a^4*tan(1/2*d*x + 1/2*c)^7 + 30*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 480*a^2*b^2
*tan(1/2*d*x + 1/2*c)^7 + 120*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 120*b^4*tan(1/
2*d*x + 1/2*c)^7 + 116*a^4*tan(1/2*d*x + 1/2*c)^5 + 600*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^5 + 180*b^4*tan(1/2*d*x + 1/2*c)^5 - 40*a^4*tan(1/2*d*x + 1/2*c)^
3 - 30*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 480*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 -
120*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 120*b^4*tan(1/2*d*x + 1/2*c)^3 + 30*a^4*
tan(1/2*d*x + 1/2*c) + 75*a^3*b*tan(1/2*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 60*a*b^3*tan(1/2*d*x + 1/2*c) + 30*b^4*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

3.448 $\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx$

Optimal. Leaf size=222

$$\frac{4ab\left(4a^2+5b^2\right)\tan^3(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\tan(c+dx)}{5d}+\frac{\left(36a^2b^2+5a^4+8b^4\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^2\left(5a^2+32\right.}{}$$

```
[Out] ((5*a^4 + 36*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a*b*(4*a^2
 + 5*b^2)*Tan[c + d*x])/(5*d) + ((5*a^4 + 36*a^2*b^2 + 8*b^4)*Sec[c + d*x]*
Tan[c + d*x])/(16*d) + (a^2*(5*a^2 + 32*b^2)*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) + (7*a^3*b*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (a^2*(a + b*Cos[c +
d*x])^2*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a*b*(4*a^2 + 5*b^2)*Tan[c +
 d*x]^3)/(15*d)
```

Rubi [A] time = 0.379992, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3031, 3021, 2748, 3767, 3768, 3770}

$$\frac{4ab\left(4a^2+5b^2\right)\tan^3(c+dx)}{15d}+\frac{4ab\left(4a^2+5b^2\right)\tan(c+dx)}{5d}+\frac{\left(36a^2b^2+5a^4+8b^4\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^2\left(5a^2+32\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```


```
[Out] ((5*a^4 + 36*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a*b*(4*a^2
 + 5*b^2)*Tan[c + d*x])/(5*d) + ((5*a^4 + 36*a^2*b^2 + 8*b^4)*Sec[c + d*x]*
Tan[c + d*x])/(16*d) + (a^2*(5*a^2 + 32*b^2)*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) + (7*a^3*b*Sec[c + d*x]^4*Tan[c + d*x])/(15*d) + (a^2*(a + b*Cos[c +
d*x])^2*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) + (4*a*b*(4*a^2 + 5*b^2)*Tan[c +
 d*x]^3)/(15*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^4\sec^7(c+dx)\,dx = \frac{a^2(a+b\cos(c+dx))^2\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{6}\int (a+b\cos(c+dx))\,(14$$

$$= \frac{7a^3b\sec^4(c+dx)\tan(c+dx)}{15d}+\frac{a^2(a+b\cos(c+dx))^2\sec^5(c+dx)\tan(c+$$

$$= \frac{a^2\left(5a^2+32b^2\right)\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{7a^3b\sec^4(c+dx)\tan(c+dx)}{15d}+$$

$$= \frac{a^2\left(5a^2+32b^2\right)\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{7a^3b\sec^4(c+dx)\tan(c+dx)}{15d}+$$

$$= \frac{\left(5a^4+36a^2b^2+8b^4\right)\sec(c+dx)\tan(c+dx)}{16d}+\frac{a^2\left(5a^2+32b^2\right)\sec^3(c+dx)}{24d}$$

$$= \frac{\left(5a^4+36a^2b^2+8b^4\right)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4ab\left(4a^2+5b^2\right)\tan(c+dx)}{5d}+$$

Mathematica [A] time = 0.982429, size = 154, normalized size = 0.69

$$\frac{15\left(36a^2b^2+5a^4+8b^4\right)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(64ab\left(5\left(2a^2+b^2\right)\tan^2(c+dx)+15\left(a^2+b^2\right)+3a^2\tan\right.\right.}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^4*Sec[c + d*x]^7,x]
```

```
[Out] (15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*(
5*a^4 + 36*a^2*b^2 + 8*b^4)*Sec[c + d*x] + 10*a^2*(5*a^2 + 36*b^2)*Sec[c +
d*x]^3 + 40*a^4*Sec[c + d*x]^5 + 64*a*b*(15*(a^2 + b^2) + 5*(2*a^2 + b^2)*T
an[c + d*x]^2 + 3*a^2*Tan[c + d*x]^4)))/(240*d)
```

Maple [A] time = 0.072, size = 302, normalized size = 1.4

$$\frac{a^4 (\sec (dx+c))^5 \tan (dx+c)}{6d}+\frac{5a^4 (\sec (dx+c))^3 \tan (dx+c)}{24d}+\frac{5a^4 \sec (dx+c) \tan (dx+c)}{16d}+\frac{5a^4 \ln (\sec (dx+c)+}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x)
```

```
[Out] 1/6*a^4*sec(d*x+c)^5*tan(d*x+c)/d+5/24*a^4*sec(d*x+c)^3*tan(d*x+c)/d+5/16*a
^4*sec(d*x+c)*tan(d*x+c)/d+5/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+32/15*a^3*b
*tan(d*x+c)/d+4/5*a^3*b*sec(d*x+c)^4*tan(d*x+c)/d+16/15*a^3*b*sec(d*x+c)^2*
tan(d*x+c)/d+3/2/d*a^2*b^2*tan(d*x+c)*sec(d*x+c)^3+9/4/d*a^2*b^2*tan(d*x+c)
*sec(d*x+c)+9/4/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c))+8/3/d*a*b^3*tan(d*x+c)+
4/3/d*a*b^3*tan(d*x+c)*sec(d*x+c)^2+1/2/d*b^4*tan(d*x+c)*sec(d*x+c)+1/2/d*b
^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.992319, size = 371, normalized size = 1.67

$$128\left(3 \tan (dx+c)^5+10 \tan (dx+c)^3+15 \tan (dx+c)\right)a^3b+640\left(\tan (dx+c)^3+3 \tan (dx+c)\right)ab^3-5a^4\left(\frac{2(15 \sin(d}{\sin(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/480*(128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3*b +
 640*(tan(d*x + c)^3 + 3*tan(d*x + c))*a*b^3 - 5*a^4*(2*(15*sin(d*x + c)^5
- 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1
)) - 180*a^2*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2
*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) -
 120*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log
(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 2.11043, size = 528, normalized size = 2.38

$$15\left(5a^4+36a^2b^2+8b^4\right)\cos (dx+c)^6 \log (\sin (dx+c)+1)-15\left(5a^4+36a^2b^2+8b^4\right)\cos (dx+c)^6 \log (-\sin (dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1)
 - 15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) +
2*(128*(4*a^3*b + 5*a*b^3)*cos(d*x + c)^5 + 192*a^3*b*cos(d*x + c) + 15*(5*
a^4 + 36*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 40*a^4 + 64*(4*a^3*b + 5*a*b^3)*
cos(d*x + c)^3 + 10*(5*a^4 + 36*a^2*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*c
os(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**4*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49498, size = 799, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^4*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
 15*(5*a^4 + 36*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(16
5*a^4*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 900*a^2
*b^2*tan(1/2*d*x + 1/2*c)^11 - 960*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*b^4*
tan(1/2*d*x + 1/2*c)^11 + 25*a^4*tan(1/2*d*x + 1/2*c)^9 + 2240*a^3*b*tan(1/
2*d*x + 1/2*c)^9 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 3520*a*b^3*tan(1/2
*d*x + 1/2*c)^9 - 360*b^4*tan(1/2*d*x + 1/2*c)^9 + 450*a^4*tan(1/2*d*x + 1/
2*c)^7 - 4992*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 360*a^2*b^2*tan(1/2*d*x + 1/2*
c)^7 - 5760*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 240*b^4*tan(1/2*d*x + 1/2*c)^7 +
 450*a^4*tan(1/2*d*x + 1/2*c)^5 + 4992*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 360*a
^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 5760*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*b^4
*tan(1/2*d*x + 1/2*c)^5 + 25*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*a^3*b*tan(1/
2*d*x + 1/2*c)^3 - 1260*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3520*a*b^3*tan(1/2
*d*x + 1/2*c)^3 - 360*b^4*tan(1/2*d*x + 1/2*c)^3 + 165*a^4*tan(1/2*d*x + 1/
2*c) + 960*a^3*b*tan(1/2*d*x + 1/2*c) + 900*a^2*b^2*tan(1/2*d*x + 1/2*c) +
960*a*b^3*tan(1/2*d*x + 1/2*c) + 120*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 - 1)^6)/d
```

3.449 $\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{a(3a^2+2b^2)\sin(c+dx)}{3b^4d}-\frac{2a^5\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+b}}+\frac{(4a^2+3b^2)\sin(c+dx)\cos(c+dx)}{8b^3d}+\frac{x(4a^2b^2+8a^4+3b^4)}{8b^5}$$

```
[Out] ((8*a^4 + 4*a^2*b^2 + 3*b^4)*x)/(8*b^5) - (2*a^5*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^5*Sqrt[a + b]*d) - (a*(3*a^2 + 2*b
^2)*Sin[c + d*x])/(3*b^4*d) + ((4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(
8*b^3*d) - (a*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(4*b*d)
```

Rubi [A] time = 0.544933, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3049, 3023, 2735, 2659, 205}

$$-\frac{a(3a^2+2b^2)\sin(c+dx)}{3b^4d}-\frac{2a^5\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d\sqrt{a-b}\sqrt{a+b}}+\frac{(4a^2+3b^2)\sin(c+dx)\cos(c+dx)}{8b^3d}+\frac{x(4a^2b^2+8a^4+3b^4)}{8b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x]),x]
```


```
[Out] ((8*a^4 + 4*a^2*b^2 + 3*b^4)*x)/(8*b^5) - (2*a^5*ArcTan[(Sqrt[a - b]*Tan[(c
 + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^5*Sqrt[a + b]*d) - (a*(3*a^2 + 2*b
^2)*Sin[c + d*x])/(3*b^4*d) + ((4*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(
8*b^3*d) - (a*Cos[c + d*x]^2*Sin[c + d*x])/(3*b^2*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(4*b*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_^2))^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\int \frac{\cos^2(c+dx)\left(3a+3b\cos(c+dx)-4a\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{4b}$$

$$= -\frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\int \frac{\cos(c+dx)\left(-8a^2+ab\cos(c+dx)+3\left(4a^2+3\right.\right.}{a+b\cos(c+dx)}}{12b^2}$$

$$= \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos^2(c+dx)\sin(c+dx)}{3b^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4bd}$$

$$= -\frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos^2(c+dx)\sin(c+a}{3b^2d}$$

$$= \frac{\left(8a^4+4a^2b^2+3b^4\right)x}{8b^5} - \frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} -$$

$$= \frac{\left(8a^4+4a^2b^2+3b^4\right)x}{8b^5} - \frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d} + \frac{\left(4a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{8b^3d} -$$

$$= \frac{\left(8a^4+4a^2b^2+3b^4\right)x}{8b^5} - \frac{2a^5\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^5\sqrt{a+b}\,d} - \frac{a\left(3a^2+2b^2\right)\sin(c+dx)}{3b^4d} + \left(4a^2+\right.$$

Mathematica [A] time = 0.645041, size = 153, normalized size = 0.79

$$\frac{12\left(4a^2b^2+8a^4+3b^4\right)(c+dx)-24ab\left(4a^2+3b^2\right)\sin(c+dx)+24b^2\left(a^2+b^2\right)\sin(2(c+dx))+\frac{192a^5\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x]),x]
```

```
[Out] (12*(8*a^4 + 4*a^2*b^2 + 3*b^4)*(c + d*x) + (192*a^5*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 24*a*b*(4*a^2 + 3*b^2)*S
in[c + d*x] + 24*b^2*(a^2 + b^2)*Sin[2*(c + d*x)] - 8*a*b^3*Sin[3*(c + d*x)
] + 3*b^4*Sin[4*(c + d*x)])/(96*b^5*d)
```

Maple [B] time = 0.115, size = 672, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^3-1/d/b^3/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*a^2-2/d/b^2/(1+tan(1/2*d*x+1/2*c)
^2)^4*tan(1/2*d*x+1/2*c)^7*a-5/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x
+1/2*c)^7-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a^3-10/3/
d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*a+3/4/d/b/(1+tan(1/2*
d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan
(1/2*d*x+1/2*c)^5*a^2+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)
^3*a^2-3/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+t
an(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a^3-10/3/d/b^2/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)*a^2+5/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-2/d/
b^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a^3-2/d/b^2/(1+tan(1/2*d*
x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*a+2/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^4+1
/d/b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+3/4/d/b*arctan(tan(1/2*d*x+1/2*c))-2/
d*a^5/b^5/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.25181, size = 1042, normalized size = 5.4

$$\left[-\frac{12\sqrt{-a^2+b^2}a^5\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)-3\left(8a^6-4a^4b^2-a^2b^4-3\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/24*(12*sqrt(-a^2 + b^2)*a^5*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos
(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 +
2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 3*(8*a^6 - 4*a^4*
b^2 - a^2*b^4 - 3*b^6)*d*x + (24*a^5*b - 8*a^3*b^3 - 16*a*b^5 - 6*(a^2*b^4
- b^6)*cos(d*x + c)^3 + 8*(a^3*b^3 - a*b^5)*cos(d*x + c)^2 - 3*(4*a^4*b^2 -
 a^2*b^4 - 3*b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d), -1/24*(2
4*sqrt(a^2 - b^2)*a^5*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
 + c))) - 3*(8*a^6 - 4*a^4*b^2 - a^2*b^4 - 3*b^6)*d*x + (24*a^5*b - 8*a^3*b
^3 - 16*a*b^5 - 6*(a^2*b^4 - b^6)*cos(d*x + c)^3 + 8*(a^3*b^3 - a*b^5)*cos(
d*x + c)^2 - 3*(4*a^4*b^2 - a^2*b^4 - 3*b^6)*cos(d*x + c))*sin(d*x + c))/((
a^2*b^5 - b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38541, size = 531, normalized size = 2.75

$$\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a^5}{\sqrt{a^2-b^2}b^5}+\frac{3\left(8a^4+4a^2b^2+3b^4\right)(dx+c)}{b^5}-\frac{2\left(24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12a^2b\tan\left(\frac{1}{2}dx+\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^5/(sqrt(a^2
 - b^2)*b^5) + 3*(8*a^4 + 4*a^2*b^2 + 3*b^4)*(d*x + c)/b^5 - 2*(24*a^3*tan(
1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 24*a*b^2*tan(1/2*d*x
 + 1/2*c)^7 + 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 72*a^3*tan(1/2*d*x + 1/2*c)^5
 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*b^
3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^2*b*tan(1/2
*d*x + 1/2*c)^3 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*b^3*tan(1/2*d*x + 1/2
*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c) - 12*a^2*b*tan(1/2*d*x + 1/2*c) + 24*a*
```

```
b^2*tan(1/2*d*x + 1/2*c) - 15*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2
*c)^2 + 1)^4*b^4))/d
```

3.450 $\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{(3a^2+2b^2)\sin(c+dx)}{3b^3d}+\frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{ax(2a^2+b^2)}{2b^4}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{\sin(c+dx)\cos^2}{3bd}$$

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3*a^2 + 2*b^2)*Sin[c + d
*x])/(3*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]^2*
Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.326544, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2793, 3049, 3023, 2735, 2659, 205}

$$\frac{(3a^2+2b^2)\sin(c+dx)}{3b^3d}+\frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\sqrt{a+b}}-\frac{ax(2a^2+b^2)}{2b^4}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{\sin(c+dx)\cos^2}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(a*(2*a^2 + b^2)*x)/(2*b^4) + (2*a^4*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) + ((3*a^2 + 2*b^2)*Sin[c + d
*x])/(3*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]^2*
Sin[c + d*x])/(3*b*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{\cos(c+dx)\left(2a+2b\cos(c+dx)-3a\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{3b}$$

$$= -\frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{-3a^2+ab\cos(c+dx)+2\left(3a^2+2b^2\right)\cos^2(c+dx)}{a+b\cos(c+dx)}\,d}{6b^2}$$

$$= \frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{3bd} + \frac{\int \frac{-3a^2b-3a(2a}{a+b\,c}}{6}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+}{3bd}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos^2(c+dx)\sin(c+}{3bd}$$

$$= -\frac{a\left(2a^2+b^2\right)x}{2b^4} + \frac{2a^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}d} + \frac{\left(3a^2+2b^2\right)\sin(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c}{2b^2d}$$

Mathematica [A] time = 0.330407, size = 122, normalized size = 0.82

$$\frac{-6a\left(2a^2+b^2\right)(c+dx)+3b\left(4a^2+3b^2\right)\sin(c+dx)-\frac{24a^4\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-3ab^2\sin(2(c+dx))+b^3\sin(3(c+dx))}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-6*a*(2*a^2 + b^2)*(c + d*x) - (24*a^4*ArcTanh[((a - b)*Tan[(c + d*x)/2])/
Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sin[c + d*x] - 3*
a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*b^4*d)
```

Maple [B] time = 0.086, size = 367, normalized size = 2.5

$$2\,{\frac { \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{5}{a}^{2}}{d{b}^{3} \left(1+ \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2} \right) ^{3}}}+{\frac {a}{{b}^{2}d} \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) \right) ^{5} \left(1+ \left(\tan \left({\frac {dx}{2}}+{\frac {c}{2}} \right) \right) ^{2} \right) ^{-3}}+2\,{\frac { \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{5}}{bd \left(1+ \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2} \right) ^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*a^2+1/d/b^2/(1+tan(
1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*a+2/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3
*tan(1/2*d*x+1/2*c)^5+4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)
^3*a^2+4/3/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+2/d/b^3/(1+t
an(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*a^2+2/d/b/(1+tan(1/2*d*x+1/2*c)^2
)^3*tan(1/2*d*x+1/2*c)-1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c
)*a-2/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^3-1/d/b^2*a*arctan(tan(1/2*d*x+1/2
*c))+2/d*a^4/b^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23132, size = 860, normalized size = 5.81

$$\left[-\frac{3\,\sqrt{-a^{2}+b^{2}}a^{4}\log\left(\frac{2\,ab\cos\left(dx+c\right)+\left(2\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+2\,\sqrt{-a^{2}+b^{2}}\left(a\cos\left(dx+c\right)+b\right)\sin\left(dx+c\right)-a^{2}+2\,b^{2}}{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}}\right)+3\,\left(2\,a^{5}-a^{3}b^{2}-ab^{4}\right)dx-\left(\right.}{6\,\left(a^{2}b^{4}-b^{6}\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d
*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*
b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(2*a^5 - a^3*b^2
- a*b^4)*d*x - (6*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^2*b^3 - b^5)*cos(d*x + c
```

```
)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d),
1/6*(6*sqrt(a^2 - b^2)*a^4*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*si
n(d*x + c))) - 3*(2*a^5 - a^3*b^2 - a*b^4)*d*x + (6*a^4*b - 2*a^2*b^3 - 4*b
^5 + 2*(a^2*b^3 - b^5)*cos(d*x + c)^2 - 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*s
in(d*x + c))/((a^2*b^4 - b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39326, size = 336, normalized size = 2.27

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a^4}{\sqrt{a^2-b^2}\,b^4}+\frac{3\left(2a^3+ab^2\right)(dx+c)}{b^4}-\frac{2\left(6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6b^2\tan\cdots\right.}{\cdots}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^4/(sqrt(a^2
 - b^2)*b^4) + 3*(2*a^3 + a*b^2)*(d*x + c)/b^4 - 2*(6*a^2*tan(1/2*d*x + 1/2
*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^5 + 12*a^
2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x
 + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

3.451 $\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)}{2b^3}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*Sin[c + d*x])/(b^2*d) + (C
os[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.184406, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2793, 3023, 2735, 2659, 205}

$$-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(2a^2+b^2\right)}{2b^3}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*a^2 + b^2)*x)/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) - (a*Sin[c + d*x])/(b^2*d) + (C
os[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{\cos(c+dx)\sin(c+dx)}{2bd}+\frac{\int \frac{a+b\cos(c+dx)-2a\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2b}\\ &= -\frac{a\sin(c+dx)}{b^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}+\frac{\int \frac{ab+\left(2a^2+b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{2b^2}\\ &= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{a\sin(c+dx)}{b^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}-\frac{a^3\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^3}\\ &= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{a\sin(c+dx)}{b^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}-\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\right.}{b^3d}\\ &= \frac{\left(2a^2+b^2\right)x}{2b^3}-\frac{2a^3\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2bd}\end{aligned}$$

Mathematica [A] time = 0.232169, size = 97, normalized size = 0.88

$$\frac{2\left(2a^2+b^2\right)(c+dx)+\frac{8a^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}-4ab\sin(c+dx)+b^2\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*(2*a^2 + b^2)*(c + d*x) + (8*a^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 4*a*b*Sin[c + d*x] + b^2*Sin[2*(c + d*x)]
)/(4*b^3*d)
```

Maple [B] time = 0.087, size = 222, normalized size = 2.

$$-2\,\frac{(\tan(1/2\,dx+c/2))^3\,a}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-\frac{1}{bd}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-2\,\frac{a\tan(1/2\,dx+c/2)}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*a-1/d/b/(1+tan(1/2
*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*ta
n(1/2*d*x+1/2*c)*a+1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+2/d/
b^3*arctan(tan(1/2*d*x+1/2*c))*a^2+1/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d*a^3
/b^3/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.11995, size = 716, normalized size = 6.51

$$\left[-\frac{\sqrt{-a^2+b^2}a^3\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)-\left(2a^4-a^2b^2-b^4\right)dx+\left(2a^3b\right.}{2\left(a^2b^3-b^5\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^3*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (2*a^4 - a^2*b^2 - b^
4)*d*x + (2*a^3*b - 2*a*b^3 - (a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/(
(a^2*b^3 - b^5)*d), -1/2*(2*sqrt(a^2 - b^2)*a^3*arctan(-(a*cos(d*x + c) + b
)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^4 - a^2*b^2 - b^4)*d*x + (2*a^3*b
- 2*a*b^3 - (a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - b^5)*d)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36231, size = 239, normalized size = 2.17

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a^3}{\sqrt{a^2-b^2}b^3}+\frac{(2a^2+b^2)(dx+c)}{b^3}-\frac{2\left(2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^3/(sqrt(a^2 -
 b^2)*b^3) + (2*a^2 + b^2)*(d*x + c)/b^3 - 2*(2*a*tan(1/2*d*x + 1/2*c)^3 +
b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

3.452 $\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sin(c+dx)}{bd}$$

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
Sqrt[a - b]*b^2*Sqrt[a + b]*d) + Sin[c + d*x]/(b*d)
```

Rubi [A] time = 0.118807, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2746, 12, 2735, 2659, 205}

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\sqrt{a+b}}-\frac{ax}{b^2}+\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*a^2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
Sqrt[a - b]*b^2*Sqrt[a + b]*d) + Sin[c + d*x]/(b*d)
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sin(c+dx)}{bd} - \frac{\int \frac{a\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}$$

$$= \frac{\sin(c+dx)}{bd} - \frac{a\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b}$$

$$= -\frac{ax}{b^2} + \frac{\sin(c+dx)}{bd} + \frac{a^2\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2}$$

$$= -\frac{ax}{b^2} + \frac{\sin(c+dx)}{bd} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2d}$$

$$= -\frac{ax}{b^2} + \frac{2a^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d} + \frac{\sin(c+dx)}{bd}$$

Mathematica [A] time = 0.140654, size = 72, normalized size = 0.95

$$\frac{-\frac{2a^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - a(c+dx) + b\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-(a*(c + d*x)) - (2*a^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + b*Sin[c + d*x])/(b^2*d)
```

Maple [A] time = 0.083, size = 102, normalized size = 1.3

$$2\,\frac{\tan(1/2\,dx+c/2)}{bd\left(1+(\tan(1/2\,dx+c/2))^2\right)} - 2\,\frac{a\arctan(\tan(1/2\,dx+c/2))}{b^2d} + 2\,\frac{a^2}{b^2d\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan(1/2\,dx+c/2)(a-b)}{\sqrt{(a-b)(a+b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-2/d/b^2*a*arctan(tan(1/2*
d*x+1/2*c))+2/d*a^2/b^2/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)
/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0802, size = 585, normalized size = 7.7

$$\left[-\frac{\sqrt{-a^2+b^2}a^2\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+2\left(a^3-ab^2\right)dx-2\left(a^2b-b^3\right)}{2\left(a^2b^2-b^4\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*a^2*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
 + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^
2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(a^3 - a*b^2)*d*x -
 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d), (sqrt(a^2 - b^2)*a^2*ar
ctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*b^2)*
d*x + (a^2*b - b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36573, size = 170, normalized size = 2.24

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)a^2}{\sqrt{a^2-b^2}b^2}+\frac{(dx+c)a}{b^2}-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a^2/(sqrt(a^2 - b^
2)*b^2) + (d*x + c)*a/b^2 - 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2
 + 1)*b))/d
```

3.453 $\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]
*b*Sqrt[a + b]*d)
```

Rubi [A] time = 0.056086, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2735, 2659, 205}

$$\frac{x}{b}-\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x]),x]
```

```
[Out] x/b - (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]
*b*Sqrt[a + b]*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{x}{b} - \frac{a\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}$$
$$= \frac{x}{b} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$
$$= \frac{x}{b} - \frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}d}$$

Mathematica [A] time = 0.0865724, size = 58, normalized size = 0.98

$$\frac{\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + c + dx}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (c + d*x + (2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[
-a^2 + b^2])/(b*d)
```

Maple [A] time = 0.079, size = 67, normalized size = 1.1

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{bd} - 2\,\frac{a}{bd\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan\left(1/2\,dx+c/2\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/b*arctan(tan(1/2*d*x+1/2*c))-2/d*a/b/((a-b)*(a+b))^(1/2)*arctan(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0909, size = 486, normalized size = 8.24

$$\left[\frac{2\left(a^2-b^2\right)dx - \sqrt{-a^2+b^2}a\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)}{2\left(a^2b-b^3\right)d}, \frac{\left(a^2-b^2\right)dx - \sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*a*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/((a^2
*b - b^3)*d), ((a^2 - b^2)*d*x - sqrt(a^2 - b^2)*a*arctan(-(a*cos(d*x + c)
+ b)/(sqrt(a^2 - b^2)*sin(d*x + c))))/((a^2*b - b^3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3971, size = 126, normalized size = 2.14

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}b}+\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a/(sqrt(a^2 - b^2)*
b) + (d*x + c)/b)/d
```

3.454 $\int \frac{1}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
+ b]*d)
```

Rubi [A] time = 0.0306421, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2659, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a
+ b]*d)
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a+b\cos(c+dx)}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] time = 0.035661, size = 48, normalized size = 0.98

$$-\frac{2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{d\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2]
*d)
```

Maple [A] time = 0.066, size = 44, normalized size = 0.9

$$2\,{\frac{1}{d\sqrt{(a-b)(a+b)}}\arctan\left({\frac{\tan(1/2\,dx+c/2)(a-b)}{\sqrt{(a-b)(a+b)}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00423, size = 404, normalized size = 8.24

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(a^2-b^2\right)d},\ \frac{\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\sin(dx+c)}\right)}{\sqrt{a^2-b^2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)
^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2))/((a^2 - b^2)*d), arctan(-(a*
cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))/(sqrt(a^2 - b^2)*d)]
```

Sympy [A] time = 10.2722, size = 172, normalized size = 3.51

$$\begin{cases} \frac{\tilde{\infty}x}{\cos(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{1}{bd\tan\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } a = -b \\ \frac{x}{a+b\cos(c)} & \text{for } d = 0 \\ \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for } a = b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} - \frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-bd\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/cos(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(b*d*tan(c/2 +
 d*x/2)), Eq(a, -b)), (x/(a + b*cos(c)), Eq(d, 0)), (tan(c/2 + d*x/2)/(b*d)
, Eq(a, b)), (log(-sqrt(-a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sq
rt(-a/(a - b) - b/(a - b)) - b*d*sqrt(-a/(a - b) - b/(a - b))) - log(sqrt(-
a/(a - b) - b/(a - b)) + tan(c/2 + d*x/2))/(a*d*sqrt(-a/(a - b) - b/(a - b)
) - b*d*sqrt(-a/(a - b) - b/(a - b))), True))
```

Giac [A] time = 1.34528, size = 105, normalized size = 2.14

$$-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*d)
```

3.455 $\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] (-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sq
rt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0722405, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2747, 3770, 2659, 205}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sq
rt[a + b]*d) + ArcTanh[Sin[c + d*x]]/(a*d)
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\int \sec(c+dx)\,dx}{a} - \frac{b\int \frac{1}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{ad} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$
$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d} + \frac{\tanh^{-1}(\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.0854344, size = 102, normalized size = 1.5

$$\frac{\frac{2b\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2
] - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]])/(a*d)
```

Maple [A] time = 0.09, size = 88, normalized size = 1.3

$$-\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + \frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - 2\,\frac{b}{da\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan(1/2\,dx+c/2)(a-b)}{\sqrt{(a-b)(a+b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/d/a*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a*ln(tan(1/2*d*x+1/2*c)+1)-2/d/a*b/((a-
b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.58225, size = 643, normalized size = 9.46

$$\left[-\frac{\sqrt{-a^2+b^2}b\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+}{2\left(a^3-ab^2\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)
/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (a^2 - b^2)*log(sin(d*x
 + c) + 1) + (a^2 - b^2)*log(-sin(d*x + c) + 1))/((a^3 - a*b^2)*d), -1/2*(2
*sqrt(a^2 - b^2)*b*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x +
c))) - (a^2 - b^2)*log(sin(d*x + c) + 1) + (a^2 - b^2)*log(-sin(d*x + c) +
1))/((a^3 - a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x)), x)
```

Giac [B] time = 1.40241, size = 161, normalized size = 2.37

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a
) - log(abs(tan(1/2*d*x + 1/2*c) + 1))/a + log(abs(tan(1/2*d*x + 1/2*c) - 1
))/a)/d
```

3.456 $\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}$$

```
[Out] (2*b^2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]
*Sqrt[a + b]*d) - (b*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a*d)
```

Rubi [A] time = 0.130359, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 12, 2747, 3770, 2659, 205}

$$\frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]
*Sqrt[a + b]*d) - (b*ArcTanh[Sin[c + d*x]])/(a^2*d) + Tan[c + d*x]/(a*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2747

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\tan(c+dx)}{ad} - \frac{\int \frac{b\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{\tan(c+dx)}{ad} - \frac{b\int \frac{\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a}$$
$$= \frac{\tan(c+dx)}{ad} - \frac{b\int \sec(c+dx)\,dx}{a^2} + \frac{b^2\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2}$$
$$= -\frac{b\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{\tan(c+dx)}{ad} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}$$
$$= \frac{2b^2\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}d} - \frac{b\tanh^{-1}(\sin(c+dx))}{a^2d} + \frac{\tan(c+dx)}{ad}$$

Mathematica [A] time = 0.373664, size = 115, normalized size = 1.35

$$\frac{-\frac{2b^2\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + a\tan(c+dx) + b\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\right.\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-2*b^2*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
b^2] + b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]]) + a*Tan[c + d*x])/(a^2*d)
```

Maple [A] time = 0.097, size = 134, normalized size = 1.6

$$-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{b}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) - \frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{b}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) + 2\,\frac{}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/d/a/(tan(1/2*d*x+1/2*c)-1)+1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)-1)-1/a/d/(tan
(1/2*d*x+1/2*c)+1)-1/d*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d/a^2*b^2/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6158, size = 887, normalized size = 10.44

$$\left[-\frac{\sqrt{-a^2+b^2}b^2\cos(dx+c)\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)+\left(a^2b-b^3\right)\cos(\cdots}{2\left(a^4-a^2b^2\right)d\cos(\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a^2 + b^2)*b^2*cos(d*x + c)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + (a^2*b -
 b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) - (a^2*b - b^3)*cos(d*x + c)*log(-
sin(d*x + c) + 1) - 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*
x + c)), 1/2*(2*sqrt(a^2 - b^2)*b^2*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2
- b^2)*sin(d*x + c)))*cos(d*x + c) - (a^2*b - b^3)*cos(d*x + c)*log(sin(d*x
 + c) + 1) + (a^2*b - b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(a^3 - a
*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x)), x)
```

Giac [B] time = 1.45407, size = 207, normalized size = 2.44

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b^2}{\sqrt{a^2-b^2}a^2}+\frac{b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{2\tan\left(\frac{1}{2}d\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\cdots\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^2/(sqrt(a^2 - b^
2)*a^2) + b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^2 + 2*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a
))/d
```

3.457 $\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=119

$$-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{b\tan(c+dx)}{a^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] (-2*b^3*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b
]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b*Tan
[c + d*x])/(a^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.324242, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{b\tan(c+dx)}{a^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*b^3*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b
]*Sqrt[a + b]*d) + ((a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - (b*Tan
[c + d*x])/(a^2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
```

```
qQ[a, 0]))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-2b+a\cos(c+dx)+b\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{2a} \\
&= -\frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(a^2+2b^2+ab\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{2a^2} \\
&= -\frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} - \frac{b^3\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^3} + \frac{\left(a^2+2b^2\right)\int \sec(c+dx)\,dx}{2a^3} \\
&= \frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\left(2b^3\right)\text{Subst}\left(\int \right.}{} \\
&= -\frac{2b^3\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}d} + \frac{\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{b\tan(c+dx)}{a^2d} + \frac{\sec(c+dx}{2}
\end{aligned}$$

Mathematica [A] time = 1.00293, size = 238, normalized size = 2.

$$\frac{\frac{8b^3\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + \frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{a^2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} - 2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((8*b^3*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b
^2] - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*b^2*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
 + 4*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 4*a*b*
Tan[c + d*x])/(4*a^3*d)
```

Maple [B] time = 0.112, size = 262, normalized size = 2.2

$$\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{b}{a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{1}{2da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c)),x)
```

```
[Out] 1/2/d/a/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)+1/d/a^2/(ta
n(1/2*d*x+1/2*c)-1)*b-1/2/d/a*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3*ln(tan(1/2*d
*x+1/2*c)-1)*b^2-1/2/d/a/(tan(1/2*d*x+1/2*c)+1)^2+1/2/a/d/(tan(1/2*d*x+1/2*
c)+1)+1/d/a^2/(tan(1/2*d*x+1/2*c)+1)*b+1/2/d/a*ln(tan(1/2*d*x+1/2*c)+1)+1/d
/a^3*ln(tan(1/2*d*x+1/2*c)+1)*b^2-2/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arctan(ta
n(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.53718, size = 1061, normalized size = 8.92

$$\left[-\frac{2\sqrt{-a^2+b^2}b^3\cos(dx+c)^2\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)-\left(a^4+a^2b^2-2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-a^2 + b^2)*b^3*cos(d*x + c)^2*log((2*a*b*cos(d*x + c) + (2*a
^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
+ c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (a^4
 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^4 + a^2*b^2 -
 2*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(a^4 - a^2*b^2 - 2*(a^3*b
 - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2), -
1/4*(4*sqrt(a^2 - b^2)*b^3*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*si
n(d*x + c)))*cos(d*x + c)^2 - (a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(si
```

```
n(d*x + c) + 1) + (a^4 + a^2*b^2 - 2*b^4)*cos(d*x + c)^2*log(-sin(d*x + c)
+ 1) - 2*(a^4 - a^2*b^2 - 2*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a
^5 - a^3*b^2)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x)), x)
```

Giac [A] time = 1.44655, size = 285, normalized size = 2.39

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b^3}{\sqrt{a^2-b^2}a^3}+\frac{(a^2+2b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{(a^2+2b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{2(\cdots}{}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^3/(sqrt(a^2 -
 b^2)*a^3) + (a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - (a^2 +
2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 2*(a*tan(1/2*d*x + 1/2*c)^3
 + 2*b*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.458 $\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d}-\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{b\tan(c+dx)\sec(c+d}{2a^2d}$$

```
[Out] (2*b^4*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]
*Sqrt[a + b]*d) - (b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + ((2*a
^2 + 3*b^2)*Tan[c + d*x])/(3*a^3*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*
d) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.522148, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d}-\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}-\frac{b\tan(c+dx)\sec(c+d}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*b^4*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]
*Sqrt[a + b]*d) - (b*(a^2 + 2*b^2)*ArcTanh[Sin[c + d*x]])/(2*a^4*d) + ((2*a
^2 + 3*b^2)*Tan[c + d*x])/(3*a^3*d) - (b*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*
d) + (Sec[c + d*x]^2*Tan[c + d*x])/(3*a*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-3b+2a\cos(c+dx)+2b\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx}{3a}$$

$$= -\frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(2\left(2a^2+3b^2\right)+ab\cos(c+dx)-3b^2\cos^2(c+dx)\right)\text{s}}{a+b\cos(c+dx)}}{6a^2}$$

$$= \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{\int \frac{\left(-3b\left(a^2+2b\right.\right.}{}}{}$$

$$= \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{\sec^2(c+dx)\tan(c+dx)}{3ad} + \frac{b^4\int \frac{1}{a+b\cos}}{a^4}$$

$$= -\frac{b\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d} - \frac{b\sec(c+dx)\tan(c+dx)}{2a^2d} + \text{se}$$

$$= \frac{2b^4\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} - \frac{b\left(a^2+2b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{\left(2a^2+3b^2\right)\tan(c+dx)}{3a^3d} -$$

Mathematica [A] time = 2.37024, size = 258, normalized size = 1.64

$$\frac{\frac{1}{2}\sec^3(c+dx)\left(4a\sin(c+dx)\left(\left(2a^2+3b^2\right)\cos(2(c+dx))+4a^2-3ab\cos(c+dx)+3b^2\right)+9b\left(a^2+2b^2\right)\cos(c+dx)\left(\text{lo}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Cos[c + d*x]),x]
```

```
[Out] ((-24*b^4*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
 b^2] + (Sec[c + d*x]^3*(9*b*(a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*b*(a
^2 + 2*b^2)*Cos[3*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 4*a*(4*a^2 + 3*b^2 - 3*a*b*Cos[c
+ d*x] + (2*a^2 + 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/2)/(12*a^4*d)
```

Maple [B] time = 0.111, size = 400, normalized size = 2.6

$$-\frac{1}{3da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{2da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{b}{2a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-\frac{1}{da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*cos(d*x+c)),x)
```

```
[Out] -1/3/d/a/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/a/(tan(1/2*d*x+1/2*c)-1)^2-1/2/d/a^
2/(tan(1/2*d*x+1/2*c)-1)^2*b-1/d/a/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^2/(tan(1/
2*d*x+1/2*c)-1)*b-1/d/a^3/(tan(1/2*d*x+1/2*c)-1)*b^2+1/2/d*b/a^2*ln(tan(1/2
*d*x+1/2*c)-1)+1/d*b^3/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a/(tan(1/2*d*x+1/
2*c)+1)^3+1/2/d/a/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)
^2*b-1/a/d/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)*b-1/d/a^
3/(tan(1/2*d*x+1/2*c)+1)*b^2-1/2/d*b/a^2*ln(tan(1/2*d*x+1/2*c)+1)-1/d*b^3/a
^4*ln(tan(1/2*d*x+1/2*c)+1)+2/d*b^4/a^4/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*
d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.57493, size = 1229, normalized size = 7.83

$$\left[-\frac{6\sqrt{-a^2+b^2}b^4\cos(dx+c)^3\log\left(\frac{2ab\cos(dx+c)+(2a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2b^2}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right)+3\left(a^4b+a^2b^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*sqrt(-a^2 + b^2)*b^4*cos(d*x + c)^3*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
```

```
 + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*(
a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(a^4*b +
a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*a^5 - 2*a^3*b
^2 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 - 3*(a^4*b - a^2*b^3)*cos
(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x + c)^3), 1/12*(12*sqrt(
a^2 - b^2)*b^4*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)))
*cos(d*x + c)^3 - 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(sin(d*x +
c) + 1) + 3*(a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
 + 2*(2*a^5 - 2*a^3*b^2 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 - 3*
(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d*cos(d*x +
c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*cos(c + d*x)), x)
```

Giac [B] time = 1.46699, size = 386, normalized size = 2.46

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)b^4}{\sqrt{a^2-b^2}\,a^4}+\frac{3\left(a^2b+2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{3\left(a^2b+2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right.\right.}{a^4}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b^4/(sqrt(a^2
 - b^2)*a^4) + 3*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3
*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(6*a^2*tan(1/2*
d*x + 1/2*c)^5 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*b^2*tan(1/2*d*x + 1/2*c)^
5 - 4*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*ta
n(1/2*d*x + 1/2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*b^2*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a^3))/d
```

3.459 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=266

$$\frac{(-7a^2b^2+12a^4-2b^4)\sin(c+dx)}{3b^4d(a^2-b^2)}+\frac{2a^4(4a^2-5b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{(4a$$

```
[Out] -((a*(4*a^2 + b^2)*x)/b^5) + (2*a^4*(4*a^2 - 5*b^2)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^5*(a + b)^(3/2)*d) + ((12*a^4
 - 7*a^2*b^2 - 2*b^4)*Sin[c + d*x])/(3*b^4*(a^2 - b^2)*d) - (a*(2*a^2 - b^2
)*Cos[c + d*x]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((4*a^2 - b^2)*Cos[c + d
*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.722972, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3049, 3023, 2735, 2659, 205}

$$\frac{(-7a^2b^2+12a^4-2b^4)\sin(c+dx)}{3b^4d(a^2-b^2)}+\frac{2a^4(4a^2-5b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}+\frac{(4a$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -((a*(4*a^2 + b^2)*x)/b^5) + (2*a^4*(4*a^2 - 5*b^2)*ArcTan[(Sqrt[a - b]*Tan
[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^5*(a + b)^(3/2)*d) + ((12*a^4
 - 7*a^2*b^2 - 2*b^4)*Sin[c + d*x])/(3*b^4*(a^2 - b^2)*d) - (a*(2*a^2 - b^2
)*Cos[c + d*x]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((4*a^2 - b^2)*Cos[c + d
*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(3a^2-ab\cos(c+dx)-\left(4a^2-b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(4a^2-b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-2a\left(4a^2\right.\right.}{}}{}$$

$$= -\frac{a\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(4a^2-b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^3(}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{3b^4\left(a^2-b^2\right)d} - \frac{a\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(4a^2-b^2\right)\cos}{3b^2(}$$

$$= -\frac{a\left(4a^2+b^2\right)x}{b^5} + \frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{3b^4\left(a^2-b^2\right)d} - \frac{a\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(4a^2+b^2\right)x}{b^5} + \frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c+dx)}{3b^4\left(a^2-b^2\right)d} - \frac{a\left(2a^2-b^2\right)\cos(c+dx)\sin(c+dx}{b^3\left(a^2-b^2\right)d}$$

$$= -\frac{a\left(4a^2+b^2\right)x}{b^5} + \frac{2a^4\left(4a^2-5b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^5(a+b)^{3/2}d} + \frac{\left(12a^4-7a^2b^2-2b^4\right)\sin(c}{3b^4\left(a^2-b^2\right)d}$$

Mathematica [C] time = 0.879634, size = 176, normalized size = 0.66

$$\frac{9b\left(4a^2+b^2\right)\sin(c+dx) + \frac{24a^4\left(4a^2-5b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \frac{12a^5b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} - 6ab^2\sin(2(c+dx)) - 12a(2a-}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-12*a*(2*a - I*b)*(2*a + I*b)*(c + d*x) + (24*a^4*(4*a^2 - 5*b^2)*ArcTanh[
((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + 9*b*(4*a
^2 + b^2)*Sin[c + d*x] + (12*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Co
s[c + d*x])) - 6*a*b^2*Sin[2*(c + d*x)] + b^3*Sin[3*(c + d*x)])/(12*b^5*d)
```

Maple [A] time = 0.098, size = 504, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*a^2+2/d/b^3/(1+tan(
1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^5*a+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)
^3*tan(1/2*d*x+1/2*c)^5+12/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2
*c)^3*a^2+4/3/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^3+6/d/b^4
/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)*a^2-2/d/b^3/(1+tan(1/2*d*x+1
```

```
/2*c)^2)^3*tan(1/2*d*x+1/2*c)*a+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*
d*x+1/2*c)-8/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^3-2/d/b^3*arctan(tan(1/2*d*
x+1/2*c))*a+2/d*a^5/b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b+a+b)+8/d*a^6/b^5/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*a
rctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-10/d*a^4/b^3/(a-b)/(a+b
)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.6388, size = 1644, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(4*a^7*b - 7*a^5*b^3 + 2*a^3*b^5 + a*b^7)*d*x*cos(d*x + c) + 6*(4*
a^8 - 7*a^6*b^2 + 2*a^4*b^4 + a^2*b^6)*d*x + 3*(4*a^7 - 5*a^5*b^2 + (4*a^6*
b - 5*a^4*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*
a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x
 + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(
12*a^7*b - 19*a^5*b^3 + 5*a^3*b^5 + 2*a*b^7 + (a^4*b^4 - 2*a^2*b^6 + b^8)*c
os(d*x + c)^3 - 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^2 + 2*(3*a^6*b
^2 - 5*a^4*b^4 + a^2*b^6 + b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a
^2*b^8 + b^10)*d*cos(d*x + c) + (a^5*b^5 - 2*a^3*b^7 + a*b^9)*d), -1/3*(3*(
4*a^7*b - 7*a^5*b^3 + 2*a^3*b^5 + a*b^7)*d*x*cos(d*x + c) + 3*(4*a^8 - 7*a^
6*b^2 + 2*a^4*b^4 + a^2*b^6)*d*x - 3*(4*a^7 - 5*a^5*b^2 + (4*a^6*b - 5*a^4*
b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 -
 b^2)*sin(d*x + c))) - (12*a^7*b - 19*a^5*b^3 + 5*a^3*b^5 + 2*a*b^7 + (a^4*
b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^3 - 2*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos
(d*x + c)^2 + 2*(3*a^6*b^2 - 5*a^4*b^4 + a^2*b^6 + b^8)*cos(d*x + c))*sin(d
*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)*d*cos(d*x + c) + (a^5*b^5 - 2*a^3*b^
7 + a*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31688, size = 450, normalized size = 1.69

$$\frac{6\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^2b^4-b^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{6\left(4\,a^6-5\,a^4b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^5-b^7\right)\sqrt{a^2-b^2}}-3\left(4\,a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(6*a^5*tan(1/2*d*x + 1/2*c)/((a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 6*(4*a^6 - 5*a^4*b^2)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^5 - b^7)*sqrt(a^2 - b^2)) -
3*(4*a^3 + a*b^2)*(d*x + c)/b^5 + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a*b*t
an(1/2*d*x + 1/2*c)^5 + 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*tan(1/2*d*x +
 1/2*c)^3 + 2*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2*c) - 3*a
*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c
)^2 + 1)^3*b^4))/d
```

3.460 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=166

$$-\frac{a^4\sin(c+dx)}{b^3d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2a^3\left(3a^2-4b^2\right)\tanh^{-1}\left(\frac{(a-b)\sin(c+dx)}{\sqrt{b^2-a^2}(\cos(c+dx)+1)}\right)}{b^4d\left(b^2-a^2\right)^{3/2}}+\frac{x\left(6a^2+b^2\right)}{2b^4}-\frac{2a\sin(c+dx)}{b^3d}+\sin(c$$

```
[Out] ((6*a^2 + b^2)*x)/(2*b^4) - (2*a^3*(3*a^2 - 4*b^2)*ArcTanh[((a - b)*Sin[c +
 d*x])/(Sqrt[-a^2 + b^2]*(1 + Cos[c + d*x]))]])/(b^4*(-a^2 + b^2)^(3/2)*d) -
 (2*a*Sin[c + d*x])/(b^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (a^4*
Sin[c + d*x])/(b^3*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.428566, antiderivative size = 213, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3049, 3023, 2735, 2659, 205}

$$-\frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{b^3d\left(a^2-b^2\right)}-\frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\left(3a^2-b^2\right)\sin}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((6*a^2 + b^2)*x)/(2*b^4) - (2*a^3*(3*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) - (a*(3*a^2
 - 2*b^2)*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) + ((3*a^2 - b^2)*Cos[c + d*x]*S
in[c + d*x])/(2*b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^2*Sin[c + d*x])/(b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2a^2-ab\cos(c+dx)-\left(3a^2-b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-a\left(3a^2-b^2\right)+b\left(a^2+b \ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(\ldots}{b\left(a^2-b^2\right)d(a+b\cos \ldots}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\cos \ldots}{b\left(a^2-b \ldots\right.}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\cos(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)d} - \frac{a^2\cos \ldots}{b\left(a^2-b \ldots\right.}$$

$$= \frac{\left(6a^2+b^2\right)x}{2b^4} - \frac{2a^3\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^4(a+b)^{3/2}d} - \frac{a\left(3a^2-2b^2\right)\sin(c+dx)}{b^3\left(a^2-b^2\right)d} + \frac{(3 \ldots}{\ldots}$$

Mathematica [A] time = 0.732477, size = 144, normalized size = 0.87

$$\frac{2\left(6a^2+b^2\right)(c+dx)-\frac{8a^3\left(3a^2-4b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{4a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}-8ab\sin(c+dx)+b^2\sin(2(c+dx))}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*(6*a^2 + b^2)*(c + d*x) - (8*a^3*(3*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c
 + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) - 8*a*b*Sin[c + d*x] - (4
*a^4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])) + b^2*Sin[2*(c
+ d*x)])/(4*b^4*d)
```

Maple [B] time = 0.091, size = 358, normalized size = 2.2

$$-4\,\frac{\left(\tan\left(1/2\,dx+c/2\right)\right)^{3}a}{db^{3}\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{2}}-\frac{1}{b^{2}d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\right)^{-2}-4\,\frac{a\tan\left(1/2\,dx+c/2\right)}{db^{3}\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*a-1/d/b^2/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-4/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)*a+1/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+
6/d/b^4*arctan(tan(1/2*d*x+1/2*c))*a^2+1/d/b^2*arctan(tan(1/2*d*x+1/2*c))-2
/d*a^4/b^3/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b+a+b)-6/d*a^5/b^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+8/d*a^3/b^2/(a-b)/(a+b)/((a-b)*(a+b)
)^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.39211, size = 1428, normalized size = 8.6

$$\left[\frac{\left(6\,a^{6}b-11\,a^{4}b^{3}+4\,a^{2}b^{5}+b^{7}\right)dx\cos\left(dx+c\right)+\left(6\,a^{7}-11\,a^{5}b^{2}+4\,a^{3}b^{4}+ab^{6}\right)dx-\left(3\,a^{6}-4\,a^{4}b^{2}+\left(3\,a^{5}b-4\,a^{3}b^{3}\right)\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((6*a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + b^7)*d*x*cos(d*x + c) + (6*a^7 -
11*a^5*b^2 + 4*a^3*b^4 + a*b^6)*d*x - (3*a^6 - 4*a^4*b^2 + (3*a^5*b - 4*a^3
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - (6*a^6*b - 1
0*a^4*b^3 + 4*a^2*b^5 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2 + 3*(a^5
*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7
 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)*d), 1/2*((6*a^6*b -
11*a^4*b^3 + 4*a^2*b^5 + b^7)*d*x*cos(d*x + c) + (6*a^7 - 11*a^5*b^2 + 4*a^
3*b^4 + a*b^6)*d*x - 2*(3*a^6 - 4*a^4*b^2 + (3*a^5*b - 4*a^3*b^3)*cos(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x
+ c))) - (6*a^6*b - 10*a^4*b^3 + 4*a^2*b^5 - (a^4*b^3 - 2*a^2*b^5 + b^7)*co
s(d*x + c)^2 + 3*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/
((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c) + (a^5*b^4 - 2*a^3*b^6 + a*b^8)
*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48344, size = 354, normalized size = 2.13

$$-\frac{\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^2b^3-b^5\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{4\left(3\,a^5-4\,a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^4-b^6\right)\sqrt{a^2-b^2}}-\frac{(6\,a\cdots}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a^4*tan(1/2*d*x + 1/2*c)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) - 4*(3*a^5 - 4*a^3*b^2)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) -
 (6*a^2 + b^2)*(d*x + c)/b^4 + 2*(4*a*tan(1/2*d*x + 1/2*c)^3 + b*tan(1/2*d*
x + 1/2*c)^3 + 4*a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/((tan(1/2
*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

3.461 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$\frac{(2a^2-b^2)\sin(c+dx)}{b^2d(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{2ax}{b^3}$$

```
[Out] (-2*a*x)/b^3 + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + ((2*a^2 - b^2)*Sin[c +
 d*x])/(b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.256047, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3023, 2735, 2659, 205}

$$\frac{(2a^2-b^2)\sin(c+dx)}{b^2d(a^2-b^2)}+\frac{2a^2(2a^2-3b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)\cos(c+dx)}{bd(a^2-b^2)(a+b\cos(c+dx))}-\frac{2ax}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*a*x)/b^3 + (2*a^2*(2*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/((a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + ((2*a^2 - b^2)*Sin[c +
 d*x])/(b^2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]*Sin[c + d*x])/(b*(a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{a^2-ab\cos(c+dx)-\left(2a^2-b^2\right)\cos^2(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{a^2b+2a\left(a^2-b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{2ax}{b^3} + \frac{\left(2a^2-b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(a^2\left(2a^2-3b^2\right)\right)\int \frac{}{a+}}{b^3\left(a^2-b^2\right.}$$

$$= -\frac{2ax}{b^3} + \frac{\left(2a^2-b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(2a^2\left(2a^2-3b^2\right)\right)\text{Su}}{}$$

$$= -\frac{2ax}{b^3} + \frac{2a^2\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d} + \frac{\left(2a^2-b^2\right)\sin(c+dx)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\cos(c}{b\left(a^2-b^2\right)c}$$

Mathematica [A] time = 0.664714, size = 113, normalized size = 0.73

$$\frac{\frac{2a^2\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} + \sin(c+dx)\left(\frac{a^3b}{(a-b)(a+b)(a+b\cos(c+dx))}+b\right) - 2a(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*a*(c + d*x) + (2*a^2*(2*a^2 - 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])
/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) + (b + (a^3*b)/((a - b)*(a + b)*(a +
 b*Cos[c + d*x])))*Sin[c + d*x])/(b^3*d)
```

Maple [A] time = 0.096, size = 238, normalized size = 1.5

$$2\,\frac{\tan(1/2\,dx+c/2)}{b^2d\left(1+(\tan(1/2\,dx+c/2))^2\right)} - 4\,\frac{\arctan(\tan(1/2\,dx+c/2))\,a}{db^3} + 2\,\frac{a^3\tan(1/2\,dx+c/2)}{b^2d\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-4/d/b^3*arctan(tan(1/2*
d*x+1/2*c))*a+2/d*a^3/b^2/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^
2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)+4/d*a^4/b^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)
*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-6/d*a^2/b/(a-b)/(a+b)
/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.17531, size = 1220, normalized size = 7.87

$$\left[-\frac{4\left(a^5b-2a^3b^3+ab^5\right)dx\cos(dx+c)+4\left(a^6-2a^4b^2+a^2b^4\right)dx+\left(2a^5-3a^3b^2+\left(2a^4b-3a^2b^3\right)\cos(dx+c)\right)\sqrt{-a^2 \cdots}}{2\left(\left(a^4b^4-2a^2b^{6} \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 4*(a^6 - 2*a^4*b^2
+ a^2*b^4)*d*x + (2*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*s
qrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*
sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(
d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^5*b - 3*a^3*b^3 + a*b^5 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^4 - 2*a^2*b
^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d), -(2*(a^5*b - 2
*a^3*b^3 + a*b^5)*d*x*cos(d*x + c) + 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*d*x - (2
*a^5 - 3*a^3*b^2 + (2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^5*b - 3*a^3
*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^
4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42021, size = 412, normalized size = 2.66

$$-\frac{2\left(\frac{\left(2\,a^{4}-3\,a^{2}b^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{2}b^{3}-b^{5}\right)\sqrt{a^{2}-b^{2}}}-\frac{2\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((2*a^4 - 3*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2))
)/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b
*tan(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x +
1/2*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*
tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4
 - b*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)*(a^2*b^2
- b^4)) + (d*x + c)*a/b^3)/d
```

3.462 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=108

$$-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x}{b^2}$$

```
[Out] x/b^2 - (2*a*(a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - (a^2*Sin[c + d*x])/(b*(a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.142766, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2790, 2735, 2659, 205}

$$-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}-\frac{a^2\sin(c+dx)}{bd\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] x/b^2 - (2*a*(a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b
]])/((a - b)^(3/2)*b^2*(a + b)^(3/2)*d) - (a^2*Sin[c + d*x])/(b*(a^2 - b^2)
*d*(a + b*Cos[c + d*x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{ab+\left(a^2-b^2\right)\cos(c+dx)}{a+b\cos(c+dx)}\,dx}{b\left(a^2-b^2\right)}\\ &= \frac{x}{b^2}-\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\left(a\left(a^2-2b^2\right)\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b^2\left(a^2-b^2\right)}\\ &= \frac{x}{b^2}-\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\left(2a\left(a^2-2b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}\right.\right.}{b^2\left(a^2-b^2\right)d}\\ &= \frac{x}{b^2}-\frac{2a\left(a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d}-\frac{a^2\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.37213, size = 103, normalized size = 0.95

$$\frac{-\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}-\frac{a^2b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}+c+dx}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (c + d*x - (2*a*(a^2 - 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2
+ b^2]])/(-a^2 + b^2)^(3/2) - (a^2*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*
Cos[c + d*x])))/(b^2*d)
```

Maple [B] time = 0.089, size = 200, normalized size = 1.9

$$2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{b^2d}-2\,\frac{a^2\tan\left(1/2\,dx+c/2\right)}{bd\left(a^2-b^2\right)\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a-\left(\tan\left(1/2\,dx+c/2\right)\right)^2b+a+b\right)}-2\,\frac{}{b^2d\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d/b^2*arctan(tan(1/2*d*x+1/2*c))-2/d*a^2/b/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(
tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)-2/d*a^3/b^2/(a-b)/(a+b)/
((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+4/
d*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.29651, size = 1037, normalized size = 9.6

$$\left[\frac{2\left(a^4b-2a^2b^3+b^5\right)dx\cos\left(dx+c\right)+2\left(a^5-2a^3b^2+ab^4\right)dx-\left(a^4-2a^2b^2+\left(a^3b-2ab^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log}{2\left(\left(a^4b^3-2a^2b^5+b^7\right)d\cos\left(dx+c\right)+\left(a^5b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b - 2*a^2*b^3 + b^5)*d*x*cos(d*x + c) + 2*(a^5 - 2*a^3*b^2 + a
*b^4)*d*x - (a^4 - 2*a^2*b^2 + (a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(-a^2 +
b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 +
 b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2
+ 2*a*b*cos(d*x + c) + a^2)) - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^4*b^3
- 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^4 + a*b^6)*d), ((a^4
*b - 2*a^2*b^3 + b^5)*d*x*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d*x - (a
^4 - 2*a^2*b^2 + (a^3*b - 2*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a
*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^4*b - a^2*b^3)*sin(
d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*cos(d*x + c) + (a^5*b^2 - 2*a^3*b^
4 + a*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40347, size = 236, normalized size = 2.19

$$-\frac{\frac{2a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a^2b-b^3\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a+b\right)}-\frac{2\left(a^3-2ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2a+2b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^2-b^4\right)\sqrt{a^2-b^2}}-\frac{dx+c}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*tan(1/2*d*x + 1/2*c)/((a^2*b - b^3)*(a*tan(1/2*d*x + 1/2*c)^2 - b*t
an(1/2*d*x + 1/2*c)^2 + a + b)) - 2*(a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*
x + 1/2*c))/sqrt(a^2 - b^2)))/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) - (d*x + c)
/b^2)/d
```

3.463 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2} dx$

Optimal. Leaf size=85

$$\frac{a\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] (-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
 + b)^(3/2)*d) + (a*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.0688283, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2754, 12, 2659, 205}

$$\frac{a\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
 + b)^(3/2)*d) + (a*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{a\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} + \frac{\int \frac{b}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}$$
$$= \frac{a\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{b\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2}$$
$$= \frac{a\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d}$$
$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{a\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}$$

Mathematica [A] time = 0.228724, size = 83, normalized size = 0.98

$$\frac{\frac{a\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} - \frac{2b\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{(b^2-a^2)^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
3/2) + (a*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/d
```

Maple [A] time = 0.089, size = 116, normalized size = 1.4

$$2\frac{a\tan(1/2\,dx+c/2)}{d(a^2-b^2)\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b+a+b\right)} - 2\frac{b}{d(a-b)(a+b)\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\text{ta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b+a+b)-2/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)
*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.09125, size = 728, normalized size = 8.56

$$\left[\frac{\left(b^2\cos\left(dx+c\right)+ab\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)+2\left(a^3-ab^2\right)\sin\left(dx+c\right)}{2\left(\left(a^4b-2\,a^2b^3+b^5\right)d\cos\left(dx+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*cos(d*x + c) + a*b)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (
2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d
*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2
*(a^3 - a*b^2)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a
^5 - 2*a^3*b^2 + a*b^4)*d), -((b^2*cos(d*x + c) + a*b)*sqrt(a^2 - b^2)*arct
an(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^3 - a*b^2)*si
n(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 +
a*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31, size = 182, normalized size = 2.14

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)b}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*b/(a^2 - b^2)^(3/2)
 + a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*
c)^2 + a + b)*(a^2 - b^2)))/d
```

3.464 $\int \frac{1}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
+ b)^(3/2)*d) - (b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.0552126, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2664, 12, 2659, 205}

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-2),x]
```

```
[Out] (2*a*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a
+ b)^(3/2)*d) - (b*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2}\,dx = -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{a}{a+b\cos(c+dx)}\,dx}{-a^2+b^2}$$

$$= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{a\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{b\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}$$

$$= \frac{2a\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))}$$

Mathematica [A] time = 0.185773, size = 84, normalized size = 0.98

$$\frac{\frac{2a\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - \frac{b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-2),x]
```

```
[Out] ((2*a*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3
/2) - (b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])))/d
```

Maple [A] time = 0.078, size = 116, normalized size = 1.4

$$-2\,\frac{\tan(1/2\,dx+c/2)\,b}{d\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b+a+b\right)}+2\,\frac{a}{d\,(a-b)\,(a+b)\,\sqrt{(a-b)\,(a+b)}}\arctan\left(\frac{\tan}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/d/(a^2-b^2)*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b+a+b)+2/d*a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.98652, size = 726, normalized size = 8.44

$$\left[\frac{\left(ab\cos\left(dx+c\right)+a^2\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)-2\left(a^2b-b^3\right.}{2\left(\left(a^4b-2\,a^2b^3+b^5\right)d\cos\left(dx+c\right)+\left(a^5-2\,a^3b^2+ab^4\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cos(d*x + c) + a^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (
2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d
*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2
*(a^2*b - b^3)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a
^5 - 2*a^3*b^2 + a*b^4)*d), ((a*b*cos(d*x + c) + a^2)*sqrt(a^2 - b^2)*arcta
n(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^2*b - b^3)*sin
(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c) + (a^5 - 2*a^3*b^2 + a
*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22766, size = 182, normalized size = 2.12

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)a}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)\left(a^2-b^2\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a/(a^2 - b^2)^(3/2
) + b*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 + a + b)*(a^2 - b^2)))/d
```

3.465 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=118

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

```
[Out] (-2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*(a - b)^(3/2)*(a + b)^(3/2)*d) + ArcTanh[Sin[c + d*x]]/(a^2*d) + (b^2*Sin
[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.218543, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2802, 3001, 3770, 2659, 205}

$$-\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (-2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*(a - b)^(3/2)*(a + b)^(3/2)*d) + ArcTanh[Sin[c + d*x]]/(a^2*d) + (b^2*Sin
[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \frac{\left(a^2-b^2-ab\cos(c+dx)\right)\sec(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}+\frac{\int \sec(c+dx)\,dx}{a^2}-\frac{\left(b\left(2a^2-b^2\right)\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))}-\frac{\left(2b\left(2a^2-b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+b+(\ldots}\right.}{a^2\left(a^2-\ldots\right.}$$

$$= -\frac{2b\left(2a^2-b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos\ldots}$$

Mathematica [A] time = 0.332905, size = 146, normalized size = 1.24

$$\frac{\frac{2b\left(b^2-2a^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}}+\frac{ab^2\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))}-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right.\right.\ldots}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*b*(-2*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(3/2) - Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*Sin[c + d*x])/((a - b)*(a + b)*(a +
b*Cos[c + d*x])))/(a^2*d)
```

Maple [B] time = 0.118, size = 221, normalized size = 1.9

$$-\frac{1}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{1}{a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2\,\frac{b^2\tan(1/2\,dx+c/2)}{da\left(a^2-b^2\right)\left((\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)+2/d*b^2/
a/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b+a+b)-4/d*b/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-
b)/((a-b)*(a+b))^(1/2))+2/d*b^3/a^2/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(
tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.15265, size = 1331, normalized size = 11.28

$$\left[-\frac{\left(2a^3b-ab^3+\left(2a^2b^2-b^4\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2-2\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+}{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*
log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)
*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a
*b*cos(d*x + c) + a^2)) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b
^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b
 - 2*a^2*b^3 + b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^3*b^2 - a*b
^4)*sin(d*x + c))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*
a^5*b^2 + a^3*b^4)*d), -1/2*(2*(2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cos(d*x
 + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*
x + c))) - (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c
))*log(sin(d*x + c) + 1) + (a^5 - 2*a^3*b^2 + a*b^4 + (a^4*b - 2*a^2*b^3 +
b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^3*b^2 - a*b^4)*sin(d*x + c
))/((a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cos(d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b
^4)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**2, x)
```

Giac [A] time = 1.36417, size = 267, normalized size = 2.26

$$\frac{\frac{2\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^3-ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}-\frac{2\left(2\,a^2b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4-a^2b^2\right)\sqrt{a^2-b^2}}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*b^2*tan(1/2*d*x + 1/2*c)/((a^3 - a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*ta
n(1/2*d*x + 1/2*c)^2 + a + b)) - 2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + log(abs(tan(
1/2*d*x + 1/2*c) + 1))/a^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2)/d
```

3.466 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$\frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2-2b^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2b\tanh^{-1}(\sin(c+$$

```
[Out] (2*b^2*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - (2*b*ArcTanh[Sin[c + d*x]])/(a^3*d) +
 ((a^2 - 2*b^2)*Tan[c + d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.407008, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(a^2-2b^2\right)\tan(c+dx)}{a^2d\left(a^2-b^2\right)}+\frac{b^2\tan(c+dx)}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{2b\tanh^{-1}(\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b^2*(3*a^2 - 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^3*(a - b)^(3/2)*(a + b)^(3/2)*d) - (2*b*ArcTanh[Sin[c + d*x]])/(a^3*d) +
 ((a^2 - 2*b^2)*Tan[c + d*x])/(a^2*(a^2 - b^2)*d) + (b^2*Tan[c + d*x])/(a*(
a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2-2b^2-ab\cos(c+dx)+b^2\cos^2(c+dx)\right)\sec^2(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-2b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-2b\left(a^2-b^2\right)+ab^2\cos(c+dx)\right)\sec(c}{a+b\cos(c+dx)}}{a^2\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-2b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{(2b)\int \sec(c+dx)\,dx}{a^3} + \frac{\left(b^2\right.}{}$$

$$= -\frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(a^2-2b^2\right)\tan(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + $$

$$= \frac{2b^2\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d} - \frac{2b\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{\left(a^2-2b^2\right)\tan(c+}{a^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.891426, size = 163, normalized size = 1.05

$$\frac{-\frac{2b^2\left(2b^2-3a^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{3/2}} - \frac{ab^3\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))} + a\tan(c+dx) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*b^2*(-3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^
2]])/(-a^2 + b^2)^(3/2) + 2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2*
b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (a*b^3*Sin[c + d*x])/((a - b)*
(a + b)*(a + b*Cos[c + d*x])) + a*Tan[c + d*x])/(a^3*d)
```

Maple [A] time = 0.115, size = 271, normalized size = 1.8

$$2\,\frac{b\ln\left(\tan\left(1/2\,dx+c/2\right)-1\right)}{da^{3}}-\frac{1}{a^{2}d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{1}{a^{2}d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-2\,\frac{b\ln\left(\tan\left(1/2\,dx+c/2\right)+1\right)}{da^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 2/d*b/a^3*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)-1/d/a^2/(
tan(1/2*d*x+1/2*c)+1)-2/d*b/a^3*ln(tan(1/2*d*x+1/2*c)+1)-2/d*b^3/a^2/(a^2-b
^2)*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)+
6/d*b^2/a/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/(
(a-b)*(a+b))^(1/2))-4/d*b^4/a^3/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(
1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.83338, size = 1689, normalized size = 10.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(((3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^3*b^2 - 2*a*b^4)*cos(d*x
+ c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)
^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*((a^4*b^2 - 2*a^2*b^4 +
b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x
 + c) + 1) - 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^3
*b^3 + a*b^5)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(a^6 - 2*a^4*b^2 + a
^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b
```

```
- 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos
(d*x + c)), (((3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^3*b^2 - 2*a*b^4)*co
s(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*s
in(d*x + c))) - ((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^
3*b^3 + a*b^5)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((a^4*b^2 - 2*a^2*b^4
+ b^6)*cos(d*x + c)^2 + (a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*log(-sin(
d*x + c) + 1) + (a^6 - 2*a^4*b^2 + a^2*b^4 + (a^5*b - 3*a^3*b^3 + 2*a*b^5)*
cos(d*x + c))*sin(d*x + c))/((a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*x + c)^2
 + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**2, x)
```

Giac [B] time = 1.27567, size = 448, normalized size = 2.89

$$-\frac{2\left(\frac{\left(3a^2b^2-2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.\right.}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((3*a^2*b^2 - 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2))
)/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*t
an(1/2*d*x + 1/2*c)^3 - a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*b^3*tan(1/2*d*x +
1/2*c)^3 + a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) - a*b^2*ta
n(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4
 - b*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^4 - a^
2*b^2)) + b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - b*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/a^3)/d
```

3.467 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$-\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\left(2a^2-3b^2\right)\tan(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{\left(a^2-3b^2\right)\text{ta}}{2a^2}$$

```
[Out] (-2*b^3*(4*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]]
)/(2*a^4*d) - (b*(2*a^2 - 3*b^2)*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2
- 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*
x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.681478, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}-\frac{b\left(2a^2-3b^2\right)\tan(c+dx)}{a^3d\left(a^2-b^2\right)}+\frac{\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{\left(a^2-3b^2\right)\text{ta}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^3*(4*a^2 - 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])
/(a^4*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((a^2 + 6*b^2)*ArcTanh[Sin[c + d*x]]
)/(2*a^4*d) - (b*(2*a^2 - 3*b^2)*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2
- 3*b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*
x]*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2-3b^2-ab\cos(c+dx)+2b^2\cos^2(c+dx)\right)\sec^3(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \int \frac{\left(-2b\left(2a^2-3b^2\right)+a(\ldots}{\ldots}$$

$$= -\frac{b\left(2a^2-3b^2\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\sec(c+dx)\tan(\ldots}{a\left(a^2-b^2\right)d(a+b\cos \ldots}$$

$$= -\frac{b\left(2a^2-3b^2\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b^2\sec(c+dx)\tan(\ldots}{a\left(a^2-b^2\right)d(a+b\cos \ldots}$$

$$= \frac{\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{b\left(2a^2-3b^2\right)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-3b^2\right)\sec(c+dx)\tan \ldots}{2a^2\left(a^2-b^2\right)d}$$

$$= -\frac{2b^3\left(4a^2-3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d} + \frac{\left(a^2+6b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^4d} - \frac{b\left(2a^2-3 \ldots}{a^3(\ldots}$$

Mathematica [A] time = 5.53266, size = 285, normalized size = 1.31

$$\frac{\frac{8b^3(3b^2-4a^2)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{(b^2-a^2)^{3/2}}+\frac{a^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{a^2}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2}-2a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((8*b^3*(-4*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2
]])/(-a^2 + b^2)^(3/2) - 2*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1
2*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*a^2*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + 12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^
2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - a^2/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + (4*a*b^4*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])
) - 8*a*b*Tan[c + d*x])/(4*a^4*d)
```

Maple [A] time = 0.123, size = 401, normalized size = 1.9

$$\frac{1}{2\,a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{1}{2\,a^2d}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\,\frac{b}{da^3\left(\tan\left(1/2\,dx+c/2\right)-1\right)}-\frac{1}{2\,a^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)+2/d/a^3
/(tan(1/2*d*x+1/2*c)-1)*b-1/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)-3/d/a^4*ln(tan
(1/2*d*x+1/2*c)-1)*b^2-1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^2/(tan(1/
2*d*x+1/2*c)+1)+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b+1/2/d/a^2*ln(tan(1/2*d*x+1
/2*c)+1)+3/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)*b^2+2/d*b^4/a^3/(a^2-b^2)*tan(1/2
*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)-8/d*b^3/a^2
/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+
b))^(1/2))+6/d*b^5/a^4/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1
/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.45971, size = 2009, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*((4*a^2*b^4 - 3*b^6)*cos(d*x + c)^3 + (4*a^3*b^3 - 3*a*b^5)*cos(d*
x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^6*b + 4*a^4*b^3 -
11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^
6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b + 4*a^4*b^3 - 11*a^2*b^5
 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^7 - 2*a^5*b^2 + a^3*b^4 - 2*(2*a^5*b
^2 - 5*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2 - 3*(a^6*b - 2*a^4*b^3 + a^2*b^5)*
cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x + c)^3
 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2), -1/4*(4*((4*a^2*b^4 - 3*b
^6)*cos(d*x + c)^3 + (4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*
arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((a^6*b + 4*
a^4*b^3 - 11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^
4 + 6*a*b^6)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((a^6*b + 4*a^4*b^3 -
11*a^2*b^5 + 6*b^7)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 11*a^3*b^4 + 6*a*b^
6)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^7 - 2*a^5*b^2 + a^3*b^4 -
2*(2*a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2 - 3*(a^6*b - 2*a^4*b^3 +
 a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(
d*x + c)^3 + (a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**2, x)
```

Giac [A] time = 1.4547, size = 396, normalized size = 1.82

$$\frac{\frac{4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^5-a^3b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}+\frac{4\left(4\,a^2b^3-3\,b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{\left(a^2+6\cdots\right.}{\cdots}}{2\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^4*tan(1/2*d*x + 1/2*c)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 4*(4*a^2*b^3 - 3*b^5)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*t
an(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*sqrt(a^2 - b^2)) +
(a^2 + 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - (a^2 + 6*b^2)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/
```

```
2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) - 4*b*tan(1/2*d*x + 1/2*c))/((tan
(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.468 $\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=270

$$\frac{2b^4\left(5a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(7a^2b^2+2a^4-12b^4\right)\tan(c+dx)}{3a^4d\left(a^2-b^2\right)}-\frac{b\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{(a}{}$$

```
[Out] (2*b^4*(5*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]
])/(a^5*d) + ((2*a^4 + 7*a^2*b^2 - 12*b^4)*Tan[c + d*x])/(3*a^4*(a^2 - b^2)
*d) - (b*(a^2 - 2*b^2)*Sec[c + d*x]*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a
^2 - 4*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.967372, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{2b^4\left(5a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}+\frac{\left(7a^2b^2+2a^4-12b^4\right)\tan(c+dx)}{3a^4d\left(a^2-b^2\right)}-\frac{b\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d}+\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (2*b^4*(5*a^2 - 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) - (b*(a^2 + 4*b^2)*ArcTanh[Sin[c + d*x]
])/(a^5*d) + ((2*a^4 + 7*a^2*b^2 - 12*b^4)*Tan[c + d*x])/(3*a^4*(a^2 - b^2)
*d) - (b*(a^2 - 2*b^2)*Sec[c + d*x]*Tan[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a
^2 - 4*b^2)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c
 + d*x]^2*Tan[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sec^2(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(a^2-4b^2-ab\cos(c+dx)+3b^2\cos^2(c+dx)\right)\sec^4(c+dx)}{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2-4b^2\right)\sec^2(c+dx)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^2(c+dx)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\left(-6b\left(a^2-2b^2\right)+a(\ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{b\left(a^2-2b^2\right)\sec(c+dx)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-4b^2\right)\sec^2(c+dx)\tan(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^2(\ldots}{a\left(a^2-b^2\ldots\right.}$$

$$= \frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\sec(c+dx)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-4b^2\right)\sec\ldots}{3a^2\ldots}$$

$$= \frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\sec(c+dx)\tan(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2-4b^2\right)\sec\ldots}{3a^2\ldots}$$

$$= -\frac{b\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{\left(2a^4+7a^2b^2-12b^4\right)\tan(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{b\left(a^2-2b^2\right)\sec\ldots}{a^3(\ldots}$$

$$= \frac{2b^4\left(5a^2-4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\left(a^2+4b^2\right)\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{\left(2a^4+7a\ldots\right.}{\ldots}$$

Mathematica [A] time = 6.1516, size = 499, normalized size = 1.85

$$-\frac{b^5\sin(c+dx)}{a^4d(a-b)(a+b)(a+b\cos(c+dx))} + \frac{2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)}{3a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(\ldots\right.}{3a^4d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (-2*b^4*(5*a^2 - 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]
])/(a^5*(a^2 - b^2)*Sqrt[-a^2 + b^2]*d) + ((a^2*b + 4*b^3)*Log[Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]])/(a^5*d) + ((-(a^2*b) - 4*b^3)*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]])/(a^5*d) + (a - 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^2) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])^3) + Sin[(c + d*x)/2]/(6*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3) + (-a + 6*b)/(12*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2
*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2])/(3*a^4*d*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])) + (2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2])/(3
*a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^5*Sin[c + d*x])/(a^4*(a
- b)*(a + b)*d*(a + b*Cos[c + d*x]))
```

Maple [B] time = 0.13, size = 535, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -1/3/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/d/
a^3/(tan(1/2*d*x+1/2*c)-1)^2*b-1/d/a^2/(tan(1/2*d*x+1/2*c)-1)-1/d/a^3/(tan(
1/2*d*x+1/2*c)-1)*b-3/d/a^4/(tan(1/2*d*x+1/2*c)-1)*b^2+1/d*b/a^3*ln(tan(1/2
*d*x+1/2*c)-1)+4/d*b^3/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/3/d/a^2/(tan(1/2*d*x+
1/2*c)+1)^3+1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+1/d/a^3/(tan(1/2*d*x+1/2*c)+
1)^2*b-1/d/a^2/(tan(1/2*d*x+1/2*c)+1)-1/d/a^3/(tan(1/2*d*x+1/2*c)+1)*b-3/d/
a^4/(tan(1/2*d*x+1/2*c)+1)*b^2-1/d*b/a^3*ln(tan(1/2*d*x+1/2*c)+1)-4/d*b^3/a
^5*ln(tan(1/2*d*x+1/2*c)+1)-2/d*b^5/a^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)+10/d*b^4/a^3/(a-b)/(a+b)/((a-
b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-8/d*b^
6/a^5/(a-b)/(a+b)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b
)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 8.08479, size = 2218, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(3*((5*a^2*b^5 - 4*b^7)*cos(d*x + c)^4 + (5*a^3*b^4 - 4*a*b^6)*cos(d*
x + c)^3)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 3*((a^6*b^2 + 2*a^4*b^
4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*
a*b^7)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((a^6*b^2 + 2*a^4*b^4 - 7*
a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^5 + 4*a*b^7)
*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 2*(a^8 - 2*a^6*b^2 + a^4*b^4 + (2
*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 + 2*(a^8 + a^6*b
^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^2 - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5
)*cos(d*x + c))*sin(d*x + c))/((a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)
^4 + (a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^3), 1/6*(6*((5*a^2*b^5 - 4
*b^7)*cos(d*x + c)^4 + (5*a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3)*sqrt(a^2 - b^2
)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*((a^6*b^
2 + 2*a^4*b^4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*
a^3*b^5 + 4*a*b^7)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) + 3*((a^6*b^2 + 2*
a^4*b^4 - 7*a^2*b^6 + 4*b^8)*cos(d*x + c)^4 + (a^7*b + 2*a^5*b^3 - 7*a^3*b^
5 + 4*a*b^7)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) + 2*(a^8 - 2*a^6*b^2 +
a^4*b^4 + (2*a^7*b + 5*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 + 2*
(a^8 + a^6*b^2 - 5*a^4*b^4 + 3*a^2*b^6)*cos(d*x + c)^2 - 2*(a^7*b - 2*a^5*b
^3 + a^3*b^5)*cos(d*x + c))*sin(d*x + c))/((a^9*b - 2*a^7*b^3 + a^5*b^5)*d*
```

```
cos(d*x + c)^4 + (a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a + b*cos(c + d*x))**2, x)
```

Giac [A] time = 1.39282, size = 497, normalized size = 1.84

$$-\frac{\frac{6\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^6-a^4b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)}+\frac{6\left(5\,a^2b^4-4\,b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-a^5b^2\right)\sqrt{a^2-b^2}}+\frac{3\,(a}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(6*b^5*tan(1/2*d*x + 1/2*c)/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^2
 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)) + 6*(5*a^2*b^4 - 4*b^6)*(pi*floor(1/2
*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*
tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^7 - a^5*b^2)*sqrt(a^2 - b^2)) +
 3*(a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 3*(a^2*b + 4*b^
3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^5 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5
 + 3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^2*tan(1/2*d*x + 1/2*c)^5 - 2*a^2*tan(
1/2*d*x + 1/2*c)^3 - 18*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/
2*c) - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 - 1)^3*a^4))/d
```

3.469 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=300

$$-\frac{3a\left(-7a^2b^2+4a^4+2b^4\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}-\frac{a^3\left(-29a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\left(4a^2-7b^2\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cdots}$$

```
[Out] ((12*a^2 + b^2)*x)/(2*b^5) - (a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(Sq
rt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*
d) - (3*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^2*d)
 + ((6*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2
)^2*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])^2) - (a^2*(4*a^2 - 7*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(2*b^2*(a^2
- b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.783223, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3047, 3049, 3023, 2735, 2659, 205}

$$-\frac{3a\left(-7a^2b^2+4a^4+2b^4\right)\sin(c+dx)}{2b^4d\left(a^2-b^2\right)^2}-\frac{a^3\left(-29a^2b^2+12a^4+20b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\left(4a^2-7b^2\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((12*a^2 + b^2)*x)/(2*b^5) - (a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTan[(Sq
rt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a + b)^(5/2)*
d) - (3*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sin[c + d*x])/(2*b^4*(a^2 - b^2)^2*d)
 + ((6*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2
)^2*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c
+ d*x])^2) - (a^2*(4*a^2 - 7*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(2*b^2*(a^2
- b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(3a^2-2ab\cos(c+dx)-2\left(2a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(4a^2-7b^2\right)\cos^2(c+dx)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{\cos(c+dx)(-}{}}{}$$

$$= \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(4a^2-\right.}{2b^2\left(a^2\right.}$$

$$= -\frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2 d} - \frac{a^2}{2b\left(a^2\right.}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c}{2b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{3a\left(4a^4-7a^2b^2+2b^4\right)\sin(c+dx)}{2b^4\left(a^2-b^2\right)^2 d} + \frac{\left(6a^4-10a^2b^2+b^4\right)\cos(c+dx)\sin(c}{2b^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(12a^2+b^2\right)x}{2b^5} - \frac{a^3\left(12a^4-29a^2b^2+20b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^5(a+b)^{5/2}d} - \frac{3a\left(4a^4-7a^2b^2+2b^4\right.}{2b^4\left(a^2-b^2\right.}$$

Mathematica [A] time = 1.99726, size = 199, normalized size = 0.66

$$\frac{2\left(12a^2+b^2\right)(c+dx) + \frac{2a^4b\left(10b^2-7a^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{4a^3\left(-29a^2b^2+12a^4+20b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} + \frac{2a^5b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2} - 12a}{4b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(12*a^2 + b^2)*(c + d*x) + (4*a^3*(12*a^4 - 29*a^2*b^2 + 20*b^4)*ArcTanh
[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 12*a*b*
Sin[c + d*x] + (2*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])
^2) + (2*a^4*b*(-7*a^2 + 10*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*
Cos[c + d*x])) + b^2*Sin[2*(c + d*x)])/(4*b^5*d)
```

Maple [B] time = 0.101, size = 802, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*a-1/d/b^3/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-6/d/b^4/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)*a+1/d/b^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+
```

```
12/d/b^5*arctan(tan(1/2*d*x+1/2*c))*a^2+1/d/b^3*arctan(tan(1/2*d*x+1/2*c))-
6/d*a^6/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+10/d*a^
4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*
b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*a^6/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*a^5/b^3/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)+10/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-12/d*a^7/b^5/(a^4-2*a^2*b
^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(
1/2))+29/d*a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-20/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a-b)*
(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48709, size = 2525, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 - b^10)*d*x*cos(d
*x + c)^2 + 4*(12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b^7 - a*b^9)*d*x*
cos(d*x + c) + 2*(12*a^10 - 35*a^8*b^2 + 33*a^6*b^4 - 9*a^4*b^6 - a^2*b^8)*
d*x - (12*a^9 - 29*a^7*b^2 + 20*a^5*b^4 + (12*a^7*b^2 - 29*a^5*b^4 + 20*a^3
*b^6)*cos(d*x + c)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)*cos(d*x + c))
*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 -
2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(12*a^9*b - 33*a^7*b^3 + 27*a
^5*b^5 - 6*a^3*b^7 - (a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(d*x + c)^
3 + 4*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*cos(d*x + c)^2 + (18*a^8*b^
2 - 50*a^6*b^4 + 43*a^4*b^6 - 11*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6
*b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5
*b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c) + (a^8*b^5 - 3*a^6*b^7 + 3*a^4*b
^9 - a^2*b^11)*d), 1/2*((12*a^8*b^2 - 35*a^6*b^4 + 33*a^4*b^6 - 9*a^2*b^8 -
 b^10)*d*x*cos(d*x + c)^2 + 2*(12*a^9*b - 35*a^7*b^3 + 33*a^5*b^5 - 9*a^3*b
^7 - a*b^9)*d*x*cos(d*x + c) + (12*a^10 - 35*a^8*b^2 + 33*a^6*b^4 - 9*a^4*b
^6 - a^2*b^8)*d*x - (12*a^9 - 29*a^7*b^2 + 20*a^5*b^4 + (12*a^7*b^2 - 29*a^
5*b^4 + 20*a^3*b^6)*cos(d*x + c)^2 + 2*(12*a^8*b - 29*a^6*b^3 + 20*a^4*b^5)
*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2
)*sin(d*x + c))) - (12*a^9*b - 33*a^7*b^3 + 27*a^5*b^5 - 6*a^3*b^7 - (a^6*b
^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*cos(d*x + c)^3 + 4*(a^7*b^3 - 3*a^5*b^5
+ 3*a^3*b^7 - a*b^9)*cos(d*x + c)^2 + (18*a^8*b^2 - 50*a^6*b^4 + 43*a^4*b^6
```

```
 - 11*a^2*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^
11 - b^13)*d*cos(d*x + c)^2 + 2*(a^7*b^6 - 3*a^5*b^8 + 3*a^3*b^10 - a*b^12)
*d*cos(d*x + c) + (a^8*b^5 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38984, size = 1030, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(12*a^7 - 29*a^5*b^2 + 20*a^3*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^4*b^5 - 2*a^2*b^7 + b^9)*sqrt(a^2 - b^2)) - 2*(12*a
^7*tan(1/2*d*x + 1/2*c)^7 - 18*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 17*a^5*b^2*ta
n(1/2*d*x + 1/2*c)^7 + 33*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 2*a^3*b^4*tan(1/
2*d*x + 1/2*c)^7 - 13*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 4*a*b^6*tan(1/2*d*x
+ 1/2*c)^7 + b^7*tan(1/2*d*x + 1/2*c)^7 + 36*a^7*tan(1/2*d*x + 1/2*c)^5 - 1
8*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 67*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 29*a^4
*b^3*tan(1/2*d*x + 1/2*c)^5 + 26*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 5*a^2*b^5
*tan(1/2*d*x + 1/2*c)^5 - 4*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 3*b^7*tan(1/2*d*
x + 1/2*c)^5 + 36*a^7*tan(1/2*d*x + 1/2*c)^3 + 18*a^6*b*tan(1/2*d*x + 1/2*c
)^3 - 67*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 29*a^4*b^3*tan(1/2*d*x + 1/2*c)^3
 + 26*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4
*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 3*b^7*tan(1/2*d*x + 1/2*c)^3 + 12*a^7*tan(1
/2*d*x + 1/2*c) + 18*a^6*b*tan(1/2*d*x + 1/2*c) - 17*a^5*b^2*tan(1/2*d*x +
1/2*c) - 33*a^4*b^3*tan(1/2*d*x + 1/2*c) - 2*a^3*b^4*tan(1/2*d*x + 1/2*c) +
 13*a^2*b^5*tan(1/2*d*x + 1/2*c) + 4*a*b^6*tan(1/2*d*x + 1/2*c) - b^7*tan(1
/2*d*x + 1/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^4 - b
*tan(1/2*d*x + 1/2*c)^4 + 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (12*a^2
+ b^2)*(d*x + c)/b^5)/d
```

3.470 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{(3a^2-2b^2)\sin(c+dx)}{2b^3d(a^2-b^2)}+\frac{3a^2(-5a^2b^2+2a^4+4b^4)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+\ldots$$

```
[Out] (-3*a*x)/b^4 + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) + ((3*a^2 -
2*b^2)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^2*Sin[c + d*
x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*a^3*(a^2 - 2*b^2)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.487519, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3031, 3023, 2735, 2659, 205}

$$\frac{(3a^2-2b^2)\sin(c+dx)}{2b^3d(a^2-b^2)}+\frac{3a^2(-5a^2b^2+2a^4+4b^4)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (-3*a*x)/b^4 + (3*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) + ((3*a^2 -
2*b^2)*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^2*Sin[c + d*
x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*a^3*(a^2 - 2*b^2)*Sin[c
 + d*x])/(2*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
```

```
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(2a^2-2ab\cos(c+dx)-\left(3a^2-2b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \int \frac{3a^2b\left(a^2-2b^2\right)+a(3$$

$$= \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\sin(c+dx}{2b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+$$

$$= -\frac{3ax}{b^4} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\mathrm{si}}{2b^3\left(a^2-b^2\right)^2 d(a+}$$

$$= -\frac{3ax}{b^4} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a^3\left(a^2-2b^2\right)\mathrm{si}}{2b^3\left(a^2-b^2\right)^2 d(a+}$$

$$= -\frac{3ax}{b^4} + \frac{3a^2\left(2a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}d} + \frac{\left(3a^2-2b^2\right)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{a^2}{2b(a}$$

Mathematica [A] time = 1.52214, size = 177, normalized size = 0.8

$$\frac{\frac{a^3b(5a^2-8b^2)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))}-\frac{6a^2(-5a^2b^2+2a^4+4b^4)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{(b^2-a^2)^{5/2}}-\frac{a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^2}-6a(c+dx)+2b\sin(c+dx)}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-6*a*(c + d*x) - (6*a^2*(2*a^4 - 5*a^2*b^2 + 4*b^4)*ArcTanh[((a - b)*Tan[(
c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) + 2*b*Sin[c + d*x] - (a^
4*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^2) + (a^3*b*(5*a^2
- 8*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])))/(2*b^4*d
)
```

Maple [B] time = 0.094, size = 679, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-6/d/b^4*a*arctan(tan(1/
2*d*x+1/2*c))+4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+
b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/d*a^4/b^2/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/
2*c)^3-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b
)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+4/d*a^5/b^3/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d*
a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c)-8/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+6/d*a^6/b^4/(a^4-
2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(
a+b))^(1/2))-15/d*a^4/b^2/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(ta
n(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+12/d*a^2/(a^4-2*a^2*b^2+b^4)/((
a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.42042, size = 2209, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(12*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c)^2 + 24
*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*x*cos(d*x + c) + 12*(a^9 - 3*a
^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*x + 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (2*a
^6*b^2 - 5*a^4*b^4 + 4*a^2*b^6)*cos(d*x + c)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4
*a^3*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
 b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
 - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*a^8
*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b
^7 - b^9)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*a*b^8)*
cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12)*d*co
s(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)
+ (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d), -1/2*(6*(a^7*b^2 - 3*a^5
*b^4 + 3*a^3*b^6 - a*b^8)*d*x*cos(d*x + c)^2 + 12*(a^8*b - 3*a^6*b^3 + 3*a^
4*b^5 - a^2*b^7)*d*x*cos(d*x + c) + 6*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^
6)*d*x - 3*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 + (2*a^6*b^2 - 5*a^4*b^4 + 4*a^2*
b^6)*cos(d*x + c)^2 + 2*(2*a^7*b - 5*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c))*sqr
t(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) -
 (6*a^8*b - 17*a^6*b^3 + 13*a^4*b^5 - 2*a^2*b^7 + 2*(a^6*b^3 - 3*a^4*b^5 +
3*a^2*b^7 - b^9)*cos(d*x + c)^2 + (9*a^7*b^2 - 25*a^5*b^4 + 20*a^3*b^6 - 4*
a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^1
2)*d*cos(d*x + c)^2 + 2*(a^7*b^5 - 3*a^5*b^7 + 3*a^3*b^9 - a*b^11)*d*cos(d*
x + c) + (a^8*b^4 - 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45234, size = 478, normalized size = 2.16

$$-\frac{\frac{3\left(2a^6-5a^4b^2+4a^2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4-2a^2b^6+b^8\right)\sqrt{a^2-b^2}}-\frac{4a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7a^4b^2\tan\left(\frac{1}{2}d\right.}{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(2*a^6 - 5*a^4*b^2 + 4*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) - (4*a^6*tan(1/
2*d*x + 1/2*c)^3 - 5*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 7*a^4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*tan(1/2*d*x + 1/2*c) +
```

```
 5*a^5*b*tan(1/2*d*x + 1/2*c) - 7*a^4*b^2*tan(1/2*d*x + 1/2*c) - 8*a^3*b^3*
tan(1/2*d*x + 1/2*c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + 3*(d*x + c)*a/b^4 - 2*tan(1/2*d*
x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*b^3))/d
```

3.471 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=179

$$-\frac{a\left(-5a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx)}$$

```
[Out] x/b^3 - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*Cos[c + d*x]*Sin
[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(2*a^2 - 5*b^2
)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.303562, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2792, 3021, 2735, 2659, 205}

$$-\frac{a\left(-5a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{2b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd\left(a^2-b^2\right)(a+b\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] x/b^3 - (a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/((a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*Cos[c + d*x]*Sin
[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(2*a^2 - 5*b^2
)*Sin[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{a^2-2ab\cos(c+dx)-2\left(a^2-b^2\right)\cos^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{ab\left(a^2-4b^2\right)+2\cdots}{a+b\cdots}}{2b^2\left(\cdots\right.}$$

$$= \frac{x}{b^3} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\left(a\left(2a^4-\cdots\right.\right.}{\cdots}$$

$$= \frac{x}{b^3} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(2a^2-5b^2\right)\sin(c+dx)}{2b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\left(a\left(2a^4-\cdots\right.\right.}{\cdots}$$

$$= \frac{x}{b^3} - \frac{a\left(2a^4-5a^2b^2+6b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}$$

Mathematica [A] time = 1.09943, size = 149, normalized size = 0.83

$$\frac{-\frac{a^2b\sin(c+dx)\left(3b\left(a^2-2b^2\right)\cos(c+dx)+2a^3-5ab^2\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}+\frac{2a\left(-5a^2b^2+2a^4+6b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}+2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (2*(c + d*x) + (2*a*(2*a^4 - 5*a^2*b^2 + 6*b^4)*ArcTanh[((a - b)*Tan[(c + d
*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - (a^2*b*(2*a^3 - 5*a*b^2 + 3
*b*(a^2 - 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Co
s[c + d*x])^2))/(2*b^3*d)
```

Maple [B] time = 0.094, size = 639, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d/b^3*arctan(tan(1/2*d*x+1/2*c))-2/d*a^4/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d*a^
3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+
b^2)*tan(1/2*d*x+1/2*c)^3+6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-2/d*a^4/b^2/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/
2*d*x+1/2*c)-1/d*a^3/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^
2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+6/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d*
a^5/b^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(
a-b)/((a-b)*(a+b))^(1/2))+5/d*a^3/b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)
*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-6/d*a*b/(a^4-2*a^2*b^
2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1
/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21593, size = 1944, normalized size = 10.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 + 8*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + 4*(a^8 - 3*a^6*b^2 +
 3*a^4*b^4 - a^2*b^6)*d*x - (2*a^7 - 5*a^5*b^2 + 6*a^3*b^4 + (2*a^5*b^2 - 5
*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(2*a^6*b - 5*a^4*b^3 + 6*a^2*b^5)*co
s(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b
^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^7*b - 7*a^5*b
^3 + 5*a^3*b^5 + 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sin(d*x
+ c))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b
^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5
+ 3*a^4*b^7 - a^2*b^9)*d), 1/2*(2*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d
*x*cos(d*x + c)^2 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x +
 c) + 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x - (2*a^7 - 5*a^5*b^2 +
```

```
6*a^3*b^4 + (2*a^5*b^2 - 5*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(2*a^6*b -
 5*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x +
c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^7*b - 7*a^5*b^3 + 5*a^3*b^5
+ 3*(a^6*b^2 - 3*a^4*b^4 + 2*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^6*b^5
 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6
+ 3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a
^2*b^9)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46025, size = 431, normalized size = 2.41

$$\frac{\frac{\left(2\,a^5-5\,a^3b^2+6\,ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^3-2\,a^2b^5+b^7\right)\sqrt{a^2-b^2}}-\frac{2\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^3b^2\tan\left(\frac{1}{2}\,d\right.}{(}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^5 - 5*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
- b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(a^2 - b^2)) - (2*a^5*tan(1/2*d*x
 + 1/2*c)^3 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 5*a^3*b^2*tan(1/2*d*x + 1/2*
c)^3 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c) + 3*a^
4*b*tan(1/2*d*x + 1/2*c) - 5*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*a^2*b^3*tan(1
/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + (d*x + c)/b^3)/d
```

3.472 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$\frac{(a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{a(a^2-4b^2)\sin(c+dx)}{2bd(a^2-b^2)^2(a+b\cos(c+dx))}$$

```
[Out] ((a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(5/2)*(a + b)^(5/2)*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2) + (a*(a^2 - 4*b^2)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*C
os[c + d*x]))
```

Rubi [A] time = 0.173072, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2790, 2754, 12, 2659, 205}

$$\frac{(a^2+2b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^2\sin(c+dx)}{2bd(a^2-b^2)(a+b\cos(c+dx))^2}+\frac{a(a^2-4b^2)\sin(c+dx)}{2bd(a^2-b^2)^2(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(5/2)*(a + b)^(5/2)*d) - (a^2*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^2) + (a*(a^2 - 4*b^2)*Sin[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*C
os[c + d*x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{\int \frac{2ab+\left(a^2-2b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2-4b^2\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}+\frac{\int \frac{b\left(a^2+2b^2\right)}{a+b\cos(c+dx)}}{2b\left(a^2-b^2\right)^2}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2-4b^2\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}+\frac{\left(a^2+2b^2\right)\int \frac{}{a}}{2\left(a^2-\right.}$$

$$= -\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2-4b^2\right)\sin(c+dx)}{2b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}+\frac{\left(a^2+2b^2\right)\text{Su}}{}$$

$$= \frac{\left(a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{a^2\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{a\left(a^2-4\right.}{2b\left(a^2-b^2\right)^2 \,}$$

Mathematica [A] time = 0.536378, size = 115, normalized size = 0.77

$$\frac{\frac{a\sin(c+dx)\left(\left(a^2-4b^2\right)\cos(c+dx)-3ab\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}-\frac{2\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-
a^2 + b^2)^(5/2) + (a*(-3*a*b + (a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/(
(a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.085, size = 400, normalized size = 2.7

$$-\frac{a^2}{d\,(a-b)\left(a^2+2\,ab+b^2\right)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2b+a+b\right)^{-2}-4\,\frac{}{d\left((\tan(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2
*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b-4/d/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2
*d*x+1/2*c)*b+1/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2
/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/(a^
4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.23701, size = 1281, normalized size = 8.6

$$\left[-\frac{\left(a^4+2a^2b^2+\left(a^2b^2+2b^4\right)\cos\left(dx+c\right)^2+2\left(a^3b+2ab^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{2ab\cos(dx+c)+\left(2a^2-b^2\right)\cos(dx+c)^2+2}{b^2\cos(dx+c)^2}\right.}{4\left(\left(a^6b^2-3a^4b^4+3a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3a^5b^3+3a^3b\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((a^4 + 2*a^2*b^2 + (a^2*b^2 + 2*b^4)*cos(d*x + c)^2 + 2*(a^3*b + 2*a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(3*a^4*b -
 3*a^2*b^3 - (a^5 - 5*a^3*b^2 + 4*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*
b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3
+ 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6)*d), 1/2*((a^4 + 2*a^2*b^2 + (a^2*b^2 + 2*b^4)*cos(d*x + c)^2 + 2*(a^3*b
+ 2*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt
(a^2 - b^2)*sin(d*x + c))) - (3*a^4*b - 3*a^2*b^3 - (a^5 - 5*a^3*b^2 + 4*a*
b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d
*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c)
+ (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35381, size = 338, normalized size = 2.27

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)\left(a^2+2\,b^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a}{\left(a^4-2\,a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(a^2 + 2*b^2)/((a^4
- 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 - a^3*tan(1/2*d*x
+ 1/2*c) + 3*a^2*b*tan(1/2*d*x + 1/2*c) + 4*a*b^2*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
+ a + b)^2))/d
```

3.473 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=134

$$\frac{\left(a^2+2b^2\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{3ab\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (-3*a*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*
(a + b)^(5/2)*d) + (a*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2
) + ((a^2 + 2*b^2)*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.123146, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2754, 12, 2659, 205}

$$\frac{\left(a^2+2b^2\right)\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{a\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}-\frac{3ab\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (-3*a*b*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*
(a + b)^(5/2)*d) + (a*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2
) + ((a^2 + 2*b^2)*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{2b-a\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+2b^2\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int -\frac{3ab}{a+b\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+2b^2\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{(3ab)\int \frac{1}{a+b\cos(c+dx)}}{2\left(a^2-b^2\right)^2}$$

$$= \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+2b^2\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{(3ab)\,\text{Subst}\left(\int \frac{}{a+}\right.}{}$$

$$= -\frac{3ab\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+2b^2\right)\sin(}{2\left(a^2-b^2\right)^2 d(a+b}$$

Mathematica [A] time = 0.351407, size = 115, normalized size = 0.86

$$\frac{\frac{\sin(c+dx)\left(b\left(a^2+2b^2\right)\cos(c+dx)+a\left(2a^2+b^2\right)\right)}{(a+b\cos(c+dx))^2} + \frac{6ab\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}}{2d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((6*a*b*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b
^2] + ((a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a +
b*Cos[c + d*x])^2)/(2*(a - b)^2*(a + b)^2*d)
```

Maple [B] time = 0.082, size = 475, normalized size = 3.5

$$2\,\frac{a^2\left(\tan(1/2\,dx+c/2)\right)^3}{d\left(\left(\tan(1/2\,dx+c/2)\right)^2a-\left(\tan(1/2\,dx+c/2)\right)^2b+a+b\right)^2(a-b)\left(a^2+2\,ab+b^2\right)} + \frac{ab}{d\,(a-b)\left(a^2+2\,ab+b^2\right)}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 2/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*
a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^2*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b+2/d/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*
x+1/2*c)^3*b^2+2/d*a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^
2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b+2/d/
(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)
*tan(1/2*d*x+1/2*c)*b^2-3/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arc
```

```
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27101, size = 1207, normalized size = 9.01

$$\left[-\frac{3\left(ab^3\cos\left(dx+c\right)^2+2\,a^2b^2\cos\left(dx+c\right)+a^3b\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2-2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right.}{4\left(\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3\,a^5b^3+3\,a^3b^5-ab^7\right)d\cos\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a*b^3*cos(d*x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a^3*b)*sqrt(-a^2
+ b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2
 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(2*a^5 - a^3*b^2 - a*b^4 + (a^4*b + a^2*
b^3 - 2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d
*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d), -1/2*(3*(a*b^3*cos(d*
x + c)^2 + 2*a^2*b^2*cos(d*x + c) + a^3*b)*sqrt(a^2 - b^2)*arctan(-(a*cos(d
*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (2*a^5 - a^3*b^2 - a*b^4 + (
a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4
+ 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*
b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52567, size = 366, normalized size = 2.73

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)ab}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b^3\tan\left(\frac{1}{2}\,dx+\right.}{\left(a^4-2\,a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}\,d\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*a*b/((a^4 - 2*a^2*b
^2 + b^4)*sqrt(a^2 - b^2)) + (2*a^3*tan(1/2*d*x + 1/2*c)^3 - a^2*b*tan(1/2*
d*x + 1/2*c)^3 + a*b^2*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^
3 + 2*a^3*tan(1/2*d*x + 1/2*c) + a^2*b*tan(1/2*d*x + 1/2*c) + a*b^2*tan(1/2
*d*x + 1/2*c) + 2*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 + b^4)*(a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2))/d
```

3.474 $\int \frac{1}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

```
[Out] ((2*a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(5/2)*(a + b)^(5/2)*d) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c +
d*x])^2) - (3*a*b*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.109509, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 205}

$$\frac{(2a^2+b^2)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3ab\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}-\frac{b\sin(c+dx)}{2d\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-3),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)
^(5/2)*(a + b)^(5/2)*d) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c +
d*x])^2) - (3*a*b*Sin[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3}\,dx = -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2a+b\cos(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{3ab\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{2a^2+b^2}{a+b\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)^2}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{3ab\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(2a^2+b^2\right)\int \frac{}{a+b}}{2\left(a^2-b\right.}$$

$$= -\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{3ab\sin(c+dx)}{2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\left(2a^2+b^2\right)\text{Subs}}{}$$

$$= \frac{\left(2a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{3ab\sin}{2\left(a^2-b^2\right)^2 d(a}$$

Mathematica [A] time = 0.391753, size = 113, normalized size = 0.85

$$\frac{\frac{b\sin(c+dx)\left(-4a^2-3ab\cos(c+dx)+b^2\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} - \frac{2\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-3),x]
```

```
[Out] ((-2*(2*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-
a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*Cos[c + d*x])*Sin[c + d*x])/((a
 - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.082, size = 400, normalized size = 3.

$$-4\,\frac{a\left(\tan\left(1/2\,dx+c/2\right)\right)^3 b}{d\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2 a-\left(\tan\left(1/2\,dx+c/2\right)\right)^2 b+a+b\right)^2 (a-b)\left(a^2+2\,ab+b^2\right)} - \frac{b^2}{d\,(a-b)\left(a^2+2\,ab+b^2\right)}\left(\text{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a-b)/(a^2+2*a
*b+b^2)*tan(1/2*d*x+1/2*c)^3*b-1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2-4/d/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2*a/(a+b)/(a^2-2*a*b+b^2)*tan(1/2
*d*x+1/2*c)*b+1/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+
b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b^2+2/d*a^2/(a^4-2*a^2*b^2+b^4)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+1/d/(a^
4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.27104, size = 1276, normalized size = 9.59

$$\left[-\frac{\left(2\,a^4+a^2b^2+\left(2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+2\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2}{b^2\cos(dx+c)^2\cdot}\right.}{4\left(\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\left(a^7b-3\,a^5b^3+3\,a^3b^5\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((2*a^4 + a^2*b^2 + (2*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^3*b + a
*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(4*a^4*b -
 5*a^2*b^3 + b^5 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^
2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)
*d), 1/2*((2*a^4 + a^2*b^2 + (2*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^3*b
+ a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a
^2 - b^2)*sin(d*x + c))) - (4*a^4*b - 5*a^2*b^3 + b^5 + 3*(a^3*b^2 - a*b^4)
*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos
(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a
^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21951, size = 339, normalized size = 2.55

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)(2a^2+b^2)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))*(2*a^2 + b^2)/((a^4
- 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)) + (4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*a
*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^2*b*tan(1/2*
d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/((a
^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
+ a + b)^2))/d
```

3.475 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

```
[Out] -((b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt
[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d)) + ArcTanh[Sin[c + d*x]]/(a^3
*d) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b^2*
(5*a^2 - 2*b^2)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.4562, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{b\left(-5a^2b^2+6a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))}+\frac{b^2\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt
[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d)) + ArcTanh[Sin[c + d*x]]/(a^3
*d) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b^2*
(5*a^2 - 2*b^2)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2-b^2\right)-2ab\cos(c+dx)+b^2\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{\left(2\left(a^2-b^2\right)^2-ab\left(\cdots\right.}{\cdots}}{2\cdots}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\sin(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \sec(c+dx)\,\cdots}{a^3}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(5a^2-2b^2\right)\sin(c+d\cdots}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c\cdots}$$

$$= -\frac{b\left(6a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{b^2\,\mathrm{si}\cdots}{2a\left(a^2-b^2\right)d(\cdots}$$

Mathematica [A] time = 1.07702, size = 192, normalized size = 1.05

$$\frac{\frac{ab^2\sin(c+dx)\left(b\left(5a^2-2b^2\right)\cos(c+dx)+6a^3-3ab^2\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + \frac{2b\left(-5a^2b^2+6a^4+2b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}} - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\cdots\right.\right.}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] + 2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*(6*a^3 - 3*a*b^2 +
b*(5*a^2 - 2*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*C
os[c + d*x])^2))/(2*a^3*d)
```

Maple [B] time = 0.115, size = 660, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)+6/d/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3*b^2+1/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-2/d*b^4/a^2/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*
d*x+1/2*c)^3+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b
)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)*b^2-1/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d*
b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c)-6/d*a*b/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)
*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+5/d*b^3/a/(a^4-2*a^2*
b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^
(1/2))-2/d*b^5/a^3/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.59945, size = 2452, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*((6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*c
os(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(-a^2
 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a^
2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)
^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6
+ (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5
*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 2*(a^8 - 3*
a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos
(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(-
sin(d*x + c) + 1) - 2*(6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 + (5*a^5*b^3 - 7*a
^3*b^5 + 2*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 3*a^7*b^4 + 3*a^5
*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*
b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d), -1/2*((6
*a^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^4*b^3 - 5*a^2*b^5 + 2*b^7)*cos(d*x +
c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*ar
ctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (a^8 - 3*a^6*b
^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x
+ c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c))*log(sin(d*
x + c) + 1) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 + (a^6*b^2 - 3*a^4*b^4
 + 3*a^2*b^6 - b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b
^6 + (5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^
2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b
^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 -
a^5*b^6)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**3, x)
```

Giac [B] time = 1.42374, size = 464, normalized size = 2.55

$$\frac{\left(6a^4b-5a^2b^3+2b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-2a^5b^2+a^3b^4\right)\sqrt{a^2-b^2}}+\frac{6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab^4\tan\left(\frac{1}{2}\right.}{\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((6*a^4*b - 5*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2
- b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) + (6*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^4*tan(1/2*d*x +
1/2*c)^3 + 2*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) +
5*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5*tan(1
```

```
/2*d*x + 1/2*c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b
*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^
3 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3)/d
```

3.476 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(-11a^2b^2+2a^4+6b^4\right)\tan(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(}$$

```
[Out] (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) - (3*b*ArcTanh[Sin[c + d*x]
])/(a^4*d) + ((2*a^4 - 11*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)
^2*d) + (b^2*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*
b^2*(2*a^2 - b^2)*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])
)
```

Rubi [A] time = 0.783818, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{3b^2\left(-5a^2b^2+4a^4+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\left(-11a^2b^2+2a^4+6b^4\right)\tan(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+}{2a^2d\left(a^2-b^2\right)^2(a+b\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (3*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) - (3*b*ArcTanh[Sin[c + d*x]
])/(a^4*d) + ((2*a^4 - 11*a^2*b^2 + 6*b^4)*Tan[c + d*x])/(2*a^3*(a^2 - b^2)
^2*d) + (b^2*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*
b^2*(2*a^2 - b^2)*Tan[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])
)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{\int \frac{\left(2a^2-3b^2-2ab\cos(c+dx)+2b^2\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{3b^2\left(2a^2-b^2\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}+\frac{\int \frac{\left(2a^4-11a^2b^2+6b^4-a\right.}{}}{}$$

$$= \frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d}+\frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{3b^2\left(2a^2-b^2\right)}{2a^2\left(a^2-b^2\right)^2 d(a}$$

$$= \frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d}+\frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2}+\frac{3b^2\left(2a^2-b^2\right)}{2a^2\left(a^2-b^2\right)^2 d(a}$$

$$= -\frac{3b\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{\left(2a^4-11a^2b^2+6b^4\right)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2 d}+\frac{b^2\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+}$$

$$= \frac{3b^2\left(4a^4-5a^2b^2+2b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d}-\frac{3b\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{\left(2a^4-11a^2b^2\right.}{2a^3\left(\right.}$$

Mathematica [A] time = 4.1364, size = 205, normalized size = 0.88

$$-\frac{\frac{ab^3\sin(c+dx)\left(b\left(7a^2-4b^2\right)\cos(c+dx)+8a^3-5ab^2\right)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2}+\frac{6b^2\left(-5a^2b^2+4a^4+2b^4\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{5/2}}-2a\tan(c+dx)-6b\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((6*b^2*(4*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqr
t[-a^2 + b^2]])/(-a^2 + b^2)^(5/2) - 6*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 6*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^3*(8*a^3 - 5*a
*b^2 + b*(7*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2*(
a + b*Cos[c + d*x])^2) - 2*a*Tan[c + d*x])/(2*a^4*d)
```

Maple [B] time = 0.121, size = 712, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -1/d/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d*b/a^4*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^3/
(tan(1/2*d*x+1/2*c)+1)-3/d*b/a^4*ln(tan(1/2*d*x+1/2*c)+1)-8/d*b^3/a/(tan(1/
2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3-1/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a
+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+4/d*b^5/a^3/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1
/2*c)^3-8/d*b^3/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+
b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+1/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+4/d*b
^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a
*b+b^2)*tan(1/2*d*x+1/2*c)+12/d/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))*b^2-15/d*b^4/a^2/(a^4-2*a
^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))+6/d*b^6/a^4/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/
2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.66632, size = 2907, normalized size = 12.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(3*((4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(d*x + c)^3 + 2*(4*a^5*b^3 - 5
*a^3*b^5 + 2*a*b^7)*cos(d*x + c)^2 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*b^6)*co
s(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*
x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b
^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 6*((a^6*b^3 - 3*a^4*
b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6
- a*b^8)*cos(d*x + c)^2 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x
 + c))*log(sin(d*x + c) + 1) - 6*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*c
os(d*x + c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2
+ (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c)
 + 1) - 2*(2*a^9 - 6*a^7*b^2 + 6*a^5*b^4 - 2*a^3*b^6 + (2*a^7*b^2 - 13*a^5*
b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(d*x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4
*b^5 - 9*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/((a^10*b^2 - 3*a^8*b^4 + 3*a^
6*b^6 - a^4*b^8)*d*cos(d*x + c)^3 + 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5
*b^7)*d*cos(d*x + c)^2 + (a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*
x + c)), 1/2*(3*((4*a^4*b^4 - 5*a^2*b^6 + 2*b^8)*cos(d*x + c)^3 + 2*(4*a^5*
b^3 - 5*a^3*b^5 + 2*a*b^7)*cos(d*x + c)^2 + (4*a^6*b^2 - 5*a^4*b^4 + 2*a^2*
b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 -
 b^2)*sin(d*x + c))) - 3*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x +
 c)^3 + 2*(a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2 + (a^8*b
 - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3
*((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*cos(d*x + c)^3 + 2*(a^7*b^2 - 3*a
^5*b^4 + 3*a^3*b^6 - a*b^8)*cos(d*x + c)^2 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5
 - a^2*b^7)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*a^9 - 6*a^7*b^2 + 6*a
^5*b^4 - 2*a^3*b^6 + (2*a^7*b^2 - 13*a^5*b^4 + 17*a^3*b^6 - 6*a*b^8)*cos(d*
x + c)^2 + (4*a^8*b - 20*a^6*b^3 + 25*a^4*b^5 - 9*a^2*b^7)*cos(d*x + c))*si
n(d*x + c))/((a^10*b^2 - 3*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d*cos(d*x + c)^3
+ 2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c)^2 + (a^12 - 3
*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**3, x)
```

Giac [A] time = 1.29209, size = 513, normalized size = 2.21

$$-\frac{\frac{3\left(4a^4b^2-5a^2b^4+2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^8-2a^6b^2+a^4b^4\right)\sqrt{a^2-b^2}}+\frac{8a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-7a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5ab^5\tan\left(\ldots\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(4*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*sqrt(a^2 - b^2)) + (8*a^3*b^3*ta
n(1/2*d*x + 1/2*c)^3 - 7*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 5*a*b^5*tan(1/2*d
*x + 1/2*c)^3 + 4*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*b^3*tan(1/2*d*x + 1/2*
c) + 7*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*a*b^5*tan(1/2*d*x + 1/2*c) - 4*b^6*
tan(1/2*d*x + 1/2*c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^
2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) + 3*b*log(abs(tan(1/2*d*x + 1/2*c)
 + 1))/a^4 - 3*b*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 2*tan(1/2*d*x + 1
/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^3))/d
```

3.477 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=305

$$-\frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\tan(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(a^2+12b^2\right)\tanh^{-1}(\sin(}{2a^5d}$$

```
[Out] -((b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d)) + ((a^2 + 12*b^2)*ArcTa
nh[Sin[c + d*x]])/(2*a^5*d) - (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Tan[c + d*x]
)/(2*a^4*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^2 + 6*b^4)*Sec[c + d*x]*Tan[c
+ d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2
 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b^2*(7*a^2 - 4*b^2)*Sec[c + d*x]*Tan[c
 + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.0777, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}-\frac{3b\left(-7a^2b^2+2a^4+4b^4\right)\tan(c+dx)}{2a^4d\left(a^2-b^2\right)^2}+\frac{\left(a^2+12b^2\right)\tanh^{-1}(\sin(}{2a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -((b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])
/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d)) + ((a^2 + 12*b^2)*ArcTa
nh[Sin[c + d*x]])/(2*a^5*d) - (3*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Tan[c + d*x]
)/(2*a^4*(a^2 - b^2)^2*d) + ((a^4 - 10*a^2*b^2 + 6*b^4)*Sec[c + d*x]*Tan[c
+ d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*a*(a^2
 - b^2)*d*(a + b*Cos[c + d*x])^2) + (b^2*(7*a^2 - 4*b^2)*Sec[c + d*x]*Tan[c
 + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(2\left(a^2-2b^2\right)-2ab\cos(c+dx)+3b^2\cos^2(c+dx)\right)\sec^3(c+dx)}{(a+b\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(7a^2-4b^2\right)\sec(c+dx)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))} + \frac{\int \frac{\left(2\left(a^4-10a^2b^2+\right.\right.}{}}{}$$

$$= \frac{\left(a^4-10a^2b^2+6b^4\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{b^2\sec(c+dx)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{b^2\left(7a^2-\right.}{2a^2\left(a\right.}$$

$$= -\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{b}{2a\left(a\right.}$$

$$= -\frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{\left(a^4-10a^2b^2+6b^4\right)\sec(c+dx)\tan(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{b}{2a\left(a\right.}$$

$$= \frac{\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{3b\left(2a^4-7a^2b^2+4b^4\right)\tan(c+dx)}{2a^4\left(a^2-b^2\right)^2d} + \frac{\left(a^4-10a^2b^2+6b\right.}{2a^3}$$

$$= -\frac{b^3\left(20a^4-29a^2b^2+12b^4\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\left(a^2+12b^2\right)\tanh^{-1}(\sin(c+dx))}{2a^5d} - \frac{3}{}$$

Mathematica [A] time = 6.19163, size = 427, normalized size = 1.4

$$\frac{b^4\sin(c+dx)}{2a^3d(a-b)(a+b)(a+b\cos(c+dx))^2} + \frac{3\left(3a^2b^4\sin(c+dx)-2b^6\sin(c+dx)\right)}{2a^4d(a-b)^2(a+b)^2(a+b\cos(c+dx))} + \frac{b^3\left(-29a^2b^2+20a^4+12b^4\right)\tanh^{-1}\left(\right.}{a^5d\left(a^2-b^2\right)^2\sqrt{b^2-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (b^3*(20*a^4 - 29*a^2*b^2 + 12*b^4)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt
[-a^2 + b^2]])/(a^5*(a^2 - b^2)^2*Sqrt[-a^2 + b^2]*d) + ((-a^2 - 12*b^2)*Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*a^5*d) + ((a^2 + 12*b^2)*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]])/(2*a^5*d) + 1/(4*a^3*d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^2) - (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])) - 1/(4*a^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) -
 (3*b*Sin[(c + d*x)/2])/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^
4*Sin[c + d*x])/(2*a^3*(a - b)*(a + b)*d*(a + b*Cos[c + d*x])^2) + (3*(3*a^
2*b^4*Sin[c + d*x] - 2*b^6*Sin[c + d*x]))/(2*a^4*(a - b)^2*(a + b)^2*d*(a +
 b*Cos[c + d*x]))
```

Maple [B] time = 0.133, size = 845, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2+1/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d/a^4
/(tan(1/2*d*x+1/2*c)-1)*b-1/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)-6/d/a^5*ln(tan
(1/2*d*x+1/2*c)-1)*b^2-1/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/a^3/(tan(1/
2*d*x+1/2*c)+1)+3/d/a^4/(tan(1/2*d*x+1/2*c)+1)*b+1/2/d/a^3*ln(tan(1/2*d*x+1
/2*c)+1)+6/d/a^5*ln(tan(1/2*d*x+1/2*c)+1)*b^2+10/d*b^4/a^2/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3+1/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-
b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*b^6/a^4/(tan(1/2*d*x+1/2*c)^2*a
-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1
0/d*b^4/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^
2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/d*b^5/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-6/d*b^6/a^4
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^2/(a+b)/(a^2-2*a*b+b^2
)*tan(1/2*d*x+1/2*c)-20/d*b^3/a/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arc
tan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+29/d*b^5/a^3/(a^4-2*a^2*b
^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(
1/2))-12/d*b^7/a^5/(a^4-2*a^2*b^2+b^4)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d
*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 16.4117, size = 3380, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(((20*a^4*b^5 - 29*a^2*b^7 + 12*b^9)*cos(d*x + c)^4 + 2*(20*a^5*b^4 -
 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c)^3 + (20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*
b^7)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^
2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) -
a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - ((a^8*b^2 +
 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x + c)^4 + 2*(a^9*b +
 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x + c)^3 + (a^10 + 9
*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*
x + c) + 1) + ((a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*co
s(d*x + c)^4 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*c
os(d*x + c)^3 + (a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*c
os(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a
^4*b^6 - 3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(d*x + c)^3 -
(11*a^8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c)^2 - 4*(a^9
*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^
```

```
2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*
b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^
4 - a^7*b^6)*d*cos(d*x + c)^2), -1/4*(2*((20*a^4*b^5 - 29*a^2*b^7 + 12*b^9)
*cos(d*x + c)^4 + 2*(20*a^5*b^4 - 29*a^3*b^6 + 12*a*b^8)*cos(d*x + c)^3 + (
20*a^6*b^3 - 29*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arcta
n(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - ((a^8*b^2 + 9*a^6
*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x + c)^4 + 2*(a^9*b + 9*a^7
*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x + c)^3 + (a^10 + 9*a^8*b
^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x + c)^2)*log(sin(d*x + c)
 + 1) + ((a^8*b^2 + 9*a^6*b^4 - 33*a^4*b^6 + 35*a^2*b^8 - 12*b^10)*cos(d*x
+ c)^4 + 2*(a^9*b + 9*a^7*b^3 - 33*a^5*b^5 + 35*a^3*b^7 - 12*a*b^9)*cos(d*x
 + c)^3 + (a^10 + 9*a^8*b^2 - 33*a^6*b^4 + 35*a^4*b^6 - 12*a^2*b^8)*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6
 - 3*(2*a^7*b^3 - 9*a^5*b^5 + 11*a^3*b^7 - 4*a*b^9)*cos(d*x + c)^3 - (11*a^
8*b^2 - 43*a^6*b^4 + 50*a^4*b^6 - 18*a^2*b^8)*cos(d*x + c)^2 - 4*(a^9*b - 3
*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*cos(d*x + c))*sin(d*x + c))/((a^11*b^2 - 3*
a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 3*a^10*b^3 +
3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c)^3 + (a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^
7*b^6)*d*cos(d*x + c)^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**3, x)
```

Giac [B] time = 1.37061, size = 1081, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(20*a^4*b^3 - 29*a^2*b^5 + 12*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)/sqrt(a^2 - b^2)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*sqrt(a^2 - b^2)) + 2*(a^7*
tan(1/2*d*x + 1/2*c)^7 + 4*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*a^5*b^2*tan(1/
2*d*x + 1/2*c)^7 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*b^4*tan(1/2*d*
x + 1/2*c)^7 - 17*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*a*b^6*tan(1/2*d*x + 1
/2*c)^7 + 12*b^7*tan(1/2*d*x + 1/2*c)^7 + 3*a^7*tan(1/2*d*x + 1/2*c)^5 + 4*
a^6*b*tan(1/2*d*x + 1/2*c)^5 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 26*a^4*b^
3*tan(1/2*d*x + 1/2*c)^5 - 29*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 67*a^2*b^5*t
an(1/2*d*x + 1/2*c)^5 + 18*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*b^7*tan(1/2*d*
x + 1/2*c)^5 + 3*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*a^6*b*tan(1/2*d*x + 1/2*c)^
3 + 5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 -
29*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*
a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*b^7*tan(1/2*d*x + 1/2*c)^3 + a^7*tan(1/2*
d*x + 1/2*c) - 4*a^6*b*tan(1/2*d*x + 1/2*c) - 13*a^5*b^2*tan(1/2*d*x + 1/2*
```

```
c) + 2*a^4*b^3*tan(1/2*d*x + 1/2*c) + 33*a^3*b^4*tan(1/2*d*x + 1/2*c) + 17*
a^2*b^5*tan(1/2*d*x + 1/2*c) - 18*a*b^6*tan(1/2*d*x + 1/2*c) - 12*b^7*tan(1
/2*d*x + 1/2*c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b
*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) + (a^2 + 1
2*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - (a^2 + 12*b^2)*log(abs(tan(
1/2*d*x + 1/2*c) - 1))/a^5)/d
```

3.478 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=307

$$\frac{\left(-23a^2b^2+12a^4+6b^4\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-28a^4b^2+35a^2b^4+8a^6-20b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(4a^2-9b^2\right)\sin}{6b^2d\left(a^2-b^2\right)^2(}$$

```
[Out] (-4*a*x)/b^5 + (a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*ArcTan[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d)
 + ((12*a^4 - 23*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - (
a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3)
 - (a^2*(4*a^2 - 9*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d
*(a + b*Cos[c + d*x])^2) + (a^3*(4*a^4 - 11*a^2*b^2 + 12*b^4)*Sin[c + d*x])
/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.899682, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2792, 3047, 3031, 3023, 2735, 2659, 205}

$$\frac{\left(-23a^2b^2+12a^4+6b^4\right)\sin(c+dx)}{6b^4d\left(a^2-b^2\right)^2}+\frac{a^2\left(-28a^4b^2+35a^2b^4+8a^6-20b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(4a^2-9b^2\right)\sin}{6b^2d\left(a^2-b^2\right)^2(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] (-4*a*x)/b^5 + (a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*ArcTan[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5*(a + b)^(7/2)*d)
 + ((12*a^4 - 23*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) - (
a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3)
 - (a^2*(4*a^2 - 9*b^2)*Cos[c + d*x]^2*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^2*d
*(a + b*Cos[c + d*x])^2) + (a^3*(4*a^4 - 11*a^2*b^2 + 12*b^4)*Sin[c + d*x])
/(2*b^4*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(3a^2-3ab\cos(c+dx)-\left(4a^2-3b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\cos^2(c+dx)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{\cos(c+dx)(-}{}}{}$$

$$= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\cos^2(c+dx)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a^3\left(4a^4-11\right.}{2b^4\left(a^2-b\right.}$$

$$= \frac{\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(4a^2-9b^2\right)\cos}{6b^2\left(a^2-b^2\right)^2 d(}$$

$$= -\frac{4ax}{b^5} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(4a^2-\right.}{6b^2\left(a^2-\right.}$$

$$= -\frac{4ax}{b^5} + \frac{\left(12a^4-23a^2b^2+6b^4\right)\sin(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(4a^2-\right.}{6b^2\left(a^2-\right.}$$

$$= -\frac{4ax}{b^5} + \frac{a^2\left(8a^6-28a^4b^2+35a^2b^4-20b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^5(a+b)^{7/2}d} + \frac{\left(12a^4-23a^2b^2+6b^4\right.}{6b^4\left(a^2-b^2\right.}$$

Mathematica [A] time = 5.65826, size = 240, normalized size = 0.78

$$\frac{\frac{5a^4b\left(3b^2-2a^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + \frac{a^3b\left(-71a^2b^2+26a^4+60b^4\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))} + \frac{6a^2\left(-28a^4b^2+35a^2b^4+8a^6-20b^6\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} + \frac{2a^5b\sin(c+}{(a-b)(a+b)(a+b\cos}}{6b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (-24*a*(c + d*x) + (6*a^2*(8*a^6 - 28*a^4*b^2 + 35*a^2*b^4 - 20*b^6)*ArcTan
h[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + 6*b*Si
n[c + d*x] + (2*a^5*b*Sin[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3
) + (5*a^4*b*(-2*a^2 + 3*b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos
[c + d*x])^2) + (a^3*b*(26*a^4 - 71*a^2*b^2 + 60*b^4)*Sin[c + d*x])/((a - b
)^3*(a + b)^3*(a + b*Cos[c + d*x])))/(6*b^5*d)
```

Maple [B] time = 0.101, size = 1396, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d/b^4*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-8/d/b^5*a*arctan(tan(1/
2*d*x+1/2*c))+6/d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+
```

```
b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/d*a^6/b^3/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(1/2*d*x+1/2*c)^5-18/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+5/d
*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a
^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+20/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*
c)^5+12/d*a^7/b^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^
2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-116/3/d*a^5/b^2/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2
)*tan(1/2*d*x+1/2*c)^3+40/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+6/d*a^7/b^4
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3
*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-18
/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3
-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-5/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1
/2*c)+20/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/
(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+8/d*a^8/b^5/(a^6-3*a^4*b^2+3*a
^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b
))^(1/2))-28/d*a^6/b^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*ar
ctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+35/d*a^4/b/(a^6-3*a^4*b^
2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)
*(a+b))^(1/2))-20/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)
*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.85581, size = 3557, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="fricas")

[Out] [-1/12*(48*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*d*x*cos(d
*x + c)^3 + 144*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d
*x*cos(d*x + c)^2 + 144*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b
^9)*d*x*cos(d*x + c) + 48*(a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*
b^8)*d*x + 3*(8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a^5*b^6 + (8*a^8*b^3 -
28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c)^3 + 3*(8*a^9*b^2 - 28*a^
7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + 3*(8*a^10*b - 28*a^8*b^3
+ 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*
x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c)
+ b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
```

```
a^2)) - 2*(24*a^11*b - 92*a^9*b^3 + 133*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9 +
 6*(a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)^3 + (
44*a^9*b^3 - 169*a^7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x +
 c)^2 + 3*(20*a^10*b^2 - 77*a^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10
)*cos(d*x + c))*sin(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b
^14 + b^16)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*
b^13 + a*b^15)*d*cos(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*
a^4*b^12 + a^2*b^14)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4
*a^5*b^11 + a^3*b^13)*d), -1/6*(24*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3
*b^9 + a*b^11)*d*x*cos(d*x + c)^3 + 72*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 -
4*a^4*b^8 + a^2*b^10)*d*x*cos(d*x + c)^2 + 72*(a^11*b - 4*a^9*b^3 + 6*a^7*b
^5 - 4*a^5*b^7 + a^3*b^9)*d*x*cos(d*x + c) + 24*(a^12 - 4*a^10*b^2 + 6*a^8*
b^4 - 4*a^6*b^6 + a^4*b^8)*d*x - 3*(8*a^11 - 28*a^9*b^2 + 35*a^7*b^4 - 20*a
^5*b^6 + (8*a^8*b^3 - 28*a^6*b^5 + 35*a^4*b^7 - 20*a^2*b^9)*cos(d*x + c)^3
+ 3*(8*a^9*b^2 - 28*a^7*b^4 + 35*a^5*b^6 - 20*a^3*b^8)*cos(d*x + c)^2 + 3*(
8*a^10*b - 28*a^8*b^3 + 35*a^6*b^5 - 20*a^4*b^7)*cos(d*x + c))*sqrt(a^2 - b
^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (24*a^11
*b - 92*a^9*b^3 + 133*a^7*b^5 - 71*a^5*b^7 + 6*a^3*b^9 + 6*(a^8*b^4 - 4*a^6
*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)^3 + (44*a^9*b^3 - 169*a^
7*b^5 + 239*a^5*b^7 - 132*a^3*b^9 + 18*a*b^11)*cos(d*x + c)^2 + 3*(20*a^10*
b^2 - 77*a^8*b^4 + 110*a^6*b^6 - 59*a^4*b^8 + 6*a^2*b^10)*cos(d*x + c))*sin
(d*x + c))/((a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b^14 + b^16)*d*cos(d
*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^13 + a*b^15)*d*co
s(d*x + c)^2 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a^2*b^14
)*d*cos(d*x + c) + (a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^1
3)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2901, size = 760, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(8*a^8 - 28*a^6*b^2 + 35*a^4*b^4 - 20*a^2*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*
sqrt(a^2 - b^2)) - (18*a^9*tan(1/2*d*x + 1/2*c)^5 - 42*a^8*b*tan(1/2*d*x +
1/2*c)^5 - 24*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 117*a^6*b^3*tan(1/2*d*x + 1/
2*c)^5 - 24*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*a^4*b^5*tan(1/2*d*x + 1/2*
c)^5 + 60*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*a^9*tan(1/2*d*x + 1/2*c)^3 -
152*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 236*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 1
20*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a^9*tan(1/2*d*x + 1/2*c) + 42*a^8*b*
```

```
tan(1/2*d*x + 1/2*c) - 24*a^7*b^2*tan(1/2*d*x + 1/2*c) - 117*a^6*b^3*tan(1/
2*d*x + 1/2*c) - 24*a^5*b^4*tan(1/2*d*x + 1/2*c) + 105*a^4*b^5*tan(1/2*d*x
+ 1/2*c) + 60*a^3*b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b
^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3)
 + 12*(d*x + c)*a/b^5 - 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1
)*b^4))/d
```

3.479 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=250

$$-\frac{a\left(-7a^4b^2+8a^2b^4+2a^6-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a^3\left(3a^2-8b^2\right)\sin(}{6b^3d\left(a^2-b^2\right)^2(a+b\,\mathrm{co}}$$

```
[Out] x/b^4 - (a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - (a^2*Cos[
c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a^3*
(3*a^2 - 8*b^2)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2
) - (a^2*(9*a^4 - 28*a^2*b^2 + 34*b^4)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^3*d
*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.571181, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3031, 3021, 2735, 2659, 205}

$$-\frac{a\left(-7a^4b^2+8a^2b^4+2a^6-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a^3\left(3a^2-8b^2\right)\sin(}{6b^3d\left(a^2-b^2\right)^2(a+b\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^4,x]
```


```
[Out] x/b^4 - (a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) - (a^2*Cos[
c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^3) + (a^3*
(3*a^2 - 8*b^2)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2
) - (a^2*(9*a^4 - 28*a^2*b^2 + 34*b^4)*Sin[c + d*x])/(6*b^3*(a^2 - b^2)^3*d
*(a + b*Cos[c + d*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
```

```
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(2a^2-3ab\cos(c+dx)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^3}dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{\int \frac{2a^2b\left(3a^2-8b^2\right)+a \cdots}{\cdots}}{\cdots}$$

$$= -\frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{a^2\left(9a^4-28a^2b \cdots\right.}{6b^3\left(a^2-b^2\right)^3 \cdots}$$

$$= \frac{x}{b^4} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{a^2\left(9a^4-28 \cdots\right.}{6b^3\left(a^2-b \cdots\right.}$$

$$= \frac{x}{b^4} - \frac{a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a^3\left(3a^2-8b^2\right)\sin(c+dx)}{6b^3\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{a^2\left(9a^4-28 \cdots\right.}{6b^3\left(a^2-b \cdots\right.}$$

$$= \frac{x}{b^4} - \frac{a\left(2a^6-7a^4b^2+8a^2b^4-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}b^4(a+b)^{7/2}d} - \frac{a^2\cos^2(c+dx)\sin(c+dx \cdots}{3b\left(a^2-b^2\right)d(a+b\cos(c+ \cdots\right.}$$

Mathematica [A] time = 2.67099, size = 227, normalized size = 0.91

$$\frac{\frac{a^3b\left(7a^2-12b^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + \frac{a^2b\left(32a^2b^2-11a^4-36b^4\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))} - \frac{6a\left(-7a^4b^2+8a^2b^4+2a^6-8b^6\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - \frac{2a^4b\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx \cdots}}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (6*(c + d*x) - (6*a*(2*a^6 - 7*a^4*b^2 + 8*a^2*b^4 - 8*b^6)*ArcTanh[((a - b
)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - (2*a^4*b*Sin[c
+ d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + (a^3*b*(7*a^2 - 12*b^2)*
Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + (a^2*b*(-11*a^
4 + 32*a^2*b^2 - 36*b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Cos[c +
d*x])))/(6*b^4*d)
```

Maple [B] time = 0.092, size = 1356, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d/b^4*arctan(tan(1/2*d*x+1/2*c))-2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c
)^5+1/d*a^5/b^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)
/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+6/d*a^4/b/(tan(1/2*d*x+1/2*
c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/
```

```
2*d*x+1/2*c)^5-4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-12/d*a^2*b/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3
)*tan(1/2*d*x+1/2*c)^5-4/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*
c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+44/3/d*a
^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/
(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-24/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*
c)^3-2/d*a^6/b^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b
)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-1/d*a^5/b^2/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c)+6/d*a^4/b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)
^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+4/d*a^3/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
n(1/2*d*x+1/2*c)-12/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-2/d*a^7/b^4/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))+7/d*a^5/b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+
b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-8/d*a^3/(a^6
-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-
b)/((a-b)*(a+b))^(1/2))+8/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b
))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.58953, size = 3158, normalized size = 12.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="fricas")

[Out] [1/12*(12*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*x*cos(d*x
+ c)^3 + 36*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*x*cos(
d*x + c)^2 + 36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*
cos(d*x + c) + 12*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x
- 3*(2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6 + (2*a^7*b^3 - 7*a^5*b^5 +
8*a^3*b^7 - 8*a*b^9)*cos(d*x + c)^3 + 3*(2*a^8*b^2 - 7*a^6*b^4 + 8*a^4*b^6
- 8*a^2*b^8)*cos(d*x + c)^2 + 3*(2*a^9*b - 7*a^7*b^3 + 8*a^5*b^5 - 8*a^3*b^
7)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*c
os(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2
+ 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(6*a^10*b - 2
3*a^8*b^3 + 43*a^6*b^5 - 26*a^4*b^7 + (11*a^8*b^3 - 43*a^6*b^5 + 68*a^4*b^7
 - 36*a^2*b^9)*cos(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3
*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^
2*b^13 + b^15)*d*cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a
```

```
^3*b^12 + a*b^14)*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 -
4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 -
 4*a^5*b^10 + a^3*b^12)*d), 1/6*(6*(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2
*b^9 + b^11)*d*x*cos(d*x + c)^3 + 18*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*d*x*cos(d*x + c)^2 + 18*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c) + 6*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4
*a^5*b^6 + a^3*b^8)*d*x - 3*(2*a^10 - 7*a^8*b^2 + 8*a^6*b^4 - 8*a^4*b^6 + (
2*a^7*b^3 - 7*a^5*b^5 + 8*a^3*b^7 - 8*a*b^9)*cos(d*x + c)^3 + 3*(2*a^8*b^2
- 7*a^6*b^4 + 8*a^4*b^6 - 8*a^2*b^8)*cos(d*x + c)^2 + 3*(2*a^9*b - 7*a^7*b^
3 + 8*a^5*b^5 - 8*a^3*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x
 + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - (6*a^10*b - 23*a^8*b^3 + 43*a^
6*b^5 - 26*a^4*b^7 + (11*a^8*b^3 - 43*a^6*b^5 + 68*a^4*b^7 - 36*a^2*b^9)*co
s(d*x + c)^2 + 15*(a^9*b^2 - 4*a^7*b^4 + 7*a^5*b^6 - 4*a^3*b^8)*cos(d*x + c
))*sin(d*x + c))/((a^8*b^7 - 4*a^6*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d*
cos(d*x + c)^3 + 3*(a^9*b^6 - 4*a^7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)
*d*cos(d*x + c)^2 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*
b^13)*d*cos(d*x + c) + (a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8 - 4*a^5*b^10 + a^3
*b^12)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31204, size = 717, normalized size = 2.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^7 - 7*a^5*b^2 + 8*a^3*b^4 - 8*a*b^6)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^6*b^4 - 3*a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(a
^2 - b^2)) - (6*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^
5 - 6*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 + 45*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 -
6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 60*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a
^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 12*a^8*tan(1/2*d*x + 1/2*c)^3 - 56*a^6*b^2*
tan(1/2*d*x + 1/2*c)^3 + 116*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 - 72*a^2*b^6*ta
n(1/2*d*x + 1/2*c)^3 + 6*a^8*tan(1/2*d*x + 1/2*c) + 15*a^7*b*tan(1/2*d*x +
1/2*c) - 6*a^6*b^2*tan(1/2*d*x + 1/2*c) - 45*a^5*b^3*tan(1/2*d*x + 1/2*c) -
 6*a^4*b^4*tan(1/2*d*x + 1/2*c) + 60*a^3*b^5*tan(1/2*d*x + 1/2*c) + 36*a^2*
b^6*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*(d*x + c)/b^4)
/d
```

3.480 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4} dx$

Optimal. Leaf size=222

$$-\frac{b\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+$$

```
[Out] -((b*(3*a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(7/2)*(a + b)^(7/2)*d)) - (a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^3) - (a^2*(2*a^2 - 7*b^2)*Sin[c + d*x])/(6*b^
2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4)
*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.337224, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2792, 3021, 2754, 12, 2659, 205}

$$-\frac{b\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{a^2\sin(c+dx)\cos(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b*(3*a^2 + 2*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(7/2)*(a + b)^(7/2)*d)) - (a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2
- b^2)*d*(a + b*Cos[c + d*x])^3) - (a^2*(2*a^2 - 7*b^2)*Sin[c + d*x])/(6*b^
2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^2) + (a*(2*a^4 - 5*a^2*b^2 + 18*b^4)
*Sin[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
```

```
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{a^2-3ab\cos(c+dx)-\left(2a^2-3b^2\right)\cos^2(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{2ab\left(a^2-6b^2\right)+(2a\cdots}{(a+b\cdots}}{6b^2\cdots}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^4-5a^2b^2\cdots\right.}{6b^2\left(a^2-b^2\right)^3 d\cdots}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^4-5a^2b^2\cdots\right.}{6b^2\left(a^2-b^2\right)^3 d\cdots}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2-7b^2\right)\sin(c+dx)}{6b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^4-5a^2b^2\cdots\right.}{6b^2\left(a^2-b^2\right)^3 d\cdots}$$

$$= -\frac{b\left(3a^2+2b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{a^2\left(2a^2\cdots\right.}{6b^2\left(a^2-b^2\right)\cdots}$$

Mathematica [A] time = 1.1703, size = 158, normalized size = 0.71

$$\frac{\frac{a\sin(c+dx)\left(\left(-5a^2b^2+2a^4+18b^4\right)\cos^2(c+dx)+3ab\left(a^2+9b^2\right)\cos(c+dx)+11a^2b^2+4a^4\right)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}-\frac{6b\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-6*b*(3*a^2 + 2*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(-a^2 + b^2)^(7/2) + (a*(4*a^4 + 11*a^2*b^2 + 3*a*b*(a^2 + 9*b^2)*Cos[c +
 d*x] + (2*a^4 - 5*a^2*b^2 + 18*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)
^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

Maple [B] time = 0.089, size = 776, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+3/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2+4/3/d*a^3/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1
/2*d*x+1/2*c)^3+12/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*
a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3*b^2+2/d*a^3/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^
3)*tan(1/2*d*x+1/2*c)-3/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+6/d/(tan(1/2*
d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b
^3)*tan(1/2*d*x+1/2*c)*b^2-3/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(
a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-2/d*b^3/(a
^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(
a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33069, size = 1958, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*a^5*b + 2*a^3*b^3 + (3*a^2*b^4 + 2*b^6)*cos(d*x + c)^3 + 3*(3*a
^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 + 3*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c))*
sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2
*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos
(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(4*a^7 + 7*a^5*b^2 - 11*a^3*b^
4 + (2*a^7 - 7*a^5*b^2 + 23*a^3*b^4 - 18*a*b^6)*cos(d*x + c)^2 + 3*(a^6*b +
 8*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 +
 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 +
6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*(3*a^5*b + 2*a^3*b^3 + (3*a^2*b^4 +
 2*b^6)*cos(d*x + c)^3 + 3*(3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 + 3*(3*a^4*
b^2 + 2*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)
/(sqrt(a^2 - b^2)*sin(d*x + c))) - (4*a^7 + 7*a^5*b^2 - 11*a^3*b^4 + (2*a^7
 - 7*a^5*b^2 + 23*a^3*b^4 - 18*a*b^6)*cos(d*x + c)^2 + 3*(a^6*b + 8*a^4*b^3
 - 9*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7
 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6
- 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5
- 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a
^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36458, size = 539, normalized size = 2.43

$$\frac{3\left(3a^2b+2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{6a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(3*a^2*b + 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
+ arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (6*a^5*tan(1/2*d
```

```
*x + 1/2*c)^5 - 3*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^3*b^2*tan(1/2*d*x + 1/
2*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^
5 + 4*a^5*tan(1/2*d*x + 1/2*c)^3 + 32*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 36*a
*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*d*x + 1/2*c) + 3*a^4*b*tan(1/2*
d*x + 1/2*c) + 6*a^3*b^2*tan(1/2*d*x + 1/2*c) + 27*a^2*b^3*tan(1/2*d*x + 1/
2*c) + 18*a*b^4*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.481 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=206

$$\frac{a\left(a^2+4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\left(-10a^2\right.}{6bd\left(a^2\right.}$$

```
[Out] (a*(a^2 + 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Co
s[c + d*x])^3) + (a*(a^2 - 6*b^2)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b
*Cos[c + d*x])^2) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Sin[c + d*x])/(6*b*(a^2 - b
^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.281021, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2790, 2754, 12, 2659, 205}

$$\frac{a\left(a^2+4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^3}+\frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{\left(-10a^2\right.}{6bd\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (a*(a^2 + 4*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a -
b)^(7/2)*(a + b)^(7/2)*d) - (a^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Co
s[c + d*x])^3) + (a*(a^2 - 6*b^2)*Sin[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b
*Cos[c + d*x])^2) + ((a^4 - 10*a^2*b^2 - 6*b^4)*Sin[c + d*x])/(6*b*(a^2 - b
^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{3ab+\left(a^2-3b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{\int \frac{-2b\left(2a^2+3b^2\right)}{(a+b}}{6b}$$

$$= -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4-10a^2b^2\right.}{6b\left(a^2-b^2\right)^3}$$

$$= -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4-10a^2b^2\right.}{6b\left(a^2-b^2\right)^3}$$

$$= -\frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-6b^2\right)\sin(c+dx)}{6b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\left(a^4-10a^2b^2\right.}{6b\left(a^2-b^2\right)^3}$$

$$= \frac{a\left(a^2+4b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{a\left(a^2-\right.}{6b\left(a^2-b^2\right)^2}$$

Mathematica [A] time = 1.13827, size = 162, normalized size = 0.79

$$\frac{\frac{\sin(c+dx)\left(b\left(-10a^2b^2+a^4-6b^4\right)\cos^2(c+dx)+3a\left(-9a^2b^2+a^4-2b^4\right)\cos(c+dx)-2a^2b^3-13a^4b\right)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3} + \frac{6a\left(a^2+4b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((6*a*(a^2 + 4*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(
-a^2 + b^2)^(7/2) + ((-13*a^4*b - 2*a^2*b^3 + 3*a*(a^4 - 9*a^2*b^2 - 2*b^4)
*Cos[c + d*x] + b*(a^4 - 10*a^2*b^2 - 6*b^4)*Cos[c + d*x]^2)*Sin[c + d*x])/
((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

Maple [B] time = 0.091, size = 930, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -1/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3
*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-6/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(
a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2-2/d/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5*b^3-28/3/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+
a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-4/d/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b
^2)*tan(1/2*d*x+1/2*c)^3+1/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)
^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-6/d*a^2*b/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tan(1/2*d*x+1/2*c)+2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2-2/d/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c)*b^3+1/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)
*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+4/d*a*b^
2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.30246, size = 1959, normalized size = 9.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^6 + 4*a^4*b^2 + (a^3*b^3 + 4*a*b^5)*cos(d*x + c)^3 + 3*(a^4*b^2
 + 4*a^2*b^4)*cos(d*x + c)^2 + 3*(a^5*b + 4*a^3*b^3)*cos(d*x + c))*sqrt(-a^
2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5 - (
a^6*b - 11*a^4*b^3 + 4*a^2*b^5 + 6*b^7)*cos(d*x + c)^2 - 3*(a^7 - 10*a^5*b^
2 + 7*a^3*b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5
+ 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
```

```
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 +
 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^
7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(a^6 + 4*a^4*b^2 + (a^3*b^3 + 4*a*b
^5)*cos(d*x + c)^3 + 3*(a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^2 + 3*(a^5*b + 4*
a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a
^2 - b^2)*sin(d*x + c))) - (13*a^6*b - 11*a^4*b^3 - 2*a^2*b^5 - (a^6*b - 11
*a^4*b^3 + 4*a^2*b^5 + 6*b^7)*cos(d*x + c)^2 - 3*(a^7 - 10*a^5*b^2 + 7*a^3*
b^4 + 2*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^
7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6
 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5
 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*
a^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**4,x)

[Out] Timed out
```

Giac [B] time = 1.35974, size = 576, normalized size = 2.8

$$-\frac{\frac{3\left(a^{3}+4ab^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\left(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6}\right)\sqrt{a^{2}-b^{2}}}+\frac{3a^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+12a^{4}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-27a^{3}b^{2}\tan\left(\frac{1}{2}dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="giac")

[Out] -1/3*(3*(a^3 + 4*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
 arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2))
)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (3*a^5*tan(1/2*d*
x + 1/2*c)^5 + 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1
/2*c)^5 + 12*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^
5 + 6*b^5*tan(1/2*d*x + 1/2*c)^5 + 28*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 16*a^2
*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*b^5*tan(1/2*d*x + 1/2*c)^3 - 3*a^5*tan(1/2
*d*x + 1/2*c) + 12*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 12*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*a*b^4*tan(1/2*d*x + 1/2*c) + 6*b
^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*
x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.482 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=192

$$-\frac{b\left(4a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{}{3d\left(a^2-\right.}$$

```
[Out] -((b*(4*a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(7/2)*(a + b)^(7/2)*d)) + (a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos
[c + d*x])^3) + ((2*a^2 + 3*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Co
s[c + d*x])^2) + (a*(2*a^2 + 13*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rubi [A] time = 0.225992, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2754, 12, 2659, 205}

$$-\frac{b\left(4a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a\left(2a^2+13b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+\frac{}{3d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b*(4*a^2 + b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(7/2)*(a + b)^(7/2)*d)) + (a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos
[c + d*x])^3) + ((2*a^2 + 3*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Co
s[c + d*x])^2) + (a*(2*a^2 + 13*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
b*Cos[c + d*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{3b-2a\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{-10ab+\left(2a^2+3b^2\right)\cdots}{(a+b\cos(c+d\cdots}}{6\left(a^2-b^{\cdots}\right.}$$

$$= \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2+13b^2\right.\cdots}{6\left(a^2-b^2\right)^3 d(a+\cdots}$$

$$= \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2+13b^2\right.\cdots}{6\left(a^2-b^2\right)^3 d(a+\cdots}$$

$$= \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+3b^2\right)\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{a\left(2a^2+13b^2\right.\cdots}{6\left(a^2-b^2\right)^3 d(a+\cdots}$$

$$= -\frac{b\left(4a^2+b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\left(2a^2+\cdots\right.}{6\left(a^2-b^2\right)^2\cdots}$$

Mathematica [A] time = 1.04692, size = 164, normalized size = 0.85

$$\frac{\frac{\sin(c+dx)\left(ab^2\left(2a^2+13b^2\right)\cos^2(c+dx)-3b\left(-9a^2b^2-2a^4+b^4\right)\cos(c+dx)+10a^3b^2+6a^5-ab^4\right)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}-\frac{6b\left(4a^2+b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((-6*b*(4*a^2 + b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/
(-a^2 + b^2)^(7/2) + ((6*a^5 + 10*a^3*b^2 - a*b^4 - 3*b*(-2*a^4 - 9*a^2*b^2
 + b^4)*Cos[c + d*x] + a*b^2*(2*a^2 + 13*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])
/((a - b)^3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

Maple [B] time = 0.089, size = 931, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] 2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-t
```

```
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/
2*c)^5+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2+1/d/(tan(1/2*d*x+1/2*c)^2*
a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x
+1/2*c)^5*b^3+4/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3
/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+28/3/d/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3*b^2+2/d*a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^
2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-2/d*a^2*b/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b
^2-b^3)*tan(1/2*d*x+1/2*c)+6/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2-1/d/(tan
(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c)*b^3-4/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b
)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/d*b^3
/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.23402, size = 1960, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(4*a^5*b + a^3*b^3 + (4*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(4*a^3*b
^3 + a*b^5)*cos(d*x + c)^2 + 3*(4*a^4*b^2 + a^2*b^4)*cos(d*x + c))*sqrt(-a^
2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a
^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + a^2)) + 2*(6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^
6 + (2*a^5*b^2 + 11*a^3*b^4 - 13*a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4
*b^3 - 10*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5
+ 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 +
 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^
7*b^4 - 4*a^5*b^6 + a^3*b^8)*d), -1/6*(3*(4*a^5*b + a^3*b^3 + (4*a^2*b^4 +
b^6)*cos(d*x + c)^3 + 3*(4*a^3*b^3 + a*b^5)*cos(d*x + c)^2 + 3*(4*a^4*b^2 +
 a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(
a^2 - b^2)*sin(d*x + c))) - (6*a^7 + 4*a^5*b^2 - 11*a^3*b^4 + a*b^6 + (2*a^
5*b^2 + 11*a^3*b^4 - 13*a*b^6)*cos(d*x + c)^2 + 3*(2*a^6*b + 7*a^4*b^3 - 10
*a^2*b^5 + b^7)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b
^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^
6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^
5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4
```

```
*a^5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35768, size = 576, normalized size = 3.

$$\frac{\frac{3\left(4a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{6a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) +
arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2)))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (6*a^5*tan(1/2*d*x
 + 1/2*c)^5 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*a^3*b^2*tan(1/2*d*x + 1/2
*c)^5 - 27*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12*a*b^4*tan(1/2*d*x + 1/2*c)^5
 + 3*b^5*tan(1/2*d*x + 1/2*c)^5 + 12*a^5*tan(1/2*d*x + 1/2*c)^3 + 16*a^3*b^
2*tan(1/2*d*x + 1/2*c)^3 - 28*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 6*a^5*tan(1/2*
d*x + 1/2*c) + 6*a^4*b*tan(1/2*d*x + 1/2*c) + 12*a^3*b^2*tan(1/2*d*x + 1/2*
c) + 27*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a*b^4*tan(1/2*d*x + 1/2*c) - 3*b^
5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x
 + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.483 $\int \frac{1}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=184

$$\frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-\frac{5ab\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{}{3d\left(a^2-\right.}$$

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(7/2)*(a + b)^(7/2)*d) - (b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^3) - (5*a*b*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])
^2) - (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Cos[c + d
*x]))
```

Rubi [A] time = 0.217287, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2664, 2754, 12, 2659, 205}

$$\frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{b\left(11a^2+4b^2\right)\sin(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}-\frac{5ab\sin(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}-\frac{}{3d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-4),x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
- b)^(7/2)*(a + b)^(7/2)*d) - (b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[
c + d*x])^3) - (5*a*b*Sin[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])
^2) - (b*(11*a^2 + 4*b^2)*Sin[c + d*x])/(6*(a^2 - b^2)^3*d*(a + b*Cos[c + d
*x]))
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^4}\,dx = -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{\int \frac{-3a+2b\cos(c+dx)}{(a+b\cos(c+dx))^3}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{2\left(3a^2+2b^2\right)-5ab\ldots}{(a+b\cos(c+\ldots}}{6\left(a^2-b\ldots\right.}$$

$$= -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{b\left(11a^2+4b\ldots\right.}{6\left(a^2-b^2\right)^3 d(a\ldots}$$

$$= -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{b\left(11a^2+4b\ldots\right.}{6\left(a^2-b^2\right)^3 d(a\ldots}$$

$$= -\frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\sin(c+dx)}{6\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} - \frac{b\left(11a^2+4b\ldots\right.}{6\left(a^2-b^2\right)^3 d(a\ldots}$$

$$= \frac{a\left(2a^2+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} - \frac{5ab\ldots}{6\left(a^2-b^2\right)^2\ldots}$$

Mathematica [A] time = 0.886346, size = 159, normalized size = 0.86

$$\frac{\frac{6a\left(2a^2+3b^2\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}} - \frac{b\sin(c+dx)\left(b^2\left(11a^2+4b^2\right)\cos^2(c+dx)+3ab\left(9a^2+b^2\right)\cos(c+dx)-5a^2b^2+18a^4+2b^4\right)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 + 3*b^2)*ArcTanh[((a - b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])
/(-a^2 + b^2)^(7/2) - (b*(18*a^4 - 5*a^2*b^2 + 2*b^4 + 3*a*b*(9*a^2 + b^2)*
Cos[c + d*x] + b^2*(11*a^2 + 4*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/((a - b)^
3*(a + b)^3*(a + b*Cos[c + d*x])^3))/(6*d)
```

Maple [B] time = 0.087, size = 776, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -6/d*a^2*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3
+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2
*c)^5*b^2-2/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(
a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3-12/d*a^2*b/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3-4/3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)
^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-6/d*a^2*b/(tan(
1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2
-b^3)*tan(1/2*d*x+1/2*c)+3/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^2-2/d/(tan(1
/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-
b^3)*tan(1/2*d*x+1/2*c)*b^3+2/d*a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a
+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+3/d*a*b^2/(
a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*
(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.20035, size = 1956, normalized size = 10.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*a^6 + 3*a^4*b^2 + (2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 + 3*(2*a
^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + 3*(2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*
sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 - 2
*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos
(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 2*(18*a^6*b - 23*a^4*b^3 + 7*a^2
*b^5 - 2*b^7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b
^2 - 8*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 +
 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 +
6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 +
6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7
*b^4 - 4*a^5*b^6 + a^3*b^8)*d), 1/6*(3*(2*a^6 + 3*a^4*b^2 + (2*a^3*b^3 + 3*
a*b^5)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + 3*(2*a^5
*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/
(sqrt(a^2 - b^2)*sin(d*x + c))) - (18*a^6*b - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^
7 + (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos(d*x + c)^2 + 3*(9*a^5*b^2 - 8*a^3*
```

```
b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7
- 4*a^2*b^9 + b^11)*d*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
 4*a^3*b^8 + a*b^10)*d*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 -
 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c) + (a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^
5*b^6 + a^3*b^8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.319, size = 539, normalized size = 2.93

$$-\frac{\frac{3\left(2a^3+3ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{18a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-27a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^2b^3\tan\left(\frac{1}{2}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(2*a^3 + 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b)
 + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(a^2 - b^2
)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + (18*a^4*b*tan(1
/2*d*x + 1/2*c)^5 - 27*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^3*tan(1/2*d
*x + 1/2*c)^5 - 3*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*b^5*tan(1/2*d*x + 1/2*c)
^5 + 36*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 32*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 -
4*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*a^4*b*tan(1/2*d*x + 1/2*c) + 27*a^3*b^2*t
an(1/2*d*x + 1/2*c) + 6*a^2*b^3*tan(1/2*d*x + 1/2*c) + 3*a*b^4*tan(1/2*d*x
+ 1/2*c) + 6*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3))/d
```

3.484 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=251

$$-\frac{b\left(-8a^4b^2+7a^2b^4+8a^6-2b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-17a^2b^2+26a^4+6b^4\right)\sin(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{b^2\left(8a^2-3b^2\right)}{6a^2d\left(a^2-b^2\right)^2(a}$$

```
[Out] -((b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d)) + ArcTanh[Sin[c
+ d*x]]/(a^4*d) + (b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^3) + (b^2*(8*a^2 - 3*b^2)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos
[c + d*x])^2) + (b^2*(26*a^4 - 17*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(6*a^3*(a^
2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.787525, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$-\frac{b\left(-8a^4b^2+7a^2b^4+8a^6-2b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{b^2\left(-17a^2b^2+26a^4+6b^4\right)\sin(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\cos(c+dx))}+\frac{b^2\left(8a^2-3b^2\right)}{6a^2d\left(a^2-b^2\right)^2(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a^4*(a - b)^(7/2)*(a + b)^(7/2)*d)) + ArcTanh[Sin[c
+ d*x]]/(a^4*d) + (b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
)^3) + (b^2*(8*a^2 - 3*b^2)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Cos
[c + d*x])^2) + (b^2*(26*a^4 - 17*a^2*b^2 + 6*b^4)*Sin[c + d*x])/(6*a^3*(a^
2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^4}dx = \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3\left(a^2-b^2\right)-3ab\cos(c+dx)+2b^2\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^3}dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(6\left(a^2-b^2\right)^2-2ab(6a\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2} + \frac{b^2\left(26a^4-17a^2b\ldots\right.}{6a^3\left(a^2-b^2\right)^3d\ldots}$$

$$= \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+dx))^2} + \frac{b^2\left(26a^4-17a^2b\ldots\right.}{6a^3\left(a^2-b^2\right)^3d\ldots}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(8a^2-3b^2\right)\sin(c+dx)}{6a^2\left(a^2-b^2\right)^2d(a+b\cos(c+d\ldots}$$

$$= -\frac{b\left(8a^6-8a^4b^2+7a^2b^4-2b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d} + \frac{\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{\ldots}{3a\left(a^2-\ldots\right.}$$

Mathematica [A] time = 2.95232, size = 274, normalized size = 1.09

$$\frac{\frac{2a^3b^2\sin(c+dx)}{(a-b)(a+b)(a+b\cos(c+dx))^3} + \frac{a^2b^2\left(8a^2-3b^2\right)\sin(c+dx)}{(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + \frac{ab^2\left(-17a^2b^2+26a^4+6b^4\right)\sin(c+dx)}{(a-b)^3(a+b)^3(a+b\cos(c+dx))} + \frac{6b\left(8a^4b^2-7a^2b^4-8a^6+2b^6\right)\tanh^{-1}\left(\frac{(a-b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)}{\left(b^2-a^2\right)^{7/2}}\ldots}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] ((6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTanh[((a - b)*Tan[(c + d*
x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) - 6*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] + 6*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*a^3*b^2*Si
n[c + d*x])/((a - b)*(a + b)*(a + b*Cos[c + d*x])^3) + (a^2*b^2*(8*a^2 - 3*
b^2)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Cos[c + d*x])^2) + (a*b^2*(2
6*a^4 - 17*a^2*b^2 + 6*b^4)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Cos[c
 + d*x])))/(6*a^4*d)
```

Maple [B] time = 0.121, size = 1377, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -1/d/a^4*ln(tan(1/2*d*x+1/2*c)-1)+1/d/a^4*ln(tan(1/2*d*x+1/2*c)+1)+12/d/(ta
n(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a-b)/(a^3+3*a^2*b+3*a
*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^2+4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5*b^3-
```

```
6/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+
3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-1/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*
x+1/2*c)^5+2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^
3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+24/d/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3*b^2-44/3/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2
*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+4/d*b^6
/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/
(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+12/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d
*x+1/2*c)^2*b+a+b)^3*a/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b
^2-4/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^3-6/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1
/2*c)+1/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+
b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+2/d*b^6/a^3/(tan(1/2*d*x+1/
2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(
1/2*d*x+1/2*c)-8/d*a^2*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*
arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+8/d*b^3/(a^6-3*a^4*b^2
+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))-7/d*b^5/a^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)
*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*b^7/a^4/(a^6-3*a^
4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.7338, size = 3962, normalized size = 15.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7 + (8*a^6*b^4 - 8*a^4
*b^6 + 7*a^2*b^8 - 2*b^10)*cos(d*x + c)^3 + 3*(8*a^7*b^3 - 8*a^5*b^5 + 7*a^
3*b^7 - 2*a*b^9)*cos(d*x + c)^2 + 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 - 2*
a^2*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log((2*a*b*cos(d*x + c) + (2*a^2 -
b^2)*cos(d*x + c)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c)
- a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)) - 6*(a^11 -
 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4
*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^
6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5
- 4*a^4*b^7 + a^2*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + 6*(a^11 - 4*a^
9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7
- 4*a^2*b^9 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4
```

```
*a^3*b^8 + a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a
^4*b^7 + a^2*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(36*a^9*b^2 - 68
*a^7*b^4 + 43*a^5*b^6 - 11*a^3*b^8 + (26*a^7*b^4 - 43*a^5*b^6 + 23*a^3*b^8
- 6*a*b^10)*cos(d*x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^
9)*cos(d*x + c))*sin(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*
b^9 + a^4*b^11)*d*cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4
*a^7*b^8 + a^5*b^10)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5
 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^4 -
4*a^9*b^6 + a^7*b^8)*d), -1/6*(3*(8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b
^7 + (8*a^6*b^4 - 8*a^4*b^6 + 7*a^2*b^8 - 2*b^10)*cos(d*x + c)^3 + 3*(8*a^7
*b^3 - 8*a^5*b^5 + 7*a^3*b^7 - 2*a*b^9)*cos(d*x + c)^2 + 3*(8*a^8*b^2 - 8*a
^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*co
s(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c))) - 3*(a^11 - 4*a^9*b^2 + 6*a
^7*b^4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9
 + b^11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 +
a*b^10)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^
2*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + 3*(a^11 - 4*a^9*b^2 + 6*a^7*b^
4 - 4*a^5*b^6 + a^3*b^8 + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^
11)*cos(d*x + c)^3 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^1
0)*cos(d*x + c)^2 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9
)*cos(d*x + c))*log(-sin(d*x + c) + 1) - (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*
b^6 - 11*a^3*b^8 + (26*a^7*b^4 - 43*a^5*b^6 + 23*a^3*b^8 - 6*a*b^10)*cos(d*
x + c)^2 + 15*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)*cos(d*x + c))*s
in(d*x + c))/((a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d*
cos(d*x + c)^3 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^1
0)*d*cos(d*x + c)^2 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6
*b^9)*d*cos(d*x + c) + (a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^
8)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.477, size = 748, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(a^2 - b^2)))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a
^2 - b^2)) + (36*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*b^3*tan(1/2*d*x +
1/2*c)^5 - 6*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*b^5*tan(1/2*d*x + 1/2*
c)^5 - 6*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 - 15*a*b^7*tan(1/2*d*x + 1/2*c)^5 +
 6*b^8*tan(1/2*d*x + 1/2*c)^5 + 72*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 - 116*a^4
```

```
*b^4*tan(1/2*d*x + 1/2*c)^3 + 56*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*b^8*ta
n(1/2*d*x + 1/2*c)^3 + 36*a^6*b^2*tan(1/2*d*x + 1/2*c) + 60*a^5*b^3*tan(1/2
*d*x + 1/2*c) - 6*a^4*b^4*tan(1/2*d*x + 1/2*c) - 45*a^3*b^5*tan(1/2*d*x + 1
/2*c) - 6*a^2*b^6*tan(1/2*d*x + 1/2*c) + 15*a*b^7*tan(1/2*d*x + 1/2*c) + 6*
b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3) + 3*log(abs(tan(1/
2*d*x + 1/2*c) + 1))/a^4 - 3*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4)/d
```

3.485 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx$

Optimal. Leaf size=308

$$\frac{b^2\left(-35a^4b^2+28a^2b^4+20a^6-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-65a^4b^2+68a^2b^4+6a^6-24b^6\right)\tan(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b^2(-11\ldots}{2a^3d(\ldots}$$

```
[Out] (b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) - (4*b*ArcTanh
[Sin[c + d*x]])/(a^5*d) + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Tan[c
 + d*x])/(6*a^4*(a^2 - b^2)^3*d) + (b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x])^3) + (b^2*(9*a^2 - 4*b^2)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])^2) + (b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Tan[c +
 d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.27018, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2802, 3055, 3001, 3770, 2659, 205}

$$\frac{b^2\left(-35a^4b^2+28a^2b^4+20a^6-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(-65a^4b^2+68a^2b^4+6a^6-24b^6\right)\tan(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b^2(-11\ldots}{2a^3d(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTan[(Sqrt[a - b]*Tan[(c
+ d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) - (4*b*ArcTanh
[Sin[c + d*x]])/(a^5*d) + ((6*a^6 - 65*a^4*b^2 + 68*a^2*b^4 - 24*b^6)*Tan[c
 + d*x])/(6*a^4*(a^2 - b^2)^3*d) + (b^2*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x])^3) + (b^2*(9*a^2 - 4*b^2)*Tan[c + d*x])/(6*a^2*(a^2 - b^
2)^2*d*(a + b*Cos[c + d*x])^2) + (b^2*(12*a^4 - 11*a^2*b^2 + 4*b^4)*Tan[c +
 d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Cos[c + d*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^4}\,dx = \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{\int \frac{\left(3a^2-4b^2-3ab\cos(c+dx)+3b^2\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^3}dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(9a^2-4b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{\int \frac{\left(6a^4-23a^2b^2+12b^4\right.}{}}{}$$

$$= \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2\left(9a^2-4b^2\right)\tan(c+dx)}{6a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^2} + \frac{b^2\left(12a^4-11a^2b\right.}{2a^3\left(a^2-b^2\right)^3 d}$$

$$= \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2(}{6a^2(a^2}$$

$$= \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^3} + \frac{b^2(}{6a^2(a^2}$$

$$= -\frac{4b\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{\left(6a^6-65a^4b^2+68a^2b^4-24b^6\right)\tan(c+dx)}{6a^4\left(a^2-b^2\right)^3 d} + \frac{b^2\tan(}{3a\left(a^2-b^2\right)d(a}$$

$$= \frac{b^2\left(20a^6-35a^4b^2+28a^2b^4-8b^6\right)\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4b\tanh^{-1}(\sin(c+dx))}{a^5d} + \frac{(6a}{}$$

Mathematica [A] time = 6.25589, size = 416, normalized size = 1.35

$$-\frac{b^3\sin(c+dx)}{3a^2d(a-b)(a+b)(a+b\cos(c+dx))^3} + \frac{6b^5\sin(c+dx)-11a^2b^3\sin(c+dx)}{6a^3d(a-b)^2(a+b)^2(a+b\cos(c+dx))^2} + \frac{50a^2b^5\sin(c+dx)-47a^4b^3\sin(c+}{6a^4d(a-b)^3(a+b)^3(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^4,x]
```

```
[Out] -((b^2*(20*a^6 - 35*a^4*b^2 + 28*a^2*b^4 - 8*b^6)*ArcTanh[((a - b)*Tan[(c +
 d*x)/2])/Sqrt[-a^2 + b^2]])/(a^5*(a^2 - b^2)^3*Sqrt[-a^2 + b^2]*d)) + (4*b
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a^5*d) - (4*b*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]])/(a^5*d) + Sin[(c + d*x)/2]/(a^4*d*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])) - (b^3*Sin[c + d*x])/(3*a^2*(a - b)*(a + b)*d*(a + b*Cos[c + d
*x])^3) + (-11*a^2*b^3*Sin[c + d*x] + 6*b^5*Sin[c + d*x])/(6*a^3*(a - b)^2*
(a + b)^2*d*(a + b*Cos[c + d*x])^2) + (-47*a^4*b^3*Sin[c + d*x] + 50*a^2*b^
5*Sin[c + d*x] - 18*b^7*Sin[c + d*x])/(6*a^4*(a - b)^3*(a + b)^3*d*(a + b*C
os[c + d*x]))
```

Maple [B] time = 0.129, size = 1429, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x)
```

```
[Out] -1/d/a^4/(tan(1/2*d*x+1/2*c)-1)+4/d*b/a^5*ln(tan(1/2*d*x+1/2*c)-1)-1/d/a^4/
(tan(1/2*d*x+1/2*c)+1)-4/d*b/a^5*ln(tan(1/2*d*x+1/2*c)+1)-20/d/(tan(1/2*d*x
+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*t
an(1/2*d*x+1/2*c)^5*b^3-5/d*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c
)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+18/d*b^5/
a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*
b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*
c)^5-6/d*b^7/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a-b
)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-40/d/(tan(1/2*d*x+1/2*c)^2
*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3*b^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/
2*d*x+1/2*c)^3+116/3/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-12/d*b^7/a^4
/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a^2-2*a*b+b^2)/(a^2
+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-20/d/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1
/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)*b^3+5/d
*b^4/a/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a
^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+18/d*b^5/a^2/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/
2*c)-2/d*b^6/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b
)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)-6/d*b^7/a^4/(tan(1/2*d*x+1/2
*c)^2*a-tan(1/2*d*x+1/2*c)^2*b+a+b)^3/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1
/2*d*x+1/2*c)+20/d*a*b^2/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*
arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-35/d*b^4/a/(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-
b)*(a+b))^(1/2))+28/d*b^6/a^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(
1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-8/d*b^8/a^5/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a-b)*(a+b))^(1/2)*arctan(tan(1/2*d*x+1/2*c)*(a-b
)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 19.2746, size = 4566, normalized size = 14.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b^11)*cos(d*x + c)^4 +
 3*(20*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*cos(d*x + c)^3 + 3*(20
*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*x + c)^2 + (20*a^9*b^
2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
```

```
((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a
*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + a^2)) + 24*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 +
b^12)*cos(d*x + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b
^11)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2
*b^10)*cos(d*x + c)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b
^9)*cos(d*x + c))*log(sin(d*x + c) + 1) - 24*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*
b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^
7 - 4*a^3*b^9 + a*b^11)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^
6 - 4*a^4*b^8 + a^2*b^10)*cos(d*x + c)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5
- 4*a^5*b^7 + a^3*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(6*a^12 - 2
4*a^10*b^2 + 36*a^8*b^4 - 24*a^6*b^6 + 6*a^4*b^8 + (6*a^9*b^3 - 71*a^7*b^5
+ 133*a^5*b^7 - 92*a^3*b^9 + 24*a*b^11)*cos(d*x + c)^3 + 3*(6*a^10*b^2 - 59
*a^8*b^4 + 110*a^6*b^6 - 77*a^4*b^8 + 20*a^2*b^10)*cos(d*x + c)^2 + (18*a^1
1*b - 132*a^9*b^3 + 239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c))*s
in(d*x + c))/((a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d*
cos(d*x + c)^4 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^
10)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^
7*b^9)*d*cos(d*x + c)^2 + (a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^
8*b^8)*d*cos(d*x + c)), 1/6*(3*((20*a^6*b^5 - 35*a^4*b^7 + 28*a^2*b^9 - 8*b
^11)*cos(d*x + c)^4 + 3*(20*a^7*b^4 - 35*a^5*b^6 + 28*a^3*b^8 - 8*a*b^10)*c
os(d*x + c)^3 + 3*(20*a^8*b^3 - 35*a^6*b^5 + 28*a^4*b^7 - 8*a^2*b^9)*cos(d*
x + c)^2 + (20*a^9*b^2 - 35*a^7*b^4 + 28*a^5*b^6 - 8*a^3*b^8)*cos(d*x + c))
*sqrt(a^2 - b^2)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d*x + c)
)) - 12*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 + b^12)*cos(d*x + c)
^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*b^11)*cos(d*x + c)^
3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*cos(d*x + c
)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*cos(d*x + c))*
log(sin(d*x + c) + 1) + 12*((a^8*b^4 - 4*a^6*b^6 + 6*a^4*b^8 - 4*a^2*b^10 +
 b^12)*cos(d*x + c)^4 + 3*(a^9*b^3 - 4*a^7*b^5 + 6*a^5*b^7 - 4*a^3*b^9 + a*
b^11)*cos(d*x + c)^3 + 3*(a^10*b^2 - 4*a^8*b^4 + 6*a^6*b^6 - 4*a^4*b^8 + a^
2*b^10)*cos(d*x + c)^2 + (a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*
b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (6*a^12 - 24*a^10*b^2 + 36*a^8*
b^4 - 24*a^6*b^6 + 6*a^4*b^8 + (6*a^9*b^3 - 71*a^7*b^5 + 133*a^5*b^7 - 92*a
^3*b^9 + 24*a*b^11)*cos(d*x + c)^3 + 3*(6*a^10*b^2 - 59*a^8*b^4 + 110*a^6*b
^6 - 77*a^4*b^8 + 20*a^2*b^10)*cos(d*x + c)^2 + (18*a^11*b - 132*a^9*b^3 +
239*a^7*b^5 - 169*a^5*b^7 + 44*a^3*b^9)*cos(d*x + c))*sin(d*x + c))/((a^13*
b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d*cos(d*x + c)^4 + 3*(
a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(d*x + c)^3
 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x + c
)^2 + (a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46913, size = 792, normalized size = 2.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*(20*a^6*b^2 - 35*a^4*b^4 + 28*a^2*b^6 - 8*b^8)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))/sqrt(a^2 - b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*
sqrt(a^2 - b^2)) + (60*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 105*a^5*b^4*tan(1/2
*d*x + 1/2*c)^5 - 24*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 117*a^3*b^6*tan(1/2*d
*x + 1/2*c)^5 - 24*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 - 42*a*b^8*tan(1/2*d*x +
1/2*c)^5 + 18*b^9*tan(1/2*d*x + 1/2*c)^5 + 120*a^6*b^3*tan(1/2*d*x + 1/2*c)
^3 - 236*a^4*b^5*tan(1/2*d*x + 1/2*c)^3 + 152*a^2*b^7*tan(1/2*d*x + 1/2*c)^
3 - 36*b^9*tan(1/2*d*x + 1/2*c)^3 + 60*a^6*b^3*tan(1/2*d*x + 1/2*c) + 105*a
^5*b^4*tan(1/2*d*x + 1/2*c) - 24*a^4*b^5*tan(1/2*d*x + 1/2*c) - 117*a^3*b^6
*tan(1/2*d*x + 1/2*c) - 24*a^2*b^7*tan(1/2*d*x + 1/2*c) + 42*a*b^8*tan(1/2*
d*x + 1/2*c) + 18*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a^6*b^4
- a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 + a + b)^3)
 + 12*b*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^5 - 12*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^5 + 6*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^
4))/d
```

3.486 $\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=264

$$\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}-\frac{2\left(17a^2b^2+8a^4-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(8a^2+19b^2\right)}{}$$

```
[Out] (2*a*(8*a^2 + 19*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^4 + 17*
a^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(105*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*a^2 + 25*b^2)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^2*d) - (8*a*(a + b*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^
(3/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.411567, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105b^2d}-\frac{2\left(17a^2b^2+8a^4-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(8a^2+19b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*a*(8*a^2 + 19*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(105*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(8*a^4 + 17*
a^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*b)/(a + b)])/(105*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(8*a^2 + 25*b^2)
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b^2*d) - (8*a*(a + b*Cos[c + d
*x])^(3/2)*Sin[c + d*x])/(35*b^2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^
(3/2)*Sin[c + d*x])/(7*b*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \frac{2\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7bd} + \frac{2\int \sqrt{a+b\cos(c+dx)}\left(a+\frac{5}{2}b\right.}{}$$

$$= -\frac{8a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35b^2d} + \frac{2\cos(c+dx)(a+b\cos(c+dx))^{3/2}\sin(c}{7bd}$$

$$= \frac{2\left(8a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} - \frac{8a(a+b\cos(c+dx))^{3/2}\sin(c+}{35b^2d}$$

$$= \frac{2\left(8a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} - \frac{8a(a+b\cos(c+dx))^{3/2}\sin(c+}{35b^2d}$$

$$= \frac{2\left(8a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105b^2d} - \frac{8a(a+b\cos(c+dx))^{3/2}\sin(c+}{35b^2d}$$

$$= \frac{2a\left(8a^2+19b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(8a^4+17a^2b^2-25b^4\right)}{105b^3d\sqrt{a}}$$

Mathematica [A] time = 1.12696, size = 214, normalized size = 0.81

$$\frac{b\sin(c+dx)\left(\left(145b^3-4a^2b\right)\cos(c+dx)-16a^3+36ab^2\cos(2(c+dx))+136ab^2+15b^3\cos(3(c+dx))\right)-4\left(17a^2b^2+8a\right.}{210b^3d\sqrt{a+b\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (4*a*(8*a^3 + 8*a^2*b + 19*a*b^2 + 19*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(8*a^4 + 17*a^2*b^2 - 25*b^4)*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] +
b*(-16*a^3 + 136*a*b^2 + (-4*a^2*b + 145*b^3)*Cos[c + d*x] + 36*a*b^2*Cos[2
*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*b^3*d*Sqrt[a + b*
Cos[c + d*x]])
```

Maple [B] time = 2.851, size = 827, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos
(1/2*d*x+1/2*c)^9*b^4+144*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)
^7*b^4-4*cos(1/2*d*x+1/2*c)^5*a^2*b^2-288*cos(1/2*d*x+1/2*c)^5*a*b^3+640*co
s(1/2*d*x+1/2*c)^5*b^4-8*cos(1/2*d*x+1/2*c)^3*a^3*b+6*cos(1/2*d*x+1/2*c)^3*
a^2*b^2+230*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4-8*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-17*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/
```

```
2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*b^4+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-8*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+19*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^2*b^2-19*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^3+8*cos(1/2*d*x+1/2*c)*a^3*b-2*cos(1/2*d*x+1/2*c)*a^2*b^2-86*cos(
1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/b^3/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/
2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)+a}\cos(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.487 $\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=207

$$\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(2a^2-9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)(a+}{5b}$$

```
[Out] (-2*(2*a^2 - 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(a^2 - b^2)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(1
5*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(15*b*d) + (2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.28356, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\cos(c+dx)}}-\frac{2\left(2a^2-9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)(a+}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(2*a^2 - 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (4*a*(a^2 - b^2)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(1
5*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(15*b*d) + (2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{2\int\left(\frac{3b}{2}-a\cos(c+dx)\right)\sqrt{a+b\cos(c+dx)}}{5b}$$

$$= -\frac{4a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{4\int}{}$$

$$= -\frac{4a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{1}{15}\left(9\right.$$

$$= -\frac{4a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15bd} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{\left(\left(9-\right.\right.}{}$$

$$= \frac{2\left(9-\frac{2a^2}{b^2}\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\right.}{15b^2d\sqrt{a+b\cos(c+\right.}}$$

Mathematica [A] time = 0.791228, size = 180, normalized size = 0.87

$$\frac{b\sin(c+dx)\left(2a^2+8ab\cos(c+dx)+3b^2\cos(2(c+dx))+3b^2\right)+4a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2\left(2a^2b\right.}{15b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*(2*a^3 + 2*a^2*b - 9*a*b^2 - 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 4*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c +
 d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(2*a^2 + 3*b^2 +
8*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*b^2*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Maple [B] time = 2.934, size = 665, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1
/2*d*x+1/2*c)^7*b^3+16*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*b
^3+2*cos(1/2*d*x+1/2*c)^3*a^2*b-24*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d*
x+1/2*c)^3*b^3+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^3+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-2*cos(1/2*d*x+1/2*c)*a^2*b+8*c
os(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/b^2/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.488 $\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=162

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*Sqrt[a + b*Cos[
c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.173445, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*d*Sqrt[a + b*Cos[
c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{a+b\cos(c+dx)}\,dx &= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{b}{2}+\frac{1}{2}a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx\\
&= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{a\int \sqrt{a+b\cos(c+dx)}\,dx}{3b}-\frac{\left(a^2-b^2\right)\int \frac{}{\sqrt{a+b\cos}}}{3b}\\
&= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{\left(a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\
&= \frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+}\right)}{3bd\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.551547, size = 137, normalized size = 0.85

$$\frac{-2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2b\sin(c+dx)(a+b\cos(c+dx))+2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*b*d*Sq
rt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.043, size = 452, normalized size = 2.8

$$-\frac{2}{3\,bd}\sqrt{\left(2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\Bigg(4\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{5}b^{2}+2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{3}ab-6\,(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/2*c)
*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\cos(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos(dx+c)+a}\cos(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\cos(c+dx)}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\cos(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c), x)
```

3.489 $\int \sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])
```

Rubi [A] time = 0.0388896, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2655, 2653}

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\cos(c+dx)}\,dx &= \frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\\ &= \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}\end{aligned}$$

Mathematica [A] time = 0.0687594, size = 57, normalized size = 1.

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])
```

Maple [B] time = 2.441, size = 170, normalized size = 3.

$$-2\,\frac{\sqrt{\left(2\,b\,(\cos(1/2\,dx+c/2))^2+a-b\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2\,(a-b)}}{\sqrt{-2\,b\,(\sin(1/2\,dx+c/2))^4+(a+b)\,(\sin(1/2\,dx+c/2))^2}\,\sin(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^2\,b+a+bd}}\sqrt{\frac{2\,b(}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(a-b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a), x)
```

3.490 $\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=118

$$\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.226215, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2803, 2663, 2661, 2807, 2805}

$$\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b
)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx = a\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + b\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{\left(a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}} + \frac{\left(b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}$$
$$= \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 2.18378, size = 81, normalized size = 0.69

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(bF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*EllipticF[(c + d*x)/2, (2*b)/(a +
b)] + a*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)]))/(d*Sqrt[a + b*Cos[c + d
*x]])
```

Maple [A] time = 2.727, size = 194, normalized size = 1.6

$$-2\frac{\sqrt{\left(2b(\cos(1/2\,dx+c/2))^2+a-b\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}}{\sqrt{-2b(\sin(1/2\,dx+c/2))^4+(a+b)(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^2b+a+b}d}\sqrt{2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b-EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2))*a)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\cos{(c + dx)}}\sec{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx + c) + a}\sec(dx + c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c), x)
```

3.491 $\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$

Optimal. Leaf size=197

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{b\sqrt{\frac{a+b\cos}{a}}}{d}$$

```
[Out] -((Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])) + (a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (b*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d
*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.497798, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2796, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}+\frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{b\sqrt{\frac{a+b\cos}{a}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] -((Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[
(a + b*Cos[c + d*x])/(a + b)])) + (a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (b*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(d
*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx = \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(\frac{b}{2}-\frac{1}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}\int \sqrt{a+b\cos(c+dx)}\,dx - \int \frac{\left(-\frac{b^2}{2}-\frac{1}{2}ab\cos\right.}{\sqrt{a+b}}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{2}a\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx + \frac{1}{2}b\int \frac{\mathrm{s}}{\sqrt{a+}}$$

$$= -\frac{\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \cdots$$

$$= -\frac{\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} + \cdots$$

Mathematica [C] time = 7.24165, size = 307, normalized size = 1.56

$$\frac{4\tan(c+dx)\sqrt{a+b\cos(c+dx)} + \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{\cdots}\right.\right.\right.\right.}{\cdots}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] ((2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(
a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(
a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*E
llipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
+ d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(
-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] time = 2.977, size = 622, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a-2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)+2*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b-Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)*s
in(1/2*d*x+1/2*c)^2-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
```

```
1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b)/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^2, x)
```

3.492 $\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx$

Optimal. Leaf size=262

$$\frac{(4a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -(b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) +
((4*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2
, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*x]]) + (b*Sqrt[a + b*Cos[c +
d*x]]*Tan[c + d*x])/(4*a*d) + (Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c
+ d*x])/(2*d)
```

Rubi [A] time = 0.731488, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2796, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}}+\frac{b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4ad}+\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] -(b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) +
((4*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2
, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*x]]) + (b*Sqrt[a + b*Cos[c +
d*x]]*Tan[c + d*x])/(4*a*d) + (Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c
+ d*x])/(2*d)
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sec^3(c+dx)\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{\left(\frac{b}{2}+a\cos(c+dx)+\frac{1}{2}b\,\mathrm{c}\right.}{\sqrt{a+b\,\mathrm{co}}}$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx}{2d}$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx}{2d}$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx}{2d}$$

$$= -\frac{b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4ad} +$$

$$= -\frac{b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 6.5095, size = 515, normalized size = 1.97

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{b\tan(c+dx)}{4a}+\frac{1}{2}\tan(c+dx)\sec(c+dx)\right)}{d} + \frac{\frac{2\left(8a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2ib^2\sin(c+dx)\cos(2(c+}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] ((8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c +
d*x]] + ((2*I)*b^2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c +
 d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos
[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c +
d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d
*x])^2)))/(16*a*d) + (Sqrt[a + b*Cos[c + d*x]]*((b*Tan[c + d*x])/(4*a) + (S
ec[c + d*x]*Tan[c + d*x])/2))/d
```

Maple [B] time = 3.535, size = 977, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*b^2*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(12*a*b+8*b^2)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-4*a^2-6*a*b-2*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2
*c)+4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2
))*a^2+EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)*sin(1/2*d*x
+1/2*c)^4-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^2-4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b)
)^(1/2))*a^2+EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)*sin(1
/2*d*x+1/2*c)^2+3*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))*b^2)/a/(2*cos(1/2*d*x+1/2*c)^2-1)^2/(-2*b*sin(1/2*d*x+1/2*
c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\cos(c + dx)}\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx + c) + a}\sec(dx + c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^3, x)
```

3.493 $\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=314

$$\frac{2(8a^2+49b^2)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}+\frac{2a(8a^2+39b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315b^2d}-\frac{2a(31a^2b^2+8a^4-3}{315b^3}$$

```
[Out] (2*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*
a*(8*a^4 + 31*a^2*b^2 - 39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*
(8*a^2 + 39*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^2*d) + (2*(8
*a^2 + 49*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^2*d) - (8*a*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b^2*d) + (2*Cos[c + d*x]*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.517743, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2(8a^2+49b^2)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315b^2d}+\frac{2a(8a^2+39b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315b^2d}-\frac{2a(31a^2b^2+8a^4-3}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*(8*a^4 + 33*a^2*b^2 + 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*
a*(8*a^4 + 31*a^2*b^2 - 39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*b)/(a + b)])/(315*b^3*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*
(8*a^2 + 39*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b^2*d) + (2*(8
*a^2 + 49*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b^2*d) - (8*a*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b^2*d) + (2*Cos[c + d*x]*(a +
b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(9*b*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
```

```
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{2\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{9bd} + \frac{2\int(a+b\cos(c+dx))^{3/2}\left(a+\right.}{}$$

$$= -\frac{8a(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63b^2d} + \frac{2\cos(c+dx)(a+b\cos(c+dx))^{5/2}\sin(}{9bd}$$

$$= \frac{2\left(8a^2+49b^2\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315b^2d} - \frac{8a(a+b\cos(c+dx))^{5/2}\sin}{63b^2d}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a+b\cos(c+}{315b^2d}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a+b\cos(c+}{315b^2d}$$

$$= \frac{2a\left(8a^2+39b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315b^2d} + \frac{2\left(8a^2+49b^2\right)(a+b\cos(c+}{315b^2d}$$

$$= \frac{2\left(8a^4+33a^2b^2+147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\left(8a^4+31a\right.}{}$$

Mathematica [A] time = 1.38139, size = 262, normalized size = 0.83

$$\frac{b\sin(c+dx)\left(\left(1606ab^3-8a^3b\right)\cos(c+dx)+4\left(53a^2b^2+84b^4\right)\cos(2(c+dx))+916a^2b^2-32a^4+170ab^3\cos(3(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*(8*a^5 + 8*a^4*b + 33*a^3*b^2 + 33*a^2*b^3 + 147*a*b^4 + 147*b^5)*Sqrt[(
a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 8*a*(8
*a^4 + 31*a^2*b^2 - 39*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)] + b*(-32*a^4 + 916*a^2*b^2 + 301*b^4 + (-8*a^3*b
+ 1606*a*b^3)*Cos[c + d*x] + 4*(53*a^2*b^2 + 84*b^4)*Cos[2*(c + d*x)] + 170
*a*b^3*Cos[3*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)])*Sin[c + d*x])/(1260*b^3*
d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.608, size = 995, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b
^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1360*a*b^4+2240*b^5)*sin(1/2*d
*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-424*a^2*b^3-2040*a*b^4-2072*b^5)*sin(1/2*d
*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-4*a^3*b^2+424*a^2*b^3+1568*a*b^4+952*b^5)*
sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(8*a^4*b+2*a^3*b^2-282*a^2*b^3-444*
```

```
a*b^4-168*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-8*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-31*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*a^3*b^2+39*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a*b^4+8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^5-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+33*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2-33*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3+147*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*b^5)/b^3/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cos(dx+c)^4+a\cos(dx+c)^3\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 + a*cos(d*x + c)^3)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x)
```

3.494 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=258

$$-\frac{2\left(6a^2-25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}+\frac{2\left(-31a^2b^2+6a^4+25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4a\left(3a^2\right.}{}$$

```
[Out] (-4*a*(3*a^2 - 41*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(6*a^4 - 31
*a^2*b^2 + 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(105*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(6*a^2 - 25*b^2
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b*d) - (4*a*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(35*b*d) + (2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*
x])/(7*b*d)
```

Rubi [A] time = 0.390561, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(6a^2-25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105bd}+\frac{2\left(-31a^2b^2+6a^4+25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4a\left(3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-4*a*(3*a^2 - 41*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)])/(105*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(6*a^4 - 31
*a^2*b^2 + 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2
, (2*b)/(a + b)])/(105*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*(6*a^2 - 25*b^2
)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*b*d) - (4*a*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(35*b*d) + (2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*
x])/(7*b*d)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{2\int \left(\frac{5b}{2} - a\cos(c+dx)\right)(a+b\cos(c+dx)}{7b}$$

$$= -\frac{4a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35bd} + \frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{4}{}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} - \frac{4a(a+b\cos(c+dx))^{3/2}\sin(}{35bd}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} - \frac{4a(a+b\cos(c+dx))^{3/2}\sin(}{35bd}$$

$$= -\frac{2\left(6a^2-25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105bd} - \frac{4a(a+b\cos(c+dx))^{3/2}\sin(}{35bd}$$

$$= \frac{4a\left(41-\frac{3a^2}{b^2}\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(6a^4-31a^2b^2+25b^4\right)}{105b^2d\sqrt{a}}$$

Mathematica [A] time = 1.14585, size = 214, normalized size = 0.83

$$\frac{b\sin(c+dx)\left(b\left(108a^2+145b^2\right)\cos(c+dx)+12a^3+78ab^2\cos(2(c+dx))+178ab^2+15b^3\cos(3(c+dx))\right)+4\left(-31a^2b^2\right.}{210b^2d\sqrt{a+b\cos}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(3*a^3 + 3*a^2*b - 41*a*b^2 - 41*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 4*(6*a^4 - 31*a^2*b^2 + 25*b^4)
*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] +
 b*(12*a^3 + 178*a*b^2 + b*(108*a^2 + 145*b^2)*Cos[c + d*x] + 78*a*b^2*Cos[
2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*b^2*d*Sqrt[a + b
*Cos[c + d*x]])
```

Maple [B] time = 3.057, size = 827, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos
(1/2*d*x+1/2*c)^9*b^4+312*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)
^7*b^4+108*cos(1/2*d*x+1/2*c)^5*a^2*b^2-624*cos(1/2*d*x+1/2*c)^5*a*b^3+640*
cos(1/2*d*x+1/2*c)^5*b^4+6*cos(1/2*d*x+1/2*c)^3*a^3*b-162*cos(1/2*d*x+1/2*c
)^3*a^2*b^2+440*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4+6*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-31*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b^4-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+6*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+82*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-82*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos
(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^3-6*cos(1/2*d*x+1/2*c)*a^3*b+54*cos(1/2*d*x+1/2*c)*a^2*b^2-12
8*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/b^2/(-2*b*sin(1/2*d*x
+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*
d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^3 + a*cos(d*x + c)^2)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.495 $\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=199

$$-\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)(a+}{5d}$$

```
[Out] (2*(a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(5*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(a^2 - b^2)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(5*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d
) + (2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.247547, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)(a+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(5*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(a^2 - b^2)*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(5*b*d*S
qrt[a + b*Cos[c + d*x]]) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d
) + (2*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int \left(\frac{3b}{2}+\frac{3}{2}a\cos(c+dx)\right)\sqrt{a+b\cos(c}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{4}{15}\int$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} - \frac{\left(a\left(a^2\right.\right.}{}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d} + \frac{2(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(\left(a^2+\right.\right.}{}$$

$$= \frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F}{5bd\sqrt{a+b\cos(c}}$$

Mathematica [A] time = 0.777986, size = 174, normalized size = 0.87

$$\frac{b\sin(c+dx)\left(4a^2+6ab\cos(c+dx)+b^2\cos(2(c+dx))+b^2\right)-2a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2\left(a^2b+a^3\right.}{5bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^3 + a^2*b + 3*a*b^2 + 3*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(4*a^2 + b^2 + 6*a*b*C
os[c + d*x] + b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(5*b*d*Sqrt[a + b*Cos[c +
 d*x]])
```

Maple [B] time = 2.925, size = 663, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*cos(1/2
*d*x+1/2*c)^7*b^3+12*cos(1/2*d*x+1/2*c)^5*a*b^2-16*cos(1/2*d*x+1/2*c)^5*b^3
+4*cos(1/2*d*x+1/2*c)^3*a^2*b-18*cos(1/2*d*x+1/2*c)^3*a*b^2+10*cos(1/2*d*x+
1/2*c)^3*b^3-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^3-(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^
2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-4*cos(1/2*d*x+1/2*c)*a^2*b+6*cos(1/2*d*x
+1/2*c)*a*b^2-2*cos(1/2*d*x+1/2*c)*b^3)/b/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(3/2),x)
```

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.496 $\int (a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=157

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.168686, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2656, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\right.}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*d*S
qrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^{3/2}\,dx &= \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}\left(3a^2+b^2\right)+2ab\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx\\ &= \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{1}{3}(4a)\int \sqrt{a+b\cos(c+dx)}\,dx+\frac{1}{3}\left(-a^2+b^2\right)\int \frac{}{\sqrt{a+}}\\ &= \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{\left(4a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(\left(-a\right.\right.}{}\\ &= \frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{}}{}\end{aligned}$$

Mathematica [A] time = 0.548281, size = 134, normalized size = 0.85

$$\frac{-2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2b\sin(c+dx)(a+b\cos(c+dx))+8a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (8*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b
)/(a + b)] - 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
+ d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Sqrt
[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.059, size = 450, normalized size = 2.9

$$-\frac{2}{3\,d}\sqrt{\left(2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^2+a-b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,\left(\cos\left(1/2\,dx+c/2\right)\right)^5b^2+2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^3ab-6\,\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2-(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b^2+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+
1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))
*a^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+1/
2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2
*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/
2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\cos(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2), x)
```

3.497 $\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

Optimal. Leaf size=179

$$\frac{2a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.314484, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2804, 2655, 2653, 2803, 2663, 2661, 2807, 2805}

$$\frac{2a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*
a^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a
+ b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2804

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = a\int \sqrt{a+b\cos(c+dx)}\sec(c+dx)\,dx + b\int \sqrt{a+b\cos(c+dx)}\,dx$$

$$= a^2\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx + (ab)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx + \frac{\left(b\sqrt{a+b\cos(c+\right.}{}$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(a^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} +$$

Mathematica [A] time = 2.30971, size = 107, normalized size = 0.6

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b(a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\left(bF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(a + b)*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + a*(b*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*EllipticPi[2,
(c + d*x)/2, (2*b)/(a + b)])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 2.746, size = 249, normalized size = 1.4

$$-2\,\frac{\sqrt{\left(2\,b\,(\cos(1/2\,dx+c/2))^{2}+a-b\right)(\sin(1/2\,dx+c/2))^{2}}\sqrt{(\sin(1/2\,dx+c/2))^{2}}}{\sqrt{-2\,b\,(\sin(1/2\,dx+c/2))^{4}+(a+b)\,(\sin(1/2\,dx+c/2))^{2}}\,\sin(1/2\,dx+c/2)\,\sqrt{-2\,(\sin(1/2\,dx+c/2))^{2}\,b+a+b}\,d}\sqrt{2\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x)
```

```
[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(EllipticF(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-Ell
ipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2)/(-2*b*sin(1/2*d*x+1/2
*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+
1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

3.498 $\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=209

$$\frac{(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}-\frac{a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] -((a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2 + 2*b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + (3*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/d
```

Rubi [A] time = 0.541808, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2799, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(a^2+2b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{a\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}-\frac{a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] -((a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + ((a^2 + 2*b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x
]]) + (3*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
(2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (a*Sqrt[a + b*Cos[c + d*x]]*
Tan[c + d*x])/d
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(\frac{3ab}{2}+b^2\cos(c+dx)-\frac{1}{2}ab\cos^2(c+\right.}{\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{1}{2}a\int\sqrt{a+b\cos(c+dx)}\,dx - \int\frac{\left(-\frac{3ab^2}{2}-\right.}{}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{2}(3ab)\int\frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx - \frac{1}{2}\left(-\right.$$

$$= -\frac{a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d}$$

$$= -\frac{a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(a^2+2b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 10.9327, size = 363, normalized size = 1.74

$$\frac{4a\tan(c+dx)\sqrt{a+b\cos(c+dx)} + b\left(-\frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)-2aF\right.\right.}{b^2\sqrt{-\frac{1}{a+}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (b*((8*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
 + b)])/Sqrt[a + b*Cos[c + d*x]] + (10*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*
I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a
 + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*S
qrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticP
i[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)])))/(b^2*Sqrt[-(a + b)^(-1)])) + 4*a*Sqrt[a + b*Cos[c + d*x]]*Ta
n[c + d*x])/(4*d)
```

Maple [B] time = 3.527, size = 740, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x)
```

```
[Out] -((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a^2-2*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+
2*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2-3*EllipticPi(cos(1/2
```

```
*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a^2+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b)*sin(1
/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+
2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-3*a*b*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a)/(2*cos(1/2*d*x+1/2*c)^2-1)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2
*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x)
```

3.499 $\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=255

$$\frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4d}+\frac{7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-5*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (7*a*b*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]])
+ ((4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (5*b*Sqrt[a + b*Cos[
c + d*x]]*Tan[c + d*x])/(4*d) + (a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Ta
n[c + d*x])/(2*d)
```

Rubi [A] time = 0.762961, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2799, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{5b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4d}+\frac{7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```


```
[Out] (-5*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*d*
Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (7*a*b*Sqrt[(a + b*Cos[c + d*x])/(a +
 b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]])
+ ((4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*
x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (5*b*Sqrt[a + b*Cos[
c + d*x]]*Tan[c + d*x])/(4*d) + (a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Ta
n[c + d*x])/(2*d)
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
```

```
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx = \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{\left(\frac{5ab}{2}+\left(a^2+2b^2\right)\cos(c+\right.}{\sqrt{a-}}$$

$$= \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+d}{2d}$$

$$= \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+d}{2d}$$

$$= \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d} + \frac{a\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+d}{2d}$$

$$= -\frac{5b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{5b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}$$

$$= -\frac{5b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{7ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 6.38495, size = 508, normalized size = 1.99

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}a\tan(c+dx)\sec(c+dx)+\frac{5}{4}b\tan(c+dx)\right)}{d} + \frac{\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{10ib^2\sin(c+dx)\cos(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] ((8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*
x]] + ((10*I)*b^2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c +
d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*
x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[
c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d
*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*
x])^2)))/(16*d) + (Sqrt[a + b*Cos[c + d*x]]*((5*b*Tan[c + d*x])/4 + (a*Sec[
```

```
c + d*x]*Tan[c + d*x])/2))/d
```

Maple [B] time = 3.326, size = 980, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x)
```

```
[Out] -1/4*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-40*b^2*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(28*a*b+40*b^2)*sin(1/2*d*x+1/2*c)^4
*cos(1/2*d*x+1/2*c)+(-4*a^2-14*a*b-10*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c)-4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2+
3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-7*EllipticF(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+5*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a*b-5*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^2)*s
in(1/2*d*x+1/2*c)^4+4*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))*a^2+3*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2-7*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b+5*EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))*a*b-5*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*b^2)*sin(1/2*d*x+1/2*c)^2-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2
*b/(a-b))^(1/2))*a^2-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x
+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))*b^2+7*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2
+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-5*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a+5*b^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)^2/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x)
```

3.500 $\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=371

$$\frac{2\left(8a^2+81b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2+67b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{693b^2d}+\frac{2\left(57a^2b^2+8a^4\right.}{}$$

```
[Out] (2*a*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (
2*(8*a^6 + 49*a^4*b^2 + 78*a^2*b^4 - 135*b^6)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(693*b^2*d) + (2*a*(8*a^2 + 67*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(693*b^2*d) + (2*(8*a^2 + 81*b^2)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(693*b^2*d) - (8*a*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^
2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```

Rubi [A] time = 0.629158, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2793, 3023, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(8a^2+81b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{5/2}}{693b^2d}+\frac{2a\left(8a^2+67b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{693b^2d}+\frac{2\left(57a^2b^2+8a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*a*(8*a^4 + 51*a^2*b^2 + 741*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (
2*(8*a^6 + 49*a^4*b^2 + 78*a^2*b^4 - 135*b^6)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(693*b^3*d*Sqrt[a + b*Cos[c +
d*x]]) + (2*(8*a^4 + 57*a^2*b^2 + 135*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(693*b^2*d) + (2*a*(8*a^2 + 67*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c
 + d*x])/(693*b^2*d) + (2*(8*a^2 + 81*b^2)*(a + b*Cos[c + d*x])^(5/2)*Sin[c
 + d*x])/(693*b^2*d) - (8*a*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(99*b^
2*d) + (2*Cos[c + d*x]*(a + b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(11*b*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{2\cos(c+dx)(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{11bd} + \frac{2\int(a+b\cos(c+dx))^{5/2}\left(\right.}{}$$

$$= -\frac{8a(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{99b^2d} + \frac{2\cos(c+dx)(a+b\cos(c+dx))^{7/2}\text{s}}{11bd}$$

$$= \frac{2\left(8a^2+81b^2\right)(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{693b^2d} - \frac{8a(a+b\cos(c+dx))^{7/2}\text{s}}{99b^2d}$$

$$= \frac{2a\left(8a^2+67b^2\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{693b^2d} + \frac{2\left(8a^2+81b^2\right)(a+b\,\text{cc}}{69}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{693b^2d} + \frac{2a\left(8a^2+67b^2\right)}{}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{693b^2d} + \frac{2a\left(8a^2+67b^2\right)}{}$$

$$= \frac{2\left(8a^4+57a^2b^2+135b^4\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{693b^2d} + \frac{2a\left(8a^2+67b^2\right)}{}$$

$$= \frac{2a\left(8a^4+51a^2b^2+741b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{693b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(8a^6+\right.}{}$$

Mathematica [A] time = 1.18883, size = 268, normalized size = 0.72

$$\frac{16\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left(b\left(663a^2b^3+2a^4b+135b^5\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a\left(51a^2b^2+8a^4+741b^4\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (16*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*(b*(2*a^4*b + 663*a^2*b^3 + 135*b^5)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*(8*a^4 + 51*a^2*b^2 + 741*b^4)*(
(a + b)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - a*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])) - b*(a + b*Cos[c + d*x])*((64*a^4 - 3732*a^2*b^2 - 2610*b^4)
*Sin[c + d*x] - b*(4*(6*a^3 + 619*a*b^2)*Sin[2*(c + d*x)] + b*((452*a^2 + 5
13*b^2)*Sin[3*(c + d*x)] + 7*b*(46*a*Sin[4*(c + d*x)] + 9*b*Sin[5*(c + d*x)
])))))/(5544*b^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.718, size = 1140, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/693*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4032*b^
6*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-7168*a*b^5-10080*b^6)*sin(1/2*
```

```
d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(4384*a^2*b^4+14336*a*b^5+11376*b^6)*sin(1
/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-928*a^3*b^3-6576*a^2*b^4-13232*a*b^5-6
984*b^6)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-4*a^4*b^2+928*a^3*b^3+50
24*a^2*b^4+6064*a*b^5+2772*b^6)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(8*
a^5*b+2*a^4*b^2-642*a^3*b^3-1416*a^2*b^4-1338*a*b^5-558*b^6)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/
2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))
^(1/2))*a^6-49*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b
^2-78*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(
a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^4+135*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^6+8*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^6-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^5*b+51*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^4*b^2-51*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^3*b^3+741*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2
*b^4-741*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^5)/b^3/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^5+2\,ab\cos(dx+c)^4+a^2\cos(dx+c)^3\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3)*s
qrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x)
```

3.501 $\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=308

$$-\frac{2\left(10a^2-49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}-\frac{4a\left(5a^2-57b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315bd}+\frac{4a\left(-62a^2b^2+5a^4\right.}{31}$$

```
[Out] (-2*(10*a^4 - 279*a^2*b^2 - 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(4*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4
*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) - (2*(
10*a^2 - 49*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d) - (4*a*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2*(a + b*Cos[c + d*x])
^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.509187, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2791, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2\left(10a^2-49b^2\right)\sin(c+dx)(a+b\cos(c+dx))^{3/2}}{315bd}-\frac{4a\left(5a^2-57b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315bd}+\frac{4a\left(-62a^2b^2+5a^4\right.}{31}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2),x]
```



```
[Out] (-2*(10*a^4 - 279*a^2*b^2 - 147*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) +
(4*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticF[(c + d*x)/2, (2*b)/(a + b)])/(315*b^2*d*Sqrt[a + b*Cos[c + d*x]]) - (4
*a*(5*a^2 - 57*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*b*d) - (2*(
10*a^2 - 49*b^2)*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(315*b*d) - (4*a*
(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(63*b*d) + (2*(a + b*Cos[c + d*x])
^(7/2)*Sin[c + d*x])/(9*b*d)
```


Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{2(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{2\int\left(\frac{7b}{2}-a\cos(c+dx)\right)(a+b\cos(c+$$

$$= -\frac{4a(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{63bd} + \frac{2(a+b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

$$= -\frac{2\left(10a^2-49b^2\right)(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{315bd} - \frac{4a(a+b\cos(c+dx))^{5}}{63bd}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)(a+b\,c}{31}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)(a+b\,c}{31}$$

$$= -\frac{4a\left(5a^2-57b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{315bd} - \frac{2\left(10a^2-49b^2\right)(a+b\,c}{31}$$

$$= -\frac{2\left(10a^4-279a^2b^2-147b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{315b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{4a\left(5a\right.}{}$$

Mathematica [A] time = 1.3869, size = 263, normalized size = 0.85

$$\frac{b\sin(c+dx)\left(4ab\left(160a^2+619b^2\right)\cos(c+dx)+8\left(85a^2b^2+42b^4\right)\cos(2(c+dx))+1984a^2b^2+40a^4+260ab^3\cos(3(c+\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-8*(10*a^5 + 10*a^4*b - 279*a^3*b^2 - 279*a^2*b^3 - 147*a*b^4 - 147*b^5)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 1
6*a*(5*a^4 - 62*a^2*b^2 + 57*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*b)/(a + b)] + b*(40*a^4 + 1984*a^2*b^2 + 301*b^4 + 4*a*
b*(160*a^2 + 619*b^2)*Cos[c + d*x] + 8*(85*a^2*b^2 + 42*b^4)*Cos[2*(c + d*x
)] + 260*a*b^3*Cos[3*(c + d*x)] + 35*b^4*Cos[4*(c + d*x)])*Sin[c + d*x])/(1
260*b^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.139, size = 995, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/315*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b
^5*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2080*a*b^4+2240*b^5)*sin(1/2*d
*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-1360*a^2*b^3-3120*a*b^4-2072*b^5)*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(320*a^3*b^2+1360*a^2*b^3+2408*a*b^4+952*b^
5)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-10*a^4*b-160*a^3*b^2-666*a^2*b
^3-684*a*b^4-168*b^5)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-10*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+10*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b+279*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^3*b^2-279*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a^2*b^3+147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^4-147*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)
^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^5+
10*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-124*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+114*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4
+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c
)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^4+2\,ab\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*s
qrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\cos\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x)
```

3.502 $\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=249

$$\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}-\frac{2\left(2a^2b^2+3a^4-5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{21bd\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(3a^2+29b^2\right)\sqrt{\cdots}}{2\cdots}$$

```
[Out] (2*a*(3*a^2 + 29*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(21*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a^4 + 2*a^2*
b^2 - 5*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b
)/(a + b)])/(21*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(3*a^2 + 5*b^2)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(a + b*Cos[c + d*x])^(3/2)*Sin
[c + d*x])/(7*d) + (2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.359508, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}-\frac{2\left(2a^2b^2+3a^4-5b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{21bd\sqrt{a+b\cos(c+dx)}}+\frac{2a\left(3a^2+29b^2\right)\sqrt{\cdots}}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*a*(3*a^2 + 29*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(21*b*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(3*a^4 + 2*a^2*
b^2 - 5*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b
)/(a + b)])/(21*b*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(3*a^2 + 5*b^2)*Sqrt[a +
 b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(a + b*Cos[c + d*x])^(3/2)*Sin
[c + d*x])/(7*d) + (2*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\cos(c+dx))^{5/2}\,dx = \frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+\frac{2}{7}\int\left(\frac{5b}{2}+\frac{5}{2}a\cos(c+dx)\right)(a+b\cos($$

$$=\frac{2a(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{7d}+\frac{2(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}+$$

$$=\frac{2\left(3a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2a(a+b\cos(c+dx))^{3/2}\sin(c}{7d}$$

$$=\frac{2\left(3a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2a(a+b\cos(c+dx))^{3/2}\sin(c}{7d}$$

$$=\frac{2\left(3a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2a(a+b\cos(c+dx))^{3/2}\sin(c}{7d}$$

$$=\frac{2a\left(3a^2+29b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{21bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2\left(3a^4+2a^2b^2-5b^4\right.}{21bd\sqrt{}}$$

Mathematica [A] time = 0.886519, size = 214, normalized size = 0.86

$$\frac{b\sin(c+dx)\left(b\left(72a^2+29b^2\right)\cos(c+dx)+36a^3+24ab^2\cos(2(c+dx))+44ab^2+3b^3\cos(3(c+dx))\right)-4\left(2a^2b^2+3\right.}{42bd\sqrt{a+b\cos(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*a*(3*a^3 + 3*a^2*b + 29*a*b^2 + 29*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b
)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(3*a^4 + 2*a^2*b^2 - 5*b^4)*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*
(36*a^3 + 44*a*b^2 + b*(72*a^2 + 29*b^2)*Cos[c + d*x] + 24*a*b^2*Cos[2*(c +
 d*x)] + 3*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(42*b*d*Sqrt[a + b*Cos[c + d
```

```
*x]])
```

Maple [B] time = 2.899, size = 827, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/21*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*cos(1
/2*d*x+1/2*c)^9*b^4+96*cos(1/2*d*x+1/2*c)^7*a*b^3-120*cos(1/2*d*x+1/2*c)^7*
b^4+72*cos(1/2*d*x+1/2*c)^5*a^2*b^2-192*cos(1/2*d*x+1/2*c)^5*a*b^3+128*cos(
1/2*d*x+1/2*c)^5*b^4+18*cos(1/2*d*x+1/2*c)^3*a^3*b-108*cos(1/2*d*x+1/2*c)^3
*a^2*b^2+130*cos(1/2*d*x+1/2*c)^3*a*b^3-72*cos(1/2*d*x+1/2*c)^3*b^4+3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*a^3*b+29*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b^2-29*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3-3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^2*b^2+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*b^4-18*cos(1/2*d*x+1/2*c)*a^3*b+36*cos(1/2*d*x+1/2*c)*a^2*b^2-34*cos(
1/2*d*x+1/2*c)*a*b^3+16*cos(1/2*d*x+1/2*c)*b^4)/b/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*
c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^3+2\,ab\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.503 $\int (a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=197

$$-\frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2b\sin(c+dx)(a}{5}$$

```
[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d
*Sqrt[a + b*Cos[c + d*x]]) + (16*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(15*d) + (2*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.261172, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$-\frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2b\sin(c+dx)(a}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*(a^2 - b^2)*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*d
*Sqrt[a + b*Cos[c + d*x]]) + (16*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(15*d) + (2*b*(a + b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\,dx = \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\cos(c+dx)}\left(\frac{1}{2}\left(5a^2+3b^2\right)+4ab\cos$$

$$= \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{4}{15}\int \frac{\frac{1}{4}a\,(1}{}$$

$$= \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} - \frac{1}{15}\left(8a\left(a^2\right.\right.$$

$$= \frac{16ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d} + \frac{2b(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(\left(23a^2+9\right.\right.}{}$$

$$= \frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c\right.}{15d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.783861, size = 177, normalized size = 0.9

$$\frac{b\sin(c+dx)\left(22a^2+28ab\cos(c+dx)+3b^2\cos(2(c+dx))+3b^2\right)-16a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2}{15d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 16*a*(a^2 - b^2)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2
```

```
+ 28*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*d*Sqrt[a
+ b*Cos[c + d*x]])
```

Maple [B] time = 3.25, size = 662, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1
/2*d*x+1/2*c)^7*b^3+56*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*b
^3+22*cos(1/2*d*x+1/2*c)^3*a^2*b-84*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d
*x+1/2*c)^3*b^3-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+8*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+23*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^3-23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/
2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-22*cos(1/2*d*x+1/2*c)*a^2*b
+28*cos(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x
+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.504 $\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

Optimal. Leaf size=222

$$\frac{2b\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

```
[Out] (14*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*b*(2*a^2 + b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*C
os[c + d*x]]) + (2*a^3*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
+ d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.585709, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2793, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}}+\frac{2a^3\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```


```
[Out] (14*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*b*(2*a^2 + b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*d*Sqrt[a + b*C
os[c + d*x]]) + (2*a^3*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
+ d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (2*b^2*Sqrt[a + b*
Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec(c+dx)\,dx = \frac{2b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{\left(\frac{3a^3}{2}+\frac{1}{2}b\left(9a^2+b^2\right)\cos(c+dx)+\frac{7}{2}a\right.}{\sqrt{a+b\cos(c}}$$

$$= \frac{2b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} - \frac{2\int \frac{\left(-\frac{3a^3b}{2}-\frac{1}{2}b^2\left(2a^2+b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{3b} +$$

$$= \frac{2b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + a^3\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx + \frac{1}{3}\left(b\left(2a^2+\right.\right.$$

$$= \frac{14ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d}$$

$$= \frac{14ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\right.}{3d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 1.63996, size = 379, normalized size = 1.71

$$\frac{\frac{4b\left(9a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2a\left(6a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + 4b^2\sin(c+dx)\sqrt{a+b\cos(c+dx)} + \frac{14i\csc(c+dx)}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] ((4*b*(9*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*a*(6*a^2 + 7*b^2)*Sqrt[(a
+ b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[
a + b*Cos[c + d*x]] + ((14*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt
[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*Ar
cSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(
-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a
+ b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[
a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/Sqrt[-(a + b)^(-1)] + 4*b^2*Sqrt[
a + b*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

Maple [A] time = 3.405, size = 528, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x)
```

```
[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^3+2*cos(1/2*d*x+1/2*c)^3*a*b^2-6*cos(1/2*d*x+1/2*c)^3*b^3+2
*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2
```

```
*d*x+1/2*c),(-2*b/(a-b))^(1/2))+7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/
2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^
(1/2))*a^2*b-7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-3*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2*cos(1/2*d*x+1/2*c)*a*b
^2+2*cos(1/2*d*x+1/2*c)*b^3)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

3.505 $\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=222

$$\frac{a\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

```
[Out] -(((a^2 - 2*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (a*(a^2 + 4*b^2)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
 b*Cos[c + d*x]]) + (5*a^2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*Sqrt[a
+ b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.592406, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2792, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{a\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a^2\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] -(((a^2 - 2*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (a*(a^2 + 4*b^2)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a +
 b*Cos[c + d*x]]) + (5*a^2*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) + (a^2*Sqrt[a
+ b*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = \frac{a^2\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(\frac{5a^2b}{2}+3ab^2\cos(c+dx)-\frac{1}{2}b\left(a^2-2b^2\right.\right.}{\sqrt{a+b\cos(c+}}$$

$$= \frac{a^2\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} - \frac{\int \frac{\left(-\frac{5}{2}a^2b^2-\frac{1}{2}ab\left(a^2+4b^2\right)\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{b} +$$

$$= \frac{a^2\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{2}\left(5a^2b\right)\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx + \frac{1}{2}\left(a\right.$$

$$= -\frac{\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a^2\sqrt{a+b\cos(c+dx)}\tan(}{d}$$

$$= -\frac{\left(a^2-2b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{a\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,I}{d\sqrt{a+b\cos(c}}$$

Mathematica [C] time = 2.19818, size = 390, normalized size = 1.76

$$\frac{\frac{2b\left(9a^2+2b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(a^2-2b^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\cos(c+dx)}\right)|\frac{a+b}{a-b}\right)+\right.}{ab\sqrt{-\frac{1}{a+b}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] ((24*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(9*a^2 + 2*b^2)*Sqrt[(a + b*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos
[c + d*x]] + ((2*I)*(a^2 - 2*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*
Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*EllipticE
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]
+ b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]],
 (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*S
qrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(-1)]) + 4
*a^2*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d)
```

Maple [B] time = 3.174, size = 960, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x)
```

```
[Out] -((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a^2*b*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a^3-2*a^2*b)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*
a^3+4*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-EllipticE(cos(
```

```
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a^2*b+2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2
-2*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-5*EllipticPi(cos(1/
2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+4*a*b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)
*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*a^3+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a
^2*b+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-5*a^2*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2)))/(2*cos(1/2*d*x+1/2*c)^2-1)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x)
```

3.506 $\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=270

$$\frac{b\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a^2\tan(c+dx)\,\mathrm{se}}{}$$

```
[Out] (-9*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (b*(11*a^2 + 8*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*
Cos[c + d*x]]) + (a*(4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (9
*a*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a^2*Sqrt[a + b*Cos[c +
 d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.876703, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2792, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\cos(c+dx)}}+\frac{a^2\tan(c+dx)\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```


```
[Out] (-9*a*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*
d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (b*(11*a^2 + 8*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*
Cos[c + d*x]]) + (a*(4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*d*Sqrt[a + b*Cos[c + d*x]]) + (9
*a*b*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*d) + (a^2*Sqrt[a + b*Cos[c +
 d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = \frac{a^2\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \frac{\left(\frac{9a^2b}{2}+a\left(a^2+6b^2\right)\mathrm{c}\right.}{}$$

$$=\frac{9ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+\frac{a^2\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan}{2d}$$

$$=\frac{9ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+\frac{a^2\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan}{2d}$$

$$=\frac{9ab\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4d}+\frac{a^2\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan}{2d}$$

$$=-\frac{9ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{9ab\sqrt{a+b\cos(c+dx)}\tan(c+}{4d}$$

$$=-\frac{9ab\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{b\left(11a^2+8b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\right.}{4d\sqrt{a+b\cos(c+}}$$

Mathematica [C] time = 2.67647, size = 395, normalized size = 1.46

$$\frac{\frac{4b\left(a^2+4b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+\frac{a\left(8a^2+21b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+2a\tan(c+dx)\sec(c+dx)\sqrt{a+b\cos(c+d}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] ((4*b*(a^2 + 4*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (a*(8*a^2 + 21*b^2)*Sqrt[(a +
 b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a
 + b*Cos[c + d*x]] - ((9*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2
*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Cos[c + d*x]]], (a + b)/(a - b)])))/Sqrt[-(a + b)^(-1)] + 2*a*Sqrt[a +
b*Cos[c + d*x]]*(2*a + 9*b*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(8*d)
```

Maple [B] time = 3.832, size = 1134, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x)
```

```
[Out] -1/4*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-72*a*b^2
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(44*a^2*b+72*a*b^2)*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+(-4*a^3-22*a^2*b-18*a*b^2)*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/
2*c)^2+(a+b)/(a-b))^(1/2)*(11*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/
2))*a^2*b+8*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-4*Elliptic
Pi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-15*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))*a*b^2-9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a^2*b+9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2)*s
in(1/2*d*x+1/2*c)^4-4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+
1/2*c)^2+(a+b)/(a-b))^(1/2)*(11*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^2*b+8*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-4*Ellipt
icPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-15*EllipticPi(cos(1/2*d*x
+1/2*c),2,(-2*b/(a-b))^(1/2))*a*b^2-9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a^2*b+9*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2)
*sin(1/2*d*x+1/2*c)^2+11*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*a^2*b+8*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c
)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^3-15*a*b^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-9*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a*b^2)/(2*cos(1/2*d*x+1/2*c)^2-1)^2/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2
*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x)
```

3.507 $\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

Optimal. Leaf size=323

$$\frac{(16a^2+33b^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{a(16a^2+59b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}-\frac{(16a^2+33b^2)\sqrt{a+b\,c}}{24d\sqrt{}}$$

```
[Out] -((16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (a*(16*a^2 + 59*b^2)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(2
4*d*Sqrt[a + b*Cos[c + d*x]]) + (5*b*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c
 + d*x]]) + ((16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*d
) + (13*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d) + (a
^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 1.16942, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2792, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(16a^2+33b^2)\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{24d}+\frac{a(16a^2+59b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{a+b\cos(c+dx)}}-\frac{(16a^2+33b^2)\sqrt{a+b\,c}}{24d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] -((16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(24*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (a*(16*a^2 + 59*b^2)*S
qrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(2
4*d*Sqrt[a + b*Cos[c + d*x]]) + (5*b*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(8*d*Sqrt[a + b*Cos[c
 + d*x]]) + ((16*a^2 + 33*b^2)*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(24*d
) + (13*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(12*d) + (a
^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx = \frac{a^2\sqrt{a+b\cos(c+dx)}\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \frac{\left(\frac{13a^2b}{2}+a\left(2a^2+9b^2\right)\mathrm{c}\ldots\right.}{\ldots}$$

$$=\frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{12d}+\frac{a^2\sqrt{a+b\cos(c+dx)}\sec^2(c\ldots}{3d}$$

$$=\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d}+\frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c\ldots}{12d}$$

$$=\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d}+\frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c\ldots}{12d}$$

$$=\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{24d}+\frac{13ab\sqrt{a+b\cos(c+dx)}\sec(c\ldots}{12d}$$

$$=-\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(16a^2+33b^2\right)\sqrt{a+b\,\mathrm{c}\ldots}}{24\ldots}$$

$$=-\frac{\left(16a^2+33b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{24d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{a\left(16a^2+59b^2\right)\sqrt{\frac{a+b\,\mathrm{c}\ldots}{\ldots}}}{24d\sqrt{a+b\ldots}}$$

Mathematica [C] time = 4.15978, size = 434, normalized size = 1.34

$$\frac{\frac{2b\left(104a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}+4\sec^2(c+dx)\sqrt{a+b\cos(c+dx)}\left(\left(8a^2+\frac{33b^2}{2}\right)\sin(2(c+dx))+8a^2\tan(c+dx)\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] ((104*a*b^2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)
/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*b*(104*a^2 - 3*b^2)*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*
Cos[c + d*x]] - ((2*I)*(16*a^2 + 33*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a
+ b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(-2*a*(a - b)*E
llipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(
```

```
a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
+ d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^(
-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^2*(26*a*b*Sin[c + d*x] + (8
*a^2 + (33*b^2)/2)*Sin[2*(c + d*x)] + 8*a^2*Tan[c + d*x]))/(96*d)
```

Maple [B] time = 4.29, size = 1742, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x)
```

```
[Out] -1/24*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((256*a^2
*b+528*b^3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-128*a^3-384*a^2*b-472
*a*b^2-792*b^3)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(128*a^3+328*a^2*b+
472*a*b^2+396*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-48*a^3-100*a^2
*b-118*a*b^2-66*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+8*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(60*El
lipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b+15*EllipticPi(cos(1
/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^3-16*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*a^3-59*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*
b^2+16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-16*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+33*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a*b^2-33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*b^3)*sin(1/2*d*x+1/2*c)^6-12*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(60*EllipticPi(cos(1/2*d*x+1/2*c),2,(
-2*b/(a-b))^(1/2))*a^2*b+15*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1
/2))*b^3-16*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-59*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+16*EllipticE(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))*a^3-16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^2*b+33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-33*El
lipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3)*sin(1/2*d*x+1/2*c)^4+6*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(60*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*a^2*b+15*Elli
pticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^3-16*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-59*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*a*b^2+16*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-16*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b+33*EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-33*EllipticE(cos(1/2*d*x+1/2*c),(-2*b
/(a-b))^(1/2))*b^3)*sin(1/2*d*x+1/2*c)^2-60*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-15*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,2,(-2*b/(a-b))^(1/2))+16*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a^3+59*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2
*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1
6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+16*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b-33*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+33*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),(-2*b/(a-b))^(1/2))*b^3)/(2*cos(1/2*d*x+1/2*c)^2-1)^3/(-2*b*sin(1/2*d*x+1
```

```
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*
x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^4, x)
```

3.508 $\int (a + b\cos(c + dx))^{7/2}\,dx$

Optimal. Leaf size=246

$$\frac{2b\left(71a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(-46a^2b^2+71a^4-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{a+b\cos(c+dx)}}+\frac{32a(1}{}$$

```
[Out] (32*a*(11*a^2 + 13*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)])/(105*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(71*a^4 - 46*a
^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*b)/(a + b)])/(105*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(71*a^2 + 25*b^2)*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (24*a*b*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*d) + (2*b*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x]
)/(7*d)
```

Rubi [A] time = 0.374696, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2656, 2753, 2752, 2663, 2661, 2655, 2653}

$$\frac{2b\left(71a^2+25b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105d}-\frac{2\left(-46a^2b^2+71a^4-25b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{a+b\cos(c+dx)}}+\frac{32a(1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(7/2),x]
```


```
[Out] (32*a*(11*a^2 + 13*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)])/(105*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*(71*a^4 - 46*a
^2*b^2 - 25*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*b)/(a + b)])/(105*d*Sqrt[a + b*Cos[c + d*x]]) + (2*b*(71*a^2 + 25*b^2)*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(105*d) + (24*a*b*(a + b*Cos[c + d*x]
)^(3/2)*Sin[c + d*x])/(35*d) + (2*b*(a + b*Cos[c + d*x])^(5/2)*Sin[c + d*x]
)/(7*d)
```


Rule 2656

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[1/n, Int[(a + b*Sin
[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x], x
], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && In
tegerQ[2*n]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{7/2}\,dx = \frac{2b(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{2}{7}\int (a+b\cos(c+dx))^{3/2}\left(\frac{1}{2}\left(7a^2+5b^2\right)+6ab\cos(c$$

$$= \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} + \frac{2b(a+b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{4}{35}\int \sqrt{a+$$

$$= \frac{2b\left(71a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} +$$

$$= \frac{2b\left(71a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} +$$

$$= \frac{2b\left(71a^2+25b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{105d} + \frac{24ab(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{35d} +$$

$$= \frac{32a\left(11a^2+13b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{105d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\left(71a^4-46a^2b^2-25b^4\right)\sqrt{\frac{a+b\cos}{a+}}}{105d\sqrt{a+b\cos(}}$$

Mathematica [A] time = 1.08187, size = 211, normalized size = 0.86

$$\frac{b\sin(c+dx)\left(b\left(752a^2+145b^2\right)\cos(c+dx)+488a^3+162ab^2\cos(2(c+dx))+262ab^2+15b^3\cos(3(c+dx))\right)-4\left(-46a\right.}{210d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (64*a*(11*a^3 + 11*a^2*b + 13*a*b^2 + 13*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 4*(71*a^4 - 46*a^2*b^2 - 25*b
^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)
] + b*(488*a^3 + 262*a*b^2 + b*(752*a^2 + 145*b^2)*Cos[c + d*x] + 162*a*b^2
*Cos[2*(c + d*x)] + 15*b^3*Cos[3*(c + d*x)])*Sin[c + d*x])/(210*d*Sqrt[a +
b*Cos[c + d*x]])
```

Maple [B] time = 3.351, size = 824, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(7/2),x)
```

```
[Out] -2/105*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*cos
(1/2*d*x+1/2*c)^9*b^4+648*cos(1/2*d*x+1/2*c)^7*a*b^3-600*cos(1/2*d*x+1/2*c)
^7*b^4+752*cos(1/2*d*x+1/2*c)^5*a^2*b^2-1296*cos(1/2*d*x+1/2*c)^5*a*b^3+640
*cos(1/2*d*x+1/2*c)^5*b^4+244*cos(1/2*d*x+1/2*c)^3*a^3*b-1128*cos(1/2*d*x+1
/2*c)^3*a^2*b^2+860*cos(1/2*d*x+1/2*c)^3*a*b^3-360*cos(1/2*d*x+1/2*c)^3*b^4
-71*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+46*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2
*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b^4+176*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*
c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4
-176*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+208*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-208*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))*a*b^3-244*cos(1/2*d*x+1/2*c)*a^3*b+376*cos(1/2*d*x+1/2
*c)*a^2*b^2-212*cos(1/2*d*x+1/2*c)*a*b^3+80*cos(1/2*d*x+1/2*c)*b^4)/(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sqrt{b\cos(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.509 $\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{59F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{60\sqrt{7}d}+\frac{47E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}d}+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}-\frac{3\sin(c+dx)(4\cos(c+dx)\cdot}{70d}$$

```
[Out] (47*EllipticE[(c + d*x)/2, 8/7])/(20*Sqrt[7]*d) + (59*EllipticF[(c + d*x)/2
, 8/7])/(60*Sqrt[7]*d) + (59*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(105*d)
 - (3*(3 + 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(70*d) + (Cos[c + d*x]*(3 +
4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(14*d)
```

Rubi [A] time = 0.185166, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3023, 2753, 2752, 2661, 2653}

$$\frac{59F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{60\sqrt{7}d}+\frac{47E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}d}+\frac{\sin(c+dx)\cos(c+dx)(4\cos(c+dx)+3)^{3/2}}{14d}-\frac{3\sin(c+dx)(4\cos(c+dx)\cdot}{70d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[3 + 4*Cos[c + d*x]],x]
```


```
[Out] (47*EllipticE[(c + d*x)/2, 8/7])/(20*Sqrt[7]*d) + (59*EllipticF[(c + d*x)/2
, 8/7])/(60*Sqrt[7]*d) + (59*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(105*d)
 - (3*(3 + 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(70*d) + (Cos[c + d*x]*(3 +
4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(14*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \frac{\cos(c+dx)(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{14d} + \frac{1}{14}\int \sqrt{3+4\cos(c+dx)}\left(3+10\right.$$

$$= -\frac{3(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} + \frac{\cos(c+dx)(3+4\cos(c+dx))^{3/2}\sin(c+}{14d}$$

$$= \frac{59\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{105d} - \frac{3(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} + \frac{\cos(c-}{}$$

$$= \frac{59\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{105d} - \frac{3(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} + \frac{\cos(c-}{}$$

$$= \frac{47E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20\sqrt{7}d} + \frac{59F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{60\sqrt{7}d} + \frac{59\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{105d} - \frac{3(}{}$$

Mathematica [A] time = 0.245166, size = 92, normalized size = 0.67

$$\frac{59\sqrt{7}F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)+141\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)+(212\sin(c+dx)+9\sin(2(c+dx))+30\sin(3(c+dx)))\sqrt{4\cos(c+dx)+}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (141*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + 59*Sqrt[7]*EllipticF[(c + d*x)/2
, 8/7] + Sqrt[3 + 4*Cos[c + d*x]]*(212*Sin[c + d*x] + 9*Sin[2*(c + d*x)] +
30*Sin[3*(c + d*x)]))/(420*d)
```

Maple [A] time = 2.513, size = 275, normalized size = 2.

$$-\frac{1}{420\,d}\sqrt{\left(8\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(7680\,\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^8-14976\,(\sin(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/420*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7680*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-14976*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/
2*c)+12344*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+413*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^
(1/2))-141*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-4480*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x
+1/2*c))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x
+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos(dx+c)+3} \cos(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{4 \cos(dx+c)+3} \cos(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos(dx+c)+3} \cos(dx+c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

3.510 $\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{5d}$$

```
[Out] (21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(20*d) - (Sqrt[7]*EllipticF[(c + d
*x)/2, 8/7])/(20*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(5*d) + ((3 +
 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d)
```

Rubi [A] time = 0.137643, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2791, 2753, 2752, 2661, 2653}

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{\sin(c+dx)(4\cos(c+dx)+3)^{3/2}}{10d}-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(20*d) - (Sqrt[7]*EllipticF[(c + d
*x)/2, 8/7])/(20*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(5*d) + ((3 +
 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
```

```
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} + \frac{1}{10}\int (6-3\cos(c+dx))\sqrt{3+4\cos(c+dx)}\,d$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{5d} + \frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} + \frac{1}{15}\int \frac{21}{\sqrt{3}}$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{5d} + \frac{(3+4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} - \frac{7}{40}\int \frac{}{\sqrt{3}}$$

$$= \frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20d} - \frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{20d} - \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{5d} + \frac{(}{}$$

Mathematica [A] time = 0.139423, size = 81, normalized size = 0.77

$$\frac{-\sqrt{7}F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right) + 21\sqrt{7}E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right) + 2(\sin(c+dx)+2\sin(2(c+dx)))\sqrt{4\cos(c+dx)+3}}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] - Sqrt[7]*EllipticF[(c + d*x)/2, 8/
7] + 2*Sqrt[3 + 4*Cos[c + d*x]]*(Sin[c + d*x] + 2*Sin[2*(c + d*x)]))/(20*d)
```

Maple [A] time = 2.188, size = 253, normalized size = 2.4

$$-\frac{1}{20\,d}\sqrt{\left(8\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-256\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+384\,(\sin(1/2\,dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/20*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-256*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+384*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)-7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2*2^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d
*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-140*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos(dx+c)+3}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{4 \cos(dx+c)+3}\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos(dx+c)+3}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

3.511 $\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{6d}+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) + (Sqrt[7]*EllipticF[(c + d*x)/
2, 8/7])/(6*d) + (2*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0836391, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2753, 2752, 2661, 2653}

$$\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{6d}+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}+\frac{2\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) + (Sqrt[7]*EllipticF[(c + d*x)/
2, 8/7])/(6*d) + (2*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{3+4\cos(c+dx)}\,dx = \frac{2\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{2+\frac{3}{2}\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}dx$$

$$= \frac{2\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{4}\int \sqrt{3+4\cos(c+dx)}\,dx + \frac{7}{12}\int \frac{}{\sqrt{3+4}}$$

$$= \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d} + \frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{6d} + \frac{2\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.0604662, size = 69, normalized size = 0.88

$$\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right) + 3\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right) + 4\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (3*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + Sqrt[7]*EllipticF[(c + d*x)/2, 8/7
] + 4*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

Maple [A] time = 2.113, size = 231, normalized size = 3.

$$-\frac{1}{6d}\sqrt{\left(8\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(64\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+7\sqrt{(\sin(1/2\,dx+c/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/6*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x
+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-3*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2*2^(1/2))-56*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-8*sin(1/2*d*x+1/
2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos(dx+c)+3}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{4 \cos\left(dx+c\right)+3}\cos\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos\left(c+dx\right)+3}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos\left(dx+c\right)+3}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

3.512 $\int \sqrt{3+4\cos(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

```
[Out] (2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d
```

Rubi [A] time = 0.011538, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2653}

$$\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \sqrt{3+4\cos(c+dx)}\,dx = \frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

Mathematica [A] time = 0.0236297, size = 23, normalized size = 1.

$$\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d
```

Maple [B] time = 1.635, size = 137, normalized size = 6.

$$2\,\frac{\sqrt{\left(8\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-8\,(\cos(1/2\,dx+c/2))^2+1}\text{EllipticE}\left(\text{co}\right.}{\sqrt{-8\,(\sin(1/2\,dx+c/2))^4+7\,(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{8\,(\cos(1/2\,dx+c/2))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2*2^(1/2))/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos(dx+c)+3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{4 \cos(dx+c)+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos(c+dx)+3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(4*cos(c + d*x) + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos(dx+c)+3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3), x)
```

3.513 $\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{8F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

```
[Out] (8*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d) + (6*EllipticPi[2, (c + d*x)/2,
 8/7])/(Sqrt[7]*d)
```

Rubi [A] time = 0.0870154, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2803, 2661, 2805}

$$\frac{8F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (8*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d) + (6*EllipticPi[2, (c + d*x)/2,
 8/7])/(Sqrt[7]*d)
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+4\cos(c+dx)}\sec(c+dx)\,dx &= 3\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx+4\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx\\ &= \frac{8F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}\end{aligned}$$

Mathematica [A] time = 0.0491347, size = 41, normalized size = 0.85

$$\frac{8F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (8*EllipticF[(c + d*x)/2, 8/7] + 6*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7
]*d)
```

Maple [A] time = 2.178, size = 158, normalized size = 3.3

$$-2\,\frac{\sqrt{\left(8\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-8\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\left(4\,\mathrm{EllipticF}\left(\mathrm{c}\right.\right.}{\sqrt{-8\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+7\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{8\,\left(\mathrm{c}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(4*EllipticF(cos(1/2*d*x+1/2
*c),2*2^(1/2))-3*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/(-8*sin(1/2*d*
x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos(c + dx) + 3}\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos(dx + c) + 3}\sec(dx + c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

3.514 $\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx$

Optimal. Leaf size=95

$$\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

```
[Out] -((Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d) + (3*EllipticF[(c + d*x)/2, 8/7]
)/(Sqrt[7]*d) + (4*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d) + (Sqrt[3 +
 4*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.245654, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2796, 3060, 2653, 3002, 2661, 2805}

$$\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] -((Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/d) + (3*EllipticF[(c + d*x)/2, 8/7]
)/(Sqrt[7]*d) + (4*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d) + (Sqrt[3 +
 4*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3+4\cos(c+dx)}\sec^2(c+dx)\,dx &= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d}+\int \frac{\left(2-2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx\\ &= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d}-\frac{1}{4}\int \frac{(-8-6\cos(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx -\\ &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{d}+\frac{3}{2}\int \frac{1}{\sqrt{3+4\cos(c\cdot}}\\ &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{d}+\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{\sqrt{3+4\cos}}{}\end{aligned}$$

Mathematica [C] time = 1.09799, size = 157, normalized size = 1.65

$$\frac{6\sqrt{7}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)+21\sqrt{4\cos(c+dx)+3}\tan(c+dx)+\frac{i\sqrt{7}\sin(c+dx)\left(-12F\left(i\sinh^{-1}\left(\sqrt{4\cos(c+dx)+3}\right)|-\frac{1}{7}\right)+21E\left(i\sinh^{-1}\left(\sqrt{4\cos}\right.\right.\right.}{\sqrt{\sin^2(c+dx)}}}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (6*Sqrt[7]*EllipticPi[2, (c + d*x)/2, 8/7] + (I*Sqrt[7]*(21*EllipticE[I*Arc
Sinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*C
os[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]
]], -1/7])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2] + 21*Sqrt[3 + 4*Cos[c + d*x]]
*Tan[c + d*x])/(21*d)
```

Maple [B] time = 3.312, size = 350, normalized size = 3.7

$$-\frac{1}{d}\sqrt{-\left(-8\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\frac{\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-8\left(\cos(1/2\,dx+c/2)\right)^2+1}\text{Ellipti}}{\sqrt{-8\left(\sin(1/2\,dx+c/2)\right)^4+7\left(\sin(1/2\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-8*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4
+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2))-2
*cos(1/2*d*x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/
(2*cos(1/2*d*x+1/2*c)^2-1)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos(dx+c)+3}\sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{4 \cos(dx+c)+3}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos(c+dx)+3}\sec^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4 \cos(dx+c)+3}\sec(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

3.515 $\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{5\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}+\frac{\sqrt{4\cos(c+dx)+3}\,\mathrm{ta}}{2}$$

```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) + (3*EllipticF[(c + d*x)/2, 8/
7])/(Sqrt[7]*d) + (5*EllipticPi[2, (c + d*x)/2, 8/7])/(3*Sqrt[7]*d) + (Sqrt
[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.363618, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2796, 3055, 3059, 2653, 3002, 2661, 2805}

$$\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{5\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}+\frac{\sqrt{4\cos(c+dx)+3}\,\mathrm{ta}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) + (3*EllipticF[(c + d*x)/2, 8/
7])/(Sqrt[7]*d) + (5*EllipticPi[2, (c + d*x)/2, 8/7])/(3*Sqrt[7]*d) + (Sqrt
[3 + 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sec[c
+ d*x]*Tan[c + d*x])/(2*d)
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \sqrt{3+4\cos(c+dx)}\sec^3(c+dx)\,dx = \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \frac{(2+3\cos(c+dx)+2\cos}{\sqrt{3+4\cos}}$$

$$= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx}{2d}$$

$$= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx}{2d}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}}{}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} + \frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d} + \frac{5\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d} + \frac{\sqrt{3+4\cos}}{}$$

Mathematica [C] time = 1.24727, size = 194, normalized size = 1.44

$$\frac{\frac{12F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{\sqrt{7}}+\frac{6\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{\sqrt{7}}+(2\cos(c+dx)+3)\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)+\frac{2i\sin(c+dx)\left(-12F\left(i\sinh^{-1}(\sqrt{4}\right.\right.}{}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] ((12*EllipticF[(c + d*x)/2, 8/7])/Sqrt[7] + (6*EllipticPi[2, (c + d*x)/2, 8
/7])/Sqrt[7] + (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]]
, -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*Ellip
ticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt
[7]*Sqrt[Sin[c + d*x]^2]) + (3 + 2*Cos[c + d*x])*Sqrt[3 + 4*Cos[c + d*x]]*S
ec[c + d*x]*Tan[c + d*x])/(6*d)
```

Maple [B] time = 3.497, size = 408, normalized size = 3.

$$-\frac{1}{d}\sqrt{-\left(-8\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+7\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-8*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/
2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)^2-2/3*cos(1/2*d*x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))+1/3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-5/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1
/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(
1/2)))/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos\left(c+dx\right)+3}\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(4*cos(c + d*x) + 3)*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

3.516 $\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx$

Optimal. Leaf size=140

$$-\frac{59F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{60\sqrt{7}d}-\frac{47E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20\sqrt{7}d}-\frac{\sin(c+dx)\cos(c+dx)(3-4\cos(c+dx))^{3/2}}{14d}-\frac{3\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{70d}$$

```
[Out] (-47*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (59*EllipticF[(c +
Pi + d*x)/2, 8/7])/(60*Sqrt[7]*d) + (59*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*
x])/(105*d) - (3*(3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(70*d) - ((3 - 4*
Cos[c + d*x])^(3/2)*Cos[c + d*x]*Sin[c + d*x])/(14*d)
```

Rubi [A] time = 0.187183, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3023, 2753, 2752, 2662, 2654}

$$-\frac{59F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{60\sqrt{7}d}-\frac{47E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20\sqrt{7}d}-\frac{\sin(c+dx)\cos(c+dx)(3-4\cos(c+dx))^{3/2}}{14d}-\frac{3\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{70d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^3,x]
```


```
[Out] (-47*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (59*EllipticF[(c +
Pi + d*x)/2, 8/7])/(60*Sqrt[7]*d) + (59*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*
x])/(105*d) - (3*(3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(70*d) - ((3 - 4*
Cos[c + d*x])^(3/2)*Cos[c + d*x]*Sin[c + d*x])/(14*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
```

```
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3-4\cos(c+dx)}\cos^3(c+dx)\,dx &= -\frac{(3-4\cos(c+dx))^{3/2}\cos(c+dx)\sin(c+dx)}{14d} - \frac{1}{14}\int \sqrt{3-4\cos(c+dx)}\,\big(3 \\
&= -\frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} - \frac{(3-4\cos(c+dx))^{3/2}\cos(c+dx)\sin(c \\
&= \frac{59\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{105d} - \frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} - (3- \\
&= \frac{59\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{105d} - \frac{3(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{70d} - (3- \\
&= -\frac{47E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{20\sqrt{7}d} - \frac{59F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{60\sqrt{7}d} + \frac{59\sqrt{3-4\cos(c+dx)}\sin}{105d}
\end{aligned}$$

Mathematica [A] time = 0.203248, size = 114, normalized size = 0.81

$$\frac{654\sin(c+dx) - 511\sin(2(c+dx)) + 108\sin(3(c+dx)) - 60\sin(4(c+dx)) - 413\sqrt{4\cos(c+dx)-3}F\left(\frac{1}{2}(c+dx)\middle|8\right)}{420d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^3,x]
```

```
[Out] (141*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] - 413*Sqrt[-3 + 4*
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 654*Sin[c + d*x] - 511*Sin[2*(c +
 d*x)] + 108*Sin[3*(c + d*x)] - 60*Sin[4*(c + d*x)])/(420*d*Sqrt[3 - 4*Cos[
c + d*x]])
```

Maple [A] time = 2.907, size = 276, normalized size = 2.

$$\frac{1}{420\,d}\sqrt{-\left(8\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(7680\,\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}-8064\,\left(\sin\left(1/2\,dx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/420*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(7680*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8-8064*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2
*c)+5432*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+59*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14
^(1/2))+141*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-568*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-4\,\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-4\,\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4 \cos(dx + c) + 3} \cos(dx + c)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3, x)
```

3.517 $\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}+\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}+\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{5d}$$

```
[Out] (21*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*d) - (Sqrt[7]*EllipticF[(
c + Pi + d*x)/2, 8/7])/(20*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(5*
d) - ((3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d)
```

Rubi [A] time = 0.137994, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2791, 2753, 2752, 2662, 2654}

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}+\frac{21\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}-\frac{\sin(c+dx)(3-4\cos(c+dx))^{3/2}}{10d}+\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^2,x]
```

```
[Out] (21*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*d) - (Sqrt[7]*EllipticF[(
c + Pi + d*x)/2, 8/7])/(20*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(5*
d) - ((3 - 4*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(10*d)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
```

```
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\cos^2(c+dx)\,dx = -\frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} - \frac{1}{10}\int \sqrt{3-4\cos(c+dx)}(-6-3\cos(c+$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{5d} - \frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} - \frac{1}{15}\int$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{5d} - \frac{(3-4\cos(c+dx))^{3/2}\sin(c+dx)}{10d} - \frac{7}{40}\int$$

$$= \frac{21\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{20d} - \frac{\sqrt{7}F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{20d} + \frac{\sqrt{3-4\cos(c+dx)}\sin}{5d}$$

Mathematica [A] time = 0.20503, size = 104, normalized size = 0.97

$$-\frac{14\sin(c+dx)-16\sin(2(c+dx))+8\sin(3(c+dx))+7\sqrt{4\cos(c+dx)-3}F\left(\frac{1}{2}(c+dx)\middle|8\right)+21\sqrt{4\cos(c+dx)-3}E}{20d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]^2,x]
```

```
[Out] -(21*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 7*Sqrt[-3 + 4*Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 8] + 14*Sin[c + d*x] - 16*Sin[2*(c + d*x
)] + 8*Sin[3*(c + d*x)])/(20*d*Sqrt[3 - 4*Cos[c + d*x]])
```

Maple [A] time = 2.527, size = 253, normalized size = 2.4

$$\frac{1}{20\,d}\sqrt{-\left(8\,(\cos(1/2\,dx+c/2))^2-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-256\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+128\,(\sin(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/20*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-256*sin(1/2
*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+128*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c
)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2/7*14^(1/2))-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/
2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-12*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2
*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4 \cos(dx+c)+3}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-4 \cos(dx+c)+3}\cos(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4 \cos(dx+c)+3}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2, x)
```

3.518 $\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{6d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}$$

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) - (Sqrt[7]*EllipticF[(c +
 Pi + d*x)/2, 8/7])/(6*d) + (2*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0838027, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2753, 2752, 2662, 2654}

$$-\frac{\sqrt{7}F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{6d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{2d}+\frac{2\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) - (Sqrt[7]*EllipticF[(c +
 Pi + d*x)/2, 8/7])/(6*d) + (2*Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2753

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
 + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\cos(c+dx)\,dx = \frac{2\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{-2+\frac{3}{2}\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}dx$$

$$= \frac{2\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{3d} - \frac{1}{4}\int \sqrt{3-4\cos(c+dx)}\,dx - \frac{7}{12}\int \frac{1}{\sqrt{3-4\cos}}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{2d} - \frac{\sqrt{7}F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{6d} + \frac{2\sqrt{3-4\cos(c+dx)}\sin(c+}{3d}$$

Mathematica [A] time = 0.0956601, size = 94, normalized size = 1.18

$$\frac{12\sin(c+dx)-8\sin(2(c+dx))-7\sqrt{4\cos(c+dx)-3}F\left(\frac{1}{2}(c+dx)\middle|8\right)+3\sqrt{4\cos(c+dx)-3}E\left(\frac{1}{2}(c+dx)\middle|8\right)}{6d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x],x]
```

```
[Out] (3*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] - 7*Sqrt[-3 + 4*Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 8] + 12*Sin[c + d*x] - 8*Sin[2*(c + d*x)])
/(6*d*Sqrt[3 - 4*Cos[c + d*x]])
```

Maple [A] time = 2.482, size = 231, normalized size = 2.9

$$\frac{1}{6d}\sqrt{-\left(8\left(\cos(1/2\,dx+c/2)\right)^2-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(64\left(\sin(1/2\,dx+c/2)\right)^4\cos(1/2\,dx+c/2)+\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{56}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(64*sin(1/2*d*
x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+
1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2/7*14^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(8*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1
/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4\cos(dx+c)+3}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-4\cos\left(dx+c\right)+3}\cos\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{3-4\cos\left(c+dx\right)}\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*cos(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-4\cos\left(dx+c\right)+3}\cos\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c), x)
```

3.519 $\int \sqrt{3-4\cos(c+dx)}\,dx$

Optimal. Leaf size=24

$$\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}$$

```
[Out] (2*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d
```

Rubi [A] time = 0.0113837, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2654}

$$\frac{2\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\,dx=\frac{2\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{d}$$

Mathematica [A] time = 0.0317568, size = 44, normalized size = 1.83

$$-\frac{2\sqrt{4\cos(c+dx)-3}E\left(\frac{1}{2}(c+dx)\middle|8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos[
c + d*x]])
```

Maple [B] time = 1.914, size = 138, normalized size = 5.8

$$-2\frac{\sqrt{-\left(8\,(\cos(1/2\,dx+c/2))^2-7\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{56\,(\sin(1/2\,dx+c/2))^2-7}\text{EllipticE}\left(\cos\right.}{\sqrt{8\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{-8\,(\cos(1/2\,dx+c/2))^2+7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2/7*14^(1/2))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2
*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4 \cos(dx+c)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-4 \cos(dx+c)+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3-4\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3 - 4*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4 \cos(dx+c)+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3), x)
```

3.520 $\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=50

$$-\frac{8F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{6\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}$$

```
[Out] (-8*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d) - (6*EllipticPi[2, (c + P
i + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rubi [A] time = 0.0875074, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2803, 2662, 2806}

$$-\frac{8F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{6\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (-8*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d) - (6*EllipticPi[2, (c + P
i + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx &= 3\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx-4\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx\\ &= -\frac{8F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{6\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}d}\end{aligned}$$

Mathematica [A] time = 0.0595939, size = 61, normalized size = 1.22

$$\frac{2\sqrt{4\cos(c+dx)-3}\left(3\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)-4F\left(\frac{1}{2}(c+dx)\middle|8\right)\right)}{d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*(-4*EllipticF[(c + d*x)/2, 8] + 3*EllipticPi[2
, (c + d*x)/2, 8]))/(d*Sqrt[3 - 4*Cos[c + d*x]])
```

Maple [A] time = 2.533, size = 159, normalized size = 3.2

$$\frac{2}{7d}\sqrt{-\left(8\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{56\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-7}\left(4\,\mathrm{EllipticF}\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/7*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*(4*EllipticF(cos(1/2*d*x+1
/2*c),2/7*14^(1/2))+3*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/(8*sin
(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/
2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3-4\cos(c+dx)}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4\cos(dx+c)+3}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c), x)
```

3.521 $\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{3F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}$$

```
[Out] -((Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d) + (3*EllipticF[(c + Pi + d*
x)/2, 8/7])/(Sqrt[7]*d) + (4*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.248646, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2796, 3060, 2654, 3002, 2662, 2806}

$$\frac{3F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{d}+\frac{4\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] -((Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/d) + (3*EllipticF[(c + Pi + d*
x)/2, 8/7])/(Sqrt[7]*d) + (4*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/d
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
```

```
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rubi steps

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx = \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d} + \int \frac{\left(-2+2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d} + \frac{1}{4}\int \frac{(-8+6\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx - \frac{1}{2}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{d} + \frac{3}{2}\int \frac{1}{\sqrt{3-4\cos(c}}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{d} + \frac{3F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}d} + \frac{4\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}d} +$$

Mathematica [C] time = 1.41136, size = 178, normalized size = 1.82

$$\frac{21\sqrt{3-4\cos(c+dx)}\tan(c+dx) - \frac{42\sqrt{4\cos(c+dx)-3}\Pi\left(2;\frac{1}{2}(c+dx)|8\right)}{\sqrt{3-4\cos(c+dx)}} - \frac{i\sqrt{7}\sin(c+dx)\left(-12F\left(i\sinh^{-1}\left(\sqrt{3-4\cos(c+dx)}\right)|-\frac{1}{7}\right)+21E\left(i\sinh^{-1}\left(\sqrt{3-}\right.\right.\right.}{\sqrt{\sin^2(c+dx)}}}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] ((-42*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*C
os[c + d*x]] - (I*Sqrt[7]*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]]
, -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*Ellip
ticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/Sqrt[
Sin[c + d*x]^2] + 21*Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(21*d)
```

Maple [B] time = 4.174, size = 351, normalized size = 3.6

$$-\frac{1}{d}\sqrt{-\left(8\left(\cos(1/2\,dx+c/2)\right)^2-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(\frac{4}{7}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{56\left(\sin(1/2\,dx+c/2)\right)^2-7}\,\mathrm{EllipticPi}\left(\cos\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/7*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))-2
*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)+3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/
2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/
2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2
*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4 \cos(dx+c)+3}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-4 \cos(dx+c)+3}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3-4\cos(c+dx)}\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4 \cos(dx+c)+3}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2, x)
```

3.522 $\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$

Optimal. Leaf size=138

$$-\frac{3F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{5\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}d}-\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{\sqrt{3-}}{}$$

```
[Out] (Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) - (3*EllipticF[(c + Pi + d
*x)/2, 8/7])/(Sqrt[7]*d) - (5*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3*Sqrt
[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 - 4*Cos[c
+ d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.373832, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2796, 3055, 3059, 2654, 3002, 2662, 2806}

$$-\frac{3F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{5\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}d}-\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{\sqrt{3-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) - (3*EllipticF[(c + Pi + d
*x)/2, 8/7])/(Sqrt[7]*d) - (5*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3*Sqrt
[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 - 4*Cos[c
+ d*x]]*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rule steps

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx = \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int\frac{(-2+3\cos(c+dx)-2\cos^2(}{\sqrt{3-4\cos(c}}$$

$$= -\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}-$$

$$= -\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2d}-$$

$$= \frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{\sqrt{3-4\cos(c+dx)}\,s}{}$$

$$= \frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d}-\frac{3F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{5\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}-\frac{\sqrt{3}}{}$$

Mathematica [C] time = 1.82903, size = 237, normalized size = 1.72

$$-\frac{\frac{12\sqrt{4\cos(c+dx)-3}F\left(\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}}+\frac{6\sqrt{4\cos(c+dx)-3}\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}}-\sqrt{3-4\cos(c+dx)}(2\cos(c+dx)-3)\tan(c+dx)\sec(c+d}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 4*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] ((-12*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c
 + d*x]] + (6*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt
[3 - 4*Cos[c + d*x]] + (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c
+ d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] -
 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x
])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) - Sqrt[3 - 4*Cos[c + d*x]]*(-3 + 2*Cos[c
+ d*x])*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

Maple [B] time = 3.997, size = 408, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x+1/2
*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^2+2/3*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)
^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-3/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*s
in(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-5/21*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)
))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4\cos(dx+c)+3}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{-4\cos(dx+c)+3}\sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3-4\cos(c+dx)}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(3 - 4*cos(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4\cos(dx+c)+3}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3, x)
```

3.523 $\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=215

$$-\frac{2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-8a\sin(c+dx$$

```
[Out] (2*(8*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(8*a^2 + 7*b^2
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (8*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(15*b^2*d) + (2*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(5*b*d)
```

Rubi [A] time = 0.287047, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2793, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\cos(c+dx)}}+\frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-8a\sin(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(8*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
 + b)])/(15*b^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*(8*a^2 + 7*b^2
)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(15*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (8*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(15*b^2*d) + (2*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(5*b*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} + \frac{2\int \frac{a+\frac{3}{2}b\cos(c+dx)-2a\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{5b}$$

$$= -\frac{8a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} + \frac{4\int \frac{\frac{ab}{2}+\cdots}{\cdots}}{\cdots}$$

$$= -\frac{8a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} - \frac{\left(a\left(8a^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{8a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15b^2d} + \frac{2\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5bd} + \frac{\left(\left(8a^2+\cdots\right.\right.}{\cdots}$$

$$= \frac{2\left(8a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx\cdots\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.932382, size = 182, normalized size = 0.85

$$\frac{b\sin(c+dx)\left(-8a^2-2ab\cos(c+dx)+3b^2\cos(2(c+dx))+3b^2\right)-2a\left(8a^2+7b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2\left(8\cdots\right.}{15b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(8*a^3 + 8*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*E
llipticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(8*a^2 + 7*b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + b*(-8*a^2 + 3*b^
2 - 2*a*b*Cos[c + d*x] + 3*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(15*b^3*d*Sq
rt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.274, size = 665, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*cos(1
/2*d*x+1/2*c)^7*b^3-4*cos(1/2*d*x+1/2*c)^5*a*b^2-48*cos(1/2*d*x+1/2*c)^5*b^
3-8*cos(1/2*d*x+1/2*c)^3*a^2*b+6*cos(1/2*d*x+1/2*c)^3*a*b^2+30*cos(1/2*d*x+
1/2*c)^3*b^3-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)
/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-7*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2+8*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a^3-8*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x
+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)*a^2*b+9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^2-9*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3+8*cos(1/2*d*x+1/2*c)*a^2*b-2*cos
(1/2*d*x+1/2*c)*a*b^2-6*cos(1/2*d*x+1/2*c)*b^3)/b^3/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/
2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^{3}}{\sqrt{b\cos\left(dx+c\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

3.524 $\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=165

$$\frac{2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

```
[Out] (-4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^
2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*d*Sqrt[a +
 b*Cos[c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.187889, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2791, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\cos(c+dx)}}-\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*b^
2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[
c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^2*d*Sqrt[a +
 b*Cos[c + d*x]]) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2\int \frac{\frac{b}{2}-a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{3b}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{1}{3}\left(1+\frac{2a^2}{b^2}\right)\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx - \frac{(2a)\int \sqrt{a+b\cos}}{3b^2}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3bd} - \frac{\left(2a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{3b^2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\left(\left(1+\frac{2a}{b}\right.\right.}{}$$

$$= -\frac{4a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(1+\frac{2a^2}{b^2}\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\sqrt{a+b\cos(c+dx)}} + \frac{2\sqrt{}}{}$$

Mathematica [A] time = 0.633289, size = 137, normalized size = 0.83

$$\frac{2\left(2a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2b\sin(c+dx)(a+b\cos(c+dx))-4a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right.}{3b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-4*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + 2*(2*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[
(c + d*x)/2, (2*b)/(a + b)] + 2*b*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*b^2
*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.932, size = 453, normalized size = 2.8

$$-\frac{2}{3b^2d}\sqrt{\left(2b\left(\cos(1/2dx+c/2)\right)^2+a-b\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\left(\cos(1/2dx+c/2)\right)^5b^2+2\left(\cos(1/2dx+c/2)\right)^3ab-6\left(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*cos(1/2
*d*x+1/2*c)^5*b^2+2*cos(1/2*d*x+1/2*c)^3*a*b-6*cos(1/2*d*x+1/2*c)^3*b^2+2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))*b^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*
x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2
))*a^2+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b)
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b-2*cos(1/2*d*x+
1/2*c)*a*b+2*cos(1/2*d*x+1/2*c)*b^2)/b^2/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+
b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(dx+c)}^2}{\sqrt{b\cos{(dx+c)}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

3.525 $\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=122

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.107874, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2752, 2663, 2661, 2655, 2653}

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```


```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)]) - (2*a*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\int \sqrt{a+b\cos(c+dx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{\left(a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{b\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 2.28261, size = 86, normalized size = 0.7

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-aF\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*((a + b)*EllipticE[(c + d*x)/2, (2*b)
/(a + b)] - a*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(b*d*Sqrt[a + b*Cos[c
 + d*x]])
```

Maple [A] time = 3.016, size = 220, normalized size = 1.8

$$2\,\frac{\sqrt{\left(2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}b\sin\left(1/2\,dx+c/2\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b}\,d}\sqrt{\frac{2\,b\,(c}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*(EllipticF(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)/(-2*b*sin
(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)/(-
2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.526 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.0365285, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2663, 2661}

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx &= \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}\\ &= \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0502258, size = 57, normalized size = 1.

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
)/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [C] time = 0.417, size = 75, normalized size = 1.3

$$2\frac{1}{d\sqrt{2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b}}\sqrt{\frac{2\,b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b}{a+b}}\mathrm{InverseJacobiAM}\left(1/2\,dx+c/2,\frac{\sqrt{2}\sqrt{b}}{\sqrt{a+b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a
+b))^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2^(1/2)/(a+b)^(1/2)*b^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c) + a), x)
```

3.527 $\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.126057, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2807, 2805}

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx &= \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int \frac{\sec(c+dx)}{\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}}\,dx}{\sqrt{a+b\cos(c+dx)}}\\ &= \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0837801, size = 58, normalized size = 1.

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
 b)])/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 3.099, size = 166, normalized size = 2.9

$$2\frac{\sqrt{\left(2b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}}{\sqrt{-2b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+bd}}\sqrt{\frac{2b(\mathrm{co}}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2,(-2*b/(a-b))^(1/2))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c) + a), x)
```

3.528 $\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=206

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{b\sqrt{\frac{a+b\cos(c+d}{a+b}}}{ad\sqrt{a}}$$

```
[Out] -((Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + (Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a
*d*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(a*d
)
```

Rubi [A] time = 0.49247, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2802, 3060, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{ad}+\frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}}-\frac{\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{b\sqrt{\frac{a+b\cos(c+d}{a+b}}}{ad\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])) + (Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ell
ipticF[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqrt[a + b*Cos[c + d*x]]) - (b*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a
*d*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(a*d
)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} + \frac{\int \frac{\left(-\frac{b}{2}-\frac{1}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{a}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} - \frac{\int \sqrt{a+b\cos(c+dx)}\,dx}{2a} - \frac{\int \frac{\left(\frac{b^2}{2}-\frac{1}{2}ab\cos(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{ab}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} + \frac{1}{2}\int \frac{1}{\sqrt{a+b\cos(c+dx)}}dx - \frac{b\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}dx}{2a} - \frac{\sqrt{a+}}{}$$

$$= -\frac{\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{ad} + \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\int}{2\sqrt{a+b\,\mathrm{c}}}$$

$$= -\frac{\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\cos(c+dx)}} - \frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi}{ad\sqrt{a+b}}$$

Mathematica [C] time = 8.31197, size = 310, normalized size = 1.5

$$\frac{4\tan(c+dx)\sqrt{a+b\cos(c+dx)} - \frac{6b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+}}\right.\right.\right.\right.}{}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((-6*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/
(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*
EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/
(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
 + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b*Sqrt[-(a + b)^
(-1)]) + 4*Sqrt[a + b*Cos[c + d*x]]*Tan[c + d*x])/(4*a*d)
```

Maple [A] time = 4.924, size = 532, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
2*cos(1/2*d*x+1/2*c)^2-1)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(
```

```
-2*b/(a-b))^(1/2))+1/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/a*b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2,(-2*b/(a-b))^(1/2)))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a
+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c) + a), x)
```

3.529 $\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=268

$$\frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{3b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (3*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2
*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (b*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*x]])
 + ((4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d
*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) - (3*b*Sqrt[a + b
*Cos[c + d*x]]*Tan[c + d*x])/(4*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.712854, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{(4a^2+3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}}-\frac{3b\tan(c+dx)\sqrt{a+b\cos(c+dx)}}{4a^2d}+\frac{3b\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (3*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(4*a^2
*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (b*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*a*d*Sqrt[a + b*Cos[c + d*x]])
 + ((4*a^2 + 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d
*x)/2, (2*b)/(a + b)])/(4*a^2*d*Sqrt[a + b*Cos[c + d*x]]) - (3*b*Sqrt[a + b
*Cos[c + d*x]]*Tan[c + d*x])/(4*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]*Tan[c + d*x])/(2*a*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
```

```
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(-\frac{3b}{2}+a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)\right)\sec^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{2a}$$

$$= -\frac{3b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} + \frac{\int \frac{\left(\frac{1}{4}(4a^2 \ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{3b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} - \frac{\int \frac{\left(-\frac{1}{4}b(4 \ldots\right.}{\ldots}}{\ldots}$$

$$= -\frac{3b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{2ad} - \frac{b\int \frac{\ldots}{\sqrt{a+b \ldots}}}{\ldots}$$

$$= \frac{3b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{3b\sqrt{a+b\cos(c+dx)}\tan(c+dx)}{4a^2d} + \frac{\sqrt{a+b\cos(c \ldots}}{\ldots}$$

$$= \frac{3b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\cos(c+dx)}} + \frac{\left(4+\frac{3b^2}{a^2}\right)\sqrt{\frac{a+b \ldots}{\ldots}}}{4d\sqrt{\ldots}}$$

Mathematica [C] time = 6.38542, size = 518, normalized size = 1.93

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{\tan(c+dx)\sec(c+dx)}{2a}-\frac{3b\tan(c+dx)}{4a^2}\right)}{d} + \frac{\frac{2(8a^2+9b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{6ib^2\sin(c+dx)\cos(2(c+dx))\sqrt{\frac{b-b\cos(c \ldots}{a+b}}}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] ((8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a
+ b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^2 + 9*b^2)*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c +
d*x]] - ((6*I)*b^2*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c +
 d*x])/(a - b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[
I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] -
 b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d
*x]]], (a + b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos
[c + d*x]^2]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c +
```

```
d*x])^2)/b^2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d
*x])^2)))/(16*a^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((-3*b*Tan[c + d*x])/(4*a^
2) + (Sec[c + d*x]*Tan[c + d*x])/(2*a)))/d
```

Maple [B] time = 5.033, size = 710, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*
x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)^2+3/2/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/4*b/a
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/
(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/4/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b
*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)
)-3/4/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/
(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2
)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(
a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b
))^(1/2))-3/4/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)/sin(1/2*d*x+1/2
*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c) + a), x)
```

3.530 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2a\left(8a^2-3b^2\right)\sin(c+d}{5b^3d\left(a^2\right.}$$

```
[Out] (2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(5*b^4*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) - (8*a*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(5*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c +
d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a*(8*a
^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^3*(a^2 - b^2)*d) +
(2*(6*a^2 - b^2)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2
*(a^2 - b^2)*d)
```

Rubi [A] time = 0.513377, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2792, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(6a^2-b^2\right)\sin(c+dx)\cos(c+dx)\sqrt{a+b\cos(c+dx)}}{5b^2d\left(a^2-b^2\right)}-\frac{2a\left(8a^2-3b^2\right)\sin(c+d}{5b^3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*(16*a^4 - 8*a^2*b^2 - 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(5*b^4*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]) - (8*a*(4*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(5*b^4*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c +
d*x]^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a*(8*a
^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^3*(a^2 - b^2)*d) +
(2*(6*a^2 - b^2)*Cos[c + d*x]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*b^2
*(a^2 - b^2)*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\cos(c+dx)\left(2a^2-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}\left(6a^2-b^2\right)\cos^2(c+dx)\right)}{\sqrt{a+b\cos(c+dx)}}dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(6a^2-b^2\right)\cos(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5b^3\left(a^2-b^2\right)d} + \frac{2(}{}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5b^3\left(a^2-b^2\right)d} + \frac{2(}{}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2a\left(8a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5b^3\left(a^2-b^2\right)d} + \frac{2(}{}$$

$$= \frac{2\left(16a^4-8a^2b^2-3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{5b^4\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{8a\left(4a^2+b^2\right)\sqrt{\frac{a+b\cos(c+}{a+b}}}{5b^4d\sqrt{a+b\cos}}$$

Mathematica [A] time = 1.27169, size = 242, normalized size = 0.74

$$\frac{-b\sin(c+dx)\left(4ab\left(a^2-b^2\right)\cos(c+dx)+\left(b^4-a^2b^2\right)\cos(2(c+dx))-7a^2b^2+16a^4+b^4\right)-8a\left(-3a^2b^2+4a^4-b^4\right)}{5b^4d(a-b)(a+b)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(16*a^5 + 16*a^4*b - 8*a^3*b^2 - 8*a^2*b^3 - 3*a*b^4 - 3*b^5)*Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] - 8*a*(4*a^4
 - 3*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/
2, (2*b)/(a + b)] - b*(16*a^4 - 7*a^2*b^2 + b^4 + 4*a*b*(a^2 - b^2)*Cos[c +
 d*x] + (-(a^2*b^2) + b^4)*Cos[2*(c + d*x)])*Sin[c + d*x])/(5*(a - b)*b^4*(
a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 4.046, size = 1285, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5*(-8*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(
a^2-b^2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-8*(-2*b*sin(1/2*d*x+1/2*c)
^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(a^3-a^2*b-a*b^2+b^3)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*b*(8*a^4+2*a^3*b-4*a^2*b^2-2*a*b^3+b^4)*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-16*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)
```

```
^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+
b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5+12*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b^2+4*(-2*b*sin(1/2*d*x+1/2*c)^4+
(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*
sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))*a*b^4+16*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b
)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^5-16*(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4*b-8*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*a^3*b^2+8*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/
(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^3-3*(-2
*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^4+3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a
+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b^5)/b^4/(a-b)/(a+b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^4}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^4/(b^2*cos(d*x + c)^2 + 2*a*
b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(3/2), x)
```

3.531 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=257

$$-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)}+\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{3b^3d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*a*(8*a^2 - 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(8
*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)
/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c +
 d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.34389, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\cos(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(4a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^2d\left(a^2-b^2\right)}+\frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{3b^3d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-2*a*(8*a^2 - 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(8
*a^2 + b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)
/(a + b)])/(3*b^3*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c +
 d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{a^2-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}\left(4a^2-b^2\right)\cos^2(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{4\int \frac{1}{4}\ldots}{\ldots}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{\left(a\left(8\ldots\right.\right.}{\ldots}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(4a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{\left(a\left(8\ldots\right.\right.}{\ldots}$$

$$= -\frac{2a\left(8a^2-5b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(8a^2+b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(\ldots\right.}{3b^3d\sqrt{a+b\cos(c+\ldots}}$$

Mathematica [A] time = 0.883189, size = 197, normalized size = 0.77

$$\frac{-2b\sin(c+dx)\left(\left(b^3-a^2b\right)\cos(c+dx)-4a^3+ab^2\right)+2\left(-7a^2b^2+8a^4-b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2a\left(8a^2b+\right.}{3b^3d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(8*a^3 + 8*a^2*b - 5*a*b^2 - 5*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)
]*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + 2*(8*a^4 - 7*a^2*b^2 - b^4)*Sqrt[
(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] - 2*b*(
-4*a^3 + a*b^2 + (-(a^2*b) + b^3)*Cos[c + d*x])*Sin[c + d*x])/(3*(a - b)*b^
3*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.94, size = 984, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(4*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(a
^2-b^2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*a^3+a^2*b-a*b^2-b^3)*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)+8*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^
2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4-7
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2*b^2-(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b
)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*b^4-8*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)
/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^4+8*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3*b+5*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1
/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b)
)^(1/2))*a^2*b^2-5*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a*b^3)/b^3/(a-b
)/(a+b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/
2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^3}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3/(b^2*cos(d*x + c)^2 + 2*a*
b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

3.532 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=186

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b^2*d*S
qrt[a + b*Cos[c + d*x]]) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```

Rubi [A] time = 0.231586, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2790, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(b^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (4*a*Sqrt[(a
 + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b^2*d*S
qrt[a + b*Cos[c + d*x]]) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b
*Cos[c + d*x]])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{2\int\frac{\frac{ab}{2}+\frac{1}{2}\left(2a^2-b^2\right)\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{(2a)\int\frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b^2}+\frac{\left(2a^2-b^2\right)\int\sqrt{a+b\cos(c}{b^2\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}+\frac{\left(\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\right)\int\sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx}{a+b}}}{b^2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= \frac{2\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b^2d\sqrt{a+b\cos(c+dx)}}-$$

Mathematica [A] time = 0.670255, size = 159, normalized size = 0.85

$$\frac{2\left(2a^2b+2a^3-ab^2-b^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2a\left(2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+ab\sin(c+\right.}{b^2d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*Ellip
ticE[(c + d*x)/2, (2*b)/(a + b)] - 2*a*(2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)] + a*b*Sin[c + d*x]))/((
a - b)*b^2*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 3.902, size = 530, normalized size = 2.9

$$2\,\frac{1}{(a+b)\,(a-b)\,b^2\sin(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^2\,b+a+b}\,d}\left(2\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,\frac{(\sin(1/2\,dx+}{a-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a
-b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3-2*a*b^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^3+2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/
(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
(-2*b/(a-b))^(1/2))*a^2*b+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*
d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))*a*b^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b^3-2*a^
2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/b^2/(a-b)/(a+b)/sin(1/2*d*x+1/
2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos{(dx+c)}^2}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos{(dx+c)}+a}\cos{(dx+c)}^2}{b^2\cos{(dx+c)}^2+2\,ab\cos{(dx+c)}+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2/(b^2*cos(d*x + c)^2 + 2*a*
b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

3.533 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*(a
^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
 d*x]]) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.182183, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2a\sin(c+dx)}{d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{2a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(b*(a
^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(b*d*Sqrt[a + b*Cos[c +
 d*x]]) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx &= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{b}{2}+\frac{1}{2}a\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2} \\
&= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} - \frac{a\int \sqrt{a+b\cos(c+dx)}\,dx}{b\,(a^2-b^2)} \\
&= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{\left(a\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{b\,(a^2-b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{a+b}\ldots}{\ldots} \\
&= -\frac{2a\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{b\,(a^2-b^2)\,d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd\sqrt{a+b\cos(c+dx)}} + \frac{\ldots}{(a^2-b^2\ldots}
\end{aligned}$$

Mathematica [A] time = 0.507921, size = 137, normalized size = 0.81

$$\frac{2\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2ab\sin(c+dx)-2a(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{bd(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*
b)/(a + b)] + 2*(a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)] + 2*a*b*Sin[c + d*x])/((a - b)*b*(a + b)*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Maple [A] time = 3.72, size = 373, normalized size = 2.2

$$-2\,\frac{1}{(a+b)\,(a-b)\,b\sin(1/2\,dx+c/2)\,\sqrt{-2\,(\sin(1/2\,dx+c/2))^2\,b+a+bd}}\left(\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,\frac{(\sin(1/2\,dx+c/\ldots}{a-b}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a^2-b^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))*a^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))*a-2*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/b/(a-b)/(a+b)
/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)/(b^2*cos(d*x + c)^2 + 2*a*b*
cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx + c)}{(b \cos (dx + c) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.534 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2 -
b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.0650911, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2664, 21, 2655, 2653}

$$\frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/((a^2 -
b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*b*Sin[c + d*x])/((a^2 - b^2
)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$
$$= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\int \sqrt{a+b\cos(c+dx)}\,dx}{a^2-b^2}$$
$$= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\sqrt{a+b\cos(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$= \frac{2\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.205861, size = 83, normalized size = 0.78

$$\frac{2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)-2b\sin(c+dx)}{d(a-b)(a+b)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*b)/
(a + b)] - 2*b*Sin[c + d*x])/((a - b)*(a + b)*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 3.772, size = 217, normalized size = 2.1

$$-2\frac{1}{(a-b)(a+b)\sin(1/2\,dx+c/2)\sqrt{-2\left(\sin(1/2\,dx+c/2)\right)^2b+a+b}d}\left(\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\frac{(\sin(1/2\,dx+c/2}{a-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-
b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d
*x+1/2*c)^2)/(a-b)/(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)
^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
+ a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(a+b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-3/2), x)
```

3.535 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*(a
^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos
[c + d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rubi [A] time = 0.394094, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2802, 3059, 2655, 2653, 12, 2807, 2805}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-2*b*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(a*(a
^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*Sqrt[(a + b*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a + b*Cos
[c + d*x]]) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(\frac{1}{2}\left(a^2-b^2\right)-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}b^2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int -\frac{b\left(a^2-b^2\right)\sec(c+dx)}{2\sqrt{a+b\cos(c+dx)}}\,dx}{ab\left(a^2-b^2\right)} - \frac{b\int \sqrt{a+b\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\ldots}}{a\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

$$= -\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\sqrt{\frac{a+b\cos(c+d\ldots}{a+b}}\ldots}{a\sqrt{a\ldots}}$$

$$= -\frac{2b\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\ldots}{a\left(a^2-\ldots\right.}$$

Mathematica [C] time = 5.06386, size = 402, normalized size = 2.28

$$\frac{\frac{4b^2\sin(c+dx)}{(a^2-b^2)\sqrt{a+b\cos(c+dx)}}-\frac{\frac{2(2a^2-3b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{4ab\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}}-\frac{2i\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\left(b\left(b\Pi\left(\right.\right.\right.}{(b-a)(}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-(((-4*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)
/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(2*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c
+ d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[
c + d*x]] - ((2*I)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Co
s[c + d*x]))/(-a + b)]*Csc[c + d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[
-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ellipt
icF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b
)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
 + d*x]]], (a + b)/(a - b)])))/(a*Sqrt[-(a + b)^(-1)]))/((-a + b)*(a + b)))
 + (4*b^2*Sin[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(2*a*d)
```

Maple [A] time = 3.403, size = 376, normalized size = 2.1

$$2\,\frac{1}{(a+b)(a-b)a\sin(1/2\,dx+c/2)\sqrt{-2\,(\sin(1/2\,dx+c/2))^2b+a+b}d}\left(\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,\frac{(\sin(1/2\,dx+c/2}{a-b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b
))^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),2,(-2*b/(a-b))^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*s
in(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*
b/(a-b))^(1/2))*b^2+2*b^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/a/(a-b)/
(a+b)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec{(c+dx)}}{(a+b\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec{(dx+c)}}{(b\cos{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.536 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{a^2d\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -(((a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a
 + b*Cos[c + d*x]]) - (3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(a^2 -
3*b^2)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + Tan[c +
 d*x]/(a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.780175, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2802, 3056, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}-\frac{3b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\right.}{a^2d\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a +
 b)])/(a^2*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)])) + (Sqrt[(a +
b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(a*d*Sqrt[a
 + b*Cos[c + d*x]]) - (3*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2,
 (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(a^2 -
3*b^2)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + Tan[c +
 d*x]/(a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
```

```
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] && !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{3b}{2}+\frac{1}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{a}$$

$$= \frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\left(-\frac{3}{4}b\left(a^2-b^2\right)+\frac{1}{2}ab^2\cos(c+dx)\cdots}{\sqrt{a+\cdots}}}{a^2\left(\cdots\right.}$$

$$= \frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\left(\frac{3}{4}b^2\left(a^2-b^2\right)-\frac{1}{4}ab\left(a^2-b^2\right)\cos\cdots}{\sqrt{a+b\cos(c+d\cdots}}}{a^2b\left(a^2-b^2\cdots\right.}$$

$$= \frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c+dx)}{ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{a+b\cos(c+dx)}}\,dx}{2a} - \frac{(3b)\int\cdots}{\cdots}$$

$$= -\frac{\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b\left(a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \cdots$$

$$= -\frac{\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\cos(c+dx)}} - \cdots 3\cdots$$

Mathematica [C] time = 4.00339, size = 441, normalized size = 1.59

$$\frac{\frac{4\tan(c+dx)\left(b\left(a^2-3b^2\right)\cos(c+dx)+a^3-ab^2\right)}{\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}} - \frac{b\left(\frac{2\left(7a^2-9b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} - \frac{2i\left(a^2-3b^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}\left(2a(a-b)E\left(i\cdots\right.\right.}{\cdots}\cdots\right.}{\cdots}}{4\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-((b*((-8*a*b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(7*a^2 - 9*b^2)*Sqrt[(a + b*Cos
[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*C
os[c + d*x]] - ((2*I)*(a^2 - 3*b^2)*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))
]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c + d*x]*(2*a*(a - b)*Ellipti
cE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)
] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]
], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/(a*b^2*Sqrt[-(a + b)^(-1)])
))/((a - b)*(a + b))) + (4*(a^3 - a*b^2 + b*(a^2 - 3*b^2)*Cos[c + d*x])*Tan
```

```
[c + d*x])/((a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]))/(4*a^2*d)
```

Maple [B] time = 8.279, size = 894, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/a^2*b^2/s
in(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2/a^2*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))+2/a*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1
/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(
-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos
(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((
2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)
*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(
1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(3/2), x)
```

3.537 $\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=345

$$-\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Gamma}{4a^3d\sqrt{a+b\cos(}}$$

```
[Out] (b*(7*a^2 - 15*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (5*b*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*
a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*
Cos[c + d*x]]) - (b^2*(7*a^2 - 15*b^2)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]]) - (5*b*Tan[c + d*x])/(4*a^2*d*Sqrt[a + b*Cos[c + d
*x]]) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.08172, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$-\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3d\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\left(4a^2+15b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Gamma}{4a^3d\sqrt{a+b\cos(}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (b*(7*a^2 - 15*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(4*a^3*(a^2 - b^2)*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (5*b*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(4*
a^2*d*Sqrt[a + b*Cos[c + d*x]]) + ((4*a^2 + 15*b^2)*Sqrt[(a + b*Cos[c + d*x
])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(4*a^3*d*Sqrt[a + b*
Cos[c + d*x]]) - (b^2*(7*a^2 - 15*b^2)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)*d*S
qrt[a + b*Cos[c + d*x]]) - (5*b*Tan[c + d*x])/(4*a^2*d*Sqrt[a + b*Cos[c + d
*x]]) + (Sec[c + d*x]*Tan[c + d*x])/(2*a*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(-\frac{5b}{2}+a\cos(c+dx)+\frac{3}{2}b\cos^2(c+dx)\right)\sec^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{2a}$$

$$= -\frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\left(\frac{1}{4}\left(4a^2+15b^2\right)+\frac{3}{2}ab\cos(c+dx)-\frac{5}{4}b^2\cos^2(c\cdot}{(a+b\cos(c+dx))^{3/2}}}{2a^2}$$

$$= -\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} +$$

$$= -\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} -$$

$$= -\frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{5b\tan(c+dx)}{4a^2d\sqrt{a+b\cos(c+dx)}} + \frac{\sec(c+dx)\tan(c+dx)}{2ad\sqrt{a+b\cos(c+dx)}} -$$

$$= \frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{b^2\left(7a^2-15b^2\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} -$$

$$= \frac{b\left(7a^2-15b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^3\left(a^2-b^2\right)d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{5b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\cos(c+dx)}} +$$

Mathematica [C] time = 6.4756, size = 597, normalized size = 1.73

$$\frac{\sqrt{a+b\cos(c+dx)}\left(\frac{2b^4\sin(c+dx)}{a^3\left(a^2-b^2\right)(a+b\cos(c+dx))} - \frac{7b\tan(c+dx)}{4a^3} + \frac{\tan(c+dx)\sec(c+dx)}{2a^2}\right)}{d} - \frac{\frac{2\left(4a^3b-20ab^3\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2(29a^2}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -((2*(4*a^3*b - 20*a*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c +
 d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(8*a^4 + 29*a^2*b^2
- 45*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(7*a^2*b^2 - 15*b^4)*Sqrt[(b
```

```
 - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a - b))]*Cos[2*(c
+ d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos
[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I
*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)]))*
Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[-((a^2 -
 b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^2)]*(2*a^2 - b^
2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2)))/(16*a^3*(-a + b)
*(a + b)*d) + (Sqrt[a + b*Cos[c + d*x]]*((2*b^4*Sin[c + d*x])/(a^3*(a^2 - b
^2)*(a + b*Cos[c + d*x])) - (7*b*Tan[c + d*x])/(4*a^3) + (Sec[c + d*x]*Tan[
c + d*x])/(2*a^2)))/d
```

Maple [B] time = 10.874, size = 1542, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3*b^3/
sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/
2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*si
n(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a
-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+2/a*(-1/2*cos(1/
2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*cos(1/2*d*x+1/2*c)^2-1)^2+3/4/a^2*b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/8
*b/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+3/8/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(
1/2))-3/8/a^2*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a
-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/
2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-
2*b/(a-b))^(1/2))-3/8/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/
2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))*b^2)-2/a^3*b
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2
)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2,(-2*b/(a-b))^(1/2))-2/a^2*b*(-cos(1/2*d*x+1/2*c)/a*(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2
*c)^2-1)+1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^
4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))+1/2/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b
)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)
*b*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*
x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),
```

```
2,(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c) + a)^(3/2), x)
```

3.538 $\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=436

$$-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{8a^2\left(2a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-71a^2b^2+48a^4+3b^4\right)\sin(c+d}{15b^3d\left(a^2\right.}$$

```
[Out] (2*(128*a^6 - 212*a^4*b^2 + 55*a^2*b^4 + 9*b^6)*Sqrt[a + b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - (2*a*(128*a^4 - 116*a^2*b^2 - 17*b^4)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (8*a^2*(2*a^2 - 3*b^2)*Cos[c
+ d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
4*a*(32*a^4 - 49*a^2*b^2 + 7*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1
5*b^4*(a^2 - b^2)^2*d) + (2*(48*a^4 - 71*a^2*b^2 + 3*b^4)*Cos[c + d*x]*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.862177, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2792, 3047, 3049, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\cos^3(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{8a^2\left(2a^2-3b^2\right)\sin(c+dx)\cos^2(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(-71a^2b^2+48a^4+3b^4\right)\sin(c+d}{15b^3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(128*a^6 - 212*a^4*b^2 + 55*a^2*b^4 + 9*b^6)*Sqrt[a + b*Cos[c + d*x]]*El
lipticE[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 - b^2)^2*d*Sqrt[(a + b*Co
s[c + d*x])/(a + b)]) - (2*a*(128*a^4 - 116*a^2*b^2 - 17*b^4)*Sqrt[(a + b*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15*b^5*(a^2 -
 b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(3*
b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (8*a^2*(2*a^2 - 3*b^2)*Cos[c
+ d*x]^2*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (
4*a*(32*a^4 - 49*a^2*b^2 + 7*b^4)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(1
5*b^4*(a^2 - b^2)^2*d) + (2*(48*a^4 - 71*a^2*b^2 + 3*b^4)*Cos[c + d*x]*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*b^3*(a^2 - b^2)^2*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)\left(3a^2-\frac{3}{2}ab\cos(c+dx)-\frac{1}{2}\left(8a^2-3b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + 4\int$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + 2(48$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - 4a(3$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - 4a(3$$

$$= -\frac{2a^2\cos^3(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(2a^2-3b^2\right)\cos^2(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - 4a(3$$

$$= \frac{2\left(128a^6-212a^4b^2+55a^2b^4+9b^6\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15b^5\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - 2a\left(128a^4-\right.$$

Mathematica [A] time = 1.90669, size = 272, normalized size = 0.62

$$\frac{b\left(\frac{10a^5\sin(c+dx)}{a^2-b^2} - \frac{10a^4\left(11a^2-15b^2\right)\sin(c+dx)(a+b\cos(c+dx))}{\left(a^2-b^2\right)^2} - 28a\sin(c+dx)(a+b\cos(c+dx))^2 + 3b\sin(2(c+dx))(a+b\cos\right.}{15b^5d(a+b\,c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*((128*a^6 - 212*a^4*b^2 + 55*a^2*b
^4 + 9*b^6)*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-128*a^5 + 128*a^4*b
 + 116*a^3*b^2 - 116*a^2*b^3 + 17*a*b^4 - 17*b^5)*EllipticF[(c + d*x)/2, (2
*b)/(a + b)]))/(a - b)^2 + b*((10*a^5*Sin[c + d*x])/(a^2 - b^2) - (10*a^4*(
11*a^2 - 15*b^2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 28*a*(a
 + b*Cos[c + d*x])^2*Sin[c + d*x] + 3*b*(a + b*Cos[c + d*x])^2*Sin[2*(c + d
*x)]))/(15*b^5*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 17.893, size = 1684, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16/b^2*(-1/
10/b*cos(1/2*d*x+1/2*c)^3*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)-1/60/b^2*(-4*a+12*b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+1/60/b^2*(-4*a+12*b)*(a-b)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1
/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),(-2*b/(a-b))^(1/2))-1/60*(4*a^2-15*a*b+27*b^2)/b^3*(a-b)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d
*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-8/b
^3*(2*a+3*b)*(-1/6/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/1
2/b^2*(-2*a+6*b)*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c
)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2
*d*x+1/2*c),(-2*b/(a-b))^(1/2))))-2/b^5*(3*a^2+4*a*b+3*b^2)*(a-b)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))
-2*(4*a^3+3*a^2*b+2*a*b^2+b^3)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1
/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+10/b^5
*a^4/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2/b^5*a^5*(
1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d
*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)
^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^5}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^5}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^5/(b^3*cos(d*x + c)^3 + 3*a*
b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^5}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c) + a)^(5/2), x)
```

3.539 $\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=345

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}\cdot$$

```
[Out] (-8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]) + (2*(16*a^4 - 16*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[
c + d*x]]) - (2*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^(3/2)) + (4*a^3*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3*b^3*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.570046, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2792, 3031, 3023, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)\cos^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3b^3d\left(a^2-b^2\right)}\cdot$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-8*a*(4*a^4 - 7*a^2*b^2 + 2*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a
+ b)]) + (2*(16*a^4 - 16*a^2*b^2 - b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*
EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b^4*(a^2 - b^2)*d*Sqrt[a + b*Cos[
c + d*x]]) - (2*a^2*Cos[c + d*x]^2*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^(3/2)) + (4*a^3*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3*b^3*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\cos(c+dx)\left(2a^2-\frac{3}{2}ab\cos(c+dx)-\frac{3}{2}\left(2a^2-b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{4\int \frac{\frac{1}{2}a^2b\left(3a^2-5\right.}{}}{}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(2a^2-b^2\right)}{}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(2a^2-b^2\right)}{}$$

$$= -\frac{2a^2\cos^2(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a^3\left(3a^2-5b^2\right)\sin(c+dx)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(2a^2-b^2\right)}{}$$

$$= -\frac{8a\left(4a^4-7a^2b^2+2b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^4\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(16a^4-16a^2b^2-b^4\right)\sqrt{}}{3b^4\left(a^2-b^2\right)d\sqrt{}}$$

Mathematica [A] time = 1.56711, size = 237, normalized size = 0.69

$$\frac{2\left(\frac{b\sin(c+dx)\left(4ab\left(-8a^2b^2+5a^4+b^4\right)\cos(c+dx)+\left(b^3-a^2b\right)^2\cos(2(c+dx))-25a^4b^2+16a^6+b^6\right)}{2\left(a^2-b^2\right)^2}+\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(-16a^3b^2+16a^2b^3-16a^4b+16a^5-ab^4+b^5\right)F\left(\right.\right.}{(a-b)^2}\right.}{3b^4d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(-4*(4*a^5 - 7*a^3*b^2 + 2*a*b^4)
*EllipticE[(c + d*x)/2, (2*b)/(a + b)] + (16*a^5 - 16*a^4*b - 16*a^3*b^2 +
16*a^2*b^3 - a*b^4 + b^5)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2
 + (b*(16*a^6 - 25*a^4*b^2 + b^6 + 4*a*b*(5*a^4 - 8*a^2*b^2 + b^4)*Cos[c +
d*x] + (-(a^2*b) + b^3)^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)
))/(3*b^4*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 13.386, size = 1291, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8/b^2*(-1/6
/b*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2
)^(1/2)+1/6*(a-b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2
+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/12/b^2*(-2*a+6*b)*(
```

```
a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1
/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/
(a-b))^(1/2))))+4*(a+b)/b^4*(a-b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/
2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d
*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ellipt
icE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))+2*(3*a^2+2*a*b+b^2)/b^4*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*si
n(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),(-2*b/(a-b))^(1/2))-8/b^4*a^3/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)+2/b^4*a^4*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2
*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)
/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b
)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^4}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^4/(b^3*cos(d*x + c)^3 + 3*a*
b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^4}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c) + a)^(5/2), x)
```

3.540 $\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{8a^2(a^2-2b^2)\sin(c+dx)}{3b^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}-\frac{2a(8a^2-9b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right)}{3b^3d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]) - (2*a*(8*a^2 - 9*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) -
 (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)) - (8*a^2*(a^2 - 2*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a +
 b*Cos[c + d*x]])
```

Rubi [A] time = 0.378858, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3021, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8a^2(a^2-2b^2)\sin(c+dx)}{3b^2d(a^2-b^2)^2\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{3bd(a^2-b^2)(a+b\cos(c+dx))^{3/2}}-\frac{2a(8a^2-9b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right)}{3b^3d(a^2-b^2)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(8*a^4 - 15*a^2*b^2 + 3*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]) - (2*a*(8*a^2 - 9*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*b)/(a + b)])/(3*b^3*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) -
 (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)) - (8*a^2*(a^2 - 2*b^2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a +
 b*Cos[c + d*x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
```

```
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{a^2-\frac{3}{2}ab\cos(c+dx)-\frac{1}{2}\left(4a^2-3b^2\right)\cos^2(c+dx)}{(a+b\cos(c+dx))^{3/2}}dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{1}{2}ab(a^2-$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{\left(a\left(8a^2-9\right.\right.}{}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8a^2\left(a^2-2b^2\right)\sin(c+dx)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\left(8a^4-15\right.\right.}{}$$

$$= \frac{2\left(8a^4-15a^2b^2+3b^4\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2a\left(8a^2-9b^2\right)\sqrt{\frac{a+b\cos(c}{a+b}}}{3b^3\left(a^2-b^2\right)d\sqrt{a}}$$

Mathematica [A] time = 1.22305, size = 188, normalized size = 0.67

$$\frac{2\left(\frac{a^2b\sin(c+dx)\left(\left(9b^3-5a^2b\right)\cos(c+dx)-4a^3+8ab^2\right)}{\left(a^2-b^2\right)^2} + \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(a\left(8a^2b-8a^3+9ab^2-9b^3\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+\left(-15a^2b^2+8a^4+3b^4\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{(a-b)^2}\right)}{3b^3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((8*a^4 - 15*a^2*b^2 + 3*b^4)*Ell
ipticE[(c + d*x)/2, (2*b)/(a + b)] + a*(-8*a^3 + 8*a^2*b + 9*a*b^2 - 9*b^3)
*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2 + (a^2*b*(-4*a^3 + 8*a*b
^2 + (-5*a^2*b + 9*b^3)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^3*
d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 12.308, size = 907, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*
b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*(3*EllipticF
(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-EllipticE(cos(1/2*d*x+1/2*c),(-2*
b/(a-b))^(1/2))*a+EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b)+6/b^3
*a^2/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*s
in(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-
```

```
b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2/b^3*a^3*(
1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d
*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)
^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c
)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(
a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*
x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^3}{b^3\cos(dx+c)^3+3ab^2\cos(dx+c)^2+3a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^3/(b^3*cos(d*x + c)^3 + 3*a*
b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c) + a)^(5/2), x)
```

3.541 $\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=263

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] (-4*a*(a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2
*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Sin[c
+ d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (4*a*(a^2 - 3*b^2)
*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.336677, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2790, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-4*a*(a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*(2
*a^2 - 3*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*
b)/(a + b)])/(3*b^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (2*a^2*Sin[c
+ d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (4*a*(a^2 - 3*b^2)
*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] - Dist[1/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(2
*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1) +
 c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3ab}{2}+\frac{1}{2}\left(2a^2-3b^2\right)\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}b\left(a^2+\right.}{}}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2a\left(a^2-3\right.\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4a\left(a^2-3b^2\right)\sin(c+dx)}{3b\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{\left(2a\left(a^2-3\right.\right.}{}$$

$$= -\frac{4a\left(a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2\left(2a^2-3b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}\right.}{3b^2\left(a^2-b^2\right)d\sqrt{a+b\cos(\right.}}$$

Mathematica [A] time = 1.08111, size = 175, normalized size = 0.67

$$\frac{2\left(\frac{ab\sin(c+dx)\left(2b\left(a^2-3b^2\right)\cos(c+dx)+a^3-5ab^2\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(2a^2b-2a^3+3ab^2-3b^3\right)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+2\left(a^3-3ab^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{(a-b)^2}\right)}{3b^2d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*(2*(a^3 - 3*a*b^2)*EllipticE[(c
 + d*x)/2, (2*b)/(a + b)] + (-2*a^3 + 2*a^2*b + 3*a*b^2 - 3*b^3)*EllipticF[
(c + d*x)/2, (2*b)/(a + b)]))/(a - b)^2) + (a*b*(a^3 - 5*a*b^2 + 2*b*(a^2 -
 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*b^2*d*(a + b*Cos[c +
 d*x])^(3/2))
```

Maple [B] time = 10.696, size = 846, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))-4/b^2*a/sin(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1
/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c
)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(
a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)+2*a^2/b^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2
*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)
/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b
)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^2}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^2/(b^3*cos(d*x + c)^3 + 3*a*
b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

3.542 $\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=243

$$\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2+3b^2\right)}{3\ldots}$$

```
[Out] (-2*(a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*
(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2 + 3*b^2)*Sin[c + d*x])/(3*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.271597, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2754, 2752, 2663, 2661, 2655, 2653}

$$\frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\left(a^2+3b^2\right)}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-2*(a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a
+ b)])/(3*b*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*a*Sqrt
[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*b*
(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)
*d*(a + b*Cos[c + d*x])^(3/2)) + (2*(a^2 + 3*b^2)*Sin[c + d*x])/(3*(a^2 - b
^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
```

```
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{3b}{2}-\frac{1}{2}a\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{-ab-\frac{1}{4}\left(a^2+3b\right.}{\sqrt{a+b\cos}}}{3\left(a^2-\right.}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{a\int \frac{1}{\sqrt{a+b\cos(c+d}}}{3b\left(a^2-b^2\right.}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(a^2+3b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{\left(\left(a^2+3b^2\right)\sqrt{a}\right.}{3}$$

$$= -\frac{2\left(a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 0.944148, size = 154, normalized size = 0.63

$$\frac{2\left(\frac{\sin(c+dx)\left(b\left(a^2+3b^2\right)\cos(c+dx)+2a\left(a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2} - \frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}\left(\left(a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+a(b-a)F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)\right)}{b(a-b)^2}\right)}{3d(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-((((a + b*Cos[c + d*x])/(a + b))^(3/2)*((a^2 + 3*b^2)*EllipticE[(c + d
*x)/2, (2*b)/(a + b)] + a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]))/
((a - b)^2*b)) + ((2*a*(a^2 + b^2) + b*(a^2 + 3*b^2)*Cos[c + d*x])*Sin[c +
```

```
d*x])/(a^2 - b^2)^2))/(3*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 10.819, size = 742, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b/sin(1/2
*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*d*x+1
/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(1/2*d
*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2*a/b*(1/6/b/(a-b)/(a+b
)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b
)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*
d*x+1/2*c)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2
*b/(a-b))^(1/2))-4/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos
(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ell
ipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin
(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)}{b^3\cos(dx+c)^3+3ab^2\cos(dx+c)^2+3a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)/(b^3*cos(d*x + c)^3 + 3*a*b^
2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.543 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{8a\sqrt{a+b\,\mathrm{co}}}{3d\left(a^2\right.}$$

```
[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^
2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^2 - b^2)*d*Sqrt[
a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rubi [A] time = 0.231784, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{8a\sqrt{a+b\,\mathrm{co}}}{3d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (8*a*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^
2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (2*Sqrt[(a + b*Cos[c + d
*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(3*(a^2 - b^2)*d*Sqrt[
a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*
x]])
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}\left(3a^2+b^2\right)+}{\sqrt{a+b\,\mathrm{co}}}}{3\left(a^2-\right.}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{(4a)\int\sqrt{a+b}}{3\left(a^2\right.}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left(4a\sqrt{a+b\,\mathrm{co}}\right.}{3\left(\right.}$$

$$= \frac{8a\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{2\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{}{3\left(a^2-b^2\right.}$$

Mathematica [A] time = 0.902029, size = 158, normalized size = 0.71

$$\frac{2b\sin(c+dx)\left(-5a^2-4ab\cos(c+dx)+b^2\right)-2(a-b)(a+b)^2\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{3/2}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+8a(a+b)^2\left(\frac{a+b\cos}{a+}\right.}{3d(a-b)^2(a+b)^2(a+b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (8*a*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*EllipticE[(c + d*x)/2,
(2*b)/(a + b)] - 2*(a - b)*(a + b)^2*((a + b*Cos[c + d*x])/(a + b))^(3/2)*E
llipticF[(c + d*x)/2, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cos[c + d*
x])*Sin[c + d*x])/(3*(a - b)^2*(a + b)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 6.918, size = 489, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/3/b/(a-b)
/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*
c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+16/3*b*sin(1/2*d*x+1/2*c)^
2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*si
n(1/2*d*x+1/2*c)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),(-2*b/(a-b))^(1/2))-8/3*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
(2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1
/2))-EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/
(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/(b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c
)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-5/2), x)
```

3.544 $\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=320

$$\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-2b\left(7a^2-\right.$$

```
[Out] (-2*b*(7*a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos
[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)) + (2*b^2*(7*a^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Rubi [A] time = 0.87123, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2802, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-2b\left(7a^2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*b*(7*a^2 - 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)
/(a + b)])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) + (2*
b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])
/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) + (2*Sqrt[(a + b*Cos[c + d*x]
)/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a + b)])/(a^2*d*Sqrt[a + b*Cos
[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)) + (2*b^2*(7*a^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a
 + b*Cos[c + d*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(\frac{3}{2}\left(a^2-b^2\right)-\frac{3}{2}ab\cos(c+dx)+\frac{1}{2}b^2\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{4\int \frac{\left(\frac{3}{4}\left(a^2-b^2\right)^2-\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{4\int \frac{\left(-\frac{3}{4}b\left(a^2-b^2\right)^2\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2b^2\left(7a^2-3b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{\sec(c+dx)}{\sqrt{a+b\cos(c+dx)}}\ldots}{a^2}$$

$$= -\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}}$$

$$= -\frac{2b\left(7a^2-3b^2\right)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{2b\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [C] time = 4.41959, size = 464, normalized size = 1.45

$$\frac{\frac{4b^2\sin(c+dx)\left(b\left(7a^2-3b^2\right)\cos(c+dx)+8a^3-4ab^2\right)}{\left(a^3-ab^2\right)^2(a+b\cos(c+dx))^{3/2}} + \frac{-\frac{8ab\left(3a^2-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2\left(-19a^2b^2+6a^4+9b^4\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{\sqrt{a+b\cos(c+dx)}} + \frac{2i\left(3b^2-7a^2\right)\ldots}{\ldots}}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (((-8*a*b*(3*a^2 - b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(6*a^4 - 19*a^2*b^2 +
9*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/
(a + b)])/Sqrt[a + b*Cos[c + d*x]] + ((2*I)*(-7*a^2 + 3*b^2)*Sqrt[-((b*(-1
+ Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c +
d*x]*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c
```

```
 + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*
ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)])))/
(a*Sqrt[-(a + b)^(-1)]))/(a^2*(a - b)^2*(a + b)^2) + (4*b^2*(8*a^3 - 4*a*b^
2 + b*(7*a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/((a^3 - a*b^2)^2*(a + b*C
os[c + d*x])^(3/2)))/(6*d)
```

Maple [B] time = 10.338, size = 845, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^2*b/si
n(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2*
d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin(
1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b
))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)-2/a^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2,(-2*b/(a-b))^(1/2))-2/a*b*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*
sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*d*x+1/2
*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+(3*a-b)
/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d
*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3*a/(a-b
)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b
))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(Elli
pticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),(
-2*b/(a-b))^(1/2)))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.545 $\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=380

$$\frac{b\left(-26a^2b^2+3a^4+15b^4\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2-5b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2-5b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

```
[Out] -((3*a^4 - 26*a^2*b^2 + 15*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]) + ((3*a^2 - 5*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (5*
b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^3*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3
*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4 - 26*a^2*b^2 + 1
5*b^4)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + Tan
[c + d*x]/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Rubi [A] time = 1.0999, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2802, 3056, 3055, 3059, 2655, 2653, 3002, 2663, 2661, 2807, 2805}

$$\frac{b\left(-26a^2b^2+3a^4+15b^4\right)\sin(c+dx)}{3a^3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{b\left(3a^2-5b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{\left(3a^2-5b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\right|}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] -((3*a^4 - 26*a^2*b^2 + 15*b^4)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x
)/2, (2*b)/(a + b)])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(a + b*Cos[c + d*x])/(a +
b)]) + ((3*a^2 - 5*b^2)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d
*x)/2, (2*b)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]) - (5*
b*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*b)/(a +
b)])/(a^3*d*Sqrt[a + b*Cos[c + d*x]]) + (b*(3*a^2 - 5*b^2)*Sin[c + d*x])/(3
*a^2*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (b*(3*a^4 - 26*a^2*b^2 + 1
5*b^4)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + Tan
[c + d*x]/(a*d*(a + b*Cos[c + d*x])^(3/2))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\left(-\frac{5b}{2}+\frac{3}{2}b\cos^2(c+dx)\right)\sec(c+dx)}{(a+b\cos(c+dx))^{5/2}}dx}{a}$$

$$= \frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{\tan(c+dx)}{ad(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\left(-\frac{15}{4}b(a^2-b^2)+\frac{3}{2}ab^2\cos(c\cdots\right.}{(a\cdots}}{3\cdots}$$

$$= \frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{b(3a^4-26a^2b^2+15b^4)\sin(c+dx)}{3a^3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c\cdots}{ad(a+b\cos\cdots}$$

$$= \frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{b(3a^4-26a^2b^2+15b^4)\sin(c+dx)}{3a^3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c\cdots}{ad(a+b\cos\cdots}$$

$$= \frac{b(3a^2-5b^2)\sin(c+dx)}{3a^2(a^2-b^2)d(a+b\cos(c+dx))^{3/2}} + \frac{b(3a^4-26a^2b^2+15b^4)\sin(c+dx)}{3a^3(a^2-b^2)^2d\sqrt{a+b\cos(c+dx)}} + \frac{\tan(c\cdots}{ad(a+b\cos\cdots}$$

$$= -\frac{(3a^4-26a^2b^2+15b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{b(3a^2-5b^2)\sin(c+\cdots}{3a^2(a^2-b^2)d(a+b\cos(c\cdots}$$

$$= -\frac{(3a^4-26a^2b^2+15b^4)\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{3a^3(a^2-b^2)^2d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{(3a^2-5b^2)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\cdots}{3a^2(a^2-b^2)d\sqrt{a+b\cdots}}$$

Mathematica [C] time = 6.52516, size = 638, normalized size = 1.68

$$\frac{\sqrt{a+b\cos(c+dx)}\left(-\frac{2b^3\sin(c+dx)}{3a^2(a^2-b^2)(a+b\cos(c+dx))^2}-\frac{4(5a^2b^3\sin(c+dx)-3b^5\sin(c+dx))}{3a^3(a^2-b^2)^2(a+b\cos(c+dx))}+\frac{\tan(c+dx)}{a^3}\right)}{d} - \frac{b\left(\frac{2(20ab^3-36a^3b)\sqrt{\frac{a+b\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+\cdots\right.}{\sqrt{a+b\cos(c+dx)}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] -(b*((2*(-36*a^3*b + 20*a*b^3)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] + (2*(33*a^4 - 86*a^
2*b^2 + 45*b^4)*Sqrt[(a + b*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/
2, (2*b)/(a + b)])/Sqrt[a + b*Cos[c + d*x]] - ((2*I)*(3*a^4 - 26*a^2*b^2 +
15*b^4)*Sqrt[(b - b*Cos[c + d*x])/(a + b)]*Sqrt[-((b + b*Cos[c + d*x])/(a -
 b))]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[S
qrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a + b)/(a - b)] - b*Elliptic
Pi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Cos[c + d*x]]], (a +
 b)/(a - b)]))*Sin[c + d*x])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Cos[c + d*x]^2
]*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Cos[c + d*x]) + (a + b*Cos[c + d*x])^2)/b^
2)]*(2*a^2 - b^2 - 4*a*(a + b*Cos[c + d*x]) + 2*(a + b*Cos[c + d*x])^2))))/
(12*a^3*(-a + b)^2*(a + b)^2*d) + (Sqrt[a + b*Cos[c + d*x]]*((-2*b^3*Sin[c
+ d*x])/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (4*(5*a^2*b^3*Sin[c +
d*x] - 3*b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + Ta
n[c + d*x]/a^3))/d
```

Maple [B] time = 14.145, size = 1320, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/a^3*b^2/s
in(1/2*d*x+1/2*c)^2/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)/(a^2-b^2)*(-2*b*sin(1/2
*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*b/(a-b)*sin(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),(-2*b/(a-b))^(1/2))*a-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b/(a-b)*sin
(1/2*d*x+1/2*c)^2+(a+b)/(a-b))^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-
b))^(1/2))*b+2*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)+4/a^3*b*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(
1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2,(-2*b/(a-b))^(1/2))+2/a^2*b^2*(1/6/b/(a-b)/(a+b)*cos(1/2*d*x+1/2*c)
*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+
1/2*c)^2+1/2*(a-b)/b)^2+8/3*b*sin(1/2*d*x+1/2*c)^2/(a-b)^2/(a+b)^2*cos(1/2*
d*x+1/2*c)*a/(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+
(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-4/3
*a/(a-b)/(a+b)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-
b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2
)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-EllipticE(cos(1/2*d*x+1
/2*c),(-2*b/(a-b))^(1/2))))+2/a^2*(-cos(1/2*d*x+1/2*c)/a*(-2*b*sin(1/2*d*x+
1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-
2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-1/2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos
(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/
a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)
/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))+1/2/a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+
b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,(-2*b/(a-b))
^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c) + a)^(5/2), x)
```

3.546 $\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=282

$$-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{15d\left(a^2-b^2\right)^3\sqrt{a+b\cos(c+dx)}}-\frac{16ab\sin(c+dx)}{15d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^{3/2}}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}-\frac{16a\cdots}{15d\cdots}$$

```
[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*(a^2 - b^2)^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15
*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(5*(a^2 - b
^2)*d*(a + b*Cos[c + d*x])^(5/2)) - (16*a*b*Sin[c + d*x])/(15*(a^2 - b^2)^2
*d*(a + b*Cos[c + d*x])^(3/2)) - (2*b*(23*a^2 + 9*b^2)*Sin[c + d*x])/(15*(a
^2 - b^2)^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.357829, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2664, 2754, 2752, 2663, 2661, 2655, 2653}

$$-\frac{2b\left(23a^2+9b^2\right)\sin(c+dx)}{15d\left(a^2-b^2\right)^3\sqrt{a+b\cos(c+dx)}}-\frac{16ab\sin(c+dx)}{15d\left(a^2-b^2\right)^2(a+b\cos(c+dx))^{3/2}}-\frac{2b\sin(c+dx)}{5d\left(a^2-b^2\right)(a+b\cos(c+dx))^{5/2}}-\frac{16a\cdots}{15d\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(-7/2),x]
```

```
[Out] (2*(23*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(
a + b)])/(15*(a^2 - b^2)^3*d*Sqrt[(a + b*Cos[c + d*x])/(a + b)]) - (16*a*Sq
rt[(a + b*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*b)/(a + b)])/(15
*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) - (2*b*Sin[c + d*x])/(5*(a^2 - b
^2)*d*(a + b*Cos[c + d*x])^(5/2)) - (16*a*b*Sin[c + d*x])/(15*(a^2 - b^2)^2
*d*(a + b*Cos[c + d*x])^(3/2)) - (2*b*(23*a^2 + 9*b^2)*Sin[c + d*x])/(15*(a
^2 - b^2)^3*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2663

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{7/2}}\,dx = -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{2\int \frac{-\frac{5a}{2}+\frac{3}{2}b\cos(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx}{5\left(a^2-b^2\right)}$$

$$= -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} + \frac{4\int \frac{\frac{3}{4}\left(5a^2+3b^2\right)}{(a+b\,\mathrm{co}}}{15\left(a^2\right.}$$

$$= -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(23a^2+\right.}{15\left(a^2-b^2\right)^3}$$

$$= -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(23a^2+\right.}{15\left(a^2-b^2\right)^3}$$

$$= -\frac{2b\sin(c+dx)}{5\left(a^2-b^2\right)d(a+b\cos(c+dx))^{5/2}} - \frac{16ab\sin(c+dx)}{15\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))^{3/2}} - \frac{2b\left(23a^2+\right.}{15\left(a^2-b^2\right)^3}$$

$$= \frac{2\left(23a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^3 d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}} - \frac{16a\sqrt{\frac{a+b\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

Mathematica [A] time = 1.28339, size = 189, normalized size = 0.67

$$\frac{2\left(\frac{b\sin(c+dx)\left(b^2\left(23a^2+9b^2\right)\cos^2(c+dx)+2ab\left(27a^2+5b^2\right)\cos(c+dx)-5a^2b^2+34a^4+3b^4\right)}{\left(b^2-a^2\right)^3}+\frac{\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{5/2}\left(\left(23a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)+8a(b-a)F\left(\frac{1}{2}(c+\right.\right.}{(a-b)^3}}{15d(a+b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(-7/2),x]
```

```
[Out] (2*((((a + b*Cos[c + d*x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(c +
d*x)/2, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(c + d*x)/2, (2*b)/(a + b)]
))/(a - b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Cos[
c + d*x] + b^2*(23*a^2 + 9*b^2)*Cos[c + d*x]^2)*Sin[c + d*x])/(-a^2 + b^2)^
3))/(15*d*(a + b*Cos[c + d*x])^(5/2))
```

Maple [A] time = 8.799, size = 616, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(7/2),x)
```

```
[Out] -(-(-2*b*cos(1/2*d*x+1/2*c)^2-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1/10/b^2/(a
-b)/(a+b)*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1
/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^3+8/15*a/b/(a-b)^2/(a+b)^
2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*x+1/2*c)^2)
^(1/2)/(cos(1/2*d*x+1/2*c)^2+1/2*(a-b)/b)^2+4/15*b*sin(1/2*d*x+1/2*c)^2/(a-
b)^3/(a+b)^3*cos(1/2*d*x+1/2*c)*(23*a^2+9*b^2)/(-(-2*b*cos(1/2*d*x+1/2*c)^2
-a+b)*sin(1/2*d*x+1/2*c)^2)^(1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*a^4*b-3
0*a^3*b^2-30*a^2*b^3+15*a*b^4+15*b^5)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*co
s(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-2/1
5*(23*a^2+9*b^2)/(a-b)^2/(a+b)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2
*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b)*sin(1/2*d*
x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))-Ellipti
cE(cos(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))))/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*
d*x+1/2*c)^2*b+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{b^4\cos(dx+c)^4+4\,ab^3\cos(dx+c)^3+6\,a^2b^2\cos(dx+c)^2+4\,a^3b\cos(dx+c)+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/(b^4*cos(d*x + c)^4 + 4*a*b^3*cos(d*x + c
)^3 + 6*a^2*b^2*cos(d*x + c)^2 + 4*a^3*b*cos(d*x + c) + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(-7/2), x)
```

3.547 $\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=111

$$-\frac{23F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}d}+\frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)}}{10d}$$

```
[Out] (9*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(20*d) - (23*EllipticF[(c + d*x)/2,
 8/7])/(20*Sqrt[7]*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d) + (C
os[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d)
```

Rubi [A] time = 0.149713, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2793, 3023, 2752, 2661, 2653}

$$-\frac{23F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}d}+\frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d}+\frac{\sin(c+dx)\cos(c+dx)\sqrt{4\cos(c+dx)+3}}{10d}-\frac{\sin(c+dx)\sqrt{4\cos(c+dx)}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (9*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(20*d) - (23*EllipticF[(c + d*x)/2,
 8/7])/(20*Sqrt[7]*d) - (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d) + (C
os[c + d*x]*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(10*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{1}{10}\int \frac{3+6\cos(c+dx)-6\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{1}{60}\int \frac{6+5}{\sqrt{3+}}$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{\cos(c+dx)\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{9}{40}\int \sqrt{3+}$$

$$= \frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20d} - \frac{23F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{20\sqrt{7}d} - \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{10d} + \frac{\cos(c+dx)\sqrt{3}}{}$$

Mathematica [A] time = 0.170112, size = 81, normalized size = 0.73

$$\frac{-23\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+63\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+7(\sin(2(c+dx))-2\sin(c+dx))\sqrt{4\cos(c+dx)+3}}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (63*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] - 23*Sqrt[7]*EllipticF[(c + d*x)/2,
 8/7] + 7*Sqrt[3 + 4*Cos[c + d*x]]*(-2*Sin[c + d*x] + Sin[2*(c + d*x)]))/(1
40*d)
```

Maple [A] time = 2.559, size = 231, normalized size = 2.1

$$-\frac{1}{20\,d}\sqrt{\left(8\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-64\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+56\,(\sin(1/2\,dx+c/2)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/20*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-64*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+56*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-
23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2*2^(1/2))-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x
+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x
```

```
+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1
/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^3}{\sqrt{4\cos(dx+c)+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

3.548 $\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{17F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{12\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{4d}+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(4*d) + (17*EllipticF[(c + d*x)/2, 8
/7])/(12*Sqrt[7]*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

Rubi [A] time = 0.101715, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2791, 2752, 2661, 2653}

$$\frac{17F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{12\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{4d}+\frac{\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(4*d) + (17*EllipticF[(c + d*x)/2, 8
/7])/(12*Sqrt[7]*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(6*d)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx &= \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d}+\frac{1}{6}\int \frac{2-3\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx \\ &= \frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d}-\frac{1}{8}\int \sqrt{3+4\cos(c+dx)}\,dx+\frac{17}{24}\int \frac{1}{\sqrt{3+4\cos(c+dx)}} \\ &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{4d}+\frac{17F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{12\sqrt{7}d}+\frac{\sqrt{3+4\cos(c+dx)}\sin(c+dx)}{6d}\end{aligned}$$

Mathematica [A] time = 0.0794982, size = 70, normalized size = 0.9

$$\frac{17\sqrt{7}F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)-21\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)+14\sin(c+dx)\sqrt{4\cos(c+dx)+3}}{84d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (-21*Sqrt[7]*EllipticE[(c + d*x)/2, 8/7] + 17*Sqrt[7]*EllipticF[(c + d*x)/2
, 8/7] + 14*Sqrt[3 + 4*Cos[c + d*x]]*Sin[c + d*x])/(84*d)
```

Maple [A] time = 2.297, size = 231, normalized size = 3.

$$-\frac{1}{12\,d}\sqrt{\left(8\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(32\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4\cos\left(1/2\,dx+c/2\right)+17\,\sqrt{\left(\sin\left(1/2\,dx+c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/12*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(32*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+17*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*sin(1/2*d
*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))+3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(8*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2*2^(1/2))-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-8*sin(1/2*d*x+
1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^2}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos\left(dx+c\right)^2}{\sqrt{4\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^2}{\sqrt{4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

3.549 $\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2\sqrt{7}d}$$

```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) - (3*EllipticF[(c + d*x)/2, 8/7
])/(2*Sqrt[7]*d)
```

Rubi [A] time = 0.0516001, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2752, 2661, 2653}

$$\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(2*d) - (3*EllipticF[(c + d*x)/2, 8/7
])/(2*Sqrt[7]*d)
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx &= \frac{1}{4}\int \sqrt{3+4\cos(c+dx)}\,dx-\frac{3}{4}\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx\\ &= \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2d}-\frac{3F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{2\sqrt{7}d}\end{aligned}$$

Mathematica [A] time = 0.0542586, size = 43, normalized size = 0.84

$$\frac{7E\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)-3F\left(\frac{1}{2}(c+dx)\middle|\frac{8}{7}\right)}{2\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (7*EllipticE[(c + d*x)/2, 8/7] - 3*EllipticF[(c + d*x)/2, 8/7])/(2*Sqrt[7]*
d)
```

Maple [A] time = 1.96, size = 155, normalized size = 3.

$$\frac{1}{2d}\sqrt{\left(8\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{-8\left(\cos\left(1/2\,dx+c/2\right)\right)^2+1}\left(3\,\mathrm{EllipticF}\left(\cos\left(1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/
2*c),2*2^(1/2))+EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2)))/(-8*sin(1/2*d*x+1/
2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)}{\sqrt{4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)}{\sqrt{4\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{\sqrt{4\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(4*cos(c + d*x) + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

3.550 $\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=23

$$\frac{2F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

```
[Out] (2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rubi [A] time = 0.0121487, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2661}

$$\frac{2F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{2F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Mathematica [A] time = 0.0277494, size = 23, normalized size = 1.

$$\frac{2F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Maple [C] time = 0.046, size = 23, normalized size = 1.

$$\frac{2\sqrt{7}}{7d}\text{InverseJacobiAM}\left(\frac{dx}{2}+\frac{c}{2},\frac{2\sqrt{14}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/7/d*7^(1/2)*InverseJacobiAM(1/2*d*x+1/2*c,2/7*14^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{4\cos(dx+c)+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(4*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*cos(c + d*x) + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*cos(d*x + c) + 3), x)
```

3.551 $\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{2\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

```
[Out] (2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rubi [A] time = 0.0387461, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2805}

$$\frac{2\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx=\frac{2\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Mathematica [A] time = 0.0501663, size = 24, normalized size = 1.

$$\frac{2\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticPi[2, (c + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Maple [B] time = 1.839, size = 138, normalized size = 5.8

$$2\frac{\sqrt{\left(8\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-8\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\mathrm{EllipticPi}\left(\mathrm{co}\right.}{\sqrt{-8\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+7\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{8\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2*((8*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2))/sin(1/
2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)}{\sqrt{4\,\cos\left(dx+c\right)+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(c+dx\right)}{\sqrt{4\cos\left(c+dx\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(4*cos(c + d*x) + 3), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(4*cos(d*x + c) + 3), x)
```

3.552 $\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}$$

```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) + EllipticF[(c + d*x)/2, 8/7]/
(Sqrt[7]*d) - (4*EllipticPi[2, (c + d*x)/2, 8/7])/(3*Sqrt[7]*d) + (Sqrt[3 +
 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.256121, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2802, 3060, 2653, 3002, 2661, 2805}

$$\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```


```
[Out] -(Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) + EllipticF[(c + d*x)/2, 8/7]/
(Sqrt[7]*d) - (4*EllipticPi[2, (c + d*x)/2, 8/7])/(3*Sqrt[7]*d) + (Sqrt[3 +
 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx &= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}\int \frac{\left(-2-2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx\\ &= \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d}-\frac{1}{12}\int \frac{(8-6\cos(c+dx))\sec(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx-\frac{1}{6}\int \sqrt{3+4\cos}\\ &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{1}{2}\int \frac{1}{\sqrt{3+4\cos(c+dx)}}\,dx-\frac{2}{3}\int\\ &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{3+4\cos(c+dx)}\tan(c+\end{aligned}$$

Mathematica [C] time = 1.10433, size = 158, normalized size = 1.56

$$\frac{-\frac{6\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}}+\sqrt{4\cos(c+dx)+3}\tan(c+dx)+\frac{i\sin(c+dx)\left(-12F\left(i\sinh^{-1}\left(\sqrt{4\cos(c+dx)+3}\right)|-\frac{1}{7}\right)+21E\left(i\sinh^{-1}\left(\sqrt{4\cos(c+dx)+3}\right)|-\frac{1}{7}\right)-8\Pi\right.}{3\sqrt{7}\sqrt{\sin^2(c+dx)}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] ((-6*EllipticPi[2, (c + d*x)/2, 8/7])/Sqrt[7] + ((I/3)*(21*EllipticE[I*ArcS
inh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Co
s[c + d*x]]], -1/7] - 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]
], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 + 4*Cos[c +
 d*x]]*Tan[c + d*x])/(3*d)
```

Maple [B] time = 2.809, size = 350, normalized size = 3.5

$$-\frac{1}{d}\sqrt{-\left(-8\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\frac{2}{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-8\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+7\left(\sin\left(1/2\,dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-8*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3*cos(1/2*d*
x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2*2^(1/2))+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2*2^(1/2))+4/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{2}}{\sqrt{4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{2}}{\sqrt{4\,\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(c+dx\right)}{\sqrt{4\cos\left(c+dx\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(4*cos(c + d*x) + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(4*cos(d*x + c) + 3), x)
```

3.553 $\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx$

Optimal. Leaf size=137

$$-\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{7}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}+\frac{\sqrt{4\cos(c+dx)+}}{}$$

```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) - EllipticF[(c + d*x)/2, 8/7]/(
3*Sqrt[7]*d) + (Sqrt[7]*EllipticPi[2, (c + d*x)/2, 8/7])/(3*d) - (Sqrt[3 +
4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x
]*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.371309, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2653, 3002, 2661, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{7}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{4\cos(c+dx)+3}\tan(c+dx)}{3d}+\frac{\sqrt{4\cos(c+dx)+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```


```
[Out] (Sqrt[7]*EllipticE[(c + d*x)/2, 8/7])/(3*d) - EllipticF[(c + d*x)/2, 8/7]/(
3*Sqrt[7]*d) + (Sqrt[7]*EllipticPi[2, (c + d*x)/2, 8/7])/(3*d) - (Sqrt[3 +
4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 + 4*Cos[c + d*x]]*Sec[c + d*x
]*Tan[c + d*x])/(6*d)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2661

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/(d*Sqrt[a + b]), x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{3+4\cos(c+dx)}}\,dx = \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}\int \frac{\left(-6+3\cos(c+dx)+2\cos^2(c+dx)\right)\sec}{\sqrt{3+4\cos(c+dx)}}$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{18}\int \frac{(21}{}$$

$$= -\frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} - \frac{1}{72}\int \frac{(-8}{}$$

$$= \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} - \frac{\sqrt{3+4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3+4\cos(c+dx)}\sec(c+dx)\tan(}{6d}$$

$$= \frac{\sqrt{7}E\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} - \frac{F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3\sqrt{7}d} + \frac{\sqrt{7}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{3d} - \frac{\sqrt{3+4\cos(c+dx)}\tan(c+}{3d}$$

Mathematica [C] time = 1.21399, size = 195, normalized size = 1.42

$$\frac{\frac{4F\left(\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}}+\frac{18\Pi\left(2;\frac{1}{2}(c+dx)|\frac{8}{7}\right)}{\sqrt{7}}-(2\cos(c+dx)-1)\sqrt{4\cos(c+dx)+3}\tan(c+dx)\sec(c+dx)-\frac{2i\sin(c+dx)\left(-12F\left(i\sinh^{-1}\right.\right.}{}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[3 + 4*Cos[c + d*x]],x]
```

```
[Out] ((4*EllipticF[(c + d*x)/2, 8/7])/Sqrt[7] + (18*EllipticPi[2, (c + d*x)/2, 8
/7])/Sqrt[7] - (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]]
, -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7] - 8*Ellip
ticPi[-1/3, I*ArcSinh[Sqrt[3 + 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt
[7]*Sqrt[Sin[c + d*x]^2]) - (-1 + 2*Cos[c + d*x])*Sqrt[3 + 4*Cos[c + d*x]]*
Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

Maple [B] time = 3.541, size = 408, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(-8*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/3*cos(1/2*d*
x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2-1)^2+2/3*cos(1/2*d*x+1/2*c)*(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2
*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)-1/3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^4+7*sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2*2^(1/2))-1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2*d*x+1/2*c)^
4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2*2^(1/2))-7/3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-8*sin(1/2
*d*x+1/2*c)^4+7*sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
,2*2^(1/2)))/sin(1/2*d*x+1/2*c)/(8*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec(dx+c)^3}{\sqrt{4\cos(dx+c)+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3{(c + dx)}}{\sqrt{4\cos{(c + dx)} + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(3+4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(4*cos(c + d*x) + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx + c)^3}{\sqrt{4\,\cos(dx + c) + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3+4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(4*cos(d*x + c) + 3), x)
```

3.554 $\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{23F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20\sqrt{7}d}-\frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{10d}$$

```
[Out] (-9*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*d) + (23*EllipticF[(c + P
i + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/
(10*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]*Sin[c + d*x])/(10*d)
```

Rubi [A] time = 0.149902, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2793, 3023, 2752, 2662, 2654}

$$\frac{23F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20\sqrt{7}d}-\frac{9\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{20d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}\cos(c+dx)}{10d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```


```
[Out] (-9*Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(20*d) + (23*EllipticF[(c + P
i + d*x)/2, 8/7])/(20*Sqrt[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/
(10*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Cos[c + d*x]*Sin[c + d*x])/(10*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{\sqrt{3-4\cos(c+dx)}\cos(c+dx)\sin(c+dx)}{10d} - \frac{1}{10}\int \frac{3-6\cos(c+dx)-6\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

$$= -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{10d} - \frac{\sqrt{3-4\cos(c+dx)}\cos(c+dx)\sin(c+dx)}{10d} + \frac{1}{60}\int \frac{-6+}{\sqrt{3-}}$$

$$= -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{10d} - \frac{\sqrt{3-4\cos(c+dx)}\cos(c+dx)\sin(c+dx)}{10d} - \frac{9}{40}\int \sqrt{3-}$$

$$= -\frac{9\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20d} + \frac{23F\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{20\sqrt{7}d} - \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{10d} - \frac{\sqrt{3-}}{}$$

Mathematica [A] time = 0.156989, size = 102, normalized size = 0.9

$$\frac{-4\sin(c+dx)+\sin(2(c+dx))+2\sin(3(c+dx))+23\sqrt{4\cos(c+dx)-3}F\left(\frac{1}{2}(c+dx)\middle|8\right)+9\sqrt{4\cos(c+dx)-3}E\left(\frac{1}{2}(c+\right.}{20d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (9*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 23*Sqrt[-3 + 4*Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 8] - 4*Sin[c + d*x] + Sin[2*(c + d*x)] +
2*Sin[3*(c + d*x)])/(20*d*Sqrt[3 - 4*Cos[c + d*x]])
```

Maple [A] time = 3.38, size = 254, normalized size = 2.3

$$-\frac{1}{140\,d}\sqrt{-\left(8\,(\cos(1/2\,dx+c/2))^2-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-448\,(\sin(1/2\,dx+c/2))^6\cos(1/2\,dx+c/2)+504\,(\sin(1/2\,dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/140*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-448*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+504*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2
*c)+23*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-63*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*s
in(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-56*
```

```
sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-4\cos(dx+c)+3}\cos(dx+c)^3}{4\cos(dx+c)-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^3/(4*cos(d*x + c) - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^3}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

3.555 $\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{17F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{12\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{4d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(4*d) + (17*EllipticF[(c + Pi +
 d*x)/2, 8/7])/(12*Sqrt[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(6*
d)
```

Rubi [A] time = 0.10212, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2791, 2752, 2662, 2654}

$$\frac{17F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{12\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{4d}-\frac{\sin(c+dx)\sqrt{3-4\cos(c+dx)}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(4*d) + (17*EllipticF[(c + Pi +
 d*x)/2, 8/7])/(12*Sqrt[7]*d) - (Sqrt[3 - 4*Cos[c + d*x]]*Sin[c + d*x])/(6*
d)
```

Rule 2791

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*
Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a
^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{6d} - \frac{1}{6}\int \frac{-2-3\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$
$$= -\frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{6d} - \frac{1}{8}\int \sqrt{3-4\cos(c+dx)}\,dx + \frac{17}{24}\int \frac{1}{\sqrt{3-4\cos(c+dx}}$$
$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{4d} + \frac{17F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{12\sqrt{7}d} - \frac{\sqrt{3-4\cos(c+dx)}\sin(c+dx)}{6d}$$

Mathematica [A] time = 0.116625, size = 94, normalized size = 1.18

$$\frac{-6\sin(c+dx)+4\sin(2(c+dx))+17\sqrt{4\cos(c+dx)-3}F\left(\frac{1}{2}(c+dx)\middle|8\right)+3\sqrt{4\cos(c+dx)-3}E\left(\frac{1}{2}(c+dx)\middle|8\right)}{12d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (3*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 8] + 17*Sqrt[-3 + 4*Cos
[c + d*x]]*EllipticF[(c + d*x)/2, 8] - 6*Sin[c + d*x] + 4*Sin[2*(c + d*x)])
/(12*d*Sqrt[3 - 4*Cos[c + d*x]])
```

Maple [A] time = 3.379, size = 232, normalized size = 2.9

$$-\frac{1}{84\,d}\sqrt{-\left(8\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(224\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}\cos\left(1/2\,dx+c/2\right)+17\,\sqrt{(\sin(1/2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/84*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(224*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+17*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/
2*d*x+1/2*c)^2-7)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-21*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2/7*14^(1/2))-28*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(8*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1
/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{2}}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-4\,\cos\left(dx+c\right)+3}\cos\left(dx+c\right)^{2}}{4\,\cos\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)^2/(4*cos(d*x + c) - 3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{2}}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

3.556 $\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=53

$$\frac{3F\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{2\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{2d}$$

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) + (3*EllipticF[(c + Pi +
d*x)/2, 8/7])/(2*Sqrt[7]*d)
```

Rubi [A] time = 0.0501004, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2752, 2662, 2654}

$$\frac{3F\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{2\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)\middle|\frac{8}{7}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(2*d) + (3*EllipticF[(c + Pi +
d*x)/2, 8/7])/(2*Sqrt[7]*d)
```

Rule 2752

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx &= -\left(\frac{1}{4}\int \sqrt{3-4\cos(c+dx)}\,dx\right)+\frac{3}{4}\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx\\ &= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{2d}+\frac{3F\left(\frac{1}{2}(c+\pi+dx)\middle|\frac{8}{7}\right)}{2\sqrt{7}d}\end{aligned}$$

Mathematica [A] time = 0.0667908, size = 60, normalized size = 1.13

$$\frac{\sqrt{4\cos(c+dx)-3}\left(3F\left(\frac{1}{2}(c+dx)\middle|8\right)+E\left(\frac{1}{2}(c+dx)\middle|8\right)\right)}{2d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[-3 + 4*Cos[c + d*x]]*(EllipticE[(c + d*x)/2, 8] + 3*EllipticF[(c + d*
x)/2, 8]))/(2*d*Sqrt[3 - 4*Cos[c + d*x]])
```

Maple [A] time = 2.565, size = 158, normalized size = 3.

$$-\frac{1}{14d}\sqrt{-\left(8\,(\cos(1/2\,dx+c/2))^2-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\sqrt{56\,(\sin(1/2\,dx+c/2))^2-7}\left(3\,\mathrm{EllipticF}\left(\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/14*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)*(3*EllipticF(cos(1/2*d*x
+1/2*c),2/7*14^(1/2))-7*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2)))/(8*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-8*cos(1/2
*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-4\cos(dx+c)+3}\cos(dx+c)}{4\cos(dx+c)-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*cos(d*x + c)/(4*cos(d*x + c) - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (c + dx)}{\sqrt{3 - 4\cos (c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)/sqrt(3 - 4*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx + c)}{\sqrt{-4\, \cos (dx + c) + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

3.557 $\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{2F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}$$

```
[Out] (2*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rubi [A] time = 0.0119922, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2662}

$$\frac{2F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (2*EllipticF[(c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx=\frac{2F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Mathematica [A] time = 0.034354, size = 44, normalized size = 1.83

$$\frac{2\sqrt{4\cos(c+dx)-3}F\left(\frac{1}{2}(c+dx)\middle|8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/(d*Sqrt[3 - 4*Cos[c
 + d*x]])
```

Maple [C] time = 0.163, size = 54, normalized size = 2.3

$$2\frac{\sqrt{8\left(\cos\left(1/2\,dx+c/2\right)\right)^2-7}\mathrm{InverseJacobiAM}\left(1/2\,dx+c/2,2\sqrt{2}\right)}{d\sqrt{-8\left(\cos\left(1/2\,dx+c/2\right)\right)^2+7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)*(8*cos(1/2*d*x+1/2*c)^2-7)^(1/2)*Inve
rseJacobiAM(1/2*d*x+1/2*c,2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-4\cos\left(dx+c\right)+3}}{4\cos\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)/(4*cos(d*x + c) - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3-4\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(3 - 4*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-4\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-4*cos(d*x + c) + 3), x)
```

3.558 $\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{2\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}$$

```
[Out] (-2*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rubi [A] time = 0.0385874, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2806}

$$-\frac{2\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (-2*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(Sqrt[7]*d)
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = -\frac{2\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}d}$$

Mathematica [A] time = 0.0593641, size = 45, normalized size = 1.8

$$\frac{2\sqrt{4\cos(c+dx)-3}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)}{d\sqrt{3-4\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/(d*Sqrt[3 - 4*C
os[c + d*x]])
```

Maple [B] time = 2.503, size = 139, normalized size = 5.6

$$\frac{2}{7d}\sqrt{-\left(8\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\sqrt{56\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-7}\mathrm{EllipticPi}\left(\cos\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/7*(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2))/sin
(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-4\cos(dx+c)+3}\sec(dx+c)}{4\cos(dx+c)-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)/(4*cos(d*x + c) - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(3 - 4*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(-4*cos(d*x + c) + 3), x)
```

3.559 $\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=104

$$\frac{F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) + EllipticF[(c + Pi + d*x
)/2, 8/7]/(Sqrt[7]*d) - (4*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3*Sqrt[7]
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.250712, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2802, 3060, 2654, 3002, 2662, 2806}

$$\frac{F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{4\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}d}+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) + EllipticF[(c + Pi + d*x
)/2, 8/7]/(Sqrt[7]*d) - (4*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3*Sqrt[7]
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3060

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.)
 + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist
[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x], x] - Dist[1/(b*d), Int[Simp[a*c
*C - A*b*d + (b*c*C + a*C*d)*Sin[e + f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c
+ d*Sin[e + f*x])), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c
 - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{1}{3}\int \frac{\left(2+2\cos^2(c+dx)\right)\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{1}{12}\int \frac{(8+6\cos(c+dx))\sec(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx-\frac{1}{6}\int \sqrt{3-4}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{1}{2}\int \frac{1}{\sqrt{3-4\cos(c+dx)}}\,dx$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d}+\frac{F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{\sqrt{7}d}-\frac{4\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3\sqrt{7}d}+\sqrt{3-4\cos}$$

Mathematica [C] time = 1.4186, size = 179, normalized size = 1.72

$$\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)+\frac{6\sqrt{4\cos(c+dx)-3}\Pi\left(2;\frac{1}{2}(c+dx)|8\right)}{\sqrt{3-4\cos(c+dx)}}-\frac{i\sin(c+dx)\left(-12F\left(i\sinh^{-1}\left(\sqrt{3-4\cos(c+dx)}\right)|-\frac{1}{7}\right)+21E\left(i\sinh^{-1}\left(\sqrt{3-4c}\right.\right.\right.}{3\sqrt{7}\sqrt{\sin^2(c+dx)}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] ((6*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt[3 - 4*Cos
[c + d*x]] - ((I/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7
] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] - 8*EllipticPi[
-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x])/(Sqrt[7]*Sq
rt[Sin[c + d*x]^2]) + Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d)
```

Maple [B] time = 3.337, size = 351, normalized size = 3.4

$$-\frac{1}{d}\sqrt{-\left(8\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-7\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-\frac{2}{3}\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{8\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\,(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/3*cos(1/2*d*x
+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)+1/7*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1
/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2/7*14^(1/2))-1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/
2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-4/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(56*s
in(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(1/2)))/sin(1/2*d*x+1/2*c)/(-8
*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{2}}{\sqrt{-4\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-4\,\cos\left(dx+c\right)+3}\sec\left(dx+c\right)^{2}}{4\,\cos\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^2/(4*cos(d*x + c) - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^{2}\left(c+dx\right)}{\sqrt{3-4\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(3 - 4*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^2}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(-4*cos(d*x + c) + 3), x)
```

3.560 $\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{7}\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{\sqrt{3-4c}}{}$$

```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) + EllipticF[(c + Pi + d*x
)/2, 8/7]/(3*Sqrt[7]*d) - (Sqrt[7]*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 - 4*Cos[c + d
*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

Rubi [A] time = 0.366455, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2654, 3002, 2662, 2806}

$$\frac{F\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3\sqrt{7}d}-\frac{\sqrt{7}E\left(\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}-\frac{\sqrt{7}\Pi\left(2;\frac{1}{2}(c+dx+\pi)|\frac{8}{7}\right)}{3d}+\frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d}+\frac{\sqrt{3-4c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```


```
[Out] -(Sqrt[7]*EllipticE[(c + Pi + d*x)/2, 8/7])/(3*d) + EllipticF[(c + Pi + d*x
)/2, 8/7]/(3*Sqrt[7]*d) - (Sqrt[7]*EllipticPi[2, (c + Pi + d*x)/2, 8/7])/(3
*d) + (Sqrt[3 - 4*Cos[c + d*x]]*Tan[c + d*x])/(3*d) + (Sqrt[3 - 4*Cos[c + d
*x]]*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
```

```
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2654

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 - b]*EllipticE[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/d, x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2662

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Elli
pticF[(1*(c + Pi/2 + d*x))/2, (-2*b)/(a - b)])/(d*Sqrt[a - b]), x] /; FreeQ
[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a - b, 0]
```

Rule 2806

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(-2*b)/(a - b), (1*(e + P
i/2 + f*x))/2, (-2*d)/(c - d)])/(f*(a - b)*Sqrt[c - d]), x] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] && GtQ[c - d, 0]
```

Rule steps

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx = \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{6}\int \frac{\left(6+3\cos(c+dx)-2\cos^2(c+dx)\right)\mathrm{se}}{\sqrt{3-4\cos(c+dx)}}$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{18}\int \frac{(2}{}$$

$$= \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\sec(c+dx)\tan(c+dx)}{6d} + \frac{1}{72}\int \frac{(8}{}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\tan(c+dx)}{3d} + \frac{\sqrt{3-4\cos(c+dx)}\sec(c+}{6d}$$

$$= -\frac{\sqrt{7}E\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} + \frac{F\left(\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3\sqrt{7}d} - \frac{\sqrt{7}\Pi\left(2;\frac{1}{2}(c+\pi+dx)|\frac{8}{7}\right)}{3d} + \sqrt{3-4\mathrm{co}}$$

Mathematica [C] time = 1.74908, size = 236, normalized size = 1.69

$$\frac{-\frac{4\sqrt{4\cos(c+dx)-3}F\left(\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}}+\frac{18\sqrt{4\cos(c+dx)-3}\Pi\left(2;\frac{1}{2}(c+dx)\middle|8\right)}{\sqrt{3-4\cos(c+dx)}}+\sqrt{3-4\cos(c+dx)}(2\cos(c+dx)+1)\tan(c+dx)\sec(c+dx)\cdot}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[3 - 4*Cos[c + d*x]],x]
```

```
[Out] ((-4*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticF[(c + d*x)/2, 8])/Sqrt[3 - 4*Cos[c
+ d*x]] + (18*Sqrt[-3 + 4*Cos[c + d*x]]*EllipticPi[2, (c + d*x)/2, 8])/Sqrt
[3 - 4*Cos[c + d*x]] - (((2*I)/3)*(21*EllipticE[I*ArcSinh[Sqrt[3 - 4*Cos[c
+ d*x]]], -1/7] - 12*EllipticF[I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7] -
 8*EllipticPi[-1/3, I*ArcSinh[Sqrt[3 - 4*Cos[c + d*x]]], -1/7])*Sin[c + d*x
])/(Sqrt[7]*Sqrt[Sin[c + d*x]^2]) + Sqrt[3 - 4*Cos[c + d*x]]*(1 + 2*Cos[c +
 d*x])*Sec[c + d*x]*Tan[c + d*x])/(6*d)
```

Maple [B] time = 3.833, size = 408, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x)
```

```
[Out] -(-(8*cos(1/2*d*x+1/2*c)^2-7)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/3*cos(1/2*d*x
+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+
1/2*c)^2-1)^2-2/3*cos(1/2*d*x+1/2*c)*(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)+1/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-1/3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/2*c)^4-sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2/7*14^(1/2))-1/3*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(56*sin(1/2*d*x+1/2*c)^2-7)^(1/2)/(8*sin(1/2*d*x+1/
2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2,2/7*14^(
1/2)))/sin(1/2*d*x+1/2*c)/(-8*cos(1/2*d*x+1/2*c)^2+7)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-4\cos(dx+c)+3}\sec(dx+c)^3}{4\cos(dx+c)-3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-4*cos(d*x + c) + 3)*sec(d*x + c)^3/(4*cos(d*x + c) - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{\sqrt{3-4\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(3-4*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(3 - 4*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^3}{\sqrt{-4\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(3-4*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(-4*cos(d*x + c) + 3), x)
```

3.561 $\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10B\sin(c+d}{2}$$

```
[Out] (6*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*B*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.074807, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2635, 2639, 2641}

$$\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{10BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}+\frac{10B\sin(c+d}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (10*B*EllipticF[(c + d*x)/2, 2])/(2
1*d) + (10*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*d) + (2*B*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))\,dx = A\int \cos^{\frac{5}{2}}(c+dx)\,dx + B\int \cos^{\frac{7}{2}}(c+dx)\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2B\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(3A)\int \sqrt{\mathrm{co}$$

$$= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10B\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c}{5d}$$

$$= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{10BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{10B\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

Mathematica [A] time = 0.494371, size = 77, normalized size = 0.69

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+126AE\left(\frac{1}{2}(c+dx)\middle|2\right)+50BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (126*A*EllipticE[(c + d*x)/2, 2] + 50*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(105*d)
```

Maple [A] time = 2.775, size = 290, normalized size = 2.6

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-168\,A-360\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A
-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^3 + A\cos(dx+c)^2\right)\sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.562 $\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*A*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0601566, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2635, 2641, 2639}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*B*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*A*EllipticF[(c + d*x)/2, 2])/(3*
d) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*Cos[c + d*x]^(3/2)*
Sin[c + d*x])/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))\,dx = A\int \cos^{\frac{3}{2}}(c+dx)\,dx + B\int \cos^{\frac{5}{2}}(c+dx)\,dx$$
$$= \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}A\int \frac{1}{\sqrt{\cos(c+}$$
$$= \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2A\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + 2B$$

Mathematica [A] time = 0.222928, size = 66, normalized size = 0.76

$$\frac{2\left(\sin(c+dx)\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] time = 3.26, size = 262, normalized size = 3.

$$-\frac{2}{15\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^6+(20\,A+24\,B)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos\left(dx+c\right)^{2}+A\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.563 $\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0505281, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2639, 2635, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/(3*d) +
 (2*B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(A+B\cos(c+dx))\,dx = A\int \sqrt{\cos(c+dx)}\,dx + B\int \cos^{\frac{3}{2}}(c+dx)\,dx$$
$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,d$$
$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.102872, size = 53, normalized size = 0.87

$$\frac{2\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos
[c + d*x]]*Sin[c + d*x])))/(3*d)
```

Maple [B] time = 2.297, size = 229, normalized size = 3.8

$$\frac{2}{3\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,B\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^4+3\,A\sqrt{(\sin(1/2\,dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+2*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c)), x)
```

3.564 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=35

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/d
```

Rubi [A] time = 0.0388315, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2748, 2641, 2639}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/d
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}}\,dx &= A\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx+B\int \sqrt{\cos(c+dx)}\,dx\\ &= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.0618824, size = 35, normalized size = 1.

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/d
```

Maple [A] time = 2.808, size = 152, normalized size = 4.3

$$-2\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(A\mathrm{EllipticF}\left(c\right.\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\cos(dx+c)+A}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(cos(d*x + c)), x)
```

3.565 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d + (2
*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.048375, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2636, 2639, 2641}

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*A*EllipticE[(c + d*x)/2, 2])/d + (2*B*EllipticF[(c + d*x)/2, 2])/d + (2
*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx = A\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - A\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.134086, size = 51, normalized size = 0.89

$$\frac{2\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c + d*x)/2, 2] + (A*Sin[c
 + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [A] time = 2.744, size = 148, normalized size = 2.6

$$-2\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-2\,A\cos(1/2\,dx+c/2)\,(\sin}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(3/2), x)
```

3.566 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rubi [A] time = 0.0593403, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2636, 2641, 2639}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/d + (2*A*EllipticF[(c + d*x)/2, 2])/(3*d)
+ (2*A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*B*Sin[c + d*x])/(d*Sqrt[
Cos[c + d*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx = A\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + B\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{3}A\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx - B\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2A\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.39278, size = 65, normalized size = 0.78

$$\frac{\frac{2\sin(c+dx)(A+3B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} + 2AF\left(\frac{1}{2}(c+dx)\middle|2\right) - 6BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-6*B*EllipticE[(c + d*x)/2, 2] + 2*A*EllipticF[(c + d*x)/2, 2] + (2*(A + 3
*B*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2))/(3*d)
```

Maple [B] time = 6.086, size = 397, normalized size = 4.8

$$\frac{2}{3d}\sqrt{-\left(-2\left(\cos(1/2\,dx+c/2)\right)^2+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2A\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+6*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(5/2), x)
```

3.567 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6A\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-6*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*A*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0736735, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2748, 2636, 2639, 2641}

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{6A\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*A*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*B*EllipticF[(c + d*x)/2, 2])/(3
*d) + (2*A*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*B*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2)) + (6*A*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx = A\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)}\,dx + B\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3A)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}B\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6A\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}(3A)\int \sqrt{}$$
$$= -\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2A\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{6A\sin}{5d\sqrt{c}}$$

Mathematica [A] time = 0.298055, size = 95, normalized size = 0.86

$$\frac{9A\sin(2(c+dx)) + 6A\tan(c+dx) - 18A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 10B\sin(c+dx) + 10B\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c-\right.}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-18*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*B*Cos[c + d*x]^(3/
2)*EllipticF[(c + d*x)/2, 2] + 10*B*Sin[c + d*x] + 9*A*Sin[2*(c + d*x)] + 6
*A*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] time = 7.04, size = 502, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B*(-1/6*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2
*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*A/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+
1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-1
2*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos\left(dx+c\right)+A}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/cos(d*x + c)^(7/2), x)
```

3.568 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{2(9a^2+7b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(9a^2+7b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)\cos}{7d}$$

```
[Out] (2*(9*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a*b*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (20*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*(9*a^2 + 7*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a*b*Cos[c +
d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9
*d)
```

Rubi [A] time = 0.120122, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2635, 2641, 3014, 2639}

$$\frac{2(9a^2+7b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2(9a^2+7b^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)\cos}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (2*(9*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (20*a*b*EllipticF[(c
 + d*x)/2, 2])/(21*d) + (20*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*(9*a^2 + 7*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(45*d) + (4*a*b*Cos[c +
d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*b^2*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(9
*d)
```


Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^{\frac{7}{2}}(c+dx)\,dx + \int \cos^{\frac{5}{2}}(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)dx$$
$$= \frac{4ab\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2b^2\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{7}(10ab)\int \cos^{\frac{3}{2}}$$
$$= \frac{20ab\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2\left(9a^2+7b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{45d} + \frac{4ab\,c}{}$$
$$= \frac{2\left(9a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d} + \frac{20abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{20ab\sqrt{\cos(c+dx)}\sin(}{21d}$$

Mathematica [A] time = 0.754149, size = 113, normalized size = 0.71

$$\frac{84\left(9a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7\left(36a^2+43b^2\right)\cos(c+dx)+5b(36a\cos(2(c+dx))+156a\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (84*(9*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2] + 600*a*b*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*a^2 + 43*b^2)*Cos[c + d*x] + 5*b*(156*a
+ 36*a*Cos[2*(c + d*x)] + 7*b*Cos[3*(c + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 2.832, size = 398, normalized size = 2.5

$$-\frac{2}{315\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-1120\,b^2\,(\sin(1/2\,dx+c/2))^{10}\cos(1/2\,dx+c/2)+\left(1440\,ab+224\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*s
in(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(1440*a*b+2240*b^2)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-504*a^2-2160*a*b-2072*b^2)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(504*a^2+1680*a*b+952*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-126*a^2-480*a*b-168*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+150*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^4+2\,ab\cos(dx+c)^3+a^2\cos(dx+c)^2\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4 + 2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2)*s
qrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2), x)
```

3.569 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{2(7a^2+5b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)\cos^{\frac{3}{2}}(c}{5d}$$

```
[Out] (12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2 + 5*b^2)*EllipticF[(c
+ d*x)/2, 2])/(21*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (4*a*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b^2*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.103701, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2635, 2639, 3014, 2641}

$$\frac{2(7a^2+5b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)\cos^{\frac{3}{2}}(c}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (12*a*b*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(7*a^2 + 5*b^2)*EllipticF[(c
+ d*x)/2, 2])/(21*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(21*d) + (4*a*b*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*b^2*Cos[c + d*x
]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^{\frac{5}{2}}(c+dx)\,dx + \int \cos^{\frac{3}{2}}(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$
$$= \frac{4ab\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2b^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(6ab)\int \sqrt{c}$$
$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} + \frac{4ab\cos^{\frac{3}{2}}(}{}$$
$$= \frac{12abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2\left(7a^2+5b^2\right)\sqrt{\cos}}{21}$$

Mathematica [A] time = 0.578617, size = 98, normalized size = 0.73

$$\frac{10\left(7a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(70a^2+84ab\cos(c+dx)+15b^2\cos(2(c+dx))+65b^2\right)+2}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (252*a*b*EllipticE[(c + d*x)/2, 2] + 10*(7*a^2 + 5*b^2)*EllipticF[(c + d*x)
/2, 2] + Sqrt[Cos[c + d*x]]*(70*a^2 + 65*b^2 + 84*a*b*Cos[c + d*x] + 15*b^2
*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 2.5, size = 362, normalized size = 2.7

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,b^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-336\,ab-36\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-336*a*b-360*b^2)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*a^2+336*a*b+280*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-70*a^2-84*a*b-80*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-126*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^3+2\,ab\cos(dx+c)^2+a^2\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^3 + 2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c))*sqr
t(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.570 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=101

$$\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

```
[Out] (2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (4*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b^2
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0904047, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2635, 2641, 3014, 2639}

$$\frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b^2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (2*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c +
 d*x)/2, 2])/(3*d) + (4*a*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b^2
*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^{\frac{3}{2}}(c+dx)\,dx + \int \sqrt{\cos(c+dx)}\left(a^2+b^2\cos^2(c+dx)\right)\,dx$$

$$= \frac{4ab\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2b^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(2ab)\int \frac{}{\sqrt{\cos}}$$

$$= \frac{2\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{4ab\sqrt{\cos(c+dx)}\sin(c}{3d}$$

Mathematica [A] time = 0.298858, size = 79, normalized size = 0.78

$$\frac{6\left(5a^2+3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20abF\left(\frac{1}{2}(c+dx)\middle|2\right)+2b\sin(c+dx)\sqrt{\cos(c+dx)}(10a+3b\cos(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (6*(5*a^2 + 3*b^2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*EllipticF[(c + d*x)/2
, 2] + 2*b*Sqrt[Cos[c + d*x]]*(10*a + 3*b*Cos[c + d*x])*Sin[c + d*x])/(15*d
)
```

Maple [B] time = 2.661, size = 321, normalized size = 3.2

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,b^2\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^6+\left(40\,ab+24\,b^2\right)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(40*a*b+24*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-20*a*b-6*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10
*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)+a\right)^2\sqrt{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2\right)\sqrt{\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(cos(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.571 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{2(3a^2+b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] (4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0839678, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2789, 2639, 3014, 2641}

$$\frac{2(3a^2+b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a^2 + b^2)*EllipticF[(c + d*x)/
2, 2])/(3*d) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\cos(c+dx)}}\,dx = (2ab)\int \sqrt{\cos(c+dx)}\,dx + \int \frac{a^2+b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(3a^2+b^2\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$
$$= \frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3d}$$

Mathematica [A] time = 0.155084, size = 64, normalized size = 0.89

$$\frac{2\left(\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+b^2\sin(c+dx)\sqrt{\cos(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(6*a*b*EllipticE[(c + d*x)/2, 2] + (3*a^2 + b^2)*EllipticF[(c + d*x)/2,
2] + b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 2.885, size = 283, normalized size = 3.9

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,b^2\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+3\sqrt{\left(\sin(1/2\,dx+c/\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*b^2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\cos(dx+c)+a\right)^2}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2}{\sqrt{\cos\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/sqrt(cos(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\cos\left(dx+c\right)+a\right)^2}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(cos(d*x + c)), x)
```

3.572 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (-2*(a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (4*a*b*EllipticF[(c + d*x)/2
, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0856865, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2789, 2641, 3012, 2639}

$$-\frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (4*a*b*EllipticF[(c + d*x)/2
, 2])/d + (2*a^2*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{3}{2}}(c+dx)}\,dx = (2ab)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \left(a^2-b^2\right)\int \sqrt{\cos(c+dx)}\,dx$$
$$= -\frac{2\left(a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a^2\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.276616, size = 62, normalized size = 0.91

$$\frac{2\left(\left(b^2-a^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)}}+2bF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*((-a^2 + b^2)*EllipticE[(c + d*x)/2, 2] + a*(2*b*EllipticF[(c + d*x)/2,
2] + (a*Sin[c + d*x])/Sqrt[Cos[c + d*x]])))/d
```

Maple [A] time = 3.197, size = 202, normalized size = 3.

$$-2\,\frac{2\,ab\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)+\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x)
```

```
[Out] -2*(2*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-2*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2}{\cos(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/cos(d*x + c)^(3/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c) + a)^2}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(3/2), x)
```

3.573 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2 + 3*b^2)*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a*b*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0949957, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2636, 2639, 3012, 2641}

$$\frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-4*a*b*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2 + 3*b^2)*EllipticF[(c + d*x)
/2, 2])/(3*d) + (2*a^2*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (4*a*b*Sin[
c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{5}{2}}(c+dx)}\,dx = (2ab)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - (2ab)\int \sqrt{\cos(c+dx)}\,dx - \frac{1}{3}\left(-a^2-3b^2\right)\int \frac{}{\sqrt{\cos(}}$$
$$= -\frac{4abE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+d}{d\sqrt{\cos(c+d}}$$

Mathematica [A] time = 0.565431, size = 73, normalized size = 0.77

$$\frac{2\left(\left(a^2+3b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6abE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)(a+6b\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(-6*a*b*EllipticE[(c + d*x)/2, 2] + (a^2 + 3*b^2)*EllipticF[(c + d*x)/2,
 2] + (a*(a + 6*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] time = 6.148, size = 514, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+6*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2*sin(1/2*
d*x+1/2*c)^2+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2-24*a*b*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*b^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*a^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2+12*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/cos(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(5/2), x)
```

3.574 $\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$-\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (-2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a^2 + 5*b^2)*Sin[c + d*x])
/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.10623, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2789, 2636, 2641, 3012, 2639}

$$-\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2\left(3a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a*b*EllipticF[(c
+ d*x)/2, 2])/(3*d) + (2*a^2*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(3*a^2 + 5*b^2)*Sin[c + d*x])
/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
```

```
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\cos^{\frac{7}{2}}(c+dx)}dx = (2ab)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx + \int \frac{a^2+b^2\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx$$
$$= \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(2ab)\int \frac{1}{\sqrt{\cos(c+dx)}}dx - \frac{1}{5}\left(-3a^2-5b^2\right)\int \frac{}{\cos^{\frac{3}{2}}(}$$
$$= \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2\left(3a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} - \frac{1}{5}($$
$$= -\frac{2\left(3a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4abF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.362037, size = 124, normalized size = 0.92

$$\frac{-6\left(3a^2+5b^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+9a^2\sin(2(c+dx))+6a^2\tan(c+dx)+20ab\sin(c+dx)+20ab\cos^{\frac{3}{2}}(c+a}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*(3*a^2 + 5*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 20*a*b*C
os[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 20*a*b*Sin[c + d*x] + 9*a^2*S
in[2*(c + d*x)] + 15*b^2*Sin[2*(c + d*x)] + 6*a^2*Tan[c + d*x])/(15*d*Cos[c
 + d*x]^(3/2))
```

Maple [B] time = 7.757, size = 660, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*a*b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x
+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin
```

```
(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(12*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x
+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2
*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/cos(d*x + c)^(7/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/cos(d*x + c)^(7/2), x)
```

3.575 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=194

$$\frac{2a\left(7a^2+15b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\left(27a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(27a^2+7b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a}{}$$

```
[Out] (2*b*(27*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*a^2 + 15*
b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a^2 + 7*b^2)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(45*d) + (40*a*b^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b
^2*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 0.22039, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3023, 2748, 2635, 2641, 2639}

$$\frac{2a\left(7a^2+15b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b\left(27a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b\left(27a^2+7b^2\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (2*b*(27*a^2 + 7*b^2)*EllipticE[(c + d*x)/2, 2])/(15*d) + (2*a*(7*a^2 + 15*
b^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(21*d) + (2*b*(27*a^2 + 7*b^2)*Cos[c + d*x]^(3/2)*Sin[
c + d*x])/(45*d) + (40*a*b^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*b
^2*Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3\,dx &= \frac{2b^2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{9d}+\frac{2}{9}\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a\left(9a^2+\right.\right.\\ &= \frac{40ab^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}+\frac{2b^2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+d\\ &= \frac{40ab^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d}+\frac{2b^2\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+d\\ &= \frac{2a\left(7a^2+15b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2b\left(27a^2+7b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c\\ &= \frac{2b\left(27a^2+7b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a\left(7a^2+15b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a\left(7a^2\right.}{}\end{aligned}$$

Mathematica [A] time = 0.91535, size = 137, normalized size = 0.71

$$\frac{60\left(7a^3+15ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+84\left(27a^2b+7b^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\left(7b\left(108a^2+43b^2\right)\cos(\right.}{630d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (84*(27*a^2*b + 7*b^3)*EllipticE[(c + d*x)/2, 2] + 60*(7*a^3 + 15*a*b^2)*El
lipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*b*(108*a^2 + 43*b^2)*Cos[c
+ d*x] + 5*(84*a^3 + 234*a*b^2 + 54*a*b^2*Cos[2*(c + d*x)] + 7*b^3*Cos[3*(c
 + d*x)]))*Sin[c + d*x])/(630*d)
```

Maple [B] time = 3.132, size = 470, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2160*a*b^2+2240*b^3)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*a^2*b-3240*a*b^2-2072*b^3)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^3+1512*a^2*b+2520*a*b^2+952*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^3-378*a^2*b-720*a*b^2-168*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+225*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b^2-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3\cos(dx+c)^4+3\,ab^2\cos(dx+c)^3+3\,a^2b\cos(dx+c)^2+a^3\cos(dx+c)\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^4 + 3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c
)^2 + a^3*cos(d*x + c))*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2), x)
```

3.576 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=159

$$\frac{2b(21a^2+5b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5a^2+9b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(21a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{32ab}{}$$

```
[Out] (2*a*(5*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(21*a^2 + 5*b^
2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*d) + (32*a*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) + (2*b^2*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.202096, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3023, 2748, 2639, 2635, 2641}

$$\frac{2b(21a^2+5b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5a^2+9b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(21a^2+5b^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{32ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (2*a*(5*a^2 + 9*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(21*a^2 + 5*b^
2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(21*d) + (32*a*b^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d) + (2*b^2*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3\,dx = \frac{2b^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{7d} + \frac{2}{7}\int \sqrt{\cos(c+dx)}\left(\frac{1}{2}a\left(7a^2\right.\right.$$

$$= \frac{32ab^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2b^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+d}{7d}$$

$$= \frac{32ab^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2b^2\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))\sin(c+d}{7d}$$

$$= \frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(21a^2+5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{21d} +$$

$$= \frac{2a\left(5a^2+9b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(21a^2+5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{2b\left(21a^2\right.}{}$$

Mathematica [A] time = 0.710736, size = 110, normalized size = 0.69

$$\frac{10\left(21a^2b+5b^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+42\left(5a^3+9ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(c+dx)\sqrt{\cos(c+dx)}\left(210a^2+126ab\cos(c+\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (42*(5*a^3 + 9*a*b^2)*EllipticE[(c + d*x)/2, 2] + 10*(21*a^2*b + 5*b^3)*Ell
ipticF[(c + d*x)/2, 2] + b*Sqrt[Cos[c + d*x]]*(210*a^2 + 65*b^2 + 126*a*b*C
os[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] time = 3.802, size = 421, normalized size = 2.7

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,b^3\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^8+\left(-504\,ab^2-360\,b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-504*a*b^2-360*b^3)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(420*a^2*b+504*a*b^2+280*b^3)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-210*a^2*b-126*a*b^2-80*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+105*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.577 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=116

$$\frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}+\frac{8a}{}$$

```
[Out] (6*b*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(a^2 + b^2)*Elli
pticF[(c + d*x)/2, 2])/d + (8*a*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d)
+ (2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.176584, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2793, 3023, 2748, 2641, 2639}

$$\frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}{5d}+\frac{8a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (6*b*(5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(a^2 + b^2)*Elli
pticF[(c + d*x)/2, 2])/d + (8*a*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d)
+ (2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\cos(c+dx)}}\,dx = \frac{2b^2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx)}{5d}+\frac{2}{5}\int \frac{\frac{1}{2}a\left(5a^2+b^2\right)+\frac{3}{2}b\left(5a^2+b^2\right)\cos(c\ldots}{\sqrt{\cos(c+dx\ldots}}$$
$$=\frac{8ab^2\sqrt{\cos(c+dx)}\sin(c+dx)}{5d}+\frac{2b^2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx)}{5d}+\frac{4}{15}\int \frac{15}{4}\ldots$$
$$=\frac{8ab^2\sqrt{\cos(c+dx)}\sin(c+dx)}{5d}+\frac{2b^2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))\sin(c+dx)}{5d}+\left(a\left(a^2+\ldots\right.\right.$$
$$=\frac{6b\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{8ab^2\sqrt{\cos(c+dx)}\sin(c+\ldots}{5d}$$

Mathematica [A] time = 0.352705, size = 84, normalized size = 0.72

$$\frac{2\left(5a\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+3\left(5a^2b+b^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+b^2\sin(c+dx)\sqrt{\cos(c+dx)}(5a+b\cos(c+dx))\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(3*(5*a^2*b + b^3)*EllipticE[(c + d*x)/2, 2] + 5*a*(a^2 + b^2)*EllipticF
[(c + d*x)/2, 2] + b^2*Sqrt[Cos[c + d*x]]*(5*a + b*Cos[c + d*x])*Sin[c + d*
x]))/(5*d)
```

Maple [B] time = 3.181, size = 376, normalized size = 3.2

$$-\frac{2}{5d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-8\,b^3\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+\left(20\,ab^2+8\,b^3\right)\left(\sin\left(\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*b^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*a*b^2+8*b^3)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-10*a*b^2-2*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-15*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
```

```
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(cos(d*x + c)), x)
```

3.578 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(c+dx}{d\sqrt{c}}$$

```
[Out] (-2*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) - (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*d) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]])
```

Rubi [A] time = 0.187826, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2792, 3023, 2748, 2641, 2639}

$$\frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2a^2\sin(c+dx}{d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*b*(9*a^2 + b^2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) - (2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*d) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*
x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{2a^2b-\frac{1}{2}a\left(a^2-3b^2\right)\cos(c+dx)-\frac{1}{2}b\left(3a^2-b^2\right.}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{4}{3}\int \frac{\frac{1}{4}b}{}$$

$$= -\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \left(a\left(a^2-\right.\right.$$

$$= -\frac{2a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2b\left(9a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} - \frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c}}{3d}$$

Mathematica [A] time = 0.508369, size = 86, normalized size = 0.69

$$\frac{2\left(\left(9a^2b+b^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3\left(a^3-3ab^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{\sin(c+dx)\left(3a^3+b^3\cos(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(-3*(a^3 - 3*a*b^2)*EllipticE[(c + d*x)/2, 2] + (9*a^2*b + b^3)*Elliptic
F[(c + d*x)/2, 2] + ((3*a^3 + b^3*Cos[c + d*x])*Sin[c + d*x])/Sqrt[Cos[c +
d*x]]))/(3*d)
```

Maple [A] time = 3.388, size = 303, normalized size = 2.4

$$-\frac{2}{3d}\left(4b^3\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+9a^2b\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{2\left(\sin(1/2\,dx+c/2)\right)^2-1}\mathrm{EllipticF}\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*a^2*b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
```

```
,2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b^2-6*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(3/2), x)
```

3.579 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2b\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (16*a^2*b*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x
]]) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.189672, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2792, 3021, 2748, 2641, 2639}

$$\frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)(a+b\cos(c+dx))}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{16a^2b\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(a^2 + 9*b^2)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (16*a^2*b*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x
]]) + (2*a^2*(a + b*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{4a^2b+\frac{1}{2}a\left(a^2+9b^2\right)\cos(c+dx)-\frac{1}{2}b\left(a^2-3b\right.}{\cos^{\frac{3}{2}}(c+dx)}$$
$$=\frac{16a^2b\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{4}{3}\int \frac{\frac{1}{4}a\left(a^2+9b^2\right)-\frac{3}{4}b\left(3a^2-\right.}{\sqrt{\cos(c+d}}$$
$$=\frac{16a^2b\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}-\left(b\left(3a^2-b^2\right)\right)\int \sqrt{\cos(c+dx)}$$
$$=-\frac{2b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{16a^2b\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+$$

Mathematica [A] time = 1.2223, size = 85, normalized size = 0.71

$$\frac{2\left(\left(3b^3-9a^2b\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(\left(a^2+9b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)(a+9b\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*((-9*a^2*b + 3*b^3)*EllipticE[(c + d*x)/2, 2] + a*((a^2 + 9*b^2)*Ellipti
cF[(c + d*x)/2, 2] + (a*(a + 9*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(
3/2))))/(3*d)
```

Maple [B] time = 6.43, size = 631, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
```

```
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2*sin(1
/2*d*x+1/2*c)^2+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b*sin(1/2*d*x+1/2*c)^2-6*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b^3*sin(1/2*d*x+1/2*c)^2-36*a^2*b*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b^2-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+2*
a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*a^2*b*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(5/2), x)
```

3.580 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6a\left(a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)(a+b\cos(c+}{5d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] (-6*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(a^2 + b^2)*Ell
ipticF[(c + d*x)/2, 2])/d + (8*a^2*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
 + (6*a*(a^2 + 5*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a^2*(a +
b*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.210559, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2792, 3021, 2748, 2636, 2639, 2641}

$$\frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{6a\left(a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2a^2\sin(c+dx)(a+b\cos(c+}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*(a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*b*(a^2 + b^2)*Ell
ipticF[(c + d*x)/2, 2])/d + (8*a^2*b*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2))
 + (6*a*(a^2 + 5*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a^2*(a +
b*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{6a^2b+\frac{3}{2}a\left(a^2+5b^2\right)\cos(c+dx)+\frac{1}{2}b\left(a^2+5b\right.}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4}{15}\int \frac{\frac{9}{4}a\left(a^2+5b^2\right)+\frac{15}{4}b\left(a^2+\right.}{\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \left(b\left(a^2+b^2\right)\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a\left(a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+b}{}$$

$$= -\frac{6a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{8a^2b\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{6a}{}$$

Mathematica [A] time = 0.860373, size = 125, normalized size = 0.84

$$\frac{3\left(a^3+5ab^2\right)\sin(2(c+dx))+10b\left(a^2+b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6a\left(a^2+5b^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-6*a*(a^2 + 5*b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*b*(a^
2 + b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*a^2*b*Sin[c + d*
x] + 3*(a^3 + 5*a*b^2)*Sin[2*(c + d*x)] + 2*a^3*Tan[c + d*x])/(5*d*Cos[c +
d*x]^(3/2))
```

Maple [B] time = 8.242, size = 738, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a^2
*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b^2*(-(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a^3/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(7/2), x)
```

3.581 $\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{2a\left(5a^2+21b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2b\left(9a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(9a^2+5b^2\right)\sin(c\cdot}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*b*(9*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*a^2 + 21*b
^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (32*a^2*b*Sin[c + d*x])/(35*d*Cos[c
 + d*x]^(5/2)) + (2*a*(5*a^2 + 21*b^2)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/
2)) + (2*b*(9*a^2 + 5*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a^2*
(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.233322, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2792, 3021, 2748, 2636, 2641, 2639}

$$\frac{2a\left(5a^2+21b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2b\left(9a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b\left(9a^2+5b^2\right)\sin(c\cdot}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-2*b*(9*a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*a^2 + 21*b
^2)*EllipticF[(c + d*x)/2, 2])/(21*d) + (32*a^2*b*Sin[c + d*x])/(35*d*Cos[c
 + d*x]^(5/2)) + (2*a*(5*a^2 + 21*b^2)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3/
2)) + (2*b*(9*a^2 + 5*b^2)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*a^2*
(a + b*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{8a^2b+\frac{1}{2}a\left(5a^2+21b^2\right)\cos(c+dx)+\frac{1}{2}b\left(3a^2\right.}{\cos^{\frac{7}{2}}(c+dx)}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{4}{35}\int \frac{\frac{5}{4}a\left(5a^2+21b^2\right)+\frac{7}{4}b\left(9a\right.}{\cos^{\frac{5}{2}}(c+}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a^2(a+b\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{5}\left(b\left(9a^2+5b^2\right)\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+}$$

$$= \frac{32a^2b\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a\left(5a^2+21b^2\right)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b\left(9a^2+5b^2\right)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}} + \frac{2a^2(a+}{}$$

$$= -\frac{2b\left(9a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a\left(5a^2+21b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d} + \frac{32a^2b\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] time = 0.725602, size = 177, normalized size = 0.91

$$\frac{10a\left(5a^2+21b^2\right)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-42b\left(9a^2+5b^2\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+126a^2b\sin(c+dx)+}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-42*b*(9*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 10*a*
(5*a^2 + 21*b^2)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 126*a^2*b*S
in[c + d*x] + 378*a^2*b*Cos[c + d*x]^2*Sin[c + d*x] + 210*b^3*Cos[c + d*x]^
2*Sin[c + d*x] + 25*a^3*Sin[2*(c + d*x)] + 105*a*b^2*Sin[2*(c + d*x)] + 30*
a^3*Tan[c + d*x])/(105*d*Cos[c + d*x]^(5/2))
```

Maple [B] time = 9.352, size = 847, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^3*(-1/56*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6*a*b^2*(-
1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^3*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(
1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-6/5*a^2*b/(8*sin(1/2*d*x+1/2*c)
^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(
12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/
2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+
1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/cos(d*x + c)^(9/2), x)
```

3.582 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=112

$$\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2 + b^2)*EllipticF[(c +
d*x)/2, 2])/(3*b^3*d) - (2*a^3*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
b^3*(a + b)*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.390861, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2793, 3059, 2639, 3002, 2641, 2805}

$$\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*a*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*(3*a^2 + b^2)*EllipticF[(c +
d*x)/2, 2])/(3*b^3*d) - (2*a^3*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(
b^3*(a + b)*d) + (2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd}+\frac{2\int \frac{\frac{a}{2}+\frac{1}{2}b\cos(c+dx)-\frac{3}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{3b}\\
&= \frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd}-\frac{2\int \frac{-\frac{ab}{2}-\frac{1}{2}\left(3a^2+b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{3b^2}-\frac{a\int \sqrt{\cos(c+dx)}\,dx}{b^2}\\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd}-\frac{a^3\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b^3}+\frac{(3a^2 \ldots}{}\\
&= -\frac{2aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\left(3a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a+b)d}+\frac{2\sqrt{\cos(c \ldots}{}
\end{aligned}$$

Mathematica [A] time = 1.89608, size = 160, normalized size = 1.43

$$\frac{\frac{6\sin(c+dx)\left(\left(b^2-2a^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b^2\sqrt{\sin^2(c+dx)}}-\frac{6a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+4F \ldots}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (4*EllipticF[(c + d*x)/2, 2] - (6*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b) + 4*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (6*(2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] - 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (-2*a^2 + b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1
])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/(6*b*d)
```

Maple [B] time = 3.279, size = 516, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*a*b^2-4*b^
3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a*b^2+2*b^3)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-3*a^2*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-b^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^{\frac{5}{2}}}{b\cos (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

3.583 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2a^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticF[(c + d*x)/2, 2])/(b^2*
d) + (2*a^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d)
```

Rubi [A] time = 0.162545, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2804, 2639, 2803, 2641, 2805}

$$\frac{2a^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*EllipticE[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticF[(c + d*x)/2, 2])/(b^2*
d) + (2*a^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(b^2*(a + b)*d)
```

Rule 2804

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]), x_Symbol] :> Dist[b/d, Int[Sqrt[a + b*Sin[e + f*x]], x], x]
 - Dist[(b*c - a*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
```

```
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx &= \frac{\int \sqrt{\cos(c+dx)}\,dx}{b} - \frac{a\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx}{b}\\
&= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2} + \frac{a^2\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b^2}\\
&= \frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} - \frac{2aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2a^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.291408, size = 84, normalized size = 1.12

$$-\frac{2\sin(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-a\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+bE\left(\sin^{-1}\left(\sqrt{\cos(c+\right.\right.}{b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*(b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a + b)*EllipticF[ArcSin
[Sqrt[Cos[c + d*x]]], -1] - a*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]
], -1])*Sin[c + d*x])/(b^2*d*Sqrt[Sin[c + d*x]^2])
```

Maple [A] time = 3.318, size = 227, normalized size = 3.

$$2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\,b^2\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\Big(\mathrm{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^{\frac{3}{2}}}{b\cos (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

3.584 $\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

```
[Out] (2*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/(b*(a + b)*d)
```

Rubi [A] time = 0.100765, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2803, 2641, 2805}

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2])/(b*d) - (2*a*EllipticPi[(2*b)/(a + b), (c + d
*x)/2, 2])/(b*(a + b)*d)
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx &= \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b}-\frac{a\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b}\\ &= \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.0681823, size = 48, normalized size = 0.91

$$\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(a + b))/(b*d)
```

Maple [A] time = 2.827, size = 188, normalized size = 3.6

$$-2\,\frac{\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{b\left(a-b\right)\sqrt{-2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\left(\mathrm{EllipticF}\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2)))/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.585 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=29

$$\frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] (2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rubi [A] time = 0.0453543, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2805}

$$\frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = \frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a+b)d}$$

Mathematica [A] time = 0.0733196, size = 29, normalized size = 1.

$$\frac{2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a + b)*d)
```

Maple [B] time = 2.258, size = 150, normalized size = 5.2

$$-2\,{\frac {\sqrt {\left(2\, \left(\cos \left(1/2\,dx+c/2 \right) \right) ^{2}-1 \right) \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}\sqrt { \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}\sqrt {-2\, \left(\cos \left(1/2\,dx+c/2 \right) \right) ^{2}+1}}{ \left(a-b \right) \sqrt {-2\, \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{4}+ \left(\sin \left(1/2\,dx+c/2 \right) \right) ^{2}}\sin \left(1/2\,dx+c/2 \right) \sqrt {2\, \left(\cos \left(1/2\,dx+c/2 \right) \right) ^{2}-1}d}}{\rm EllipticPi} \left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.586 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=77

$$-\frac{2b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(a*(a + b)*d) + (2*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.237682, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2802, 3059, 2639, 12, 2805}

$$-\frac{2b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (-2*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*b*EllipticPi[(2*b)/(a + b), (c +
d*x)/2, 2])/(a*(a + b)*d) + (2*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{2\int \frac{-\frac{b}{2}-\frac{1}{2}a\cos(c+dx)-\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a}$$

$$= \frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{\int \sqrt{\cos(c+dx)}\,dx}{a} - \frac{2\int \frac{b^2}{2\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{ab}$$

$$= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} - \frac{b\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a}$$

$$= -\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d} + \frac{2\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Mathematica [B] time = 2.92873, size = 199, normalized size = 2.58

$$-\frac{\frac{2\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{ab\sqrt{\sin^2(c+dx)}} + \frac{6b\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2a\left(2 \right.}{}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] -((6*b*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (2*a*(2*Ellipti
cF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b
)))/b - (4*Sin[c + d*x])/Sqrt[Cos[c + d*x]] + (2*(-2*a*b*EllipticE[ArcSin[S
qrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]],
 -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*S
in[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*a*d)
```

Maple [B] time = 3.508, size = 354, normalized size = 4.6

$$-2\,\frac{1}{(a-b)\,a\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\,\sin(1/2\,dx+c/2)\,\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(-2\sqrt{-2\,(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/a/(a-b)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.587 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=128

$$\frac{2b^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*
a*d) + (2*b^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d) +
(2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*b*Sin[c + d*x])/(a^2*d*Sqr
t[Cos[c + d*x]])
```

Rubi [A] time = 0.545599, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{2b^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (2*b*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*EllipticF[(c + d*x)/2, 2])/(3*
a*d) + (2*b^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a + b)*d) +
(2*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2)) - (2*b*Sin[c + d*x])/(a^2*d*Sqr
t[Cos[c + d*x]])
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
```

```
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))}\,dx = \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-\frac{3b}{2}+\frac{1}{2}a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}dx}{3a}$$

$$= \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}(a^2+3b^2)+ab\cos(c+dx)+\frac{3}{4}b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{3a^2}$$

$$= \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{-\frac{1}{4}b(a^2+3b^2)-\frac{1}{4}ab^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}dx}{3a^2b} + \frac{b\int}{}$$

$$= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{2b\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}dx}{3a}$$

$$= \frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{2b^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d} + \frac{2\sin}{3ad\cos}$$

Mathematica [A] time = 4.39631, size = 214, normalized size = 1.67

$$\frac{\frac{2(2a^2+9b^2)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{6\sin(c+dx)\left((2a^2-b^2)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right.}{a\sqrt{\sin^2(c+dx)}}}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])),x]
```

```
[Out] ((2*(2*a^2 + 9*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 8*
a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2])/(a + b)) + (4*(a - 3*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)
 + (6*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*Ellip
ticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -Ar
cSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/(6*a
^2*d)
```

Maple [B] time = 7.319, size = 452, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/a^2*b^3/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-2/a^2*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
```

```
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.588 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=245

$$\frac{\left(-16a^2b^2+15a^4-2b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}-\frac{a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{a^3\left(5a^2-7b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d(a-b)(a+b)^2}-\frac{a}{bd}$$

```
[Out] -((a*(5*a^2 - 4*b^2)*EllipticE[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)*d)) + ((15
*a^4 - 16*a^2*b^2 - 2*b^4)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d)
 - (a^3*(5*a^2 - 7*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)
*b^4*(a + b)^2*d) + ((5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^
2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(
a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.704841, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-16a^2b^2+15a^4-2b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4d\left(a^2-b^2\right)}-\frac{a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}-\frac{a^3\left(5a^2-7b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^4d(a-b)(a+b)^2}-\frac{a}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -((a*(5*a^2 - 4*b^2)*EllipticE[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)*d)) + ((15
*a^4 - 16*a^2*b^2 - 2*b^4)*EllipticF[(c + d*x)/2, 2])/(3*b^4*(a^2 - b^2)*d)
 - (a^3*(5*a^2 - 7*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)
*b^4*(a + b)^2*d) + ((5*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^
2*(a^2 - b^2)*d) - (a^2*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(
a + b*Cos[c + d*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
```

```
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = -\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a^2}{2}-ab\cos(c+dx)-\frac{1}{2}\left(5a^2-2b^2\right)\cos^2(c+dx)\right)}{a+b\cos(c+dx)}dx}{b\left(a^2-b^2\right)}$$

$$= \frac{\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{2\int \frac{-\frac{1}{4}a\left(5a^2-2b^2\right.}{}}{}$$

$$= \frac{\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{2\int \frac{\frac{1}{4}ab\left(5a^2-2b^2\right.}{\sqrt{\mathrm{co}}}}{}$$

$$= -\frac{a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{\left(5a^2-2b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} - \frac{a^2\cos^{\frac{3}{2}}(c+}{b\left(a^2-b^2\right)d(a}$$

$$= -\frac{a\left(5a^2-4b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{\left(15a^4-16a^2b^2-2b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^4\left(a^2-b^2\right)d} - \frac{a^3\left(5a^2-7b^2\right.}{(a-}$$

Mathematica [A] time = 1.81864, size = 270, normalized size = 1.1

$$\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(\frac{3a^3}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2\right) - \frac{\frac{2\left(5a^3-8ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{8\left(2a^2+b^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} + \frac{6\left(5a^2-\right.}{}}{}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2 + (3*a^3)/((a^2 - b^2)*(a + b*Cos[c + d*x])))*Sin[
c + d*x] - ((2*(5*a^3 - 8*a*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])
/(a + b) + (8*(2*a^2 + b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*Elliptic
Pi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(5*a^2 - 4*b^2)*(-2*a*b*El
lipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt
[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c
+ d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))
/(12*b^2*d)
```

Maple [B] time = 8.191, size = 1070, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^2*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-4*(a+b)/b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
```

```
c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2)))+2*(3*a^2+2*a*b+b^2)/b^4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16/b^3*a^3/(-2*a*b+2*b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))+2/b^4*a^4*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(
a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+
2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^2, x)
```

3.589 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=185

$$-\frac{a\left(3a^2-4b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{a^2\left(3a^2-5b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a-b)(a+b)^2}-\frac{a^2\sin(c+dx)\sqrt{}}{bd\left(a^2-b^2\right)(a+}$$

```
[Out] ((3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - (a*(3*a^2
 - 4*b^2)*EllipticF[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)*d) + (a^2*(3*a^2 - 5*
b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) -
 (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
))
```

Rubi [A] time = 0.458616, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2792, 3059, 2639, 3002, 2641, 2805}

$$-\frac{a\left(3a^2-4b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}+\frac{a^2\left(3a^2-5b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a-b)(a+b)^2}-\frac{a^2\sin(c+dx)\sqrt{}}{bd\left(a^2-b^2\right)(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] ((3*a^2 - 2*b^2)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - (a*(3*a^2
 - 4*b^2)*EllipticF[(c + d*x)/2, 2])/(b^3*(a^2 - b^2)*d) + (a^2*(3*a^2 - 5*
b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) -
 (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Cos[c + d*x]
))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx &= -\frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{a^2}{2}-ab\cos(c+dx)-\frac{1}{2}\left(3a^2-2b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b\left(a^2-b^2\right)}\\
&= -\frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{a^2b}{2}-\frac{1}{2}a\left(3a^2-4b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b^2\left(a^2-b^2\right)} + \frac{\left(3a^2-2b^2\right)\int\sqrt{\cos(c \cdots}}{2b^2\left(a^2-b^2 \cdots\right)}\\
&= \frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\left(a^2\left(3a^2-5b^2\right)\right)\int\frac{\cdots}{\sqrt{\cos \cdots}}}{2b^3\left(a^2 \cdots\right)}\\
&= \frac{\left(3a^2-2b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{a\left(3a^2-4b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} + \frac{a^2\left(3a^2-5b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}\cdots\right)}{(a-b)b^3(a+b)\cdots}
\end{aligned}$$

Mathematica [A] time = 1.74727, size = 255, normalized size = 1.38

$$\frac{\frac{4a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(b^2-a^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(a^2-2b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\left(3a^2-2b^2\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-}\cdots\right.\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}\cdots}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x])
) + ((2*(a^2 - 2*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) +
4*a*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)
```

```
/2, 2])/(a + b)) + (2*(3*a^2 - 2*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a
^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x]
)/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b)))/(4*b*d)
```

Maple [B] time = 8.118, size = 815, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-12/b^2*a^2/(-2*a*b+2*b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2))-2/b^3*a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+
b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*
b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

3.590 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=163

$$\frac{\left(a^2-2b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{a\left(a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] -((a*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d)) + ((a^2 - 2*b^2)*Ellipti
cF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - (a*(a^2 - 3*b^2)*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2])/((a - b)*b^2*(a + b)^2*d) + (a*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.384421, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2799, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(a^2-2b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}-\frac{a\left(a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}+\frac{a\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((a*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d)) + ((a^2 - 2*b^2)*Ellipti
cF[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d) - (a*(a^2 - 3*b^2)*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2])/((a - b)*b^2*(a + b)^2*d) + (a*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{a}{2}+b\cos(c+dx)+\frac{1}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{-a^2+b^2}$$

$$= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{ab}{2}+\frac{1}{2}\left(a^2-2b^2\right)\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b\left(a^2-b^2\right)} - \frac{a\int \sqrt{\cos(c+dx)}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\left(a\left(a^2-3b^2\right)\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}}{2b^2\left(a^2-b^2\right)}$$

$$= -\frac{aE\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} + \frac{\left(a^2-2b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\left(a^2-b^2\right)d} - \frac{a\left(a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b^2(a+b)^2d} +$$

Mathematica [A] time = 3.08906, size = 198, normalized size = 1.21

$$\frac{\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} - \frac{\frac{2\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{b^2\sqrt{\sin^2(c+dx)}} - \frac{10a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}}{(a-b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x])) -
 (8*EllipticF[(c + d*x)/2, 2] - (10*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2])/(a + b) + (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*
(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticP
i[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c +
```

```
d*x]^2]))/((a - b)*(a + b)))/(4*d)
```

Maple [B] time = 7.099, size = 794, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8/b*a
/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2/b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x
+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)
*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

3.591 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=148

$$\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{\left(a^2+b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] EllipticE[(c + d*x)/2, 2]/((a^2 - b^2)*d) + (a*EllipticF[(c + d*x)/2, 2])/(
b*(a^2 - b^2)*d) - ((a^2 + b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
((a - b)*b*(a + b)^2*d) - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*
d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.396754, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2796, 3059, 2639, 3002, 2641, 2805}

$$\frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{\left(a^2+b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a-b)(a+b)^2}-\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] EllipticE[(c + d*x)/2, 2]/((a^2 - b^2)*d) + (a*EllipticF[(c + d*x)/2, 2])/(
b*(a^2 - b^2)*d) - ((a^2 + b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
((a - b)*b*(a + b)^2*d) - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*
d*(a + b*Cos[c + d*x]))
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{b}{2}-a\cos(c+dx)-\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{-a^2+b^2}$$

$$= -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)}\,dx}{2\left(a^2-b^2\right)} + \frac{\int \frac{-\frac{b^2}{2}+\frac{1}{2}ab\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} - \frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{a\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2b\left(a^2-b^2\right)} - \frac{\left(a^2+b^2\right)\int \frac{}{\sqrt{\cos(c}}}{2b\,(a}$$

$$= \frac{E\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} + \frac{aF\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d} - \frac{\left(a^2+b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{(a-b)b(a+b)^2d} - \frac{b\sqrt{\cos(c+dx}}{\left(a^2-b^2\right)d(a+}$$

Mathematica [A] time = 3.44337, size = 233, normalized size = 1.57

$$\frac{\frac{4b\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(b^2-a^2\right)(a+b\cos(c+dx))} - \frac{2\left(\frac{\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)}{a\sqrt{\sin^2(c+dx)}} - \frac{b^2\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right.}{a+b}}{b(b-a)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((4*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x]))
- (2*(-((b^2*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + 2*a*(2*E
llipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/
(a + b)) + ((-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)
*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a
), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2])
))/(b*(-a + b)*(a + b)))/(4*d)
```

Maple [B] time = 6.307, size = 713, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/(-2*a*b+2*b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))-2*a/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/
2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2
*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

3.592 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2-b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

```
[Out] -((b*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - EllipticF[(c + d*x)/2,
 2]/((a^2 - b^2)*d) + ((3*a^2 - b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(a*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.436418, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2802, 3059, 2639, 3002, 2641, 2805}

$$-\frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}+\frac{\left(3a^2-b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a-b)(a+b)^2}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -((b*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d)) - EllipticF[(c + d*x)/2,
 2]/((a^2 - b^2)*d) + ((3*a^2 - b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
 2])/(a*(a - b)*(a + b)^2*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^
2 - b^2)*d*(a + b*Cos[c + d*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx &= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2-b^2\right)-ab\cos(c+dx)-\frac{1}{2}b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{a\left(a^2-b^2\right)} \\
&= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}b\left(2a^2-b^2\right)+\frac{1}{2}ab^2\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx}{ab\left(a^2-b^2\right)} - \frac{b\int \sqrt{\cos \ldots}}{2a\left(a \ldots\right.} \\
&= -\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\cos(c+dx))} - \frac{\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{2\left(a^2-b^2\right)} + \ldots \\
&= -\frac{bE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{F\left(\frac{1}{2}(c+dx)\middle|2\right)}{\left(a^2-b^2\right)d} + \frac{\left(3a^2-b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a-b)(a+b)^2d} + \ldots
\end{aligned}
$$

Mathematica [A] time = 3.27017, size = 242, normalized size = 1.54

$$
\frac{\frac{4b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{\frac{2\left(4a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{2\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-2abE\left(\sin^{-1}\left(\sqrt{\cos(c \ldots\right.\right.\right.}{a\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c + d*x]))
 + ((2*(4*a^2 - 3*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) +
 8*a*(-EllipticF[(c + d*x)/2, 2] + (a*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2])/(a + b)) - (2*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a
*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*Elliptic
```

```
Pi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c +
d*x]^2]))/((a - b)*(a + b)))/(4*a*d)
```

Maple [B] time = 4.931, size = 612, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*b^2/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b/(a^2-b^2)/a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-6*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a/(a^2-b^2)/(-2*a*b
+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)
```

3.593 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{b\left(5a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b$$

```
[Out] -(((2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d)) + (b*Ell
ipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - (b*(5*a^2 - 3*b^2)*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2 - 3*b^2)
*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b^2*Sin[c + d*x])/
(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.683318, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}-\frac{b\left(5a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}+\frac{b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] -(((2*a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d)) + (b*Ell
ipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) - (b*(5*a^2 - 3*b^2)*EllipticPi[(
2*b)/(a + b), (c + d*x)/2, 2])/(a^2*(a - b)*(a + b)^2*d) + ((2*a^2 - 3*b^2)
*Sin[c + d*x])/(a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (b^2*Sin[c + d*x])/
(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x]))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2-3b^2\right)-ab\cos(c+dx)+\frac{1}{2}b^2\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} + \frac{2}{}$$

$$= \frac{\left(2a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))} - \frac{2}{}$$

$$= -\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{\left(2a^2-3b^2\right)\sin(c+dx)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{b}{a\left(a^2-b^2\right)d\sqrt{\cos}}$$

$$= -\frac{\left(2a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d} + \frac{bF\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d} - \frac{b\left(5a^2-3b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{a^2(a-b)(a+b)^2 d}$$

Mathematica [A] time = 2.92983, size = 282, normalized size = 1.3

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{b^3\sin(c+dx)}{\left(b^2-a^2\right)(a+b\cos(c+dx))}+2\tan(c+dx)\right) - \frac{\frac{2\left(9b^3-10a^2b\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{\left(8ab^2-4a^3\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b} - \frac{2\left(2a^2-3b^2\right)}{}}{}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (-(((2*(-10*a^2*b + 9*b^3)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a +
b) + ((-4*a^3 + 8*a*b^2)*(2*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*
b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b - (2*(2*a^2 - 3*b^2)*(-2*a*b*Ellip
ticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d
*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b))) +
 4*Sqrt[Cos[c + d*x]]*((b^3*Sin[c + d*x])/((-a^2 + b^2)*(a + b*Cos[c + d*x]
)) + 2*Tan[c + d*x]))/(4*a^2*d)
```

Maple [B] time = 9.122, size = 874, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/a^2*b^2/(-2*a
*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))+2/a^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c
)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/a*b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*
c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(
a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b
/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(3/2)), x)
```

3.594 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2} dx$

Optimal. Leaf size=281

$$\frac{(2a^2-5b^2) F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(a^2-b^2)}+\frac{b(4a^2-5b^2) E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)}+\frac{b^2(7a^2-5b^2)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}+\frac{}{ad(a^2-b^2)\cos}$$

```
[Out] (b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + ((2*a^2
 - 5*b^2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*(a^2 - b^2)*d) + (b^2*(7*a^2 -
5*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d)
 + ((2*a^2 - 5*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
- (b*(4*a^2 - 5*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) +
 (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
))
```

Rubi [A] time = 0.995391, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(2a^2-5b^2) F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d(a^2-b^2)}+\frac{b(4a^2-5b^2) E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)}+\frac{b^2(7a^2-5b^2)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}+\frac{}{ad(a^2-b^2)\cos}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```


```
[Out] (b*(4*a^2 - 5*b^2)*EllipticE[(c + d*x)/2, 2])/(a^3*(a^2 - b^2)*d) + ((2*a^2
 - 5*b^2)*EllipticF[(c + d*x)/2, 2])/(3*a^2*(a^2 - b^2)*d) + (b^2*(7*a^2 -
5*b^2)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d)
 + ((2*a^2 - 5*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
- (b*(4*a^2 - 5*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) +
 (b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]
))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
```

```
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^2}\,dx = \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}\left(2a^2-5b^2\right)-ab\cos(c+dx)+\frac{3}{2}b^2\cos\ldots}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx\ldots}}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(4a^2-5b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{\ldots}{a\left(a^2-b^2\right)d\cos\ldots}$$

$$= \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(4a^2-5b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} + \frac{\ldots}{a\left(a^2-b^2\right)d\cos\ldots}$$

$$= \frac{b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(4a^2-5b\ldots\right.}{a^3\left(a^2-b^2\right)\ldots}$$

$$= \frac{b\left(4a^2-5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\left(7a^2-5b\ldots\right.}{a^3(\ldots}$$

Mathematica [A] time = 3.63652, size = 298, normalized size = 1.06

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{3b^4\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}+2\tan(c+dx)(a\sec(c+dx)-6b)\right)+\frac{\frac{2\left(44a^2b^2+4a^4-45b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(7a^3-10ab^2\right)\left((a+b)F(\ldots\right.}{\ldots}}{\ldots}}{12a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^2),x]
```

```
[Out] (((2*(4*a^4 + 44*a^2*b^2 - 45*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
])/(a + b) + (8*(7*a^3 - 10*a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(4*a^2 - 5*b^2)*(-2
*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcS
in[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt
[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a +
 b)) + 4*Sqrt[Cos[c + d*x]]*((3*b^4*Sin[c + d*x])/((a^2 - b^2)*(a + b*Cos[c
 + d*x])) + 2*(-6*b + a*Sec[c + d*x])*Tan[c + d*x]))/(12*a^3*d)
```

Maple [B] time = 11.783, size = 1008, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8/a^3*b^3/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
```

```
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-4/a^3*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a^2*b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d
*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^
2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2)))+2/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^2\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*cos(d*x + c)^(5/2)), x)
```

3.595 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=346

$$\frac{\left(-223a^4b^2+128a^2b^4+105a^6+8b^6\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^5d\left(a^2-b^2\right)^2}-\frac{a\left(-65a^2b^2+35a^4+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{a^3\left(-86a^2b^2+35a\right.}{4b^5}$$

```
[Out] -(a*(35*a^4 - 65*a^2*b^2 + 24*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 -
 b^2)^2*d) + ((105*a^6 - 223*a^4*b^2 + 128*a^2*b^4 + 8*b^6)*EllipticF[(c +
d*x)/2, 2])/(12*b^5*(a^2 - b^2)^2*d) - (a^3*(35*a^4 - 86*a^2*b^2 + 63*b^4)*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^5*(a + b)^3*d) +
((35*a^4 - 61*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*b^3*(a^
2 - b^2)^2*d) - (a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x])^2) - (a^2*(7*a^2 - 13*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.03923, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2792, 3047, 3049, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-223a^4b^2+128a^2b^4+105a^6+8b^6\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^5d\left(a^2-b^2\right)^2}-\frac{a\left(-65a^2b^2+35a^4+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{a^3\left(-86a^2b^2+35a\right.}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(a + b*Cos[c + d*x])^3,x]
```



```
[Out] -(a*(35*a^4 - 65*a^2*b^2 + 24*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^4*(a^2 -
 b^2)^2*d) + ((105*a^6 - 223*a^4*b^2 + 128*a^2*b^4 + 8*b^6)*EllipticF[(c +
d*x)/2, 2])/(12*b^5*(a^2 - b^2)^2*d) - (a^3*(35*a^4 - 86*a^2*b^2 + 63*b^4)*
EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^5*(a + b)^3*d) +
((35*a^4 - 61*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(12*b^3*(a^
2 - b^2)^2*d) - (a^2*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a
 + b*Cos[c + d*x])^2) - (a^2*(7*a^2 - 13*b^2)*Cos[c + d*x]^(3/2)*Sin[c + d*
x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{5a^2}{2}-2ab\cos(c+dx)-\frac{1}{2}\left(7a^2-4b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(7a^2-13b^2\right)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \int \frac{\sqrt{\cos(c+dx}}{}$$

$$= \frac{\left(35a^4-61a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2(7}{4l}$$

$$= \frac{\left(35a^4-61a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2(7}{4l}$$

$$= -\frac{a\left(35a^4-65a^2b^2+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d} + \frac{\left(35a^4-61a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\sin(c+a}{12b^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a\left(35a^4-65a^2b^2+24b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d} + \frac{\left(105a^6-223a^4b^2+128a^2b^4+8b^6\right)F\left(\frac{1}{2}(c+\right.}{12b^5\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 3.06557, size = 358, normalized size = 1.03

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(ab\left(-83a^2b^2+49a^4+16b^4\right)\cos(c+dx)+4\left(b^3-a^2b\right)^2\cos(2(c+dx))-57a^4b^2+35a^6+4b^6\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{\frac{2\left(-73a^3b^2+35a^5+56ab^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16(-14a^2}{}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(35*a^6 - 57*a^4*b^2 + 4*b^6 + a*b*(49*a^4 - 83*a^2*
b^2 + 16*b^4)*Cos[c + d*x] + 4*(-(a^2*b) + b^3)^2*Cos[2*(c + d*x)])*Sin[c +
 d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(35*a^5 - 73*a^3*b^2 +
56*a*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(7*a^4 -
 14*a^2*b^2 - 2*b^4)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b
)/(a + b), (c + d*x)/2, 2]))/(a + b) + (6*(35*a^4 - 65*a^2*b^2 + 24*b^4)*(-
2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[Arc
Sin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqr
t[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2
*(a + b)^2))/(48*b^3*d)
```

Maple [B] time = 14.392, size = 2194, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/3/b^3*(2*sin(
1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2/b^4*(3*a+2*b)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+2*(6*a^2+3*a*b+b^2)/b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+40*a^3/b^4/(-2*a*b+2*b^2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))-2/b^5*a^5*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2
-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9
/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*
b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2
))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(
-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+10/b^5*a^4*(-1/a*b^2/(a^2-b^2)*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2
*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*
b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^{\frac{9}{2}}}{(b\cos (dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c) + a)^3, x)
```

3.596 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=282

$$-\frac{3a\left(-11a^2b^2+5a^4+8b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2b^2+15a^4+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{a^2\left(-38a^2b^2+15a^4+35b^4\right)}{4b^4d(a-b)^2(}$$

```
[Out] ((15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2
)^2*d) - (3*a*(5*a^4 - 11*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(4*b^
4*(a^2 - b^2)^2*d) + (a^2*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/(
a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^4*(a + b)^3*d) - (a^2*Cos[c + d*x]^
(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(5*a^
2 - 11*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*
Cos[c + d*x]))
```

Rubi [A] time = 0.77447, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3047, 3059, 2639, 3002, 2641, 2805}

$$-\frac{3a\left(-11a^2b^2+5a^4+8b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}+\frac{\left(-29a^2b^2+15a^4+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{a^2\left(-38a^2b^2+15a^4+35b^4\right)}{4b^4d(a-b)^2(}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] ((15*a^4 - 29*a^2*b^2 + 8*b^4)*EllipticE[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2
)^2*d) - (3*a*(5*a^4 - 11*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(4*b^
4*(a^2 - b^2)^2*d) + (a^2*(15*a^4 - 38*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/(
a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^4*(a + b)^3*d) - (a^2*Cos[c + d*x]^
(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) - (a^2*(5*a^
2 - 11*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*
Cos[c + d*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
```

```
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3a^2}{2}-2ab\cos(c+dx)-\frac{1}{2}\left(5a^2-4b^2\right)\cos^2(c+dx)\right)}{(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{4}a^2(}{}}{}$$

$$= -\frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(5a^2-11b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4b^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{1}{4}a^2b(}{}}{}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{a^2\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{a^2\left(5a^2-1\right.}{4b^2\left(a^2\right.}$$

$$= \frac{\left(15a^4-29a^2b^2+8b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{3a\left(5a^4-11a^2b^2+8b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2 d} + \frac{a^2\left(\right.}{}$$

Mathematica [A] time = 2.71434, size = 313, normalized size = 1.11

$$\frac{\frac{\frac{\left(-7a^2b^2+5a^4+8b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(a^3-4ab^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{\left(-29a^2b^2+15a^4+8b^4\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+\right.}{ab^2\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*a^2*Sqrt[Cos[c + d*x]]*(5*a^3 - 11*a*b^2 + b*(7*a^2 - 13*b^2)*Cos[c +
d*x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + (((5*a^4 - 7*a
^2*b^2 + 8*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(a^
3 - 4*a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2]))/(a + b) + ((15*a^4 - 29*a^2*b^2 + 8*b^4)*(-2*a*b*Ellip
ticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d
*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^
2))/(8*b^2*d)
```

Maple [B] time = 12.78, size = 1935, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-24/b^3*a^2/(-2*a*b+2*b^2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
```

```
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2))+2/b^4*a^4*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b
^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b
^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(
a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^
2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2
/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b
+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2)))-8/b^4*a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/
2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a
/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2
*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c) + a)^3, x)
```

3.597 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=264

$$\frac{(-5a^2b^2+3a^4+8b^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\,(a^2-b^2)^2}-\frac{3a\,(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\,(a^2-b^2)^2}-\frac{3a\,(-2a^2b^2+a^4+5b^4)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a-b)^2(a+b)^3}-$$

```
[Out] (-3*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((
3*a^4 - 5*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*
d) - (3*a*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*(a - b)^2*b^3*(a + b)^3*d) - (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.777197, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2792, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{(-5a^2b^2+3a^4+8b^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\,(a^2-b^2)^2}-\frac{3a\,(a^2-3b^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\,(a^2-b^2)^2}-\frac{3a\,(-2a^2b^2+a^4+5b^4)\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a-b)^2(a+b)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (-3*a*(a^2 - 3*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((
3*a^4 - 5*a^2*b^2 + 8*b^4)*EllipticF[(c + d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*
d) - (3*a*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2,
2])/(4*(a - b)^2*b^3*(a + b)^3*d) - (a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
2*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^2) + (3*a*(a^2 - 3*b^2)*Sqrt[Cos[c +
 d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
```

```
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = -\frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{a^2}{2}-2ab\cos(c+dx)-\frac{1}{2}\left(3a^2-4b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{1}{4}a^2\left(a^2-7b^2\right)\cdots}{\cdots}}{\cdots}$$

$$= -\frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{1}{4}a^2b\left(a^2-7b^2\right)\cdots}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} - \frac{a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3a\left(a^2-3b^2\right)\sqrt{\cos(c\cdots}}{4b\left(a^2-b^2\right)^2 d(a+\cdots}$$

$$= -\frac{3a\left(a^2-3b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} + \frac{\left(3a^4-5a^2b^2+8b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2 d} - \frac{3a\left(a^4-2a^2b^2+5\cdots\right.}{4(a-b\cdots}$$

Mathematica [A] time = 1.99403, size = 288, normalized size = 1.09

$$\frac{\frac{4a\sin(c+dx)\sqrt{\cos(c+dx)}\left(3b\left(a^2-3b^2\right)\cos(c+dx)+a^3-7ab^2\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{\frac{2\left(a^3+5ab^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{16\left(a^2+2b^2\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} + \frac{6\left(a^2-3b^2\right)\sin(c+dx)\left(\cdots\right)}{\cdots}}{(a-b)^2(\cdots}}{16bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((4*a*Sqrt[Cos[c + d*x]]*(a^3 - 7*a*b^2 + 3*b*(a^2 - 3*b^2)*Cos[c + d*x])*S
in[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((2*(a^3 + 5*a*b^2)*E
llipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) - (16*(a^2 + 2*b^2)*((a +
 b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
))/(a + b) + (6*(a^2 - 3*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]],
 -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2
)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b^2*S
qrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*b*d)
```

Maple [B] time = 11.211, size = 1914, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12/b^
2*a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/b^3*a^3*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/
```

```
2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2
*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a
^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2
*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-
3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+6/b^3*a^2*(-1/a*b^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/
(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

3.598 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=244

$$\frac{a\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}-\frac{\left(-10a^2b^2+a^4-3b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a-b)^2(a+b)^3}+\frac{\left(a^2+5b^2\right)}{4d\left(a^2-\right.}$$

```
[Out] -((a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (a*(a^2
- 7*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((a^4 - 10*a^
2*b^2 - 3*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^2*
(a + b)^3*d) + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2) + ((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*(a^2
- b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.657471, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2799, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{a\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}-\frac{\left(-10a^2b^2+a^4-3b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d(a-b)^2(a+b)^3}+\frac{\left(a^2+5b^2\right)}{4d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] -((a^2 + 5*b^2)*EllipticE[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) + (a*(a^2
- 7*b^2)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) - ((a^4 - 10*a^
2*b^2 - 3*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^2*
(a + b)^3*d) + (a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*
Cos[c + d*x])^2) + ((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*(a^2
- b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{-\frac{a}{2}+2b\cos(c+dx)-\frac{1}{2}a\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int \frac{-\frac{3}{4}a\left(a^2+b^2\right)+3a \cdots}{\sqrt{c \cdots}}}{\cdots}$$

$$= \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{\frac{3}{4}ab\left(a^2+b^2\right)+\frac{1}{4}\cdots}{\sqrt{\cos(c+dx \cdots}}}{2ab \cdots}$$

$$= -\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{a\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\left(a^2+5b^2\right)\sqrt{\cos(c+ \cdots}}{4\left(a^2-b^2\right)^2 d(a+b \cdots}$$

$$= -\frac{\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} + \frac{a\left(a^2-7b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2 d} - \frac{\left(a^4-10a^2b^2-3b^4\right)\Pi\left(\frac{2 \cdots}{a+ \cdots}\right.}{4(a-b)^2b^2(a+ \cdots}$$

Mathematica [A] time = 1.82911, size = 276, normalized size = 1.13

$$\frac{\frac{4\sin(c+dx)\sqrt{\cos(c+dx)}\left(b\left(a^2+5b^2\right)\cos(c+dx)+3a\left(a^2+b^2\right)\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} - \frac{-\frac{2\left(5a^2+b^2\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{2\left(a^2+5b^2\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F \cdots\right.}{ab^2\sqrt{\sin^2(c+dx \cdots}}}{(a-b)^2 \cdots}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(3*a*(a^2 + b^2) + b*(a^2 + 5*b^2)*Cos[c + d*x])*Sin
[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) - ((-2*(5*a^2 + b^2)*Elli
pticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + 24*a*(2*EllipticF[(c + d*x
)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)) + (2*(a^
2 + 5*b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*
EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a)
, -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b^2*Sqrt[Sin[c + d*x]^
2]))/((a - b)^2*(a + b)^2))/(16*d)
```

Maple [B] time = 10.702, size = 1836, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/b/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))+2*a^2/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2
+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8
```

```
/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b
^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-4*a/b^2*(-1/a*b^2/(a^2-b^2)*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos
(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*
b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^{\frac{3}{2}}}{(b\cos (dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

3.599 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=250

$$\frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}-\frac{\left(10a^2b^2+3a^4-b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4abd(a-b)^2(a+b)^3}-\frac{b\left(5a^2+b^2\right)\sin}{4ad\left(a^2-b^2\right)^2}$$

```
[Out] ((5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + (3*(a^2 +
 b^2)*EllipticF[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) - ((3*a^4 + 10*a^2*b
^2 - b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b*(a +
b)^3*d) - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) - (b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 -
 b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.681922, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2796, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4bd\left(a^2-b^2\right)^2}+\frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad\left(a^2-b^2\right)^2}-\frac{\left(10a^2b^2+3a^4-b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4abd(a-b)^2(a+b)^3}-\frac{b\left(5a^2+b^2\right)\sin}{4ad\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] ((5*a^2 + b^2)*EllipticE[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + (3*(a^2 +
 b^2)*EllipticF[(c + d*x)/2, 2])/(4*b*(a^2 - b^2)^2*d) - ((3*a^4 + 10*a^2*b
^2 - b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a*(a - b)^2*b*(a +
b)^3*d) - (b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cos[c
 + d*x])^2) - (b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 -
 b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
```

```
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]
) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{\int \frac{\frac{b}{2}-2a\cos(c+dx)+\frac{1}{2}b\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} - \frac{\int \frac{\frac{1}{4}b\left(7a^2-b^2\right)-a(2}{}}{}$$

$$= -\frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))} + \frac{\int \frac{-\frac{1}{4}b^2\left(7a^2-b^2\right)+}{\sqrt{\cos(c+dx}}}{2ab}$$

$$= \frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} - \frac{b\sqrt{\cos(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} - \frac{b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\,s}{4a\left(a^2-b^2\right)^2 d(a+b\cos}$$

$$= \frac{\left(5a^2+b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b\left(a^2-b^2\right)^2 d} - \frac{\left(3a^4+10a^2b^2-b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c\right.}{4a(a-b)^2b(a+b)^3d}$$

Mathematica [A] time = 2.80725, size = 295, normalized size = 1.18

$$\frac{\frac{\frac{2\left(3b^3-9a^2b\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8a\left(2a^2+b^2\right)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{b}+\frac{2\left(5a^2+b^2\right)\sin(c+dx)\left(\left(2a^2-b^2\right)\Pi\left(-\frac{b}{a};-\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2a(a+b)F\left(\sin^{-1}\left(\sqrt{\cos(c+dx)}\right)\middle|\right.\right.}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-4*b*Sqrt[Cos[c + d*x]]*(7*a^3 - a*b^2 + b*(5*a^2 + b^2)*Cos[c + d*x])*Si
n[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(-9*a^2*b + 3*b^3)
*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*a*(2*a^2 + b^2)*(2
*EllipticF[(c + d*x)/2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
)/(a + b)))/b + (2*(5*a^2 + b^2)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 -
b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*
b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a*d)
```

Maple [B] time = 10.494, size = 1736, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a/b*(-1/2/a*
b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^
2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
```

```
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2
)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2
/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2
)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)
))+2/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*
b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-
b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

3.600 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$-\frac{(7a^2-b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad(a^2-b^2)^2}-\frac{3b(3a^2-b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a^2-b^2)^2}+\frac{3(-2a^2b^2+5a^4+b^4)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a-b)^2(a+b)^3}+\frac{3b^2(3a}{4a^2d}$$

```
[Out] (-3*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((
7*a^2 - b^2)*EllipticF[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + (3*(5*a^4 -
 2*a^2*b^2 + b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)
^2*(a + b)^3*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^2) + (3*b^2*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 0.77485, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$-\frac{(7a^2-b^2)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ad(a^2-b^2)^2}-\frac{3b(3a^2-b^2)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a^2-b^2)^2}+\frac{3(-2a^2b^2+5a^4+b^4)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a-b)^2(a+b)^3}+\frac{3b^2(3a}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```


```
[Out] (-3*b*(3*a^2 - b^2)*EllipticE[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) - ((
7*a^2 - b^2)*EllipticF[(c + d*x)/2, 2])/(4*a*(a^2 - b^2)^2*d) + (3*(5*a^4 -
 2*a^2*b^2 + b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(4*a^2*(a - b)
^2*(a + b)^3*d) + (b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^2) + (3*b^2*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Sin[c + d
*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
```

```
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2-3b^2\right)-2ab\cos(c+dx)+\frac{1}{2}b^2\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx}{4a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$

$$= \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sin(c+dx}{4a^2\left(a^2-b^2\right)^2 d(a+b\cos(c+dx))}$$

$$= -\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} + \frac{b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\cos(c+dx))^2} + \frac{3b^2\,(}{4a}$$

$$= -\frac{3b\left(3a^2-b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} - \frac{\left(7a^2-b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a\left(a^2-b^2\right)^2 d} + \frac{3\left(5a^4-2a\right.}{}$$

Mathematica [A] time = 2.6346, size = 305, normalized size = 1.17

$$\frac{\frac{4b^2\sin(c+dx)\sqrt{\cos(c+dx)}\left(\left(9a^2b-3b^3\right)\cos(c+dx)+11a^3-5ab^2\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2} + \frac{\frac{2\left(-19a^2b^2+16a^4+9b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{16\left(ab^2-4a^3\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-a\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b} - 6}{}}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] ((4*b^2*Sqrt[Cos[c + d*x]]*(11*a^3 - 5*a*b^2 + (9*a^2*b - 3*b^3)*Cos[c + d*
x])*Sin[c + d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + ((2*(16*a^4 - 19
*a^2*b^2 + 9*b^4)*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*
(-4*a^3 + a*b^2)*((a + b)*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]))/(a + b) - (6*(3*a^2 - b^2)*(-2*a*b*EllipticE[ArcSi
n[Sqrt[Cos[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]
]], -1] + (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1]
)*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(16*a^2*d)
```

Maple [B] time = 6.707, size = 1176, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/a*b^2/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/2*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos
(1/2*d*x+1/2*c)^2+a-b)-7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a+b)/(a^2-b^2)/a*(sin(1
```

```
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+
3/4/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*b^3/a^2/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3/4*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/2*a^2/(a^2-b^2)
^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))-3/2/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(cos(d*x + c))), x)
```

3.601 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=328

$$\frac{b\left(11a^2-5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2b^2+8a^4+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{b\left(-38a^2b^2+35a^4+15b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{4a^3d(a-b)^2(a+b)^3}$$

```
[Out] -((8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^
2)^2*d) + (b*(11*a^2 - 5*b^2)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)
^2*d) - (b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Si
n[c + d*x])/(4*a^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b^2*Sin[c + d*x])
/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b^2*(11*a
^2 - 5*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*
Cos[c + d*x]))
```

Rubi [A] time = 1.07096, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{b\left(11a^2-5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-29a^2b^2+8a^4+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{b\left(-38a^2b^2+35a^4+15b^4\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx\right.}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```


```
[Out] -((8*a^4 - 29*a^2*b^2 + 15*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^
2)^2*d) + (b*(11*a^2 - 5*b^2)*EllipticF[(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)
^2*d) - (b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*EllipticPi[(2*b)/(a + b), (c + d*
x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Si
n[c + d*x])/(4*a^3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (b^2*Sin[c + d*x])
/(2*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^2) + (b^2*(11*a
^2 - 5*b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*(a + b*
Cos[c + d*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2-5b^2\right)-2ab\cos(c+dx)+\frac{3}{2}b^2\cos}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx)}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sin}{4a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+}$$

$$= -\frac{\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-29a^2b^2+15b^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}}$$

$$= -\frac{\left(8a^4-29a^2b^2+15b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2 d} + \frac{b\left(11a^2-5b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2 d} - \frac{b}{}$$

Mathematica [A] time = 3.45058, size = 338, normalized size = 1.03

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{b^3\sin(c+dx)\left(\left(7b^3-13a^2b\right)\cos(c+dx)-15a^3+9ab^2\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+8\tan(c+dx)\right)-\frac{\frac{2\left(-95a^2b^3+56a^4b+45b^5\right)\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8a\left(-10a^2b^2+2a^4+5b^4\right)\left(2\right.}{}}{}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] (-(((2*(56*a^4*b - 95*a^2*b^3 + 45*b^5)*EllipticPi[(2*b)/(a + b), (c + d*x)
/2, 2])/(a + b) + (8*a*(2*a^4 - 10*a^2*b^2 + 5*b^4)*(2*EllipticF[(c + d*x)/
2, 2] - (2*a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2])/(a + b)))/b + (2*(8
*a^4 - 29*a^2*b^2 + 15*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -
1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (2*a^2 - b^2)*
EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqr
t[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) + 4*Sqrt[Cos[c + d*x]]*((b^3*(-1
5*a^3 + 9*a*b^2 + (-13*a^2*b + 7*b^3)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b
^2)^2*(a + b*Cos[c + d*x])^2) + 8*Tan[c + d*x]))/(16*a^3*d)
```

Maple [B] time = 13.674, size = 1992, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2/a^3/(-2*a
*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))-2/a*b*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^
2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/
8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b
),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-
b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2/a^3*(-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/
sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/a^2*b*(-1/a*b^2/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a
^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/
(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{3}\cos\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(3/2)), x)
```

3.602 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=395

$$\frac{\left(-61a^2b^2+8a^4+35b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-65a^2b^2+24a^4+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b^2\left(-86a^2b^2+63a^4+35b^4\right)}{4a^4d(a-b)^2(}$$

```
[Out] (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 -
b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*EllipticF[(c + d*x)/2, 2])/(12*a
^3*(a^2 - b^2)^2*d) + (b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/
(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 61*a^2*
b^2 + 35*b^4)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)) - (
b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(
a + b*Cos[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sin[c + d*x])/(4*a^2*(a^2 -
b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```

Rubi [A] time = 1.35036, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2802, 3055, 3059, 2639, 3002, 2641, 2805}

$$\frac{\left(-61a^2b^2+8a^4+35b^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-65a^2b^2+24a^4+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b^2\left(-86a^2b^2+63a^4+35b^4\right)}{4a^4d(a-b)^2(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```


```
[Out] (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*EllipticE[(c + d*x)/2, 2])/(4*a^4*(a^2 -
b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*EllipticF[(c + d*x)/2, 2])/(12*a
^3*(a^2 - b^2)^2*d) + (b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*EllipticPi[(2*b)/
(a + b), (c + d*x)/2, 2])/(4*a^4*(a - b)^2*(a + b)^3*d) + ((8*a^4 - 61*a^2*
b^2 + 35*b^4)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)) - (
b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d*Sqrt[
Cos[c + d*x]]) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(
a + b*Cos[c + d*x])^2) + (b^2*(13*a^2 - 7*b^2)*Sin[c + d*x])/(4*a^2*(a^2 -
b^2)^2*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x]))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3059

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3002

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^3}\,dx = \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}\left(4a^2-7b^2\right)-2ab\cos(c+dx)+\frac{5}{2}}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^2} + \frac{b^2\left(13a^2-7b^2\right)}{4a^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c}$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} +$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} - \frac{b\left(24a^4-65a^2b^2+35b^4\right)\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}} +$$

$$= \frac{b\left(24a^4-65a^2b^2+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sin(c+}{12a^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx}$$

$$= \frac{b\left(24a^4-65a^2b^2+35b^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)F\left(\frac{1}{2}(c+\right.}{12a^3\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 5.33007, size = 353, normalized size = 0.89

$$\frac{4\sqrt{\cos(c+dx)}\left(\frac{3b^4\sin(c+dx)\left(b\left(17a^2-11b^2\right)\cos(c+dx)+19a^3-13ab^2\right)}{\left(a^2-b^2\right)^2(a+b\cos(c+dx))^2}+8\tan(c+dx)(a\sec(c+dx)-9b)\right)+\frac{\frac{2\left(328a^4b^2-641a^2b^4+16a^6+315b\right.}{a+b}}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^3),x]
```

```
[Out] (((2*(16*a^6 + 328*a^4*b^2 - 641*a^2*b^4 + 315*b^6)*EllipticPi[(2*b)/(a + b
), (c + d*x)/2, 2])/(a + b) + (16*(20*a^5 - 64*a^3*b^2 + 35*a*b^4)*((a + b)
*EllipticF[(c + d*x)/2, 2] - a*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]))/
(a + b) + (6*(24*a^4 - 65*a^2*b^2 + 35*b^4)*(-2*a*b*EllipticE[ArcSin[Sqrt[C
os[c + d*x]]], -1] + 2*a*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1]
+ (2*a^2 - b^2)*EllipticPi[-(b/a), -ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c
+ d*x])/(a*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2) + 4*Sqrt[Cos[c + d*
x]]*((3*b^4*(19*a^3 - 13*a*b^2 + b*(17*a^2 - 11*b^2)*Cos[c + d*x])*Sin[c +
d*x])/((a^2 - b^2)^2*(a + b*Cos[c + d*x])^2) + 8*(-9*b + a*Sec[c + d*x])*Ta
n[c + d*x]))/(48*a^4*d)
```

Maple [B] time = 18.621, size = 2128, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-12*b^3/a^4/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))+2/a^2*b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2
*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1
/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a
-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)
/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*
b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2
/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-6/a^4*b*(-(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/
2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+4/a^3*b^2*(-1/a*b^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/
(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2/a^3*(-1/6*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^3\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*cos(d*x + c)^(5/2)), x)
```

3.603 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=438

$$\frac{\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+}}{2}$$

```
[Out] -((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (Sqrt[a
+ b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (Sqrt[a + b]*(a
^2 - 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) + (a*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.9224, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2821, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (Sqrt[a
+ b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (Sqrt[a + b]*(a
^2 - 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d) + (a*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]]) + (Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx = \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\int \frac{\frac{ab}{2}+b^2\cos(c+dx)+\frac{1}{2}ab\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx}{2b}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4bd\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d}$$

$$= \frac{\sqrt{a+b}\left(a^2-4b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+d}{a+b}}}{4b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1-}{}}}{4bd}$$

Mathematica [C] time = 17.2838, size = 1152, normalized size = 2.63

$$-\frac{12a^2\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\sqrt{\frac{(a+b\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}}\csc(c+d}{(a+b)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+d}}$$

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + ((-12*a^
2*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]) - 16*a*b*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])) + 2*a*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c +
d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a
- b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*C
```

```
os[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(b*Sqrt[Cos[c + d*x]])))/(8*d)
```

Maple [B] time = 0.377, size = 1233, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/b*(2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*a*b-4*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^2+8*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+2*b^2*cos(d*x+c)^4+2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*
x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+8*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)+3*a*b*cos(d*x+c)^3+cos(d*x
+c)^2*a^2-cos(d*x+c)^2*a*b-2*b^2*cos(d*x+c)^2-a^2*cos(d*x+c)-2*cos(d*x+c)*a
*b)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.604 $\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=371

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-$$

```
[Out] -(((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[a
+ b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (a*Sqrt[a + b]*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.574802, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2821, 3054, 2809, 12, 2801, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[a
+ b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (a*Sqrt[a + b]*Cot[c + d*x]*Ell
ipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3054

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
```

```
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx = \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\int \frac{-\frac{ab}{2}+\frac{1}{2}ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}a\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}dx + \frac{\int -\frac{}{2\cos^{\frac{3}{2}}(c}}{}$$

$$= -\frac{a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{bd}$$

$$= -\frac{a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{bd}$$

$$= -\frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{ad}$$

Mathematica [A] time = 7.18976, size = 316, normalized size = 0.85

$$\frac{\sqrt{\cos(c+dx)}\left(-\frac{4a\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}} + \frac{2(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}} - 4a\sqrt{\frac{a+b\cos(c+}{(a+b)(\cos(c+}}\right.}{2d\sqrt{a+b\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((2*(a + b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])] - (4*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (4*a*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)
/(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + b*Sec[(c + d*x)/2]*Sin[(
3*(c + d*x))/2] + 2*a*Tan[(c + d*x)/2] - b*Tan[(c + d*x)/2]))/(2*d*Sqrt[a +
 b*Cos[c + d*x]])
```

Maple [B] time = 0.491, size = 801, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
```

```
(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+
c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(
d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*a*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*sin(d*x+c)+b*cos(d*x+c)^3+a*cos(d*x+c)^2-b*cos(d*x+c)^2-cos(d*x
+c)*a)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

3.605 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{2\csc(c+dx)\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right)|-\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

```
[Out] (-2*Sqrt[(a*(1 - Cos[c + d*x]))/(a + b*Cos[c + d*x])]*Sqrt[(a*(1 + Cos[c +
d*x]))/(a + b*Cos[c + d*x])]*(a + b*Cos[c + d*x])*Csc[c + d*x]*EllipticPi[b
/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]]]
, -((a - b)/(a + b))])/(Sqrt[a + b]*d)
```

Rubi [A] time = 0.0711268, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2811}

$$-\frac{2\csc(c+dx)\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right)|-\frac{a-b}{a+b}\right)}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-2*Sqrt[(a*(1 - Cos[c + d*x]))/(a + b*Cos[c + d*x])]*Sqrt[(a*(1 + Cos[c +
d*x]))/(a + b*Cos[c + d*x])]*(a + b*Cos[c + d*x])*Csc[c + d*x]*EllipticPi[b
/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[Cos[c + d*x]])/Sqrt[a + b*Cos[c + d*x]]]
, -((a - b)/(a + b))])/(Sqrt[a + b]*d)
```


Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx = -\frac{2\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(1+\cos(c+dx))}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\csc(c+dx)\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right)\right.}{\sqrt{a+b}d}$$

Mathematica [A] time = 1.15052, size = 139, normalized size = 1.03

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left((a-b)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2b\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*b*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.445, size = 197, normalized size = 1.5

$$-2\frac{(\sin(dx+c))^4}{d\sqrt{a+b\cos(dx+c)}\,(\cos(dx+c))^{3/2}\,(-1+\cos(dx+c))^2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\left(a\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(a+b*cos(d*x+c))^(1/2)*(a*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*b*EllipticPi((-1+cos(d*x+c))/sin(d*x+c
),-1,(-(a-b)/(a+b))^(1/2)))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

3.606 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=229

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b
)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rubi [A] time = 0.262857, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2795, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}-\frac{2(a-b)\sqrt{a+b}\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d) - (2*(a - b
)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[
a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x])
)/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
```

```
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + (-a+b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad} - \ldots$$

Mathematica [A] time = 3.19243, size = 203, normalized size = 0.89

$$-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\cos(c+dx)}\left(-\sin(c+dx)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}-\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(\ldots\right.\right.\right.\right.}{d\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] -((Sqrt[a + b*Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(Sqrt[Cos[c + d*x]]*Sqrt[1 +
 Cos[c + d*x]]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - Sqrt
[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
-a + b)/(a + b)] - Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]))
```

Maple [B] time = 0.483, size = 789, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(
```

```
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+
c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(
d*x+c)+b*cos(d*x+c)^2+cos(d*x+c)*a-b*cos(d*x+c)-a)/(a+b*cos(d*x+c))^(1/2)/s
in(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

3.607 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=271

$$\frac{2b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d) + (2*
(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.401727, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2796, 2998, 2816, 2994}

$$\frac{2b(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}+\frac{2\sin(c+dx)\sqrt{a+b\cos(c+}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d) + (2*
(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

```
&& NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}dx = \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{b}{2}+\frac{1}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}(a-b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}dx + \frac{1}{3}b$$

$$= \frac{2(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx)}{a-b}}}{3a^2d}$$

Mathematica [A] time = 7.42554, size = 247, normalized size = 0.91

$$\frac{\tan\left(\frac{1}{2}(c+dx)\right)\left(2a^2+2a(a+2b)\cos(c+dx)+b(a+b)\cos(2(c+dx))+ab+b^2\right)+2a(a+b)\sqrt{\cos(c+dx)+1}\cos^{\frac{3}{2}}(}{3a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*b*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] + 2*a*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[1 + Cos[c + d*x]]*Sqr
t[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] + (2*a^2 + a*b + b^2 + 2*a*(a + 2*b)*Cos[c
+ d*x] + b*(a + b)*Cos[2*(c + d*x)])*Tan[(c + d*x)/2])/(3*a*d*Cos[c + d*x]^
(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.542, size = 880, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d/a*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^2+a*b*cos(d*x+c)^3+cos(d*x+c)^3*b^2+cos(d*x+c)^2*a^2+cos(d*x+c)^2*a*b-b^2
*cos(d*x+c)^2-2*cos(d*x+c)*a*b-a^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d
*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(5/2), x)
```

3.608 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=329

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a-b)\sqrt{a+b}(9a}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(15*a^2*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c +
d*x]^(5/2)) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d
*x]^(3/2))
```

Rubi [A] time = 0.645671, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2796, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d}-\frac{2(a-b)\sqrt{a+b}(9a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
15*a^3*d) - (2*(a - b)*Sqrt[a + b]*(9*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(15*a^2*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c +
d*x]^(5/2)) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*Cos[c + d
*x]^(3/2))
```


Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{b}{2}+\frac{3}{2}a\cos(c+dx)+b\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}(9a^2-2b^2)+\frac{7}{4}ab}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b}}}{15a}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15ad\cos^{\frac{3}{2}}(c+dx)} - \frac{((a-b)(9a+2b))}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{15a^3d}$$

Mathematica [A] time = 12.9043, size = 453, normalized size = 1.38

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)\left(9a^2\sin(c+dx)-2b^2\sin(c+dx)\right)}{15a^2}+\frac{2b\tan(c+dx)\sec(c+dx)}{15a}+\frac{2}{5}\tan(c+dx)\sec^2(c+dx)\right)}{d}+$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(9*a^3 + 9*a^2*b - 2*a*b^2 - 2*b^3)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*
(9*a^2 + 7*a*b - 2*b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] - (9*a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[
c + d*x])^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Cos[c + d*x]]*((2*Sec[c + d*x]*(9*a^2*Sin[c + d*x] - 2*b^2*Sin[c + d*x]))/(
15*a^2) + (2*b*Sec[c + d*x]*Tan[c + d*x])/(15*a) + (2*Sec[c + d*x]^2*Tan[c
+ d*x])/5))/d
```

Maple [B] time = 0.4, size = 1557, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] 2/15/d/a^2*(3*a^3+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3-a*b^2*cos(d*x+c)^2-9*cos(d*x+c)^4*a^
2*b-cos(d*x+c)^4*a*b^2+5*cos(d*x+c)^3*a^2*b+2*cos(d*x+c)^3*a*b^2+4*cos(d*x+
c)*a^2*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*b^3-9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3+2*cos(d*x+c)^4*b^3-9*cos
(d*x+c)^3*a^3-2*cos(d*x+c)^3*b^3+6*cos(d*x+c)^2*a^3+9*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b-
2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*sin(d*x+c)*a*b^2-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b+2*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2+9*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*
sin(d*x+c)*a^2*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
```

```
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*
x+c)*a*b^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^3*sin(d*x+c)*b^3-9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3+9*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3)/
(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(7/2), x)
```

3.609 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=389

$$\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(25a^2+6ab+8b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{105a^3d}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(19*a^2 + 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 + 6*a*b + 8*b^2)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(105*a^3*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.923551, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2796, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(25a^2+6ab+8b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+}{a-}}}{105a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(19*a^2 + 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^4*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 + 6*a*b + 8*b^2)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(105*a^3*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(35*a*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]
*Sin[c + d*x])/(105*a^2*d*Cos[c + d*x]^(3/2))
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{b}{2}+\frac{5}{2}a\cos(c+dx)+2b\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}(25a^2-4b^2)+\frac{23}{4}a}{\cos^{\frac{5}{2}}(c+}}{}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2-4b^2\right)\sqrt{a}}{105a^2}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35ad\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2-4b^2\right)\sqrt{a}}{105a^2}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(19a^2+8b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{105a^4d}$$

Mathematica [C] time = 6.19678, size = 1304, normalized size = 3.35

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(25*a^4 - 17*a^2*b^2 - 8*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-19
*a^3*b - 8*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])) + 2*(-19*a^2*b^2 - 8*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a -
 b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Co
s[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(25*a^2*Sin[c + d*x] - 4*b^2*Sin[c + d*x
]))/(105*a^2) + (2*Sec[c + d*x]*(19*a^2*b*Sin[c + d*x] + 8*b^3*Sin[c + d*x]
```

```
))/(105*a^3) + (2*b*Sec[c + d*x]^2*Tan[c + d*x])/(35*a) + (2*Sec[c + d*x]^3
*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.492, size = 1826, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d/a^3*(-8*b^4*cos(d*x+c)^4+a^2*b^2*cos(d*x+c)^2-15*a^4+25*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*
x+c)*a^4+25*cos(d*x+c)^4*a^4-10*cos(d*x+c)^2*a^4+8*cos(d*x+c)^5*b^4+25*cos(
d*x+c)^5*a^3*b+19*cos(d*x+c)^5*a^2*b^2-4*cos(d*x+c)^5*a*b^3+19*cos(d*x+c)^4
*a^3*b-20*cos(d*x+c)^4*a^2*b^2+8*cos(d*x+c)^4*a*b^3-26*cos(d*x+c)^3*a^3*b-4
*cos(d*x+c)^3*a*b^3-18*cos(d*x+c)*a^3*b+19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+2*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d
*x+c)*a^2*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-19*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x
+c)*a^2*b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+19*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)
*a^2*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^3*sin(d*x+c)*a*b^3-19*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-19*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a
^2*b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^3*sin(d*x+c)*a*b^3-8*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4-8*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)^3*sin(d*x+c)*b^4)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/cos(d*x + c)^(9/2), x)
```

3.610 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=508

$$\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(3a^2+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\Big(\sin^{-}}{24abd}$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
24*a*b*d) + (Sqrt[a + b]*(a + 2*b)*(3*a + 8*b)*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(24*b*d) + (a*Sqrt[a + b]*(a^2 - 12*b^2)*Cot[c + d*x]*EllipticPi[(a
+ b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(8*b^2*d) + ((3*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + (a*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.25685, antiderivative size = 508, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2821, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{(3a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24bd\sqrt{\cos(c+dx)}}-\frac{(a-b)\sqrt{a+b}\,(3a^2+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\Big(\sin^{-}}{24abd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a
 + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*S
qrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(
24*a*b*d) + (Sqrt[a + b]*(a + 2*b)*(3*a + 8*b)*Cot[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(24*b*d) + (a*Sqrt[a + b]*(a^2 - 12*b^2)*Cot[c + d*x]*EllipticPi[(a
+ b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(8*b^2*d) + ((3*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*
Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x]]) + (a*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*
x])^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
```

```
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx = \frac{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{ab}{2}+2b^2\cos(c+dx\right.}{\sqrt{\cos(c+dx)}}}{3b}$$

$$= \frac{a\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\sqrt{\cos(c+dx)}(a+b\cos(c+d}{3d}$$

$$= \frac{\left(3a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} + \frac{a\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}{4d}$$

$$= \frac{\left(3a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24bd\sqrt{\cos(c+dx)}} + \frac{a\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}{4d}$$

$$= \frac{a\sqrt{a+b}\left(a^2-12b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\text{se}}{a}}}{8b^2d}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(3a^2+16b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{24abd}$$

Mathematica [C] time = 18.6459, size = 1189, normalized size = 2.34

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((-4*a*(17*a^2 + 16*b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 208*a^2*b*((Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
```

```
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/
2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2 + 16*b^2)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((7*a*Sin[c + d*x])/12
 + (b*Sin[2*(c + d*x)])/6))/d
```

Maple [B] time = 0.384, size = 1683, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/24/d/b*(-52*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*cos(d*x+c)^2*a^2*b-16
*cos(d*x+c)*a*b^2+72*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+16*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-6*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b*sin(d*x+c)+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-6*a*b^2*cos(d*x+c)^2+22
*cos(d*x+c)^4*a*b^2+17*cos(d*x+c)^3*a^2*b-14*cos(d*x+c)*a^2*b+3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
```

```
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*a^2*b+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*a*b^2+14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-52*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+7
2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a*b^2-3*a^3*cos(d*x+c)-16*cos(d*x+c)^2*b^3+8*cos(d*x+c)^
5*b^3+8*cos(d*x+c)^3*b^3+3*cos(d*x+c)^2*a^3)/(a+b*cos(d*x+c))^(1/2)/cos(d*x
+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\cos(dx+c)^2+a\cos(dx+c)\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^2 + a*cos(d*x + c))*sqrt(b*cos(d*x + c) + a)*sqrt(
cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.611 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=433

$$-\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{\sin(c+dx)(a+b\cos}{2d\sqrt{\cos(c}}$$

```
[Out] (-5*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) + (Sqrt[a
+ b]*(5*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(3
*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (3*a
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + ((a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.17026, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2821, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd}+\frac{\sin(c+dx)(a+b\cos}{2d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-5*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) + (Sqrt[a
+ b]*(5*a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (Sqrt[a + b]*(3
*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d) + (3*a
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + ((a + b*
Cos[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```


Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx = \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{\int \frac{\sqrt{a+b\cos(c+dx)}\left(-\frac{ab}{2}+b^2\cos(c+dx)+\frac{3}{2}ab\cos^2(c+dx\right.}{\cos^{\frac{3}{2}}(c+dx)}}{2b}$$

$$= -\frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{\int \frac{ab^2}{4}}{}$$

$$= \frac{3a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{\int -\frac{5}{4}}{}$$

$$= \frac{3a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{\int -}{\cos}$$

$$= -\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{4bd}$$

$$= -\frac{5(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d}$$

Mathematica [A] time = 12.34, size = 441, normalized size = 1.02

$$\frac{\sqrt{\cos(c+dx)}\left(\frac{-4\left(4a^2-ab+2b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-12a^2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+10a^2\sqrt{\frac{\cos}{\cos}}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*b*(a + b*Cos[c + d*x])*Sin[c + d*x] + (10*a*(a + b)*
Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(4*a^2 - a*b + 2*b^2)*Sqrt[(a + b*Co
s[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)] - 12*a^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 16
*b^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1,
 -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 5*a*b*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 10*a^2*Sqrt[Cos[c
```

```
+ d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - 5*a*b*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Tan[(c + d*x)/2])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]))/(8
*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.343, size = 1421, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/4/d*(8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-6*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^2-8*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-5*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-5*cos(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+8*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d
*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^2*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-8*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)-5*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-5*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-2*b^2*cos(d*x
+c)^4-7*a*b*cos(d*x+c)^3-5*cos(d*x+c)^2*a^2+5*cos(d*x+c)^2*a*b+2*b^2*cos(d*
x+c)^2+5*a^2*cos(d*x+c)+2*cos(d*x+c)*a*b)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)
^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)), x)
```

3.612 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=375

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{d}$$

```
[Out] -(((a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (3*a*Sqrt[a + b]*Co
t[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (b*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.642334, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2821, 3053, 2809, 2998, 2816, 2994}

$$\frac{b\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}(2a+b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)) + (Sqrt[
a + b]*(2*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (3*a*Sqrt[a + b]*Co
t[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (b*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \int \frac{-\frac{ab}{2}+a^2\cos(c+dx)+\frac{3}{2}ab\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}(3ab)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{-\frac{ab}{2}+a^2}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{3a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

$$= -\frac{(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad}$$

Mathematica [A] time = 7.05367, size = 341, normalized size = 0.91

$$-\frac{\sqrt{\cos(c+dx)}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(b\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))+4a(a-2b)\sqrt{\frac{\cos}{\cos(}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]^2*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a - 2*b)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 12*a*b*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1
 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2]))/(d*Sqrt[a + b*Cos[c + d*x]]*(-1 + Tan[(c + d*x)/2]^4)))
```

Maple [B] time = 0.463, size = 1003, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/d/(a+b*cos(d*x+c))^(1/2)*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*
x+c)*a*b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
```

```
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b+2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c
)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a
*b*sin(d*x+c)+cos(d*x+c)^3*b^2+cos(d*x+c)^2*a*b-b^2*cos(d*x+c)^2-cos(d*x+c)
*a*b)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\cos(c+dx))^{\frac{3}{2}}}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2)/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)), x)
```

3.613 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=337

$$-\frac{2(a-2b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*(a - 2*b)*
Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*b*Sqrt[a + b]*Cot[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rubi [A] time = 0.471292, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2798, 2809, 2998, 2816, 2994}

$$-\frac{2(a-2b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d}+\frac{2(a-b)\sqrt{a+b}\cot(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*(a - 2*b)*
Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d - (2*b*Sqrt[a + b]*Cot[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
```

Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
```

```
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = a\int \frac{a+2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + b^2\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+
dx))}{a-b}}}{d}$$

$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{d}$$

Mathematica [A] time = 12.8083, size = 359, normalized size = 1.07

$$\frac{\cos(c+dx)\left(\frac{2\left(a^2+2ab-b^2\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{b-a}{a+b}\right)}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}}-2a^2\tan\left(\frac{1}{2}(c+dx)\right)-\frac{4b^2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\Pi\left(-1;-\sin^{-1}\left(\tan\right.\right.}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(a + b*Cos[c + d*x])*Sin[c + d*x] + Cos[c + d*x]*((-2*a*(a + b)*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
```

```
d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (2*(a
^2 + 2*a*b - b^2)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])] - (4*b^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])] - a*b*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2]
- 2*a^2*Tan[(c + d*x)/2] + a*b*Tan[(c + d*x)/2]))/(d*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.457, size = 1183, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2
-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*a*b+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+2*
b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)+cos(d*x+c)^2*a*b+a^2*cos(d*x+c)-cos(d*x+c)*a*b-a^2)/(a+b*cos(d*x+
c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\cos(c + dx))^{\frac{3}{2}}}{\cos^{\frac{3}{2}}(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx + c) + a)^{\frac{3}{2}}}{\cos(dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2), x)
```

3.614 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=277

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+a}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{3ad}$$

```
[Out] (8*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2*(a
 - 3*b)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2
*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.435061, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2799, 2998, 2816, 2994}

$$\frac{2a\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-3b)(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+a}}{\sqrt{a+b}\sqrt{\cos(c+}}\right.\right.}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```


```
[Out] (8*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2*(a
 - 3*b)*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d) + (2
*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
```

```
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}dx &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2}{3}\int \frac{2ab+\frac{1}{2}\left(a^2+3b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx\\ &= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{1}{3}((a-3b)(a-b))\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+ \ldots}}\\ &= \frac{8(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+\ldots}{a-b}}\ldots}{3ad}\end{aligned}$$

Mathematica [A] time = 4.51187, size = 256, normalized size = 0.92

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(\left(a^2+4ab+3b^2\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-4b\tan\left(\frac{1}{2}(c+dx)\right)\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((2*(a + b*Cos[c + d*x])*(a + 4*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^
(3/2) + 2*Sqrt[Cos[(c + d*x)/2]^2]*(-4*b*(a + b)*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] + (a^2 + 4*a*b + 3*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 4*b*(
a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2])
)/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.553, size = 1075, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2+4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^2-4*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+4*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*b^2-4*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+a*b*cos(d*x+c)^3+4*co
s(d*x+c)^3*b^2+cos(d*x+c)^2*a^2+4*cos(d*x+c)^2*a*b-4*b^2*cos(d*x+c)^2-5*cos
(d*x+c)*a*b-a^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2), x)
```

3.615 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=325

$$\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{5a^2d}+\frac{4b\sin(c+dx)\sqrt{a}}{5d\cos^{\frac{3}{2}}(c}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*
a^2*d) - (2*(a - b)*(3*a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(5*a*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(
5/2)) + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)
)
```

Rubi [A] time = 0.661232, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2799, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{5a^2d}+\frac{4b\sin(c+dx)\sqrt{a}}{5d\cos^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*
a^2*d) - (2*(a - b)*(3*a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(5*a*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(
5/2)) + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Cos[c + d*x]^(3/2)
)
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{3ab+\frac{1}{2}\left(3a^2+5b^2\right)\cos(c+dx)+ab\cos^2(c+}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{3}{4}a\left(3a^2+b^2\right)+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{}}}{15}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{4b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{5}((a-b)(3a-$$

$$= \frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{5a^2d}$$

Mathematica [A] time = 12.9229, size = 443, normalized size = 1.36

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\sec(c+dx)\left(3a^2\sin(c+dx)+b^2\sin(c+dx)\right)}{5a}+\frac{2}{5}a\tan(c+dx)\sec^2(c+dx)+\frac{4}{5}b\tan(c+dx)\sec(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (8*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]*Sec[(c + d*x)/2]^2]*(-2*(3*a^3 + 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(3*a
^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))/(5*a*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])^(
3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]*((2*Sec[c + d*x]*(3*a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(5*a) + (4*b*S
ec[c + d*x]*Tan[c + d*x])/5 + (2*a*Sec[c + d*x]^2*Tan[c + d*x])/5))/d
```

Maple [B] time = 0.408, size = 1539, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d/a*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^3*sin(d*x+c)*a^3+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b+(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2-
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*sin(d*x+c)*a^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2-(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*
x+c)*b^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*a^3+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*a^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
```

```
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x
+c)*b^3+3*cos(d*x+c)^4*a^2*b+2*cos(d*x+c)^4*a*b^2+cos(d*x+c)^4*b^3+3*cos(d*
x+c)^3*a^3+cos(d*x+c)^3*a*b^2-cos(d*x+c)^3*b^3-2*cos(d*x+c)^2*a^3-3*a*b^2*c
os(d*x+c)^2-3*cos(d*x+c)*a^2*b-a^3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d
*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(7/2), x)
```

3.616 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=387

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c}{}}}{105a^2d}$$

```
[Out] (4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^3*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 - 57*a*b - 6*b^2)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(7*d*Cos[c + d*x]^(7/2)) + (16*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.949293, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2799, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c}{}}}{105a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```


```
[Out] (4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(105*a^3*d) + (2*(a - b)*Sqrt[a + b]*(25*a^2 - 57*a*b - 6*b^2)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(105*a^2*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sin[c
+ d*x])/(7*d*Cos[c + d*x]^(7/2)) + (16*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(35*d*Cos[c + d*x]^(5/2)) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(105*a*d*Cos[c + d*x]^(3/2))
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}dx = \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{4ab+\frac{1}{2}\left(5a^2+7b^2\right)\cos(c+dx)+2ab\cos^2(c}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}a\left(25a^2+3b\right.}{c}}{}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2+3b^2\right.}{1}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{16b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(25a^2+3b^2\right.}{1}$$

$$= \frac{4(a-b)b\sqrt{a+b}\left(41a^2-3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{105a^3d}$$

Mathematica [C] time = 6.21992, size = 1302, normalized size = 3.36

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(25*a^4 - 31*a^2*b^2 + 6*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
+ b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos
[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-82
*a^3*b + 6*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a
+ b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
 + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a +
b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a
/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2
*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
d*x]])) + 2*(-82*a^2*b^2 + 6*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d
*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a -
 b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Co
s[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
- (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(b*Sqrt[Cos[c + d*x]])))/(105*a^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Cos[c + d*x]]*((2*Sec[c + d*x]^2*(25*a^2*Sin[c + d*x] + 3*b^2*Sin[c + d*x
```

```
]))/(105*a) + (4*Sec[c + d*x]*(41*a^2*b*Sin[c + d*x] - 3*b^3*Sin[c + d*x]))
/(105*a^2) + (16*b*Sec[c + d*x]^2*Tan[c + d*x])/35 + (2*a*Sec[c + d*x]^3*Ta
n[c + d*x])/7))/d
```

Maple [B] time = 0.681, size = 1827, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d/a^2*(6*b^4*cos(d*x+c)^4-27*a^2*b^2*cos(d*x+c)^2-15*a^4+25*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(
d*x+c)*a^4+25*cos(d*x+c)^4*a^4-10*cos(d*x+c)^2*a^4-6*cos(d*x+c)^5*b^4+25*co
s(d*x+c)^5*a^3*b+82*cos(d*x+c)^5*a^2*b^2+3*cos(d*x+c)^5*a*b^3+82*cos(d*x+c)
^4*a^3*b-55*cos(d*x+c)^4*a^2*b^2-6*cos(d*x+c)^4*a*b^3-68*cos(d*x+c)^3*a^3*b
+3*cos(d*x+c)^3*a*b^3-39*cos(d*x+c)*a^3*b+82*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+51*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*si
n(d*x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-82*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-82*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(
d*x+c)*a^2*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+82*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+51*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*
x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-82*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-82*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+
c)*a^2*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3+6*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^
4+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)^3*sin(d*x+c)*b^4)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(9/2), x)
```

3.617 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=454

$$\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{5}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\left(-39a^2b\right.}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 33*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(315*a^4*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 39*a^2*b - 6*
a*b^2 - 8*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*a*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (20*b*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*a^2 + 3*b^
2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(5/2)) + (8
*b*(22*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Cos[c +
 d*x]^(3/2))
```

Rubi [A] time = 1.31789, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2799, 3055, 2998, 2816, 2994}

$$\frac{8b\left(22a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(49a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{5}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\left(-39a^2b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 33*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(315*a^4*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 39*a^2*b - 6*
a*b^2 - 8*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d) + (2*a*Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (20*b*Sqrt[a +
b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*a^2 + 3*b^
2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a*d*Cos[c + d*x]^(5/2)) + (8
*b*(22*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a^2*d*Cos[c +
 d*x]^(3/2))
```


Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{5ab+\frac{1}{2}\left(7a^2+9b^2\right)\cos(c+dx)+3ab\cos^2(c+d}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}a\left(49a^2+3b^2\right)+}{\cos^{\frac{7}{2}}}}{}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2+3b^2\right)\sqrt{}}{315}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2+3b^2\right)\sqrt{}}{315}$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{20b\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2+3b^2\right)\sqrt{}}{315}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4+33a^2b^2+8b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{315a^4d}$$

Mathematica [C] time = 6.26739, size = 1368, normalized size = 3.01

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] -((-4*a*(-39*a^4*b + 31*a^2*b^3 + 8*b^5)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/
(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b
*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((
a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(
c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*
(147*a^5 + 33*a^3*b^2 + 8*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])) + 2*(147*a^4*b + 33*a^2*b^3 + 8*b^5)*((I*Cos[(c + d*
x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Co
s[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec
[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
```

```
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
 + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(315*a^3*d) + (Sq
rt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(49*a^2*Sin[c
+ d*x] + 3*b^2*Sin[c + d*x]))/(315*a) + (8*Sec[c + d*x]^2*(22*a^2*b*Sin[c +
 d*x] - b^3*Sin[c + d*x]))/(315*a^2) + (2*Sec[c + d*x]*(147*a^4*Sin[c + d*x
] + 33*a^2*b^2*Sin[c + d*x] + 8*b^4*Sin[c + d*x]))/(315*a^3) + (20*b*Sec[c
+ d*x]^3*Tan[c + d*x])/63 + (2*a*Sec[c + d*x]^4*Tan[c + d*x])/9))/d
```

Maple [B] time = 0.682, size = 2504, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x)
```

```
[Out] 2/315/d/a^3*(52*cos(d*x+c)^3*a^4*b-cos(d*x+c)^3*a^2*b^3+53*cos(d*x+c)^2*a^3
*b^2+85*cos(d*x+c)*a^4*b-147*cos(d*x+c)^6*a^4*b-88*cos(d*x+c)^6*a^3*b^2-33*
cos(d*x+c)^6*a^2*b^3+4*cos(d*x+c)^6*a*b^4+10*cos(d*x+c)^5*a^4*b-33*cos(d*x+
c)^5*a^3*b^2+34*cos(d*x+c)^5*a^2*b^3-8*cos(d*x+c)^5*a*b^4+68*cos(d*x+c)^4*a
^3*b^2+4*cos(d*x+c)^4*a*b^4+147*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+35*a^5+147*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^
5*a^4*b+33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2+33*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+8*sin(d*x+c)*co
s(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^4-186*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-33*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-2*sin(d*x+c)*cos
(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^2*b^3-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^5*a*b^4+147*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+33*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*
a^3*b^2+33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b^4-186*sin(d*x+c)*cos
(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a^4*b-33*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-8*cos(d*x+c)^6*b^5-147*cos(d*x+c)^5*a^5+8*
```

```
cos(d*x+c)^5*b^5+98*cos(d*x+c)^4*a^5+14*cos(d*x+c)^2*a^5+8*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*b
^5-147*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*cos(d*x+c)^5*a^5+147*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+8*sin(d*x+c)*cos(d*x+c)^4*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-147*s
in(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a^5-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3-8*sin(d*x+c)*cos(d*x+c)^4*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4)/(a+b*cos
(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/cos(d*x + c)^(11/2), x)
```

3.618 $\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=506

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,(33a^2+26ab+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\Big(}{24d}$$

```
[Out] -((a - b)*Sqrt[a + b]*(33*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(24*a*d) + (Sqrt[a + b]*(33*a^2 + 26*a*b + 16*b^2)*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(24*d) - (5*a*Sqrt[a + b]*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(8*b*d) + ((33*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (13*a*b*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b^2*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 1.35589, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2793, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{(33a^2+16b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{24d\sqrt{\cos(c+dx)}}+\frac{\sqrt{a+b}\,(33a^2+26ab+16b^2)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\Big(}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(33*a^2 + 16*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(24*a*d) + (Sqrt[a + b]*(33*a^2 + 26*a*b + 16*b^2)*Cot[c + d*x]*EllipticF[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(24*d) - (5*a*Sqrt[a + b]*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[
(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(8*b*d) + ((33*a^2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]
]*Sin[c + d*x])/(24*d*Sqrt[Cos[c + d*x]]) + (13*a*b*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d) + (b^2*Cos[c + d*x]^(3/2)*Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx = \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a\left(2a^2\right.\right.}{}$$

$$= \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d} + \frac{b^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos}}{3d}$$

$$= \frac{\left(33a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} + \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}{12d}$$

$$= \frac{\left(33a^2+16b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sqrt{\cos(c+dx)}} + \frac{13ab\sqrt{\cos(c+dx)}\sqrt{a+b\cos}}{12d}$$

$$= -\frac{5a\sqrt{a+b}\left(a^2+4b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{8bd}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(33a^2+16b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{24ad}$$

Mathematica [C] time = 18.4699, size = 1203, normalized size = 2.38

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((-4*a*(59*a^2*b + 16*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt
[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])
*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c +
d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)
/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(48*a^3 + 76*a
*b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
 d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
```

```
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*
(33*a^2*b + 16*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE
[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d
*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Se
c[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a
+ b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*
x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]
*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt
[Cos[c + d*x]])))/(48*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((1
3*a*b*Sin[c + d*x])/12 + (b^2*Sin[2*(c + d*x)])/6))/d
```

Maple [B] time = 0.417, size = 1866, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/24/d*(76*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2*sin(d*x+c)+48*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-16*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-30*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+33*co
s(d*x+c)^2*a^2*b+16*cos(d*x+c)*a*b^2-120*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-33*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-16
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*b^3-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-33*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-16*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-26*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+
```

```
18*a*b^2*cos(d*x+c)^2-34*cos(d*x+c)^4*a*b^2-59*cos(d*x+c)^3*a^2*b+26*cos(d*
x+c)*a^2*b-33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-26*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*
b+76*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a*b^2-120*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+33*a^3*cos(d*x+c)+16*cos
(d*x+c)^2*b^3-8*cos(d*x+c)^5*b^3-8*cos(d*x+c)^3*b^3-33*cos(d*x+c)^2*a^3+48*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d
*x+c))/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.619 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=443

$$\frac{\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\left(15a^2+4b^2\right)}{}$$

```
[Out] (-9*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) + (Sqrt[
a + b]*(8*a^2 + 9*a*b + 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (
Sqrt[a + b]*(15*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(4*d) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*
x]]) + (b^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.00254, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2793, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d}-\frac{\sqrt{a+b}\left(15a^2+4b^2\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-9*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) + (Sqrt[
a + b]*(8*a^2 + 9*a*b + 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d) - (
Sqrt[a + b]*(15*a^2 + 4*b^2)*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(4*d) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*
x]]) + (b^2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```


Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{1}{2}\int \frac{\frac{1}{2}a\left(4a^2+b^2\right)+b\left(6a^2+b^2\right)\cos(c+ \ldots}{\sqrt{\cos(c+dx)}\sqrt{a+b\,c\ldots}}$$

$$= \frac{9ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \ldots$$

$$= \frac{9ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d} + \ldots$$

$$= -\frac{\sqrt{a+b}\left(15a^2+4b^2\right)\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\ldots}{}}}{4d}$$

$$= -\frac{9(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{4d}$$

Mathematica [A] time = 6.30982, size = 331, normalized size = 0.75

$$\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(2\left(-12a^2b+4a^3+ab^2-2b^3\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2b\left(15a^2+\ldots\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])*Sin[c + d*x] + Sqrt[Cos[(c +
 d*x)/2]^2]*(9*a*b*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 2*(4*a^3 -
12*a^2*b + a*b^2 - 2*b^3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
 b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 2*b*(15*a^2
+ 4*b^2)*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(
(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + 9*a*b*(a + b*Cos[c + d*
x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(4*d*Sqrt[a +
b*Cos[c + d*x]])
```

Maple [B] time = 0.519, size = 1629, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d/(a+b*cos(d*x+c))^(1/2)*(9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+9*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+30
```

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^2*b+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+8*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-24*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(
d*x+c)*a^2*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+9*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+9*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+30
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*
b*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b
))^(1/2))*b^3*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-24*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*cos(d*x+c)^
4*b^3+11*cos(d*x+c)^3*a*b^2+9*cos(d*x+c)^2*a^2*b-9*a*b^2*cos(d*x+c)^2-2*cos
(d*x+c)^2*b^3-9*cos(d*x+c)*a^2*b-2*cos(d*x+c)*a*b^2)/cos(d*x+c)^(1/2)/sin(d
*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\sqrt{\cos(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)), x)
```

3.620 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=445

$$-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(2a^2-6ab-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{d}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
- (5*a*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*
a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*a^2
 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 1.0102, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2792, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{a+b}\left(2a^2-6ab-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```


```
[Out] ((a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[
(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
 - (Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d
- (5*a*b*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*
a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((2*a^2
 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
```

```
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
```

```
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{3a^2b}{2} - \frac{1}{2}a\left(a^2-3b^2\right)\cos(c+dx) - \frac{1}{2}b\left(2a^2-b\right.}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\int \frac{1}{2}ab}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\left(2a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\int \frac{1}{2}ab}{\mathrm{co}}$$

$$= -\frac{5ab\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c}{a-b}}}{d}$$

$$= \frac{(a-b)\sqrt{a+b}\left(2a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{a}}{ad}$$

Mathematica [C] time = 17.7797, size = 1185, normalized size = 2.66

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*
a*(-4*a^2*b - b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + 4*a*(2*a^3 - 6*a*b^2)*((Sq
rt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[
(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/
Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sq
rt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x
])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c +
 d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) - 2*(2*a^2*b -
 b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin
[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a
 + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c +
d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*S
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(
```

```
a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-
2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c +
 d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]
)))/(2*d)
```

Maple [B] time = 0.404, size = 1623, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*a*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+6*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-6*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a*b^2+10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+6
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*si
n(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2*sin(d*x+c)+cos(d*x+c)^3*b^3+2*cos(d*x+c)^2*a^2*b+a*b^2*cos(d*x+c)^2
-cos(d*x+c)^2*b^3+2*a^3*cos(d*x+c)-2*cos(d*x+c)*a^2*b-cos(d*x+c)*a*b^2-2*a^
3)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2), x)
```

3.621 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=392

$$\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d}+\frac{2a^2\sin(c+dx)\sqrt{a}}{3d\cos^{\frac{3}{2}}(c}$$

```
[Out] (14*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) + (2*Sqr
t[a + b]*(a^2 - 7*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (
2*b^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a^2
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.739229, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2792, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d}+\frac{2a^2\sin(c+dx)\sqrt{a}}{3d\cos^{\frac{3}{2}}(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```


```
[Out] (14*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) + (2*Sqr
t[a + b]*(a^2 - 7*a*b + 9*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d) - (
2*b^2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[
c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/d + (2*a^2
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
```

```
egersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{7a^2b}{2} + \frac{1}{2}a\left(a^2+9b^2\right)\cos(c+dx) + \frac{3}{2}b^3\cos^2(c+}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{7a^2b}{2} + \frac{1}{2}a\left(a^2+9b^2\right)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + b^3\int \frac{\surd}{\sqrt{a\,\cdot}}$$

$$= -\frac{2b^2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx)}{a-b}}}{d}$$

$$= \frac{14(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx)}{a-b}}}{3d}$$

Mathematica [A] time = 6.58251, size = 330, normalized size = 0.84

$$2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}\left(\left(7a^2b+a^3+9ab^2-3b^3\right)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right) - 6b^3\sqrt{\frac{\sec^2\left(\frac{1}{2}(c\right.}{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] ((2*a*(a + b*Cos[c + d*x])*(a + 7*b*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x
]^(3/2) + 2*Sqrt[Cos[(c + d*x)/2]^2]*(-7*a*b*(a + b)*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]
^2)/(a + b)] + (a^3 + 7*a^2*b + 9*a*b^2 - 3*b^3)*EllipticF[ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] - 6*b^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - 7*a*b*(a + b*Co
s[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c + d*x)/2]))/(3*d*S
qrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.569, size = 1487, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x)
```

```
[Out] 2/3/d*(7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^2*sin(d*x+c)*a^2*b+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3-7*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^2*sin(d*x+c)*a^2*b-9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
```

```
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-6*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*s
in(d*x+c)*b^3+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-7*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a^2*b-9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-6*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+
c)*b^3-cos(d*x+c)^3*a^2*b-7*cos(d*x+c)^3*a*b^2-cos(d*x+c)^2*a^3-7*cos(d*x+c
)^2*a^2*b+7*a*b^2*cos(d*x+c)^2+8*cos(d*x+c)*a^2*b+a^3)/(a+b*cos(d*x+c))^(1/
2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2), x)
```

3.622 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=338

$$-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad}+\frac{2(a-}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(15*a*d) - (2*(a - b)*Sqrt[a + b]*(9*a^2 - 8*a*b + 15*b^2)*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(15*a*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(5*d*Cos[c + d*x]^(5/2)) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(15*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.764616, antiderivative size = 338, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2792, 3055, 2998, 2816, 2994}

$$-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15ad}+\frac{2(a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(15*a*d) - (2*(a - b)*Sqrt[a + b]*(9*a^2 - 8*a*b + 15*b^2)*Cot[c + d*x]*Ell
ipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(15*a*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(5*d*Cos[c + d*x]^(5/2)) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(15*d*Cos[c + d*x]^(3/2))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{11a^2b}{2} + \frac{3}{2}a\left(a^2+5b^2\right)\cos(c+dx) + \frac{1}{2}b\left(2a^2 +\right.}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}a^2(9a^2+}{\mathrm{c}}}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{22ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{15}\left((a-b)\right.$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2+23b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{15ad}$$

Mathematica [A] time = 11.4606, size = 386, normalized size = 1.14

$$\tan(c+dx)(a+b\cos(c+dx))\left(\left(9a^2+23b^2\right)\cos(2(c+dx))+15a^2+22ab\cos(c+dx)+23b^2\right)-4\cos^2\left(\frac{1}{2}(c+dx)\right)^{5/2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-4*(Cos[(c + d*x)/2]^2)^(5/2)*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*Sqrt
[1 + Cos[c + d*x]]*((9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((a + b*Cos[c
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^
3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2
*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (9*a^2 + 23*b^2)
*(a + b*Cos[c + d*x])*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*
Tan[(c + d*x)/2]) + (a + b*Cos[c + d*x])*(15*a^2 + 23*b^2 + 22*a*b*Cos[c +
d*x] + (9*a^2 + 23*b^2)*Cos[2*(c + d*x)])*Tan[c + d*x])/(15*d*Cos[c + d*x]^
(3/2)*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.486, size = 1750, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x)
```

```
[Out] 2/15/d*(3*a^3-15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3+34*a*b^2*cos(d*x+c
)^2-9*cos(d*x+c)^4*a^2*b-11*cos(d*x+c)^4*a*b^2-5*cos(d*x+c)^3*a^2*b-23*cos(
d*x+c)^3*a*b^2+14*cos(d*x+c)*a^2*b+23*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-9*(cos(d*x+c)/(1+c
```

```
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a
^3-23*cos(d*x+c)^4*b^3-9*cos(d*x+c)^3*a^3+23*cos(d*x+c)^3*b^3+6*cos(d*x+c)^
2*a^3+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^3*sin(d*x+c)*a^2*b+23*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2-17*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*
b-23*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)^3*sin(d*x+c)*a*b^2+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+23*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-17
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*a^2*b-23*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+23*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3-9*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*s
in(d*x+c)*a^3+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3-15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3)/(a+b*cos(d*x+c))^
(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(7/2), x)
```

3.623 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=387

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+}{a-b}}}{21ad}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(21*a^2*d) + (2*(a - b)*Sqrt[a + b]*(5*a^2 - 24*a*b + 3*b^2)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(21*a*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(7*d*Cos[c + d*x]^(5/2)) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(21*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.04434, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+}{a-b}}}{21ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```


```
[Out] (2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(21*a^2*d) + (2*(a - b)*Sqrt[a + b]*(5*a^2 - 24*a*b + 3*b^2)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])
], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Se
c[c + d*x]))/(a - b)])/(21*a*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(7*d*Cos[c + d*x]^(7/2)) + (6*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x
])/(7*d*Cos[c + d*x]^(5/2)) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*x]]*S
in[c + d*x])/(21*d*Cos[c + d*x]^(3/2))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
```

```
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{15a^2b}{2} + \frac{1}{2}a\left(5a^2+21b^2\right)\cos(c+dx) + \frac{1}{2}b\left(4a^2 + \cdots\right.}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+dx \cdots}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{4\int \frac{\frac{5}{4}a^2\left(5a^2+9b^2\right)+\cdots}{\cdots}}{\cdots}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(5a^2+9b^2\right)\sqrt{\cdots}}{21\cdots}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{6ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{7d\cos^{\frac{5}{2}}(c+dx)} + \frac{2\left(5a^2+9b^2\right)\sqrt{\cdots}}{21\cdots}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(29a^2+3b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle| -\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\cdots}}{21a^2d}$$

Mathematica [C] time = 6.24341, size = 1302, normalized size = 3.36

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((-4*a*(5*a^4 - 2*a^2*b^2 - 3*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a +
b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-29*a
^3*b - 3*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x
)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b
), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*
x]])) + 2*(-29*a^2*b^2 - 3*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x
]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b
)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[
c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
in[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) -
(a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d
*x])/(b*Sqrt[Cos[c + d*x]])))/(21*a*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]]*((2*Sec[c + d*x]^2*(5*a^2*Sin[c + d*x] + 9*b^2*Sin[c + d*x]))/21
 + (2*Sec[c + d*x]*(29*a^2*b*Sin[c + d*x] + 3*b^3*Sin[c + d*x]))/(21*a) + (
```

```
6*a*b*Sec[c + d*x]^2*Tan[c + d*x])/7 + (2*a^2*Sec[c + d*x]^3*Tan[c + d*x])/
7))/d
```

Maple [B] time = 0.51, size = 1827, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d/a*(-3*b^4*cos(d*x+c)^4-18*a^2*b^2*cos(d*x+c)^2-3*a^4+5*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+
c)*a^4+5*cos(d*x+c)^4*a^4-2*cos(d*x+c)^2*a^4+3*cos(d*x+c)^5*b^4+5*cos(d*x+c
)^5*a^3*b+29*cos(d*x+c)^5*a^2*b^2+9*cos(d*x+c)^5*a*b^3+29*cos(d*x+c)^4*a^3*
b-11*cos(d*x+c)^4*a^2*b^2+3*cos(d*x+c)^4*a*b^3-22*cos(d*x+c)^3*a^3*b-12*cos
(d*x+c)^3*a*b^3-12*cos(d*x+c)*a^3*b+29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+27*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+
c)*a^2*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-29*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)
*a^2*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^4*sin(d*x+c)*a*b^3+29*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+27*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a
^2*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^3*sin(d*x+c)*a*b^3-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-29*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2
*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^3*sin(d*x+c)*a*b^3-3*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+5*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4-3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3
*sin(d*x+c)*b^4)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(9/2), x)
```

3.624 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=454

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\,(-}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b +
 165*a*b^2 + 10*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a^
2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (38*a*b
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*
a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5
/2)) + (2*b*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a
*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.40875, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{315ad\cos^{\frac{3}{2}}(c+dx)}-\frac{2(a-b)\sqrt{a+b}\,(-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(315*a^3*d) - (2*(a - b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b +
 165*a*b^2 + 10*b^3)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]
/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d) + (2*a^
2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(9*d*Cos[c + d*x]^(9/2)) + (38*a*b
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(63*d*Cos[c + d*x]^(7/2)) + (2*(49*
a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*d*Cos[c + d*x]^(5
/2)) + (2*b*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(315*a
*d*Cos[c + d*x]^(3/2))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{2}{9}\int \frac{\frac{19a^2b}{2}+\frac{1}{2}a\left(7a^2+27b^2\right)\cos(c+dx)+\frac{3}{2}b\left(2a\right.}{\cos^{\frac{9}{2}}(c+dx)\sqrt{a+b\cos(c+}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}a^2(49a^2}{}}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2+7\right.}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2+7\right.}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{9d\cos^{\frac{9}{2}}(c+dx)} + \frac{38ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{63d\cos^{\frac{7}{2}}(c+dx)} + \frac{2\left(49a^2+7\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4+279a^2b^2-10b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{315a^3d}$$

Mathematica [C] time = 6.28619, size = 1368, normalized size = 3.01

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] -((-4*a*(-114*a^4*b + 124*a^2*b^3 - 10*b^5)*Sqrt[((a + b)*Cot[(c + d*x)/2]^
2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Si
n[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4
*a*(147*a^5 + 279*a^3*b^2 - 10*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt
[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
 + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
 d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]])) + 2*(147*a^4*b + 279*a^2*b^3 - 10*b^5)*((I*Cos[
(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/
Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a
*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*
x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]
^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a
 + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[S
```

```
qrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]
*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b +
(Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(315*a^2*d
) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^3*(49*a^2
*Sin[c + d*x] + 75*b^2*Sin[c + d*x]))/315 + (2*Sec[c + d*x]^2*(163*a^2*b*Si
n[c + d*x] + 5*b^3*Sin[c + d*x]))/(315*a) + (2*Sec[c + d*x]*(147*a^4*Sin[c
+ d*x] + 279*a^2*b^2*Sin[c + d*x] - 10*b^4*Sin[c + d*x]))/(315*a^2) + (38*a
*b*Sec[c + d*x]^3*Tan[c + d*x])/63 + (2*a^2*Sec[c + d*x]^4*Tan[c + d*x])/9)
)/d
```

Maple [B] time = 0.659, size = 2504, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x)
```

```
[Out] 2/315/d/a^2*(82*cos(d*x+c)^3*a^4*b+80*cos(d*x+c)^3*a^2*b^3+170*cos(d*x+c)^2
*a^3*b^2+130*cos(d*x+c)*a^4*b-147*cos(d*x+c)^6*a^4*b-163*cos(d*x+c)^6*a^3*b
^2-279*cos(d*x+c)^6*a^2*b^3-5*cos(d*x+c)^6*a*b^4-65*cos(d*x+c)^5*a^4*b-279*
cos(d*x+c)^5*a^3*b^2+199*cos(d*x+c)^5*a^2*b^3+10*cos(d*x+c)^5*a*b^4+272*cos
(d*x+c)^4*a^3*b^2-5*cos(d*x+c)^4*a*b^4+147*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+35*a^5+147*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*
cos(d*x+c)^5*a^4*b+279*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2+279*sin(d*x+c)*cos(d*x+c)^5*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-10
*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^4-261*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b-279*sin(d*x+c)*cos(d*x+c)^5*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-1
55*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^3+10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4+147*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+
279*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d
*x+c)*cos(d*x+c)^4*a^3*b^2+279*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3-10*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a*b
^4-261*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^4*b-279*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
```

```
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+10*cos(d*x+c)^6*b^5-
147*cos(d*x+c)^5*a^5-10*cos(d*x+c)^5*b^5+98*cos(d*x+c)^4*a^5+14*cos(d*x+c)^
2*a^5-10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^5*b^5-147*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5+147*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-1
0*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*b^5-147*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-155*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3+10*s
in(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*a*b^4)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{11}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(11/2), x)
```

3.625 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=522

$$\frac{2\left(81a^2+113b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693d\cos^{\frac{7}{2}}(c+dx)}+\frac{2b\left(229a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(205a^2b^2+135\right.}{}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(741*a^4 + 51*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(693*a^4*d) + (2*(a - b)*Sqrt[a + b]*(135*a^4 - 606*a^3*b +
 57*a^2*b^2 + 6*a*b^3 + 8*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^3*
d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2
)) + (46*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2
)) + (2*(81*a^2 + 113*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Co
s[c + d*x]^(7/2)) + (2*b*(229*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(693*a*d*Cos[c + d*x]^(5/2)) + (2*(135*a^4 + 205*a^2*b^2 - 4*b^4)*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*a^2*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 1.77457, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(81a^2+113b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693d\cos^{\frac{7}{2}}(c+dx)}+\frac{2b\left(229a^2+3b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{693ad\cos^{\frac{5}{2}}(c+dx)}+\frac{2\left(205a^2b^2+135\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```


```
[Out] (2*(a - b)*b*Sqrt[a + b]*(741*a^4 + 51*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(693*a^4*d) + (2*(a - b)*Sqrt[a + b]*(135*a^4 - 606*a^3*b +
 57*a^2*b^2 + 6*a*b^3 + 8*b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos
[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(693*a^3*
d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(11*d*Cos[c + d*x]^(11/2
)) + (46*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(99*d*Cos[c + d*x]^(9/2
)) + (2*(81*a^2 + 113*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*d*Co
s[c + d*x]^(7/2)) + (2*b*(229*a^2 + 3*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c +
 d*x])/(693*a*d*Cos[c + d*x]^(5/2)) + (2*(135*a^4 + 205*a^2*b^2 - 4*b^4)*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(693*a^2*d*Cos[c + d*x]^(3/2))
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
```

```
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{2}{11}\int \frac{\frac{23a^2b}{2}+\frac{3}{2}a\left(3a^2+11b^2\right)\cos(c+dx)+\frac{1}{2}b\left(8\right.}{\cos^{\frac{11}{2}}(c+dx)\sqrt{a+b\cos(c}}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{4\int \frac{\frac{1}{4}a^2\left(81a^2\right.}{}}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2+1\right.}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2+1\right.}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2+1\right.}{}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{11d\cos^{\frac{11}{2}}(c+dx)} + \frac{46ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{99d\cos^{\frac{9}{2}}(c+dx)} + \frac{2\left(81a^2+1\right.}{}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(741a^4+51a^2b^2+8b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{693a^4d}$$

Mathematica [C] time = 6.31775, size = 1431, normalized size = 2.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] ((-4*a*(135*a^6 - 78*a^4*b^2 - 49*a^2*b^4 - 8*b^6)*Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - 4*a*(-741*a^5*b - 51*a^3*b^3 - 8*a*b^5)*((Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x
]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
 d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-741*a^4*b^2 - 51*a^2*b^4 - 8*b^6)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
```

```
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
693*a^3*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*Sec[c + d*x]^
4*(81*a^2*Sin[c + d*x] + 113*b^2*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^3*(22
9*a^2*b*Sin[c + d*x] + 3*b^3*Sin[c + d*x]))/(693*a) + (2*Sec[c + d*x]^2*(13
5*a^4*Sin[c + d*x] + 205*a^2*b^2*Sin[c + d*x] - 4*b^4*Sin[c + d*x]))/(693*a
^2) + (2*Sec[c + d*x]*(741*a^4*b*Sin[c + d*x] + 51*a^2*b^3*Sin[c + d*x] + 8
*b^5*Sin[c + d*x]))/(693*a^3) + (46*a*b*Sec[c + d*x]^4*Tan[c + d*x])/99 + (
2*a^2*Sec[c + d*x]^5*Tan[c + d*x])/11))/d
```

Maple [B] time = 0.859, size = 2789, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x)
```

```
[Out] -2/693/d/a^3*(-140*cos(d*x+c)^5*a^3*b^3-4*cos(d*x+c)^5*a*b^5-160*cos(d*x+c)
^4*a^4*b^2+cos(d*x+c)^4*a^2*b^4-86*cos(d*x+c)^3*a^5*b+51*cos(d*x+c)^6*a^3*b
^3-52*cos(d*x+c)^6*a^2*b^4+8*cos(d*x+c)^6*a*b^5+741*cos(d*x+c)^6*a^5*b-307*
cos(d*x+c)^6*a^4*b^2+205*cos(d*x+c)^7*a^3*b^3+135*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^6-8*b^6*
cos(d*x+c)^6-63*a^6+51*cos(d*x+c)^7*a^2*b^4-4*cos(d*x+c)^7*a*b^5-116*cos(d*
x+c)^3*a^3*b^3-566*cos(d*x+c)^5*a^5*b+8*cos(d*x+c)^7*b^6+135*cos(d*x+c)^6*a
^6-54*cos(d*x+c)^4*a^6-18*cos(d*x+c)^2*a^6-224*cos(d*x+c)*a^5*b+135*cos(d*x
+c)^7*a^5*b+741*cos(d*x+c)^7*a^4*b^2-274*cos(d*x+c)^2*a^4*b^2-8*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x
+c)*b^6+135*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^6-8*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6+741*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^5*
b+663*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^6*sin(d*x+c)*a^4*b^2+51*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^3*b^3+2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^2
*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^6*sin(d*x+c)*a*b^5-741*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^5*b-741*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^4
*b^2-51*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
```

```
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^6*sin(d*x+c)*a^3*b^3-51*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a^2*b^4-8*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^6*sin(d*x+c)*a
*b^5+741*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^5*sin(d*x+c)*a^5*b+663*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a^4*b^2+51*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*
a^3*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*cos(d*x+c)^5*sin(d*x+c)*a^2*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c)*a*b^5-741*cos(d*x+c)^5*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^5*b-741*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^4*b^2-51*cos(d*x+c)^5*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3-51*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^5*sin(d*x+c
)*a^2*b^4-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^5*sin(d*x+c)*a*b^5)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x
+c)^(11/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\cos(dx+c)^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/cos(d*x + c)^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/cos(d*x + c)^(13/2), x)
```

3.626 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=379

$$\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] -(((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqrt[
a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (a*Sqrt[a + b]*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(b*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.735778, antiderivative size = 414, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {2820, 2809, 3003, 2993, 12, 2801, 2816, 2994}

$$\frac{a\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{a+b\cos(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b*d)) + (Sqrt[
a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d) + (a*Sqrt[a + b]*Cot[c + d*
x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d) + (a*Sin[c + d*x])/(b*d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2820

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> -Dist[(a*d)/(2*b), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a
 + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[(Sqrt[d*Sin[e + f*x]]*(a + 2
*b*Sin[e + f*x]))/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
```

```
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{a\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b}$$

$$= \frac{a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} +$$

$$= \frac{a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} +$$

$$= \frac{a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} +$$

$$= \frac{a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^2d} +$$

$$= -\frac{(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{abd}$$

Mathematica [C] time = 4.18681, size = 479, normalized size = 1.26

$$\frac{\sqrt{\cos(c+dx)}\left(2a\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\tan\left(\frac{1}{2}(c+dx)\right)-b\sqrt{\frac{a-b}{a+b}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\tan\left(\frac{1}{2}(c+dx)\right)+b\sqrt{\frac{a-b}{a+b}}\sin\left(\frac{3}{2}(c+dx)\right)\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*((2*I)*(a - b)*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))] - (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos
[c + d*x]))]*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -
((a + b)/(a - b))] + (4*I)*a*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(
c + d*x)/2]], -((a + b)/(a - b))] + b*Sqrt[(a - b)/(a + b)]*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*Sin[(3*(c + d*x))/2] + 2*a*Sqrt[(a
- b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2] - b*Sq
rt[(a - b)/(a + b)]*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])
)/(2*b*Sqrt[(a - b)/(a + b)]*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a
 + b*Cos[c + d*x]])
```

Maple [A] time = 0.59, size = 622, normalized size = 1.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/b*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c
)+b*cos(d*x+c)^3+a*cos(d*x+c)^2-b*cos(d*x+c)^2-cos(d*x+c)*a)/(a+b*cos(d*x+c
))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}\left(c+dx\right)}{\sqrt{a+b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.627 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=116

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rubi [A] time = 0.0684566, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2809}

$$-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
+ d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = -\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{bd}$$

Mathematica [A] time = 0.819468, size = 132, normalized size = 1.14

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*EllipticPi[-
1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]))/(d*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.499, size = 159, normalized size = 1.4

$$-2\frac{(\sin(dx+c))^2}{d\sqrt{a+b\cos(dx+c)}\,(-1+\cos(dx+c))\sqrt{\cos(dx+c)}}\left(\operatorname{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-2\operatorname{EllipticPi}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-2*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))*sin(d*x+c)^2/(a+b*c
os(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.628 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=109

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

```
[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rubi [A] time = 0.0709568, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2816}

$$\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt
[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]
))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \frac{2\sqrt{a+b}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec}{a\cdot}}}{ad}$$

Mathematica [A] time = 1.33718, size = 170, normalized size = 1.56

$$-\frac{4(a+b)\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{a-b}}\sqrt{\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a}}F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\cos(c+dx)}{a(\cos(c+dx)-1)}}\right)|\frac{2a}{a-b}\right)}{ad\sqrt{a+b\cos(c+dx)}\left(-\frac{(a+b)\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (-4*(a + b)*Cos[c + d*x]^(3/2)*Sqrt[-(((a + b)*Cot[(c + d*x)/2]^2)/(a - b))
]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[
ArcSin[Sqrt[-((a + b*Cos[c + d*x])/(a*(-1 + Cos[c + d*x])))]], (2*a)/(a - b
)])/(a*d*Sqrt[a + b*Cos[c + d*x]]*(-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]
^2)/a))^(3/2))
```

Maple [A] time = 0.453, size = 123, normalized size = 1.1

$$-2\,\frac{(\sin(dx+c))^4}{d\sqrt{a+b\cos(dx+c)}\,(\cos(dx+c))^{3/2}\,(-1+\cos(dx+c))^2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-}{a+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(a+b*cos(d*x+c))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^2 + a*
cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos{(dx+c)}+a}\sqrt{\cos{(dx+c)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))), x)
```

3.629 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=224

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(}{a+}}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt
[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```

Rubi [A] time = 0.233901, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2801, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(}{a+}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*d) - (2*Sqrt
[a + b]*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d)
```


Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
```

```
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = -\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx + \int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+d}}$$
$$= \frac{2(a-b)\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{a^2d}$$

Mathematica [A] time = 4.65348, size = 211, normalized size = 0.94

$$\frac{2\left(\tan\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))+a\sqrt{\cos(c+dx)}\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right|}{ad\sqrt{\cos(c+dx)}\sqrt{a+b\cos(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (2*(-((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]) + a*Sqrt[Cos[c + d*x]]*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + (a + b*Cos[c + d*x])*Tan[(c + d*x)/2]))/(a*d*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.494, size = 612, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d/a*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+b*cos(d*x+
c)^2+cos(d*x+c)*a-b*cos(d*x+c)-a)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x
```

```
+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b\cos(dx+c)^3+a\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^3 + a*
cos(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)), x)
```

3.630 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=274

$$\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{4b(a-b)\sqrt{a+b}\cot(c+}{}$$

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*d) + (2
*Sqrt[a + b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d) + (2*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.403616, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2802, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(a+2b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d}-\frac{4b(a-b)\sqrt{a+b}\cot(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```


```
[Out] (-4*(a - b)*b*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c +
d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*d) + (2
*Sqrt[a + b]*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d) + (2*Sq
rt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
```

```
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx = \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{-b+\frac{1}{2}a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} - \frac{(2b)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{3a} + \frac{(a+2b)\int}{}$$

$$= -\frac{4(a-b)b\sqrt{a+b}\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{3a^3d}$$

Mathematica [A] time = 13.6084, size = 371, normalized size = 1.35

$$\frac{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\tan(c+dx)\sec(c+dx)}{3a}-\frac{4b\tan(c+dx)}{3a^2}\right)}{d}+\frac{16\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\cos^2\left(\frac{1}{2}(c+dx)\right)^{7/2}\sqrt{\cos(c+dx)\sec}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] (16*(Cos[(c + d*x)/2]^2)^(7/2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[C
os[c + d*x]*Sec[(c + d*x)/2]^2]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a - 2*b)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c + d*x]*(a
+ b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*Cos[c + d*
x]^(3/2)*(1 + Cos[c + d*x])^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-4*b*Tan[c + d*x])/(3*a^2) + (2*Sec[c + d
```

```
*x]*Tan[c + d*x])/(3*a)))/d
```

Maple [B] time = 0.56, size = 883, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a^2*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^2+a*b*cos(d*x+c)^3-2*cos(d*x+c)^3*b^2+cos(d*x+c)^2*a^2-2*co
s(d*x+c)^2*a*b+2*b^2*cos(d*x+c)^2+cos(d*x+c)*a*b-a^2)/(a+b*cos(d*x+c))^(1/2
)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b*cos(d*x + c)^4 + a*
cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)), x)
```

3.631 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=465

$$-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{\left(3a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{a}$$

```
[Out] -(((3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt[a + b]*d))
+ ((3*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d) + (3*a*
Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d) - (2*a^2
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]
) + ((3*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*
d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.986229, antiderivative size = 465, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2792, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}-\frac{\left(3a^2-b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] -(((3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b^2*Sqrt[a + b]*d))
+ ((3*a + b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d) + (3*a*
Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d) - (2*a^2
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]
) + ((3*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*
d*Sqrt[Cos[c + d*x]])
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
```

```
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{2\int \frac{\frac{a^2}{2}-\frac{1}{2}ab\cos(c+dx)-\frac{1}{2}\left(3a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{\int \frac{1}{2}a(3a}{}$$

$$= -\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{b^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}} - \frac{(3a)\int}{}$$

$$= \frac{3a\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{b^3d}$$

$$= -\frac{\left(3a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ab^2\sqrt{a+b}d}$$

Mathematica [C] time = 6.21642, size = 1201, normalized size = 2.58

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(-a^2 + b^2)*d*Sqrt[a + b*Cos[c
+ d*x]]) + ((-4*a*(a^2 - b^2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c
 + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]
^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 8*a^2*b*((Sqrt[
((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c
+ d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqr
t[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[
-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*
Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*
x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2 - b^2)
*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c +
 d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c
 + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)
]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
```

```
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(
2*(a - b)*b*(a + b)*d)
```

Maple [B] time = 0.396, size = 1665, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/d/b^2/(a-b)/(a+b)*(-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*cos(d*x+c)^2*a
^2*b+cos(d*x+c)*a*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-6*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-a*b^2*cos(d*x+c)^2+cos(d*x+
c)^3*a^2*b+2*cos(d*x+c)*a^2*b+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*a^2*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-3*a^3*cos(d*x
+c)+cos(d*x+c)^2*b^3-cos(d*x+c)^3*b^3+3*cos(d*x+c)^2*a^3)/(a+b*cos(d*x+c))^
(1/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{2}}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.632 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=387

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{b^2d}$$

```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d) - (2*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b^2*d) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.502568, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2797, 2809, 2794, 2795, 2816, 2994}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b}\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d) - (2*Cot[c + d*x]*E
llipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d) - (2*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d
*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
 + Sec[c + d*x]))/(a - b)])/(b^2*d) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2797

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(3/2), x_Symbol] :> Dist[d/b, Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin
[e + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Sin[e + f*x]]/(a + b*Sin[e +
f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
```

```
2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} - \frac{a\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx}{b}$$

$$= -\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{b^2d}$$

$$= -\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx}{a-b}}}{b^2d}$$

$$= \frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b\sqrt{a+b}d} - \frac{2\cot(c+}{}$$

Mathematica [C] time = 16.4351, size = 985, normalized size = 2.55

$$\frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}}-\frac{2a\left(\frac{i\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{a+b\cos(c+dx)}E\left(i\sinh^{-1}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\middle|-\frac{2a}{-a-b}\right)\sec(c+dx)}{b\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\sqrt{\frac{(a+b\cos(c+dx))\sec(c+dx)}{a+b}}}+\frac{2a\left(a\sqrt{\frac{(a+b)\cot^2\left(\frac{1}{2}(c+dx)\right)}{b-a}}\sqrt{-\frac{(a+\cdots}{\cdots}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x
]]) - (-4*a*b*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)
*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d
*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/
2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b),
 ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/
(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]
])) + 2*a*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh
[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x]
)/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d
*x]])))/((a - b)*(a + b)*d)
```

Maple [B] time = 0.515, size = 1206, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/d/b/(a-b)/(a+b)*(-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
```

```
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*a*b-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c
)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin
(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a^2*sin(d*x+c)+2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*b-a^2*cos(d
*x+c)+cos(d*x+c)*a*b)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{3}{2}}}{b^2\cos\left(dx+c\right)^2+2\,ab\cos\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**(3/2)/(a + b*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.633 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=266

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ad\sqrt{a+b}}$$

```
[Out] (-2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) + (2*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.334565, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2794, 2795, 2816, 2994}

$$\frac{2a\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) + (2*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) + (2*a*Sin[c + d*x])/((a^2 - b^2)*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{2a\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx}{a+b} - \frac{a\int \frac{1+\mathrm{co}}{\cos^{\frac{3}{2}}(c+dx)\,\cdots}}{a^2\cdots}$$

$$= -\frac{2\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2\cot(c+\cdots}{\cdots}$$

Mathematica [A] time = 4.60895, size = 196, normalized size = 0.74

$$\frac{2\left((a-b)\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)-(a+b)\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+(a\cdots\right.}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*((a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (a
+ b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
 d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a - b)*Sq
rt[Cos[c + d*x]]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]
)
```

Maple [B] time = 0.471, size = 809, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/(a-b)/(a+b)*(-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b+(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin
(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b*sin(d*x+c)+a*cos(d*x+c)^2-b*cos(d*x+c)^2-cos(d*x+c)*a+b*cos(d*x+c)
)/(a+b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(a + b*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

3.634 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{a^2d\sqrt{a+b}}$$

```
[Out] (2*b*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*b*Sin[c + d*x])/((a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.387638, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2800, 2998, 2816, 2994}

$$-\frac{2b\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|\right.}{a^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*b*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d) - (2*b*Sin[c + d*x])/((a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2800

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[(2*b*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(b + a
*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx = -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx}{a^2-b^2}$$

$$= -\frac{2b\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx}{a+b} +$$

$$= \frac{2b\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{a^2\sqrt{a+b}d}$$

Mathematica [A] time = 5.28048, size = 202, normalized size = 0.76

$$\frac{2\left(b(b-a)\sqrt{\cos(c+dx)}\tan\left(\frac{1}{2}(c+dx)\right)+a(a+b)\sqrt{\cos(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-\right.}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(-(b*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])
+ a*(a + b)*Sqrt[1 + Cos[c + d*x]]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(
-a + b)*Sqrt[Cos[c + d*x]]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*Sqrt[a + b*C
os[c + d*x]])
```

Maple [B] time = 0.563, size = 830, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/(a+b)/(a-b)/a/(a+b*cos(d*x+c))^(1/2)*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*
b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*sin(d*x+c)*a*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-cos(d*x+c)^2*a*
b+b^2*cos(d*x+c)^2+cos(d*x+c)*a*b-cos(d*x+c)*b^2)/cos(d*x+c)^(1/2)/sin(d*x+
c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^3+2\,ab\cos(dx+c)^2+a^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^3 +
2*a*b*cos(d*x + c)^2 + a^2*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c))), x)
```

3.635 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=285

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{a^3d\sqrt{a+b}}$$

```
[Out] (2*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a + b]*d) - (
2*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*b^2
*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```

Rubi [A] time = 0.461182, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2802, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-2b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{\cos}}\right.\right.}{a^3d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```


```
[Out] (2*(a^2 - 2*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sq
rt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*
x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3*Sqrt[a + b]*d) - (
2*(a + 2*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
+ b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d) + (2*b^2
*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]
)
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2-2b^2\right)-\frac{1}{2}ab\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{(a+2b)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+}}}{a(a+b)}$$

$$= \frac{2\left(a^2-2b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+}{}}}{a^3\sqrt{a+b}d}$$

Mathematica [C] time = 6.23846, size = 1233, normalized size = 4.33

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(2*a^2*b - 2*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(a^3 - 2*a*b^2)*
((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*
Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
```

```
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(a^2*b
 - 2*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh
[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sq
rt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x]
)/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-((
(a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc
[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a
 + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(
c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a
)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticP
i[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]]
, (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos
[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d
*x]])))/(a^2*(-a + b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]]*((-2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])) + (2*Tan
[c + d*x])/a^2))/d
```

Maple [B] time = 0.609, size = 1452, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/a^2/(a-b)/(a+b)*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*
b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a^3-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*b^3+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
```

```
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+cos(d*x+c)^2*a
^2*b+a*b^2*cos(d*x+c)^2-2*cos(d*x+c)^2*b^3+a^3*cos(d*x+c)-cos(d*x+c)*a^2*b-
2*cos(d*x+c)*a*b^2+2*cos(d*x+c)*b^3-a^3+a*b^2)/(a+b*cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^4+2\,ab\cos(dx+c)^3+a^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^4 +
2*a*b*cos(d*x + c)^3 + a^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2)), x)
```

3.636 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=357

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}-\frac{2b\left(5a^2-8b^2\right)\cot(c+dx)}{}$$

```
[Out] (-2*b*(5*a^2 - 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]
*d) + (2*(a + 2*b)*(a + 4*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt
[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[
a + b*Cos[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.724071, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2802, 3055, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}-\frac{2b\left(5a^2-8b^2\right)\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```


```
[Out] (-2*b*(5*a^2 - 8*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*Sqrt[a + b]
*d) + (2*(a + 2*b)*(a + 4*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt
[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[
a + b*Cos[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*
x])/(3*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(3/2))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2-4b^2\right)-\frac{1}{2}ab\cos(c+dx)+b}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\cos(c}}{3a^2\left(a^2-b^2\right)d\cos}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\cos(c}}{3a^2\left(a^2-b^2\right)d\cos}$$

$$= -\frac{2b\left(5a^2-8b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{3a^4\sqrt{a+b}d}$$

Mathematica [C] time = 6.31621, size = 1269, normalized size = 3.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((-4*a*(a^4 + 7*a^2*b^2 - 8*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)
]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c +
 d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Co
s[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)
/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(5*a^3*b
 - 8*a*b^3)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]
^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), A
rcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-
a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
) + 2*(5*a^2*b^2 - 8*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*Ell
ipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec
[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*
x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(
b*Sqrt[Cos[c + d*x]])))/(3*a^3*(a - b)*(a + b)*d) + (Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Cos[c + d*x]]*((2*b^4*Sin[c + d*x])/(a^3*(a^2 - b^2)*(a + b*Cos[c +
 d*x])) - (10*b*Tan[c + d*x])/(3*a^3) + (2*Sec[c + d*x]*Tan[c + d*x])/(3*a^
2)))/d
```

Maple [B] time = 0.414, size = 1781, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/(a+b)/(a-b)/a^3*(-8*b^4*cos(d*x+c)^2+a^2*b^2+4*a^2*b^2*cos(d*x+c)^2-
8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^2*sin(d*x+c)*b^4-a^4-5*cos(d*x+c)^3*a^2*b^2-5*cos(d*x+c)^2*a^3*b+8*cos(
d*x+c)^2*a*b^3-4*cos(d*x+c)*a*b^3+cos(d*x+c)^2*a^4+(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4-8*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*b^4+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*a^4+cos(d*x+c)^3*a^3*b-4*cos(d*x+c)^3*a*b^3+4*co
s(d*x+c)*a^3*b+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b+5*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)
*a^2*b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*a*b^3-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b+2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2
*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)*a*b^3+8*cos(d*x+c)^3*b^4+5*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+5*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin
(d*x+c)*a^2*b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^
2*b^2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^2\cos\left(dx+c\right)^5+2\,ab\cos\left(dx+c\right)^4+a^2\cos\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^5 +
2*a*b*cos(d*x + c)^4 + a^2*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\cos\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2)), x)
```

3.637 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=433

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^3d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{}}{5a^2d\left(a^2-b^2\right)\cos}$$

```
[Out] (2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^5*S
qrt[a + b]*d) - (2*(3*a + 4*b)*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(5*a^4*Sqrt[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c +
d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^2 - 6*b^2)*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)) - (2*b*(3*a^2
- 8*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d*Cos[c
+ d*x]^(3/2))
```

Rubi [A] time = 1.05696, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2802, 3055, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)}{ad\left(a^2-b^2\right)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}-\frac{2b\left(3a^2-8b^2\right)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{5a^3d\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)}+\frac{2\left(a^2-6b^2\right)\sin(c+dx)\sqrt{}}{5a^2d\left(a^2-b^2\right)\cos}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (2*(3*a^4 + 8*a^2*b^2 - 16*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a^5*S
qrt[a + b]*d) - (2*(3*a + 4*b)*(a^2 + 4*b^2)*Cot[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(5*a^4*Sqrt[a + b]*d) + (2*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*d*Cos[c +
d*x]^(5/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^2 - 6*b^2)*Sqrt[a + b*Cos[c +
d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)) - (2*b*(3*a^2
- 8*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(5*a^3*(a^2 - b^2)*d*Cos[c
+ d*x]^(3/2))
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
```

```
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx = \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}\left(a^2-6b^2\right)-\frac{1}{2}ab\cos(c+dx)+2b^2\cos}{\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\cos(c+d}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\cos(c+dx}}{5a^2\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\cos(c+dx}}{5a^2\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c}$$

$$= \frac{2b^2\sin(c+dx)}{a\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-6b^2\right)\sqrt{a+b\cos(c+dx}}{5a^2\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c}$$

$$= \frac{2\left(3a^4+8a^2b^2-16b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c}{a+b}}}{5a^5\sqrt{a+b}d}$$

Mathematica [C] time = 6.3611, size = 1314, normalized size = 3.03

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(a + b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((a^2 + 4*b^2)*((-4*a*(4*a^2*b - 4*b^3)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
 + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
 + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(
3*a^3 - 4*a*b^2)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a +
 b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c
+ d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Cs
c[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b
)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*
x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqr
t[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/
b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*
a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d
*x]])) + 2*(3*a^2*b - 4*b^3)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*
EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*
Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c +
 d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)
/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[(
(a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[
(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*
Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Cs
c[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc
[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(b*Sqrt[Cos[c + d*x]]))))/(5*a^4*(-a + b)*(a + b)*d) + (Sqrt[Cos[c + d*x]
```

```
]*Sqrt[a + b*Cos[c + d*x]]*((-2*b^5*Sin[c + d*x])/(a^4*(a^2 - b^2)*(a + b*C
os[c + d*x])) + (2*Sec[c + d*x]*(3*a^2*Sin[c + d*x] + 11*b^2*Sin[c + d*x]))
/(5*a^4) - (6*b*Sec[c + d*x]*Tan[c + d*x])/(5*a^3) + (2*Sec[c + d*x]^2*Tan[
c + d*x])/(5*a^2)))/d
```

Maple [B] time = 0.599, size = 2478, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/5/d/(a+b)/(a-b)/a^4*(-5*cos(d*x+c)^3*a^4*b+8*cos(d*x+c)^2*a*b^4-2*cos(d*
x+c)*a^2*b^3+3*cos(d*x+c)^4*a^4*b+8*cos(d*x+c)^4*a^2*b^3+8*cos(d*x+c)^3*a^3
*b^2-16*cos(d*x+c)^3*a*b^4-6*cos(d*x+c)^3*a^2*b^3-6*cos(d*x+c)^2*a^3*b^2+2*
cos(d*x+c)*a^4*b-3*cos(d*x+c)^4*a^3*b^2+8*cos(d*x+c)^4*a*b^4-16*cos(d*x+c)^
4*b^5+3*cos(d*x+c)^3*a^5+16*cos(d*x+c)^3*b^5+a^3*b^2+3*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5-a
^5-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)^3*sin(d*x+c)*a^2*b^3+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^4-(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b+8*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)^2*sin(d*x+c)*a^3*b^2-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3-16*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4-3*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^2*sin(d*x+c)*a^4*b-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2-8*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3+16*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^
2*sin(d*x+c)*a*b^4-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4*b+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b^2-4*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(
d*x+c)*a^2*b^3-16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^4-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4*b-8*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x
```

```
+c)*a^3*b^2-2*cos(d*x+c)^2*a^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5+16*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^5+3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*a^5-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^5)/(a+b*cos(d*
x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^2\cos(dx+c)^6+2\,ab\cos(dx+c)^5+a^2\cos(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^2*cos(d*x + c)^6 +
2*a*b*cos(d*x + c)^5 + a^2*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2)), x)
```

3.638 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=497

$$-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+ab-6b^2\right)\cot(c+dx)}{}$$

```
[Out] (2*(3*a^2 - 7*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^2*(a + b
)^(3/2)*d) - (2*(3*a^2 + a*b - 6*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^2*(a + b)^(3/2)*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) - (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d
*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.06442, antiderivative size = 497, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {2792, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3bd\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+ab-6b^2\right)\cot(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*(3*a^2 - 7*b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^2*(a + b
)^(3/2)*d) - (2*(3*a^2 + a*b - 6*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^2*(a + b)^(3/2)*d) - (2*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a +
 b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(b^3*d) - (2*a^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b*(
a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (2*a^2*(3*a^2 - 7*b^2)*Sin[c + d
*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
```

```
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
```

```
&& PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{a^2}{2}-\frac{3}{2}ab\cos(c+dx)-\frac{3}{2}\left(a^2-b^2\right)\cos^2(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{3b\left(a^2-b^2\right)}$$

$$= -\frac{2a^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b^2} - \frac{2\int \frac{\frac{a^2b}{2}+\left(-\frac{3ab^2}{2}+\frac{3}{2}a\left(a^2-b^2\right)\right)\cos(c+a}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}}{3b^2\left(a^2-b^2\right)}$$

$$= -\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3d}$$

$$= -\frac{2\sqrt{a+b}\cot(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{b^3d}$$

$$= \frac{2\left(3a^2-7b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3(a-b)b^2(a+b)^{3/2}d} -$$

Mathematica [C] time = 6.28095, size = 1282, normalized size = 2.58

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*a^2*Sin[c + d*x])/(3*b*(-a
^2 + b^2)*(a + b*Cos[c + d*x])^2) + (2*(3*a^3*Sin[c + d*x] - 7*a*b^2*Sin[c
+ d*x]))/(3*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d - ((-4*a*(a^3 - a*b^
2)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]
*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*
Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d
*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-(a^2*b) - 3*b^3)*((Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a
)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[
c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*C
os[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x
)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x]
)*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^3 - 7*a*b^2)*((I*Cos
[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]
/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/
2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*
a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d
*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/
a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2
]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
```

```
+ d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b +
 (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*(a - b
)^2*b*(a + b)^2*d)
```

Maple [B] time = 0.424, size = 3911, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)^2/(a+b)^2/b^2*(-4*cos(d*x+c)^3*a^4*b+7*cos(d*x+c)^2*a*b^4+6*co
s(d*x+c)*a^2*b^3+3*cos(d*x+c)^3*a^3*b^2-7*cos(d*x+c)^3*a*b^4+6*cos(d*x+c)^2
*a^4*b-14*cos(d*x+c)^2*a^2*b^3-7*cos(d*x+c)*a^3*b^2+8*cos(d*x+c)^3*a^2*b^3+
4*cos(d*x+c)^2*a^3*b^2-2*cos(d*x+c)*a^4*b+6*EllipticPi((-1+cos(d*x+c))/sin(
d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)+6*cos(d*x+c)^
2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))
^(1/2))*b^5+3*a^5*cos(d*x+c)+6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*b-12*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a^2*b^3+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*
x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*b-12*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2-12*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a^2*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3-6*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)*a*b^4-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2+7*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+
c)*a^2*b^3+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4-3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5+6*cos(d*x+c)*sin(d*x
```

```
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a
^5+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*b^5-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-3*cos(d*x+c)*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-12*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)+6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^4*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)-3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)+7*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2*si
n(d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b^3*sin(d*x+c)+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4+2*cos(d*x+c)*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*b^2-7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4+4*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3
*b^2+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b^4-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b+14*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^3-3*co
s(d*x+c)^2*a^5)/(a+b*cos(d*x+c))^(3/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{5}{2}}}{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{5}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.639 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=342

$$-\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(a-3b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{3a}$$

```
[Out] (8*b*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) + (2*
(a - 3*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d)
+ (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.609934, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2799, 2993, 2998, 2816, 2994}

$$-\frac{8ab\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2(a-3b)\cot(c+dx)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (8*b*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d) + (2*
(a - 3*b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d)
+ (2*a*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x
])^(3/2)) - (8*a*b*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt
[a + b*Cos[c + d*x]])
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{a}{2}+\frac{3}{2}b\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{2}{}$$

$$= \frac{2a\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{8ab\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{(a}{}$$

$$= \frac{8b\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a(a-b)(a+b)^{3/2}d} + \frac{2(a-3}{}$$

Mathematica [A] time = 6.12483, size = 277, normalized size = 0.81

$$2\left(\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^3+3ab^2+4b^3\cos(c+dx)\right)-\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)}(a+b\cos(c+dx))\left(-\left(a^2+4ab+3b^2\right)\sqrt{\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(Sqrt[Cos[c + d*x]]*(a^3 + 3*a*b^2 + 4*b^3*Cos[c + d*x])*Sin[c + d*x] -
Sqrt[Cos[(c + d*x)/2]^2]*(a + b*Cos[c + d*x])*(4*b*(a + b)*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d
*x)/2]^2)/(a + b)] - (a^2 + 4*a*b + 3*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)]*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)] + 4*b*(a + b*Cos[c + d*x])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Tan[(c
+ d*x)/2])))/(3*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 0.557, size = 1782, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)^2/(a+b)^2*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-4*cos(d*x+c)^2*a^2*b-3*c
os(d*x+c)*a*b^2-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+4*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*a*b^2*co
s(d*x+c)^2-5*cos(d*x+c)^3*a*b^2+4*cos(d*x+c)*a^2*b-4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3-4*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a^2*b-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+7*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a*b^2-a^3*cos(d*x+c)-4*cos(d*x+c)^2*b^3+cos(d*x+c)^3*a^3+4*cos(d*x+c)^3*b^3
-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
```

```
x+c)^2*sin(d*x+c)*a*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+4*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*b^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3)/(a+b*cos(d*x+c))^(3/2)/cos(d*x+c)^(
1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{3}{2}}}{b^3\cos(dx+c)^3+3ab^2\cos(dx+c)^2+3a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(3/2)/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.640 $\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec}{a+}}}{}$$

```
[Out] (-2*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(a - b)*(a + b)
^(3/2)*d) + (2*(3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a
 + b)^(3/2)*d) - (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a
+ b*Cos[c + d*x])^(3/2)) + (2*(3*a^2 + b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.641685, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2796, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+b^2\right)\cot(c+dx)\sqrt{\frac{a(1-\sec}{a+}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3*a^2 + b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(a - b)*(a + b)
^(3/2)*d) + (2*(3*a - b)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d
*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a
 + b)^(3/2)*d) - (2*b*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a
+ b*Cos[c + d*x])^(3/2)) + (2*(3*a^2 + b^2)*Sin[c + d*x])/(3*(a^2 - b^2)^2*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
```

```
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{2\int \frac{\frac{b}{2}-\frac{3}{2}a\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{3\left(a^2-b^2\right)}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} - \frac{2\int}{}$$

$$= -\frac{2b\sqrt{\cos(c+dx)}\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}} + \frac{(3a}{}$$

$$= -\frac{2\left(3a^2+b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{3a^2(a-b)(a+b)^{3/2}d} +$$

Mathematica [C] time = 6.23019, size = 1273, normalized size = 3.55

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*b*Sin[c + d*x])/(3*(a^2 -
 b^2)*(a + b*Cos[c + d*x])^2) - (2*(3*a^2*b*Sin[c + d*x] + b^3*Sin[c + d*x]
))/(3*a*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(-(a^2*b) + b^3)*S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3*a^3 + a*b^2)*((Sqrt[((a + b)*Cot[(c + d
*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sq
rt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcS
in[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a +
 b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x
]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(
c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(3*a^2*b + b^3)*((I*Cos[(c + d*x
)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos
[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqr
t[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(
c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/S
qrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*S
qrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a
+ b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]]))))/(3*a*(a - b)^2*(a
+ b)^2*d)
```

Maple [B] time = 0.425, size = 2417, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)^2/(a+b)^2/a*(b^4*cos(d*x+c)^2+4*a^2*b^2*cos(d*x+c)^2-cos(d*x+c
)*a^2*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^4*sin(d*x+c)+3*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^4-3*cos(d*x+c)^3*a^2*b^2-6*cos(d
*x+c)^2*a^3*b-2*cos(d*x+c)^2*a*b^3+3*cos(d*x+c)^2*a^4+(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^4-3*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
sin(d*x+c)*a^4+2*cos(d*x+c)^3*a^3*b+2*cos(d*x+c)^3*a*b^3+4*cos(d*x+c)*a^3*b
-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
```

```
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*a*b^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2+(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin
(d*x+c)*a*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2-(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c
)*a*b^3-cos(d*x+c)^3*b^4+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+4*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a*b^3-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+3*sin(d*x+c)*cos
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
^4-4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^3*b-sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b^2+3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*a^4*cos(d*x+c))/(a+b*c
os(d*x+c))^(3/2)/sin(d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

3.641 $\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=381

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2-3ab-2b^2\right)\cot(c+dx)\sqrt{}}{}$$

```
[Out] (4*b*(3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b
)^(3/2)*d) + (2*(3*a^2 - 3*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^2*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (4*b*(3*a^2 - b^2)*Sin[c +
d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.731584, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2802, 2993, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{3ad\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2-3ab-2b^2\right)\cot(c+dx)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```


```
[Out] (4*b*(3*a^2 - b^2)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(
Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b
)^(3/2)*d) + (2*(3*a^2 - 3*a*b - 2*b^2)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[
a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*
Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/
(3*a^2*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) - (4*b*(3*a^2 - b^2)*Sin[c +
d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2-2b^2\right)-\frac{3}{2}ab\cos(c+dx)}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\,c}}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} - \frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{3a\left(a^2-b^2\right)^2 d\sqrt{\cos(c+dx)}\sqrt{a+b\,c}}$$

$$= \frac{4b\left(3a^2-b^2\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{3a^3(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.2511, size = 1296, normalized size = 3.4

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*b^2*Sin[c + d*x])/(3*a*(a^
2 - b^2)*(a + b*Cos[c + d*x])^2) + (4*(3*a^2*b^2*Sin[c + d*x] - b^4*Sin[c +
 d*x]))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + ((-4*a*(3*a^4 - 5*
a^2*b^2 + 2*b^4)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b
)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c +
d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[
(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(-6*a^3*b + 2*a*b^3)*((S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]
/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*S
qrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(-6*a^2*b
^2 + 2*b^4)*((I*Cos[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSi
nh[Sin[(c + d*x)/2]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*
x])/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(
(a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot
[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)
/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ellipti
cPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*C
os[c + d*x]])))/b + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c +
 d*x]]))))/(3*a^2*(a - b)^2*(a + b)^2*d)
```

Maple [B] time = 0.638, size = 2741, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/a^2/(a+b)^2/(a-b)^2/(a+b*cos(d*x+c))^(3/2)*(-4*cos(d*x+c)^2*a*b^4-2*
cos(d*x+c)*a^2*b^3-5*cos(d*x+c)^3*a^3*b^2+cos(d*x+c)^3*a*b^4-6*cos(d*x+c)^2
*a^4*b-4*cos(d*x+c)^2*a^2*b^3-7*cos(d*x+c)*a^3*b^2+6*cos(d*x+c)^3*a^2*b^3+1
2*cos(d*x+c)^2*a^3*b^2+6*cos(d*x+c)*a^4*b-2*cos(d*x+c)^3*b^5+3*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*a^5+3*cos(d*x+c)*a*b^4+2*cos(d*x+c)^2*b^5+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b+6*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin
(d*x+c)*a^3*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
```

```
(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4-6*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x
+c)*a^3*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4+6*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)+(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*
x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4*b*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^5+2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*sin(d*
x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
5*sin(d*x+c)+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b
^3-2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^4-12*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+4*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-6*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*si
n(d*x+c)*a^4*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^5)/cos(d*x+c)^(1/2
)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\sqrt{\cos\left(dx+c\right)}}{b^3\cos\left(dx+c\right)^4+3\,ab^2\cos\left(dx+c\right)^3+3\,a^2b\cos\left(dx+c\right)^2+a^3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^4 +
3*a*b^2*cos(d*x + c)^3 + 3*a^2*b*cos(d*x + c)^2 + a^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c))), x)
```

3.642 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(9a^2b+3a^3-\right.}{}$$

```
[Out] (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(
a - b)*(a + b)^(3/2)*d) - (2*(3*a^3 + 9*a^2*b - 6*a*b^2 - 8*b^3)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (8
*b^2*(2*a^2 - b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.813093, antiderivative size = 398, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2802, 3055, 2998, 2816, 2994}

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(9a^2b+3a^3-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```


```
[Out] (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(
a - b)*(a + b)^(3/2)*d) - (2*(3*a^3 + 9*a^2*b - 6*a*b^2 - 8*b^3)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d) + (2*b^2*Sin[c + d
*x])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Cos[c + d*x])^(3/2)) + (8
*b^2*(2*a^2 - b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Cos[c + d*x]])
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{1}{2}\left(3a^2-4b^2\right)-\frac{3}{2}ab\cos(c+\ldots}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos\ldots}}{3a\left(a^2-\ldots\right.}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-\ldots\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(\ldots}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-\ldots\right.}{3a^2\left(a^2-b^2\right)^2 d\sqrt{\cos(c\ldots}}$$

$$= \frac{2\left(3a^4-15a^2b^2+8b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\ldots}{\ldots}}}{3a^4(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.37166, size = 1321, normalized size = 3.32

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] -((-4*a*(9*a^4*b - 17*a^2*b^3 + 8*b^5)*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-
a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a
+ b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c
+ d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - 4*a*(3
*a^5 - 15*a^3*b^2 + 8*a*b^4)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*
Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d
*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[
c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2
]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (Sqrt[((a + b)
*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2
]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*Ell
ipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sq
rt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a +
 b*Cos[c + d*x]])) + 2*(3*a^4*b - 15*a^2*b^3 + 8*b^5)*((I*Cos[(c + d*x)/2]*
Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2]/Sqrt[Cos[c +
d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d
*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2*a*((a*Sqrt[((a
 + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d
*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2
]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-
(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*C
sc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*C
os[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x
)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b + (Sqrt[a + b*C
os[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^3*(a - b)^2*(a + b
)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((-2*b^3*Sin[c + d*x]
)/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) - (2*(9*a^2*b^3*Sin[c + d*x] -
 5*b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*Tan[c
 + d*x])/a^3))/d
```

Maple [B] time = 0.525, size = 3693, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d/(a-b)^2/(a+b)^2/a^3*(3*a^2*b^4-6*a^4*b^2+8*cos(d*x+c)^2*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6+3*a^6-3
*cos(d*x+c)^3*a^4*b^2+15*cos(d*x+c)^3*a^2*b^4+4*cos(d*x+c)^3*a*b^5-6*cos(d*
x+c)^2*a^5*b+30*cos(d*x+c)^2*a^3*b^3-10*cos(d*x+c)^2*a^2*b^4-16*cos(d*x+c)^
2*a*b^5+15*cos(d*x+c)*a^4*b^2-22*cos(d*x+c)*a^3*b^3-8*cos(d*x+c)*a^2*b^4+12
*cos(d*x+c)*a*b^5-8*cos(d*x+c)^3*a^3*b^3-8*cos(d*x+c)^3*b^6+8*cos(d*x+c)^2*
b^6-3*cos(d*x+c)*a^6+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5*b+3*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2-15*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2
*sin(d*x+c)*a^3*b^3-15*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^5-3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*
sin(d*x+c)*a^5*b+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b^2+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^3-2*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*s
in(d*x+c)*a^2*b^4-8*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^5+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b-12*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*a^4*b^2-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^3-7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4+16*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a
*b^5+3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^5*b+21*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2+13*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3-1
0*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
```

```
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^2*b^4-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^5+3*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^6+3*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^6+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*b^6-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)-15*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)-15*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+
c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b^4*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a*b^5*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^4*b^2-2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b^3-8*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^4+6*c
os(d*x+c)*a^5*b-6*cos(d*x+c)^2*a^4*b^2)/(a+b*cos(d*x+c))^(3/2)/sin(d*x+c)/c
os(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^5+3\,ab^2\cos(dx+c)^4+3\,a^2b\cos(dx+c)^3+a^3\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^5 +
3*a*b^2*cos(d*x + c)^4 + 3*a^2*b*cos(d*x + c)^3 + a^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2)), x)
```

3.643 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=473

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-13a^2b^2+a^4\right.}{3a^3}$$

```
[Out] (-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5
*(a - b)*(a + b)^(3/2)*d) + (2*(a^4 + 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*
b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (
2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^(3/2)) + (4*b^2*(5*a^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Co
s[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^
(3/2))
```

Rubi [A] time = 1.17324, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2802, 3055, 2998, 2816, 2994}

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-13a^2b^2+a^4\right.}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```


```
[Out] (-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^5
*(a - b)*(a + b)^(3/2)*d) + (2*(a^4 + 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*
b^4)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d) + (
2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(a + b*Cos[c + d*
x])^(3/2)) + (4*b^2*(5*a^2 - 3*b^2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Co
s[c + d*x]^(3/2)*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)*
Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d*Cos[c + d*x]^
(3/2))
```


Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx = \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{2\int \frac{\frac{3}{2}\left(a^2-2b^2\right)-\frac{3}{2}ab\cos(c+d\ldots}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos\ldots}}{3a\left(a^2-\ldots\right.}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3\ldots\right.}{3a^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c\ldots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3\ldots\right.}{3a^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c\ldots}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3\ldots\right.}{3a^2\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c\ldots}$$

$$= -\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1\ldots}{}}}{3a^5(a-b)(a+b)^{3/2}d}$$

Mathematica [C] time = 6.4626, size = 1351, normalized size = 2.86

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(a + b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] ((-4*a*(a^6 + 15*a^4*b^2 - 32*a^2*b^4 + 16*b^6)*Sqrt[((a + b)*Cot[(c + d*x)
/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])
 - 4*a*(8*a^5*b - 28*a^3*b^3 + 16*a*b^5)*((Sqrt[((a + b)*Cot[(c + d*x)/2]^2
)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a +
 b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin
[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (S
qrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc
[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[
c + d*x]*EllipticPi[-(a/b), ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)
/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Cos[c + d*x]])) + 2*(8*a^4*b^2 - 28*a^2*b^4 + 16*b^6)*((I*Co
s[(c + d*x)/2]*Sqrt[a + b*Cos[c + d*x]]*EllipticE[I*ArcSinh[Sin[(c + d*x)/2
]/Sqrt[Cos[c + d*x]]], (-2*a)/(-a - b)]*Sec[c + d*x])/(b*Sqrt[Cos[(c + d*x)
/2]^2*Sec[c + d*x]]*Sqrt[((a + b*Cos[c + d*x])*Sec[c + d*x])/(a + b)]) + (2
*a*((a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(-a + b)]*Sqrt[-(((a + b)*Cos[c +
d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)
/a]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/
2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sin[(c + d*x)/2]^4)/((a + b)*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Cos[c + d*x]]) - (a*Sqrt[((a + b)*Cot[(c + d*x)/2]^2)/(
-a + b)]*Sqrt[-(((a + b)*Cos[c + d*x]*Csc[(c + d*x)/2]^2)/a)]*Sqrt[((a + b*
Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]*Csc[c + d*x]*EllipticPi[-(a/b), ArcSin
[Sqrt[((a + b*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/a]/Sqrt[2]], (-2*a)/(-a + b
)]*Sin[(c + d*x)/2]^4)/(b*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]])))/b
+ (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(b*Sqrt[Cos[c + d*x]])))/(3*a^4*(
```

```
a - b)^2*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Cos[c + d*x]]*((2*b^
4*Sin[c + d*x])/(3*a^3*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (8*(3*a^2*b^4*
Sin[c + d*x] - 2*b^6*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(a + b*Cos[c + d*x
])) - (16*b*Tan[c + d*x])/(3*a^4) + (2*Sec[c + d*x]*Tan[c + d*x])/(3*a^3)))
/d
```

Maple [B] time = 0.512, size = 4189, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)^2/(a+b)^2/a^4*(-32*cos(d*x+c)^3*a*b^6-8*cos(d*x+c)^2*a^6*b+13*
cos(d*x+c)^2*a^5*b^2+28*cos(d*x+c)^2*a^4*b^3-42*cos(d*x+c)^2*a^3*b^4-16*cos
(d*x+c)^2*a^2*b^5+24*cos(d*x+c)^2*a*b^6+6*cos(d*x+c)*a^6*b-12*cos(d*x+c)*a^
4*b^3+6*cos(d*x+c)*a^2*b^5+cos(d*x+c)^4*a^5*b^2-8*cos(d*x+c)^4*a^4*b^3-13*c
os(d*x+c)^4*a^3*b^4+28*cos(d*x+c)^4*a^2*b^5+8*cos(d*x+c)^4*a*b^6+2*cos(d*x+
c)^3*a^6*b-16*cos(d*x+c)^3*a^5*b^2-8*cos(d*x+c)^3*a^4*b^3+56*cos(d*x+c)^3*a
^3*b^4-18*cos(d*x+c)^3*a^2*b^5-a^3*b^4-16*cos(d*x+c)^4*b^7+16*cos(d*x+c)^3*
b^7+cos(d*x+c)^2*a^7+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*b^7-a^7+2*a^5*b^2+16*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*
a*b^6-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*a^6*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5*b^2+35*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b
^3+24*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^3*b^4-20*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5-16*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^6+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^2*sin(d*x+c)*a^6*b+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^5*b^2-20*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*
b^3-56*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)*a^3*b^4+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^7+16*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^7-12*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
^2*sin(d*x+c)*a^2*b^5+32*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
```

s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^6-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*b+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b^2+28*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3-4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^4-16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^6*b+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b^2-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b^3-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^4+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^5+16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^6-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5*b^2+cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*b+7*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3+28*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^4-4*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5-16*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^6+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5*b^2+8*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b^4-28*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^7)/(a+b*cos(d*x+c))^(3/2)/sin(d*x+c)/cos(d*x+c)^(3/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\cos(dx+c)}}{b^3\cos(dx+c)^6+3\,ab^2\cos(dx+c)^5+3\,a^2b\cos(dx+c)^4+a^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(cos(d*x + c))/(b^3*cos(d*x + c)^6 +
3*a*b^2*cos(d*x + c)^5 + 3*a^2*b*cos(d*x + c)^4 + a^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2)), x)
```

3.644 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)|\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] (2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/5])/(Sqrt[5]*d)
```

Rubi [A] time = 0.0588819, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2813}

$$\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)|\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/5])/(Sqrt[5]*d)
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx = \frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}d}$$

Mathematica [B] time = 3.63154, size = 131, normalized size = 4.09

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)+2}\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+dx)\right)}\right)\middle|-\right.}{d\sqrt{\frac{-3\cos(c+dx)-2}{\cos(c+dx)-1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2]
, -4])/(d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt[Cos[c + d*x]
/(-1 + Cos[c + d*x])])
```

Maple [B] time = 0.462, size = 115, normalized size = 3.6

$$-\frac{\sqrt{2}\left(\sin\left(dx+c\right)\right)^{4}\sqrt{10}}{5d\left(-1+\cos\left(dx+c\right)\right)^{2}}\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{\frac{3}{2}}\sqrt{\frac{2+3\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\mathrm{EllipticF}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)},\frac{\sqrt{5}}{5}\right)\frac{1}{\sqrt{2+3\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(2+3*cos(d*x+c))^(1/2)*sin
(d*x+c)^4*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),1/5*5^(1/2))/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\cos\left(dx+c\right)+2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{3\cos\left(dx+c\right)+2}\sqrt{\cos\left(dx+c\right)}}{3\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 + 2*
cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\cos\left(c+dx\right)+2}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*cos(c + d*x) + 2)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

3.645 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=25

$$\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)\middle|5\right)}{d}$$

[Out] (2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5])/d

---

Rubi [A] time = 0.0525776, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2813}

$$\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]

[Out] (2*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5])/d

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx = \frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\middle|5\right)}{d}$$

Mathematica [B] time = 0.925119, size = 156, normalized size = 6.24

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-(3\cos(c+dx)-2)\csc^2\left(\frac{1}{2}(c+dx)\right)}F}{\sqrt{5}d\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]

[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]/2], 4/5]*Sin[(c + d*x)/2]^4)/(Sqrt[5]*d*Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])

Maple [B] time = 0.497, size = 107, normalized size = 4.3

$$-2\frac{(\sin(dx+c))^4}{d\sqrt{-2+3\cos(dx+c)}\,(\cos(dx+c))^{3/2}\,(-1+\cos(dx+c))^2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\mathrm{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(-2+3*cos(d*x+c))^(1/2)*sin(d*x+c)^4
*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),5^(1/2))/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)^2-2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 - 2*
cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\cos(c+dx)-2}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3*cos(c + d*x) - 2)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

3.646 $\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\sqrt{-\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])],
1/5])/(Sqrt[5]*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.116976, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2814, 2813}

$$-\frac{2\sqrt{-\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])],
1/5])/(Sqrt[5]*d*Sqrt[Cos[c + d*x]])
```

Rule 2814

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx &= \frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2\sqrt{-\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [B] time = 1.05145, size = 143, normalized size = 2.55

$$-\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{(2-3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx\right.}}{d\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*
Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[2 - 3*Co
s[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 0.499, size = 119, normalized size = 2.1

$$2\frac{\sqrt{2-3\cos(dx+c)}\,(\sin(dx+c))^4}{d\,(\cos(dx+c))^{3/2}\,(-2+3\cos(dx+c))\,(-1+\cos(dx+c))^2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{3/2}\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\mathrm{EllipticF}\left(\frac{-1}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(2-3*cos(d*x+c))^(1/2)*((-2+3*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*si
n(d*x+c)^4/cos(d*x+c)^(3/2)/(-2+3*cos(d*x+c))/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)^2-2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 -
2*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2 - 3*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))), x)
```

3.647 $\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=49

$$-\frac{2\sqrt{-\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])],
5])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.110361, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2814, 2813}

$$-\frac{2\sqrt{-\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])],
5])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2814

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx &= \frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{2\sqrt{-\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [B] time = 1.4557, size = 153, normalized size = 3.12

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+dx)\right)}F}{\sqrt{5}d\sqrt{-3\cos(c+dx)-2}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
5/2]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 4/5]*Sin[(c + d*x)/2]^4)/(Sqr
t[5]*d*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 0.447, size = 132, normalized size = 2.7

$$\frac{\sqrt{5}\sqrt{2}\sqrt{10}\left(\sin\left(dx+c\right)\right)^{4}}{5\,d\left(2+3\,\cos\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{2}}\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{\frac{3}{2}}\sqrt{-2-3\,\cos\left(dx+c\right)}\sqrt{\frac{2+3\,\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\mathrm{EllipticF}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/5/d*5^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(-2-3*cos(d*x+c))^(
1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(1/5*5^(1/2)
*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(2+3*cos
(d*x+c))/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,\cos\left(dx+c\right)-2}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\,\cos\left(dx+c\right)-2}\sqrt{\cos\left(dx+c\right)}}{3\,\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c)^2 +
2*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\cos\left(c+dx\right)-2}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*cos(c + d*x) - 2)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))), x)
```

3.648 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d
```

Rubi [A] time = 0.0529605, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2815}

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos
[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx = \frac{2\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

Mathematica [B] time = 1.08296, size = 140, normalized size = 2.41

$$\frac{4\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)+3}\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(c+dx)\right)}}{\sqrt{6}}\right)\middle|6\right)}{d\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*Sqrt[-Cot[(c + d*x)/2]^2]*Cs
c[c + d*x]*EllipticF[ArcSin[Sqrt[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/S
qrt[6]], 6])/(d*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(3 + 2*Cos[c
+ d*x])*Csc[(c + d*x)/2]^2])
```

Maple [B] time = 0.458, size = 116, normalized size = 2.

$$-\frac{\sqrt{2}\sqrt{10}\,(\sin(dx+c))^4}{5\,d\,(-1+\cos(dx+c))^2}\left(\frac{\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{i}{5}\sqrt{5}\right)\frac{1}{\sqrt{3+2\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(3+2*cos(d*x+c))^(1/2)*10^
(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),1/5*I*5^(1/2))*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}{2\cos(dx+c)^2+3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 + 3*
cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*cos(c + d*x) + 3)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

3.649 $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]
]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d)
```

Rubi [A] time = 0.0661629, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2815}

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]
]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d)
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx=\frac{2\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

Mathematica [B] time = 1.02886, size = 144, normalized size = 2.4

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{(3-2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}F}{d\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(3 - 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*S
qrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]/(-1 + Cos[c + d*x])]/Sqrt[3]], 6]*Sin[(c + d*x)/2]^4)/(d*Sqrt[
3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 0.494, size = 125, normalized size = 2.1

$$\frac{\sqrt{2}\left(\sin\left(dx+c\right)\right)^{4}}{d\left(-3+2\cos\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^{2}}\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{\frac{3}{2}}\sqrt{3-2\cos\left(dx+c\right)}\sqrt{-2\,\frac{-3+2\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\mathrm{Elliptic}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(3-2*cos(d*x+c))^(1/2)*(-2*(-
3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
I*5^(1/2))*sin(d*x+c)^4/cos(d*x+c)^(3/2)/(-3+2*cos(d*x+c))/(-1+cos(d*x+c))^
2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\cos\left(dx+c\right)+3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\cos\left(dx+c\right)+3}\sqrt{\cos\left(dx+c\right)}}{2\cos\left(dx+c\right)^{2}-3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 -
3*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3-2\cos\left(c+dx\right)}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(3 - 2*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\cos(dx+c)+3}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))), x)
```

3.650 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\tan^2(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/
(Sqrt[5]*d)
```

Rubi [A] time = 0.123599, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2817, 2815}

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\tan^2(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/
(Sqrt[5]*d)
```

Rule 2817

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[-(d*Sin[e + f*x])]/Sqrt[d*Sin[e + f*x]]
, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[-(d*Sin[e + f*x])]), x], x] /; FreeQ
[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx &= \frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}\end{aligned}$$

Mathematica [A] time = 1.22856, size = 144, normalized size = 1.71

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{2\cos(c+dx)-3}{\cos(c+dx)-1}}\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{2\cos(c+dx)-3}{\cos(c+dx)-1}}}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)-1}}\sqrt{2\cos(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt[
-Cot[(c + d*x)/2]^2]*EllipticF[ArcSin[Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[
c + d*x])]/Sqrt[3]], 6/5]*Tan[(c + d*x)/2])/(Sqrt[5]*d*Sqrt[Cos[c + d*x]/(-
1 + Cos[c + d*x])]*Sqrt[-3 + 2*Cos[c + d*x]])
```

Maple [A] time = 0.477, size = 123, normalized size = 1.5

$$\frac{\frac{i}{5}\sqrt{5}\sqrt{2}\left(\sin\left(dx+c\right)\right)^{4}}{d\left(-1+\cos\left(dx+c\right)\right)^{2}}\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{\frac{3}{2}}\sqrt{-2\frac{-3+2\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(dx+c\right)\right)\sqrt{5}}{\sin\left(dx+c\right)},\frac{i}{5}\sqrt{5}\right)\frac{1}{\sqrt{-3+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5*I/d*5^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/(-3+2*cos(d*x+c))
^(1/2)*sin(d*x+c)^4*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(I
*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2))/cos(d*x+c)^(3/2)/(-1+cos
(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}{2\cos\left(dx+c\right)^{2}-3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 - 3*
cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\cos{(c+dx)}-3}\sqrt{\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*cos(c + d*x) - 3)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

3.651 $\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\tan^2(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d
*x]^2])/d
```

Rubi [A] time = 0.114625, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2817, 2815}

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{-\tan^2(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d
*x]^2])/d
```

Rule 2817

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[-(d*Sin[e + f*x])]/Sqrt[d*Sin[e + f*x]]
, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[-(d*Sin[e + f*x])]), x], x] /; FreeQ
[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{2\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{-}}{d}$$

Mathematica [A] time = 1.0306, size = 153, normalized size = 1.87

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(c+dx\right.}}{\sqrt{5}d\sqrt{-2\cos(c+dx)-3}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
5/3]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 6/5]*Sin[(c + d*x)/2]^4)/(Sqr
t[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.458, size = 137, normalized size = 1.7

$$\frac{-\frac{i}{5}\sqrt{5}\sqrt{2}\left(\sin\left(dx+c\right)\right)^4\sqrt{10}}{d\left(3+2\cos\left(dx+c\right)\right)\left(-1+\cos\left(dx+c\right)\right)^2}\left(\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}\right)^{\frac{3}{2}}\sqrt{-3-2\cos\left(dx+c\right)}\sqrt{\frac{3+2\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\mathrm{EllipticF}\left(\frac{i}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)
```

```
[Out] -1/5*I/d*5^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*(-3-2*cos(d*x+c)
)^(1/2)*sin(d*x+c)^4*10^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/2))/cos(d*x+c)^(3/2)/(
3+2*cos(d*x+c))/(-1+cos(d*x+c))^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}{2\cos\left(dx+c\right)^2+3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c)^2 +
3*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\cos\left(c+dx\right)-3}\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-2*cos(c + d*x) - 3)*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\cos\left(dx+c\right)-3}\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))), x)
```

3.652 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=54

$$\frac{2\sqrt{\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)|\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/
5])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rubi [A] time = 0.100997, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2814, 2813}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)|\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 1/
5])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rule 2814

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{2+3\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}\end{aligned}$$

Mathematica [B] time = 0.510334, size = 150, normalized size = 2.78

$$-\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+a\right.}}{d\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[
(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sq
rt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]])
```

Maple [B] time = 0.468, size = 122, normalized size = 2.3

$$\frac{\sqrt{5}\sqrt{2}\sqrt{10}\,(\sin(dx+c))^2}{5\,d\,(-1+\cos(dx+c))}\mathrm{EllipticF}\left(\frac{\sqrt{5}\,(-1+\cos(dx+c))}{5\,\sin(dx+c)},\sqrt{5}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{2+3\,\cos(dx+c)}{1+\cos(dx+c)}}\frac{1}{\sqrt{2+3\,\cos(dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*5^(1/2)*EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*2^(
1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)/(2+3*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(-cos(d*x+
c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\,\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{3\,\cos(dx+c)+2}}{3\,\cos(dx+c)^2+2\,\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)/(3*cos(d*x + c)^2 +
2*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(3*cos(c + d*x) + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) + 2)), x)
```

3.653 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)\middle|5\right)}{d\sqrt{-\cos(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5]
)/(d*Sqrt[-Cos[c + d*x]])
```

Rubi [A] time = 0.103851, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2814, 2813}

$$\frac{2\sqrt{\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)+1}\right)\middle|5\right)}{d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[ArcSin[Sin[c + d*x]/(1 + Cos[c + d*x])], 5]
)/(d*Sqrt[-Cos[c + d*x]])
```

Rule 2814

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sign[b]*Sin[e + f*x]]/Sqrt[d*Sin[e + f*
x]], Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[Sign[b]*Sin[e + f*x]]), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && GtQ[b^2, 0] &&  !(EqQ[d^
2, 1] && GtQ[b*d, 0])
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{-2+3\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1+\cos(c+dx)}\right)\middle|5\right)}{d\sqrt{-\cos(c+dx)}}\end{aligned}$$

Mathematica [B] time = 0.389808, size = 158, normalized size = 3.36

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-(3\cos(c+dx)-2)\csc^2\left(\frac{1}{2}(c+dx)\right)}F}{\sqrt{5}d\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Sqrt[-((-
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[-((-2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]/2], 4/5]*Sin[(c + d*x)/2]^4)/
(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

Maple [B] time = 0.381, size = 109, normalized size = 2.3

$$2\,\frac{(\sin(dx+c))^2}{d\sqrt{-2+3\,\cos(dx+c)}\,(-1+\cos(dx+c))\,\sqrt{-\cos(dx+c)}}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)\sqrt{\frac{-2+3\,\cos(dx+c)}{1+\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*((-2+3*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*s
in(d*x+c)^2/(-1+cos(d*x+c))/(-cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\,\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{3\,\cos(dx+c)-2}}{3\,\cos(dx+c)^2-2\,\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)/(3*cos(d*x + c)^2 -
2*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(3*cos(c + d*x) - 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(3*cos(d*x + c) - 2)), x)
```

3.654 $\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

Optimal. Leaf size=34

$$-\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] (-2*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])], 1/5])/(Sqrt[5]*d)
```

Rubi [A] time = 0.0552737, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2813}

$$-\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (-2*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])], 1/5])/(Sqrt[5]*d)
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)|\frac{1}{5}\right)}{\sqrt{5}d}$$

Mathematica [B] time = 0.528512, size = 145, normalized size = 4.26

$$-\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{(2-3\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx\right.}}{d\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(2 - 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*
Sqrt[Cos[c + d*x]*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[Co
s[c + d*x]*Csc[(c + d*x)/2]^2]/2], -4]*Sin[(c + d*x)/2]^4)/(d*Sqrt[2 - 3*Co
s[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

Maple [B] time = 0.376, size = 121, normalized size = 3.6

$$-2\frac{\sqrt{2-3\cos(dx+c)}\,(\sin(dx+c))^2}{d\left(3(\cos(dx+c))^2-5\cos(dx+c)+2\right)\sqrt{-\cos(dx+c)}}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)\sqrt{\frac{-2+3\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\ }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))*((-2+3*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(2-3*cos(d*x+c))^(1/2)*s
in(d*x+c)^2/(3*cos(d*x+c)^2-5*cos(d*x+c)+2)/(-cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}}{3\cos(dx+c)^2-2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)/(3*cos(d*x + c)^2 -
2*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{2-3\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2 - 3*cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)), x)
```

3.655 $\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$-\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d}$$

```
[Out] (-2*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])], 5])/d
```

Rubi [A] time = 0.0553474, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2813}

$$-\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]

[Out] (-2*EllipticF[ArcSin[Sin[c + d*x]/(1 - Cos[c + d*x])], 5])/d
```

Rule 2813

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*d*EllipticF[ArcSin[Cos[e + f*x]/(1 + d*S
in[e + f*x])], -((a - b*d)/(a + b*d))])/(f*Sqrt[a + b*d]), x] /; FreeQ[{a,
b, d, e, f}, x] && LtQ[a^2 - b^2, 0] && EqQ[d^2, 1] && GtQ[b*d, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2F\left(\sin^{-1}\left(\frac{\sin(c+dx)}{1-\cos(c+dx)}\right)\middle|5\right)}{d}$$

Mathematica [B] time = 0.471152, size = 155, normalized size = 5.74

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+dx)\right)}F}{\sqrt{5}d\sqrt{-3\cos(c+dx)-2}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]

[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
5/2]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 4/5]*Sin[(c + d*x)/2]^4)/(Sqr
t[5]*d*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

Maple [B] time = 0.467, size = 129, normalized size = 4.8

$$-\frac{\sqrt{10}\,\sqrt{2}\,(\sin(dx+c))^{2}}{5d\left(3\,(\cos(dx+c))^{2}-\cos(dx+c)-2\right)}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)\sqrt{\frac{2+3\,\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))*10^(1/2)*((2+3*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-
2-3*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(3*cos(d*x+c)^2-cos(d*x+c)-2)/(-cos(d*x+
c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\,\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-\cos(dx+c)}\sqrt{-3\,\cos(dx+c)-2}}{3\,\cos(dx+c)^{2}+2\,\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)/(3*cos(d*x + c)^2 +
2*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(-3*cos(c + d*x) - 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)), x)
```

3.656 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=80

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d\sqrt{-\cos(c+dx)}}$$

```
[Out] (2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]
]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/(d*Sqrt[-Cos[c
+ d*x]])
```

Rubi [A] time = 0.104798, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2817, 2815}

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 + 2*Cos[c + d*x]
]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/(d*Sqrt[-Cos[c
+ d*x]])
```

Rule 2817

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[-(d*Sin[e + f*x])]/Sqrt[d*Sin[e + f*x]]
, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[-(d*Sin[e + f*x])]), x], x] /; FreeQ
[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{3+2\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}\\ &= \frac{2\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{-\tan^2(c+dx)}}{d\sqrt{-\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.599877, size = 154, normalized size = 1.92

$$-\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(c+dx\cdots\right.}}{d\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-4*Sqrt[-Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt
[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqr
t[(3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/Sqrt[6]], 6]*Sin[(c + d*x)/2]^4)
/(d*Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]])
```

Maple [A] time = 0.481, size = 127, normalized size = 1.6

$$\frac{-\frac{i}{5}\sqrt{5}\sqrt{2}\sqrt{10}\,(\sin(dx+c))^2}{d\,(-1+\cos(dx+c))}\text{EllipticF}\left(\frac{\frac{i}{5}(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},i\sqrt{5}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{3+2\cos(dx+c)}{1+\cos(dx+c)}}\frac{1}{\sqrt{3+2\cos\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*5^(1/2)*EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),I*5^(1/
2))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*10^(1/2)*((3+2*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)/(3+2*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/(-c
os(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)+3}}{2\cos(dx+c)^2+3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)/(2*cos(d*x + c)^2 +
3*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos{(c+dx)}}\sqrt{2\cos{(c+dx)}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2*cos(c + d*x) + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos{(dx+c)}}\sqrt{2\,\cos{(dx+c)}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) + 3)), x)
```

3.657 $\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

```
[Out] (2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 - 2*Cos[c + d*x]
]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d*Sqrt[-Cos[c
+ d*x]])
```

Rubi [A] time = 0.105928, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2817, 2815}

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\sqrt{-\tan^2(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[3 - 2*Cos[c + d*x]
]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d*Sqrt[-Cos[c
+ d*x]])
```

Rule 2817

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[-(d*Sin[e + f*x])]/Sqrt[d*Sin[e + f*x]]
, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[-(d*Sin[e + f*x])]), x], x] /; FreeQ
[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && NegQ[(a + b)/d]
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}\\ &= \frac{2\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d\sqrt{-\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.468222, size = 146, normalized size = 1.78

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{(3-2\cos(c+dx))\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx\right.}}{d\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[(3 - 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*S
qrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
Cos[c + d*x]/(-1 + Cos[c + d*x])]/Sqrt[3]], 6]*Sin[(c + d*x)/2]^4)/(d*Sqrt[
3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

Maple [A] time = 0.279, size = 107, normalized size = 1.3

$$\frac{\frac{2i}{5}\sqrt{5}\sqrt{2}}{d}\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},\frac{i}{5}\sqrt{5}\right)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{3-2\cos(dx+c)}\frac{1}{\sqrt{-2\frac{-3+2\cos(dx+c)}{1+\cos(dx+c)}}}\frac{1}{\sqrt{-\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x)
```

```
[Out] 2/5*I/d*5^(1/2)*EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^(1/2
))*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2*(-3+2*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}}{2\cos(dx+c)^2-3\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)/(2*cos(d*x + c)^2 -
3*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{3-2\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(3 - 2*cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)), x)
```

3.658 $\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=62

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] (-2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d
*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d)
```

Rubi [A] time = 0.05566, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2815}

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos[c + d
*x]]], -1/5]*Sqrt[-Tan[c + d*x]^2])/(Sqrt[5]*d)
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-\cos(c+dx)}\sqrt{-3+2\cos(c+dx)}}\,dx = -\frac{2\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{-\tan^2(c+dx)}}{\sqrt{5}d}$$

Mathematica [B] time = 0.633724, size = 160, normalized size = 2.58

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{-\cot^2\left(\frac{1}{2}(c+dx)\right)}\cot(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{-(2\cos(c+dx)-3)\csc^2\left(\frac{1}{2}(c+\right.}}{\sqrt{5}d(-\cos(c+dx))^{3/2}\sqrt{2\cos(c+dx)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-Cos[c + d*x]]*Sqrt[-3 + 2*Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[-Cot[(c + d*x)/2]^2]*Cot[c + d*x]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)
/2]^2)]*Sqrt[-((-3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2)]*EllipticF[ArcSin[
Sqrt[(-3 + 2*Cos[c + d*x])/(-1 + Cos[c + d*x])]/Sqrt[3]], 6/5]*Sin[(c + d*x
)/2]^4)/(Sqrt[5]*d*(-Cos[c + d*x])^(3/2)*Sqrt[-3 + 2*Cos[c + d*x]])
```

Maple [A] time = 0.256, size = 98, normalized size = 1.6

$$2\,\frac{\sqrt{2}\sqrt{-3+2\,\cos\left(dx+c\right)}}{d\sqrt{-\cos\left(dx+c\right)}}\mathrm{EllipticF}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)},i\sqrt{5}\right)\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\frac{1}{\sqrt{-2\,\frac{-3+2\,\cos(dx+c)}{1+\cos(dx+c)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))/(-2*(-3+2*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3+2*cos(d*
x+c))^(1/2)/(-cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{2\,\cos\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos\left(dx+c\right)}\sqrt{2\,\cos\left(dx+c\right)-3}}{2\,\cos\left(dx+c\right)^{2}-3\,\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)/(2*cos(d*x + c)^2 -
3*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos\left(c+dx\right)}\sqrt{2\cos\left(c+dx\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(2*cos(c + d*x) - 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(dx+c)}\sqrt{2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(2*cos(d*x + c) - 3)), x)
```

3.659 $\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] (-2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-
Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d
```

Rubi [A] time = 0.0556197, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2815}

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[-
Cos[c + d*x]])], -5]*Sqrt[-Tan[c + d*x]^2])/d
```

Rule 2815

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Sqrt[a^2]*Sqrt[-Cot[e + f*x]^2]*Rt[(a +
b)/d, 2]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt
[(a + b)/d, 2])], -((a + b)/(a - b))])/(a*f*Sqrt[a^2 - b^2]*Cot[e + f*x]),
x] /; FreeQ[{a, b, d, e, f}, x] && GtQ[a^2 - b^2, 0] && PosQ[(a + b)/d] &&
GtQ[a^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}\,dx = -\frac{2\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{-\tan^2(c+dx)}}{d}$$

Mathematica [B] time = 0.440684, size = 155, normalized size = 2.58

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(2\cos(c+dx)+3)\csc^2\left(\frac{1}{2}(c+dx)\right)}F}{\sqrt{5}d\sqrt{-2\cos(c+dx)-3}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]]),x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
3 + 2*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[
```

```
5/3]*Sqrt[Cos[c + d*x]/(-1 + Cos[c + d*x])]], 6/5]*Sin[(c + d*x)/2]^4)/(Sqr
t[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

Maple [B] time = 0.429, size = 128, normalized size = 2.1

$$-\frac{\sqrt{10}\sqrt{2}\left(\sin\left(dx+c\right)\right)^{2}}{5d\left(2\left(\cos\left(dx+c\right)\right)^{2}+\cos\left(dx+c\right)-3\right)}\mathrm{EllipticF}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)},\frac{i}{5}\sqrt{5}\right)\sqrt{\frac{3+2\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}\sqrt{\frac{\cos\left(dx+c\right)}{1+\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))*10^(1/2)*((3+2*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*2^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(-3-2*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(2*cos(d*x+c)^2+cos(d*x+c)-3)/(-cos(d*
x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\cos\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\cos\left(dx+c\right)-3}}{2\cos\left(dx+c\right)^{2}+3\cos\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)/(2*cos(d*x + c)^2 +
3*cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\cos\left(c+dx\right)}\sqrt{-2\cos\left(c+dx\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))**(1/2)/(-cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-cos(c + d*x))*sqrt(-2*cos(c + d*x) - 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\cos\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-2*cos(d*x+c))^(1/2)/(-cos(d*x+c))^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)), x)
```

3.660 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

```
[Out] (-4*Cot[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*S
qrt[Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(3*
d)
```

Rubi [A] time = 0.0488477, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2809}

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*S
qrt[Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(3*
d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx=-\frac{4\cot(c+dx)\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+dx)}}{3d}$$

Mathematica [B] time = 2.84075, size = 175, normalized size = 2.27

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)+2}\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\left(3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+dx)\right)}\right)\right)\right.}{3d\sqrt{\frac{-3\cos(c+dx)-2}{\cos(c+dx)-1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*Csc
[c + d*x]*(3*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]
/2], -4] - 5*EllipticPi[-2/3, ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x
)/2]^2]/2], -4]))/(3*d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sqrt
[Cos[c + d*x]/(-1 + Cos[c + d*x])])
```

Maple [B] time = 0.388, size = 142, normalized size = 1.8

$$-\frac{\sqrt{2}\sqrt{10}\,(\sin(dx+c))^2}{5\,d\,(-1+\cos(dx+c))}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,1/5\sqrt{5}\right)\right)\sqrt{\frac{2+3\cos}{1+\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*10^(1/2)*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))-
2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,1/5*5^(1/2)))*sin(d*x+c)^2/(2+3*
cos(d*x+c))^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(3*cos(c + d*x) + 2), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

3.661 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}d}$$

```
[Out] (-4*Cot[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos[
c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]
*d)
```

Rubi [A] time = 0.0590285, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2809}

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos[
c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]
*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx=-\frac{4\cot(c+dx)\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3\sqrt{5}d}$$

Mathematica [A] time = 0.6622, size = 142, normalized size = 1.89

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{3\cos(c+dx)-2}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{1}{5}\right)+2\Pi\left(-\frac{1}{5};-\sin^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right.\right.\right.\right.}{\sqrt{5}d\sqrt{\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(-2 + 3*C
os[c + d*x])/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Sqrt[5]*Tan[(c + d*x)/2]
], 1/5] + 2*EllipticPi[-1/5, -ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5]))/(Sqr
t[5]*d*Sqrt[Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

Maple [B] time = 0.428, size = 132, normalized size = 1.8

$$-2\frac{(\sin(dx+c))^2}{d\sqrt{-2+3\cos(dx+c)}(-1+\cos(dx+c))\sqrt{\cos(dx+c)}}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+}{\mathrm{si}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,5^(1/2)))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)/(-2+3*cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(
d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(3*cos(c + d*x) - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

3.662 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

```
[Out] (-4*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[2 - 3*Cos[c
 + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c
+ d*x]])/(3*Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rubi [A] time = 0.116352, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2810, 2809}

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[2 - 3*Cos[c
 + d*x]]/Sqrt[-Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c
+ d*x]])/(3*Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}\\ &= -\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c-}{3\sqrt{5}d\sqrt{-\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 1.69056, size = 147, normalized size = 1.48

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{-\frac{(2-3\cos(c+dx))^2}{(\cos(c+dx)+1)^2}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|5\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|5\right)\right)}{d\sqrt{2-3\cos(c+dx)}\sqrt{\cos(c+dx)}\sqrt{\frac{2-3\cos(c+dx)}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[-((2 - 3*Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Tan[(c + d*x)/2]],
 5] + 2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], 5]))/(d*Sqrt[2 - 3*Cos[c
+ d*x]]*Sqrt[Cos[c + d*x]]*Sqrt[(2 - 3*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

Maple [A] time = 0.399, size = 144, normalized size = 1.5

$$2\,\frac{(\sin(dx+c))^2\sqrt{2-3\cos(dx+c)}}{d\left(3(\cos(dx+c))^2-5\cos(dx+c)+2\right)\sqrt{\cos(dx+c)}}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\ldots\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,5^(1/2)))*((-2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*(2-3*cos(d*x+c))^(1/2)/(3*cos(d*x
+c)^2-5*cos(d*x+c)+2)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\cos(dx+c)+2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) + 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c) - 2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos{(c + dx)}}}{\sqrt{2 - 3\cos{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(2 - 3*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx + c)}}{\sqrt{-3\cos(dx + c) + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

3.663 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{3d\sqrt{-\cos(c+dx)}}$$

```
[Out] (-4*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[-2 - 3*Cos[
c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1
 - Sec[c + d*x]])/(3*d*Sqrt[-Cos[c + d*x]])
```

Rubi [A] time = 0.102429, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2810, 2809}

$$-\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{3d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[-2 - 3*Cos[
c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1
 - Sec[c + d*x]])/(3*d*Sqrt[-Cos[c + d*x]])
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}$$

$$= -\frac{4\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+}{3d\sqrt{-\cos(c+dx)}}$$

Mathematica [A] time = 2.17439, size = 175, normalized size = 1.73

$$-\frac{2\sqrt{-3\cos(c+dx)-2}\sqrt{\cos(c+dx)}\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\left(3F\left(\sin^{-1}\left(\frac{1}{2}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{3d\sqrt{\frac{-3\cos(c+dx)-2}{\cos(c+dx)-1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)-1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Sqrt[Cot[(c + d*x)/2]^2]*C
sc[c + d*x]*(3*EllipticF[ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^
2]/2], -4] - 5*EllipticPi[-2/3, ArcSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d
*x)/2]^2]/2], -4]))/(3*d*Sqrt[(-2 - 3*Cos[c + d*x])/(-1 + Cos[c + d*x])]*Sq
rt[Cos[c + d*x]/(-1 + Cos[c + d*x])])
```

Maple [A] time = 0.428, size = 161, normalized size = 1.6

$$\frac{\sqrt{5}\sqrt{2}\sqrt{10}\left(\sin(dx+c)\right)^2}{5\,d\left(3\left(\cos(dx+c)\right)^2-\cos(dx+c)-2\right)}\left(\mathrm{EllipticF}\left(\frac{\sqrt{5}\left(-1+\cos(dx+c)\right)}{5\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(1/5\,\frac{\sqrt{5}\left(-1+\cos(d\right.}{\sin(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*5^(1/2)*2^(1/2)*10^(1/2)*(EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d
*x+c),5^(1/2))-2*EllipticPi(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),-5,5^(1/
2)))*sin(d*x+c)^2*(-2-3*cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(3*cos(d*x+c)^2-cos(d*x+c)-2)/cos(
d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-3\cos(dx+c)-2}\sqrt{\cos(dx+c)}}{3\cos(dx+c)+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*cos(d*x + c) - 2)*sqrt(cos(d*x + c))/(3*cos(d*x + c) + 2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(-3*cos(c + d*x) - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

3.664 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=73

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] (-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*S
qrt[Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rubi [A] time = 0.0466321, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2808}

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

```
[Out] (-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*S
qrt[Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d}$$

Mathematica [A] time = 1.36494, size = 117, normalized size = 1.6

$$-\frac{2\sqrt{\cos(c+dx)}\sqrt{2\cos(c+dx)+3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-\frac{1}{5}\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+\right.\right.\right.\right.}{\sqrt{5}d\sqrt{(3\cos(c+dx)+\cos(2(c+dx))+1)\sec^4\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*(EllipticF[ArcSin[Tan[(c +
d*x)/2]], -1/5] + 2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], -1/5])*Sec[(c
 + d*x)/2]^2)/(Sqrt[5]*d*Sqrt[(1 + 3*Cos[c + d*x] + Cos[2*(c + d*x)])*Sec[(
c + d*x)/2]^4])
```

Maple [B] time = 0.374, size = 144, normalized size = 2.

$$-\frac{\sqrt{10}\sqrt{2}\left(\sin\left(dx+c\right)\right)^{2}}{5\,d\left(-1+\cos\left(dx+c\right)\right)}\left(\mathrm{EllipticF}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)},\frac{i}{5}\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos\left(dx+c\right)}{\sin\left(dx+c\right)},-1,i/5\sqrt{5}\right)\right)\sqrt{\frac{3+2\,\mathrm{co}}{1+\cos}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*10^(1/2)*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2)
)-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,1/5*I*5^(1/2)))*sin(d*x+c)^2/(
3+2*cos(d*x+c))^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(c+dx\right)}}{\sqrt{2\cos\left(c+dx\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(2*cos(c + d*x) + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

3.665 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] (3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c
 + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)
```

Rubi [A] time = 0.049012, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2808}

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] (3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c
 + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \frac{3\cot(c+dx)\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{\sqrt{5}d}$$

Mathematica [A] time = 0.813829, size = 119, normalized size = 1.59

$$-\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{3-2\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-5\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-\right.\right.}{d\sqrt{3-2\cos(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] (-4*Cos[(c + d*x)/2]^2*Sqrt[(3 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])]*Sqrt[C
os[c + d*x]/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], -5] +
```

```
2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], -5]))/(d*Sqrt[3 - 2*Cos[c + d*x
]]*Sqrt[Cos[c + d*x]])
```

Maple [B] time = 0.4, size = 153, normalized size = 2.

$$\frac{\sqrt{2}\,(\sin(dx+c))^2}{d\left(2\,(\cos(dx+c))^2-5\,\cos(dx+c)+3\right)}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},i\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},-1,i\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))-2*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,I*5^(1/2)))*(3-2*cos(d*x+c))^(1/2)*(-2*(-3+2*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)^2/(2*cos(d*x+c)^2-5*cos(d*x+c)+3)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\,\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,\cos(dx+c)+3}\sqrt{\cos(dx+c)}}{2\,\cos(dx+c)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) + 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c) - 3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(3 - 2*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

3.666 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

```
[Out] (3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[
c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c
 + d*x]])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rubi [A] time = 0.100456, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2810, 2808}

$$\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] (3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[
c + d*x]]/Sqrt[-Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c
 + d*x]])/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]])
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}\\ &= \frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec}}{\sqrt{5}d\sqrt{-\cos(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.961351, size = 135, normalized size = 1.36

$$-\frac{2i\sqrt{2\cos(c+dx)-3}\sqrt{\frac{\cos(c+dx)}{5\cos(c+dx)+5}}\left(F\left(i\sinh^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{1}{5}\right)-2\Pi\left(\frac{1}{5};i\sinh^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{1}{5}\right)\right)}{d\sqrt{\cos(c+dx)}\sqrt{\frac{3-2\cos(c+dx)}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[-3 + 2*Cos[c + d*x]]*Sqrt[Cos[c + d*x]/(5 + 5*Cos[c + d*x])]*(
EllipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5] - 2*EllipticPi[1/5, I*
ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5]))/(d*Sqrt[Cos[c + d*x]]*Sqrt[(3 -
2*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

Maple [A] time = 0.435, size = 158, normalized size = 1.6

$$\frac{-\frac{i}{5}\sqrt{5}\sqrt{2}\left(\sin\left(dx+c\right)\right)^{2}}{d\left(-1+\cos\left(dx+c\right)\right)}\left(2\,\mathrm{EllipticPi}\left(\frac{i\left(-1+\cos\left(dx+c\right)\right)\sqrt{5}}{\sin\left(dx+c\right)},1/5,i/5\sqrt{5}\right)-\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(dx+c\right)\right)\sqrt{5}}{\sin\left(dx+c\right)},\frac{i}{5}\sqrt{5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*5^(1/2)*2^(1/2)*(2*EllipticPi(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c)
,1/5,1/5*I*5^(1/2))-EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5*I*5^
(1/2)))/(-3+2*cos(d*x+c))^(1/2)*sin(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\cos\left(dx+c\right)}}{\sqrt{2\,\cos\left(dx+c\right)-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(2*cos(c + d*x) - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

3.667 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$-\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d\sqrt{-\cos(c+dx)}}$$

```
[Out] (-3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[-3 - 2*Cos[
c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1
 + Sec[c + d*x]])/(d*Sqrt[-Cos[c + d*x]])
```

Rubi [A] time = 0.102807, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {2810, 2808}

$$-\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] (-3*Cos[c + d*x]^(3/2)*Csc[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[-3 - 2*Cos[
c + d*x]]/(Sqrt[5]*Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1
 + Sec[c + d*x]])/(d*Sqrt[-Cos[c + d*x]])
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}}\\ &= -\frac{3\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+}}{d\sqrt{-\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.669417, size = 115, normalized size = 1.19

$$-\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(2\cos(c+dx)+3)\sec^4\left(\frac{1}{2}(c+dx)\right)}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{1}{5}\right)+2\Pi\left(-1;-\mathrm{s}\right.\right.}{\sqrt{5}d\sqrt{-2\cos(c+dx)-3}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*(EllipticF[ArcSin[Tan[(c + d*x)/2]], -1/5] + 2*Ellip
ticPi[-1, -ArcSin[Tan[(c + d*x)/2]], -1/5])*Sqrt[Cos[c + d*x]*(3 + 2*Cos[c
+ d*x])*Sec[(c + d*x)/2]^4])/(Sqrt[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[Cos[
c + d*x]])
```

Maple [A] time = 0.436, size = 168, normalized size = 1.7

$$\frac{-\frac{i}{5}\sqrt{5}\sqrt{2}\sqrt{10}\,(\sin(dx+c))^2}{d\left(2\,(\cos(dx+c))^2+\cos(dx+c)-3\right)}\left(\mathrm{EllipticF}\left(\frac{\frac{i}{5}(-1+\cos(dx+c))\sqrt{5}}{\sin(dx+c)},i\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{i/5\,(-1+\cos(dx\right.}{\sin(dx+c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*5^(1/2)*2^(1/2)*10^(1/2)*(EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/
sin(d*x+c),I*5^(1/2))-2*EllipticPi(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c)
,5,I*5^(1/2)))*sin(d*x+c)^2*(-3-2*cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(2*cos(d*x+c)^2+cos(d*x+
c)-3)/cos(d*x+c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos(dx+c)}}{\sqrt{-2\,\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,\cos(dx+c)-3}\sqrt{\cos(dx+c)}}{2\,\cos(dx+c)+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*cos(d*x + c) - 3)*sqrt(cos(d*x + c))/(2*cos(d*x + c) + 3)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos{(c + dx)}}}{\sqrt{-2\cos{(c + dx)} - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/sqrt(-2*cos(c + d*x) - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\cos\left(dx + c\right)}}{\sqrt{-2\,\cos\left(dx + c\right) - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

3.668 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

```
[Out] (-4*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[5/3, Arc
Sin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], 5]*Sqrt[-1 - Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(3*d)
```

Rubi [A] time = 0.0993077, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2810, 2809}

$$-\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)+2}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[5/3, Arc
Sin[Sqrt[2 + 3*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], 5]*Sqrt[-1 - Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(3*d)
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx &= \frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{2+3\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{2+3\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+}}{3d}\end{aligned}$$

Mathematica [A] time = 0.769191, size = 194, normalized size = 1.96

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+dx)\right)}\left(\right.}{3d\sqrt{-\cos(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[2 + 3*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*(3*EllipticF[ArcSin[Sq
rt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4] - 5*EllipticPi[-2/3, Ar
cSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4])*Sin[(c + d*x)/2
]^4)/(3*d*Sqrt[-Cos[c + d*x]]*Sqrt[2 + 3*Cos[c + d*x]])
```

Maple [A] time = 0.519, size = 159, normalized size = 1.6

$$-\frac{\sqrt{5}\sqrt{2}\sqrt{10}\left(\sin\left(dx+c\right)\right)^2}{5\,d\left(-1+\cos\left(dx+c\right)\right)\cos\left(dx+c\right)}\left(\mathrm{EllipticF}\left(\frac{\sqrt{5}\left(-1+\cos\left(dx+c\right)\right)}{5\,\sin\left(dx+c\right)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(1/5\,\frac{\sqrt{5}\left(-1+\cos\left(dx+c\right)\right)}{\sin\left(dx+c\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5/d*5^(1/2)*2^(1/2)*10^(1/2)*(EllipticF(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(
d*x+c),5^(1/2))-2*EllipticPi(1/5*5^(1/2)*(-1+cos(d*x+c))/sin(d*x+c),-5,5^(1
/2)))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((2+3*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)^2/(2+3*cos(d*x+c))^(1/2)*(-cos(d*x+c))^(1/2)/(-1+cos(d*x+c)
)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{3\,\cos\left(dx+c\right)+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3\cos(c+dx)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3*cos(c + d*x) + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) + 2), x)
```

3.669 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$-\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}d}$$

```
[Out] (-4*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[1/3, Arc
Sin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d
*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]*d)
```

Rubi [A] time = 0.0984587, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2810, 2809}

$$-\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{3\cos(c+dx)-2}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[1/3, Arc
Sin[Sqrt[-2 + 3*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d
*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]*d)
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx &= \frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{-2+3\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{4\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{-2+3\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+}}{3\sqrt{5}d}\end{aligned}$$

Mathematica [A] time = 0.220951, size = 144, normalized size = 1.48

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{3\cos(c+dx)-2}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{1}{5}\right)+2\Pi\left(-\frac{1}{5};-\sin^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{\sqrt{5}d\sqrt{-\cos(c+dx)}\sqrt{3\cos(c+dx)-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-2 + 3*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(-2 + 3*Co
s[c + d*x])/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Sqrt[5]*Tan[(c + d*x)/2]]
, 1/5] + 2*EllipticPi[-1/5, -ArcSin[Sqrt[5]*Tan[(c + d*x)/2]], 1/5]))/(Sqrt
[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt[-2 + 3*Cos[c + d*x]])
```

Maple [A] time = 0.394, size = 142, normalized size = 1.5

$$-2\,\frac{(\sin(dx+c))^2\sqrt{-\cos(dx+c)}}{d\sqrt{-2+3\,\cos(dx+c)}\,(-1+\cos(dx+c))\cos(dx+c)}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\mathrm{c}}{\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,5^(1/2)))*sin(d*x+c)^2*(-cos(d*x+c))^(1/2)/(-2+3*cos(d*
x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)/(-1+cos(d*x+c))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\,\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{-\cos(dx+c)}}{\sqrt{3\,\cos(dx+c)-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(c + dx)}}{\sqrt{3\cos(c + dx) - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(-2+3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3*cos(c + d*x) - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx + c)}}{\sqrt{3\cos(dx + c) - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2+3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(3*cos(d*x + c) - 2), x)
```

3.670 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}d}$$

```
[Out] (-4*Cot[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos[
c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]
*d)
```

Rubi [A] time = 0.0520857, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2809}

$$-\frac{4\cot(c+dx)\sqrt{\sec(c+dx)-1}\sqrt{\sec(c+dx)+1}\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)}{3\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*EllipticPi[1/3, ArcSin[Sqrt[2 - 3*Cos[c + d*x]]/Sqrt[-Cos[
c + d*x]]], 1/5]*Sqrt[-1 + Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(3*Sqrt[5]
*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx = -\frac{4\cot(c+dx)\Pi\left(\frac{1}{3};\sin^{-1}\left(\frac{\sqrt{2-3\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|\frac{1}{5}\right)\sqrt{-1+\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{3\sqrt{5}d}$$

Mathematica [A] time = 0.501284, size = 149, normalized size = 1.94

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{-\frac{(2-3\cos(c+dx))^2}{(\cos(c+dx)+1)^2}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|5\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\right.}{d\sqrt{2-3\cos(c+dx)}\sqrt{-\cos(c+dx)}\sqrt{\frac{2-3\cos(c+dx)}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[2 - 3*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[-((2 - 3*Cos[c + d*x])^2/(1 + Cos[c + d*x])^2)]*
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Tan[(c + d*x)/2]],
5] + 2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], 5]))/(d*Sqrt[2 - 3*Cos[c +
 d*x]]*Sqrt[-Cos[c + d*x]]*Sqrt[(2 - 3*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

Maple [B] time = 0.386, size = 154, normalized size = 2.

$$2\,\frac{(\sin(dx+c))^2\sqrt{-\cos(dx+c)}\sqrt{2-3\cos(dx+c)}}{d\left(3(\cos(dx+c))^2-5\cos(dx+c)+2\right)\cos(dx+c)}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx}{\sin(dx+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),5^(1/2))-2*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,5^(1/2)))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((-2+3*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2*(-cos(d*x+c))^(1/2)*(2-3*cos(d*x+
c))^(1/2)/(3*cos(d*x+c)^2-5*cos(d*x+c)+2)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{-3\cos(dx+c)+2}}{3\cos(dx+c)-2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) + 2)/(3*cos(d*x + c) - 2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2-3\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2 - 3*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) + 2), x)
```

3.671 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx$

Optimal. Leaf size=79

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

```
[Out] (-4*Cot[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[-2 - 3*Cos[c + d*x]]/(Sqrt[5]*
Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(
3*d)
```

Rubi [A] time = 0.0536822, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2809}

$$-\frac{4\cot(c+dx)\sqrt{-\sec(c+dx)-1}\sqrt{1-\sec(c+dx)}\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-3\cos(c+dx)-2}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*EllipticPi[5/3, ArcSin[Sqrt[-2 - 3*Cos[c + d*x]]/(Sqrt[5]*
Sqrt[-Cos[c + d*x]])], 5]*Sqrt[-1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])/(
3*d)
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2-3\cos(c+dx)}}\,dx = -\frac{4\cot(c+dx)\,\Pi\left(\frac{5}{3};\sin^{-1}\left(\frac{\sqrt{-2-3\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|5\right)\sqrt{-1-\sec(c+dx)}\sqrt{1-\sec(c+dx)}}{3d}$$

Mathematica [B] time = 0.622784, size = 194, normalized size = 2.46

$$\frac{4\sin^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\cot^2\left(\frac{1}{2}(c+dx)\right)}\csc(c+dx)\sqrt{-\cos(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{(3\cos(c+dx)+2)\csc^2\left(\frac{1}{2}(c+dx)\right)}\Big(}{3d\sqrt{-3\cos(c+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-2 - 3*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[Cot[(c + d*x)/2]^2]*Sqrt[-(Cos[c + d*x]*Csc[(c + d*x)/2]^2)]*Sqrt[(
2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]*Csc[c + d*x]*(3*EllipticF[ArcSin[Sq
rt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4] - 5*EllipticPi[-2/3, Ar
cSin[Sqrt[(2 + 3*Cos[c + d*x])*Csc[(c + d*x)/2]^2]/2], -4])*Sin[(c + d*x)/2
]^4)/(3*d*Sqrt[-2 - 3*Cos[c + d*x]]*Sqrt[-Cos[c + d*x]])
```

Maple [B] time = 0.408, size = 164, normalized size = 2.1

$$\frac{\sqrt{10}\sqrt{2}\,(\sin(dx+c))^2}{5\,d\left(3\,(\cos(dx+c))^2-\cos(dx+c)-2\right)\cos(dx+c)}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{\sqrt{5}}{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(d}{\sin(dx-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*10^(1/2)*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*5^(1/2))-2
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,1/5*5^(1/2)))*((2+3*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-2-3*cos(d*x+c))^(1
/2)*(-cos(d*x+c))^(1/2)*sin(d*x+c)^2/(3*cos(d*x+c)^2-cos(d*x+c)-2)/cos(d*x+
c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\,\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{-3\,\cos(dx+c)-2}}{3\,\cos(dx+c)+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-3*cos(d*x + c) - 2)/(3*cos(d*x + c) + 2
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3\cos(c+dx)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(-2-3*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(-3*cos(c + d*x) - 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-3\cos(dx+c)-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-2-3*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-3*cos(d*x + c) - 2), x)
```

3.672 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] (-3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[5/2, Arc
Sin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[1 - Se
c[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rubi [A] time = 0.100234, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2810, 2808}

$$-\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)+3}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```


```
[Out] (-3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[5/2, Arc
Sin[Sqrt[3 + 2*Cos[c + d*x]]/(Sqrt[5]*Sqrt[Cos[c + d*x]])], -5]*Sqrt[1 - Se
c[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```


Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3+2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= -\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{3+2\cos(c+dx)}}{\sqrt{5}\sqrt{\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+}}{d}$$

Mathematica [A] time = 0.50089, size = 119, normalized size = 1.25

$$-\frac{2\sqrt{-\cos(c+dx)}\sqrt{2\cos(c+dx)+3}\sec^2\left(\frac{1}{2}(c+dx)\right)\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-\frac{1}{5}\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{\sqrt{5}d\sqrt{(3\cos(c+dx)+\cos(2(c+dx))+1)\sec^4\left(\frac{1}{2}(c+dx)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[3 + 2*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[-Cos[c + d*x]]*Sqrt[3 + 2*Cos[c + d*x]]*(EllipticF[ArcSin[Tan[(c +
 d*x)/2]], -1/5] + 2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], -1/5])*Sec[(
c + d*x)/2]^2)/(Sqrt[5]*d*Sqrt[(1 + 3*Cos[c + d*x] + Cos[2*(c + d*x)])*Sec[
(c + d*x)/2]^4])
```

Maple [A] time = 0.385, size = 168, normalized size = 1.8

$$\frac{\frac{i}{5}\sqrt{5}\sqrt{10}\sqrt{2}\left(\sin\left(dx+c\right)\right)^2}{d\left(-1+\cos\left(dx+c\right)\right)\cos\left(dx+c\right)}\left(\text{EllipticF}\left(\frac{\frac{i}{5}\left(-1+\cos\left(dx+c\right)\right)\sqrt{5}}{\sin\left(dx+c\right)},i\sqrt{5}\right)-2\,\text{EllipticPi}\left(\frac{i/5\left(-1+\cos\left(dx+c\right)\right)\sqrt{5}}{\sin\left(dx+c\right)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5*I/d*5^(1/2)*10^(1/2)*2^(1/2)*(EllipticF(1/5*I*(-1+cos(d*x+c))*5^(1/2)/s
in(d*x+c),I*5^(1/2))-2*EllipticPi(1/5*I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),
5,I*5^(1/2)))/(3+2*cos(d*x+c))^(1/2)*(-cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(-1+co
s(d*x+c))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{2\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{2\cos\left(dx+c\right)+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2*cos(c + d*x) + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) + 3), x)
```

3.673 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] (3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[-1/2, Arc
Sin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*
x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)
```

Rubi [A] time = 0.100714, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2810, 2808}

$$\frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] (3*Sqrt[-Cos[c + d*x]]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[-1/2, Arc
Sin[Sqrt[3 - 2*Cos[c + d*x]]/Sqrt[Cos[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*
x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*d)
```

Rule 2810

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[e + f*x]]/Sqrt[-(b*Sin[e + f*x])], In
t[Sqrt[-(b*Sin[e + f*x])]/Sqrt[c + d*Sin[e + f*x]], x], x] /; FreeQ[{b, c,
d, e, f}, x] && NeQ[c^2 - d^2, 0] && NegQ[(c + d)/b]
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx = \frac{\sqrt{-\cos(c+dx)}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{3\sqrt{-\cos(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{3-2\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx}}{\sqrt{5}d}$$

Mathematica [A] time = 0.192793, size = 121, normalized size = 1.25

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{3-2\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-5\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\right|$$
$$\overline{d\sqrt{3-2\cos(c+dx)}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[3 - 2*Cos[c + d*x]],x]
```

```
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[(3 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])]*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], -5] + 2
*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], -5]))/(d*Sqrt[3 - 2*Cos[c + d*x]
]*Sqrt[-Cos[c + d*x]])
```

Maple [B] time = 0.426, size = 178, normalized size = 1.8

$$\frac{-\frac{i}{5}\sqrt{5}\sqrt{2}\left(\sin\left(dx+c\right)\right)^{2}}{d\left(2\left(\cos\left(dx+c\right)\right)^{2}-5\cos\left(dx+c\right)+3\right)\cos\left(dx+c\right)}\left(\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(dx+c\right)\right)\sqrt{5}}{\sin\left(dx+c\right)},\frac{i}{5}\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{i\,(-}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/5*I/d*5^(1/2)*2^(1/2)*(EllipticF(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/
5*I*5^(1/2))-2*EllipticPi(I*(-1+cos(d*x+c))*5^(1/2)/sin(d*x+c),1/5,1/5*I*5^
(1/2)))*(3-2*cos(d*x+c))^(1/2)*(-cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)^2/(2*cos(d
*x+c)^2-5*cos(d*x+c)+3)/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-\cos\left(dx+c\right)}}{\sqrt{-2\cos\left(dx+c\right)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos\left(dx+c\right)}\sqrt{-2\cos\left(dx+c\right)+3}}{2\cos\left(dx+c\right)-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) + 3)/(2*cos(d*x + c) - 3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{3-2\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(3 - 2*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) + 3), x)
```

3.674 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx$

Optimal. Leaf size=77

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

```
[Out] (3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos
[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*
d)
```

Rubi [A] time = 0.0532438, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2808}

$$\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{2\cos(c+dx)-3}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)}{\sqrt{5}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] (3*Cot[c + d*x]*EllipticPi[-1/2, ArcSin[Sqrt[-3 + 2*Cos[c + d*x]]/Sqrt[-Cos
[c + d*x]]], -1/5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/(Sqrt[5]*
d)
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3+2\cos(c+dx)}}\,dx = \frac{3\cot(c+dx)\Pi\left(-\frac{1}{2};\sin^{-1}\left(\frac{\sqrt{-3+2\cos(c+dx)}}{\sqrt{-\cos(c+dx)}}\right)|-\frac{1}{5}\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{\sqrt{5}d}$$

Mathematica [C] time = 0.149898, size = 140, normalized size = 1.82

$$\frac{2i\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{2\cos(c+dx)-3}\left(F\left(i\sinh^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\frac{1}{5}\right)-2\Pi\left(\frac{1}{5};i\sinh^{-1}\left(\sqrt{5}\tan\left(\frac{1}{2}(c+dx)\right)\right)|-\right.}{\sqrt{5}d\sqrt{-\cos(c+dx)}\sqrt{\frac{3-2\cos(c+dx)}{\cos(c+dx)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-3 + 2*Cos[c + d*x]],x]
```

```
[Out] ((2*I)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[-3 + 2*Cos[c + d*x]]*(Ell
ipticF[I*ArcSinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5] - 2*EllipticPi[1/5, I*Arc
Sinh[Sqrt[5]*Tan[(c + d*x)/2]], -1/5]))/(Sqrt[5]*d*Sqrt[-Cos[c + d*x]]*Sqrt
[(3 - 2*Cos[c + d*x])/(1 + Cos[c + d*x])])
```

Maple [B] time = 0.365, size = 152, normalized size = 2.

$$-\frac{\sqrt{2}\,(\sin(dx+c))^2}{d(-1+\cos(dx+c))\cos(dx+c)}\left(\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, i\sqrt{5}\right)-2\,\text{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}, -1, i\sqrt{5}\right)\right)\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),I*5^(1/2))-2*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,I*5^(1/2)))/(-3+2*cos(d*x+c))^(1/2)*(-cos(d*x
+c))^(1/2)*(-2*(-3+2*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))/cos(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{2\cos(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(-3+2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(2*cos(c + d*x) - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3+2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(2*cos(d*x + c) - 3), x)
```

3.675 $\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx$

Optimal. Leaf size=75

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

```
[Out] (-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*
Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rubi [A] time = 0.0525886, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2808}

$$-\frac{3\cot(c+dx)\sqrt{1-\sec(c+dx)}\sqrt{\sec(c+dx)+1}\,\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-2\cos(c+dx)-3}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] (-3*Cot[c + d*x]*EllipticPi[5/2, ArcSin[Sqrt[-3 - 2*Cos[c + d*x]]/(Sqrt[5]*
Sqrt[-Cos[c + d*x]])], -5]*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])/d
```

Rule 2808

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*c*Rt[b*(c + d), 2]*Tan[e + f*x]*Sqrt[1 + Csc[
e + f*x]]*Sqrt[1 - Csc[e + f*x]]*EllipticPi[(c + d)/d, ArcSin[Sqrt[c + d*Si
n[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/
(d*f*Sqrt[c^2 - d^2]), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c^2 - d^2, 0]
 && PosQ[(c + d)/b] && GtQ[c^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-3-2\cos(c+dx)}}\,dx = -\frac{3\cot(c+dx)\Pi\left(\frac{5}{2};\sin^{-1}\left(\frac{\sqrt{-3-2\cos(c+dx)}}{\sqrt{5}\sqrt{-\cos(c+dx)}}\right)\middle|-5\right)\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}{d}$$

Mathematica [A] time = 0.294639, size = 117, normalized size = 1.56

$$\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)(2\cos(c+dx)+3)\sec^4\left(\frac{1}{2}(c+dx)\right)}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|-\frac{1}{5}\right)+2\Pi\left(-1;-\sin^{-1}\right.\right.}{\sqrt{5}d\sqrt{-2\cos(c+dx)-3}\sqrt{-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-Cos[c + d*x]]/Sqrt[-3 - 2*Cos[c + d*x]],x]
```

```
[Out] (2*Cos[(c + d*x)/2]^2*(EllipticF[ArcSin[Tan[(c + d*x)/2]], -1/5] + 2*Ellipt
icPi[-1, -ArcSin[Tan[(c + d*x)/2]], -1/5])*Sqrt[Cos[c + d*x]*(3 + 2*Cos[c +
```

```
d*x])*Sec[(c + d*x)/2]^4])/(Sqrt[5]*d*Sqrt[-3 - 2*Cos[c + d*x]]*Sqrt[-Cos[
c + d*x]])
```

Maple [B] time = 0.405, size = 164, normalized size = 2.2

$$\frac{\sqrt{10}\,\sqrt{2}\,(\sin(dx+c))^2}{5d\left(2\,(\cos(dx+c))^2+\cos(dx+c)-3\right)\cos(dx+c)}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\frac{i}{5}\sqrt{5}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(}{\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/5/d*10^(1/2)*2^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),1/5*I*5^(1/2))
-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,1/5*I*5^(1/2)))*((3+2*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(-3-2*cos(d*x+c)
)^(1/2)*(-cos(d*x+c))^(1/2)*sin(d*x+c)^2/(2*cos(d*x+c)^2+cos(d*x+c)-3)/cos(
d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-\cos(dx+c)}\sqrt{-2\cos(dx+c)-3}}{2\cos(dx+c)+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-cos(d*x + c))*sqrt(-2*cos(d*x + c) - 3)/(2*cos(d*x + c) + 3
), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(c+dx)}}{\sqrt{-2\cos(c+dx)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))**(1/2)/(-3-2*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-cos(c + d*x))/sqrt(-2*cos(c + d*x) - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-\cos(dx+c)}}{\sqrt{-2\cos(dx+c)-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(d*x+c))^(1/2)/(-3-2*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(d*x + c))/sqrt(-2*cos(d*x + c) - 3), x)
```

3.676 $\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+dx)}}-\frac{b\sin(c+dx)\cos^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}}$$

```
[Out] -((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^
2 - b^2))]*Cos[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)
^(1/3))) + (a*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x
]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[
c + d*x]^(1/3))
```

Rubi [A] time = 0.199126, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+dx)}}-\frac{b\sin(c+dx)\cos^{\frac{2}{3}}(c+dx)F_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(2/3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((b*AppellF1[1/2, -1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^
2 - b^2))]*Cos[c + d*x]^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)
^(1/3))) + (a*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x
]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[
c + d*x]^(1/3))
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a+b\cos(c+dx)}\,dx = a\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx - b\int \frac{\cos^{\frac{5}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx$$

$$= -\frac{\left(b\cos^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{1-x^2}}{a^2-b^2+b^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos^2(c+dx)}} + \frac{\left(a\sqrt[6]{\cos^2(c+dx)}\right)\text{Subst}\left(\int \frac{}{\sqrt[6]{1-x^2}(}\right.}{d\sqrt[3]{\cos(c+}}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{\frac{2}{3}}(c+dx)\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[3]{\cos^2(c+dx)}} + \frac{aF_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c\right.}{}$$

Mathematica [B] time = 21.921, size = 4614, normalized size = 26.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(2/3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*
AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b
^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*
x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1
/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9
*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))
/(d*Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(5/6)*(-b^2 +
a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/6)*((a*AppellF1[1/2
, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[S
ec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF
1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*
Tan[c + d*x]^2) + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan
[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/
6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 -
 b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(
a^2 - b^2))])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^2
- b^2)*(Sec[c + d*x]^2)^(1/6)*Tan[c + d*x]^2*((a*AppellF1[1/2, 1/3, 1, 3/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2]
)/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1,
 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2
) + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(
a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -
((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1
[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*
Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2)^2 - (15*(a^2 - b^2)*Tan[c + d
```

```
*x]^2*((a*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2
)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1,
3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*Appel
lF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
 + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d
*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 5/6, 1, 3/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[
1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (
6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -(
(a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^(5/
6)*(-b^2 + a^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((a*AppellF1[
1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqr
t[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -
Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/
2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^
2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Tan[c + d*x]^2) + (a*Sqrt[Sec[c + d*x]^2]*((-2*a^2*AppellF1
[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Se
c[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c
+ d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a
^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan
[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/
2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[
c + d*x]^2) + (b*((-2*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^
2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)
) - (5*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/
(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1[1/2
, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a
^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
- b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^
2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) - (a*AppellF1[1/2, 1/3, 1,
 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*
x]^2]*(-4*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*
x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] + 9
*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Ta
n[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) -
(2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
- b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) - 2*Tan[c + d*x]^2*(3*a^2*((-12*a^
2*AppellF1[5/2, 1/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (2*AppellF1[5/2, 4/3
, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x
]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 4/3, 2, 7/2, -Tan
[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*
x])/(5*(a^2 - b^2)) - (8*AppellF1[5/2, 7/3, 1, 7/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 -
 b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*T
an[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2 - (b*Ap
pellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
))]*(2*(6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*
x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*
x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] - 9
*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Ta
n[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) -
(5*AppellF1[3/2, 11/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
```

```
 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(6*a^2*((-12*a^2
*AppellF1[5/2, 5/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - AppellF1[5/2, 11/6, 2
, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2
*Tan[c + d*x]) + 5*(a^2 - b^2)*((-6*a^2*AppellF1[5/2, 11/6, 2, 7/2, -Tan[c
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])
/(5*(a^2 - b^2)) - (11*AppellF1[5/2, 17/6, 1, 7/2, -Tan[c + d*x]^2, -((a^2*
Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(-9*(a^2 -
 b^2)*AppellF1[1/2, 5/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] + (6*a^2*AppellF1[3/2, 5/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + 5*(a^2 - b^2)*AppellF1[3/2, 11/6, 1, 5/2, -Tan
[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/((Se
c[c + d*x]^2)^(5/6)*(-b^2 + a^2*Sec[c + d*x]^2))))
```

Maple [F] time = 0.139, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\cos\left(dx+c\right)}\left(\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{2}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(2/3)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(2/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{2}{3}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(2/3)/(b*cos(d*x + c) + a), x)
```

3.677 $\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos(c+dx)}F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}}$$

```
[Out] -((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^
2 - b^2))]*Cos[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)
^(1/6))) + (a*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x
]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[
c + d*x]^(2/3))
```

Rubi [A] time = 0.185033, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos(c+dx)}F_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)\sqrt[6]{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(1/3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -((b*AppellF1[1/2, -1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^
2 - b^2))]*Cos[c + d*x]^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*(Cos[c + d*x]^2)
^(1/6))) + (a*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x
]^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[
c + d*x]^(2/3))
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx = a\int \frac{\sqrt[3]{\cos(c+dx)}}{a^2-b^2\cos^2(c+dx)}\,dx - b\int \frac{\cos^{\frac{4}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx$$

$$= -\frac{\left(b\sqrt[3]{\cos(c+dx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{1-x^2}}{a^2-b^2+b^2x^2}\,dx, x, \sin(c+dx)\right)}{d\sqrt[6]{\cos^2(c+dx)}} + \frac{\left(a\sqrt[3]{\cos^2(c+dx)}\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{1}}\right.}{d\cos^{\frac{2}{3}}}$$

$$= -\frac{bF_1\left(\frac{1}{2};-\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[6]{\cos^2(c+dx)}} + \frac{aF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin\right.}{}$$

Mathematica [B] time = 21.5322, size = 4613, normalized size = 26.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(1/3)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*
AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b
^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*
x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1
/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9
*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -
((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2))
)/(d*Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(2/3)*(-b^2 +
 a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(1/3)*((a*AppellF1[1/
2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[
Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*Appell
F1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])
*Tan[c + d*x]^2) + (b*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Ta
n[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2,
 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(a^
2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^
2 - b^2)*(Sec[c + d*x]^2)^(1/3)*Tan[c + d*x]^2*((a*AppellF1[1/2, 1/6, 1, 3/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^
2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]
^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6,
1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]
^2) + (b*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*Appe
llF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2)^2 - (12*(a^2 - b^2)*Tan[c
```

+ d*x]^2*((a*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6,
1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*App
ellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2)
)] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
+ d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 2/3, 1, 3/2, -T
an[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*Appell
F1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
+ 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^
2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)
^(2/3)*(-b^2 + a^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((a*Appel
lF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1
[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] +
(-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (a*Sqrt[Sec[c + d*x]^2]*((-2*a^2*Appe
llF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (AppellF1[3/2, 7/6, 1, 5/2
, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[
c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -(
(a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[
3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Ta
n[c + d*x]^2) + (b*((-2*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^
2)) - (4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1[1/
2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(
3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) - (a*AppellF1[1/2, 1/6,
1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c +
d*x]^2]*(2*(-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c +
 d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x]
+ 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2))
 - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(-6*a^2*((-12*a^
2*AppellF1[5/2, 1/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (AppellF1[5/2, 7/6,
2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^
2*Tan[c + d*x])/5) + (-a^2 + b^2)*((-6*a^2*AppellF1[5/2, 7/6, 2, 7/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x
])/(5*(a^2 - b^2)) - (7*AppellF1[5/2, 13/6, 1, 7/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(9*(a^2 -
 b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))] + (-6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Ta
n[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2 - (b*Ap
pellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
))]*(4*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*
x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] - 9*
(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (
4*AppellF1[3/2, 5/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -

```
 b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + 2*Tan[c + d*x]^2*(3*a^2*((-12*a^2
*AppellF1[5/2, 2/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (4*AppellF1[5/2, 5/3,
 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]
^2*Tan[c + d*x])/5) + 2*(a^2 - b^2)*((-6*a^2*AppellF1[5/2, 5/3, 2, 7/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d
*x])/(5*(a^2 - b^2)) - 2*AppellF1[5/2, 8/3, 1, 7/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x]))))/(-9*(a^2 - b
^2)*AppellF1[1/2, 2/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
 - b^2))] + 2*(3*a^2*AppellF1[3/2, 2/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + 2*(a^2 - b^2)*AppellF1[3/2, 5/3, 1, 5/2, -Tan[
c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/((Sec
[c + d*x]^2)^(2/3)*(-b^2 + a^2*Sec[c + d*x]^2))))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\cos\left(dx+c\right)}\sqrt[3]{\cos\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(dx+c\right)^{\frac{1}{3}}}{b\cos\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(1/3)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos (dx+c)^{\frac{1}{3}}}{b\cos (dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(1/3)/(b*cos(d*x + c) + a), x)
```

3.678 $\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=176

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{2/3}F_1\left(\frac{1}{2};\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{4}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c\cdots\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+dx)}}$$

```
[Out] -((b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2
 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^
(1/3))) + (a*AppellF1[1/2, 2/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]
^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c
 + d*x]^(4/3))
```

Rubi [A] time = 0.186423, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{2/3}F_1\left(\frac{1}{2};\frac{2}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{4}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[6]{\cos^2(c+dx)}F_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c\cdots\right.}{d\left(a^2-b^2\right)\sqrt[3]{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])),x]
```

```
[Out] -((b*AppellF1[1/2, 1/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2
 - b^2))]*(Cos[c + d*x]^2)^(1/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^
(1/3))) + (a*AppellF1[1/2, 2/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]
^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(2/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c
 + d*x]^(4/3))
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}(a+b\cos(c+dx))}\,dx = a\int \frac{1}{\sqrt[3]{\cos(c+dx)}\left(a^2-b^2\cos^2(c+dx)\right)}\,dx - b\int \frac{\cos^{\frac{2}{3}}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx$$

$$= -\frac{\left(b\sqrt[6]{\cos^2(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[6]{1-x^2}\left(a^2-b^2+b^2x^2\right)}\,dx, x, \sin(c+dx)\right)}{d\sqrt[3]{\cos(c+dx)}} + \frac{\left(a\cos^2(c+\right.}{}$$

$$= -\frac{bF_1\left(\frac{1}{2};\frac{1}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[6]{\cos^2(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt[3]{\cos(c+dx)}} + \frac{aF_1\left(\frac{1}{2}\right.}{}$$

Mathematica [B] time = 21.4757, size = 4605, normalized size = 26.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(1/3)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)
*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1
[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(
-9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d
*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2))
)/(d*Cos[c + d*x]^(4/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(1/3)*(-b^2 +
 a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(2/3)*((a*AppellF1[1/
2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt
[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*Appe
llF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*
Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -
Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/
2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^
2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^
2 - b^2)*(Sec[c + d*x]^2)^(2/3)*Tan[c + d*x]^2*((a*AppellF1[1/2, -1/6, 1, 3
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]
^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d
*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6
, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*
x]^2) + (b*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^
2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*Appe
llF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2)^2 - (6*(a^2 - b^2)*Tan[c +
```

```
d*x]^2*((a*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6,
 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*Ap
pellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^
2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/3, 1, 3/2, -
Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*Appel
lF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
 + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^
(1/3)*(-b^2 + a^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((a*Appell
F1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF
1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
 + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*
x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (a*Sqrt[Sec[c + d*x]^2]*((-2*a^2*App
ellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) + (AppellF1[3/2, 5/6, 1, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Ta
n[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*Appell
F1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])
*Tan[c + d*x]^2) + (b*((-2*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 -
 b^2)) - (2*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*AppellF1
[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] +
2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -(
(a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) - (a*AppellF1[1/2, -1/6
, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c
+ d*x]^2]*(2*(-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Ta
n[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c
 + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x
] + 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^
2)) + (AppellF1[3/2, 5/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(
a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(-6*a^2*((-12
*a^2*AppellF1[5/2, -1/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a
^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) + (AppellF1[5/2, 5
/6, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d
*x]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 5/6, 2, 7/2, -T
an[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c +
d*x])/(5*(a^2 - b^2)) - AppellF1[5/2, 11/6, 1, 7/2, -Tan[c + d*x]^2, -((a^2
*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x]))))/(9*(a^2 - b^
2)*AppellF1[1/2, -1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2
 - b^2))] + (-6*a^2*AppellF1[3/2, -1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 5/6, 1, 5/2, -Tan[c
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2 - (b*Appe
llF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
]*(4*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c + d*x] - 9*(a^2
 - b^2)*((-2*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) - (2*Ap
pellF1[3/2, 4/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
```

```
))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + 2*Tan[c + d*x]^2*(3*a^2*((-12*a^2*App
ellF1[5/2, 1/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2)
)]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (2*AppellF1[5/2, 4/3, 2,
7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*T
an[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 4/3, 2, 7/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(
5*(a^2 - b^2)) - (8*AppellF1[5/2, 7/3, 1, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[
c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(-9*(a^2 - b^2
)*AppellF1[1/2, 1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))] + 2*(3*a^2*AppellF1[3/2, 1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c
 + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 4/3, 1, 5/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/((Sec[c +
 d*x]^2)^(1/3)*(-b^2 + a^2*Sec[c + d*x]^2))))
```

Maple [F] time = 0.201, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\cos(dx+c)}\frac{1}{\sqrt[3]{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

3.679 $\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx$

Optimal. Leaf size=176

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{5/6}F_1\left(\frac{1}{2};\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{5}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+dx)}$$

```
[Out] -((b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2
 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^
(2/3))) + (a*AppellF1[1/2, 5/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]
^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(5/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c
 + d*x]^(5/3))
```

Rubi [A] time = 0.185417, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\cos^2(c+dx)^{5/6}F_1\left(\frac{1}{2};\frac{5}{6},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)\cos^{\frac{5}{3}}(c+dx)}-\frac{b\sin(c+dx)\sqrt[3]{\cos^2(c+dx)}F_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+\right.}{d\left(a^2-b^2\right)\cos^{\frac{2}{3}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])),x]
```

```
[Out] -((b*AppellF1[1/2, 1/3, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2
 - b^2))]*(Cos[c + d*x]^2)^(1/3)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c + d*x]^
(2/3))) + (a*AppellF1[1/2, 5/6, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]
^2)/(a^2 - b^2))]*(Cos[c + d*x]^2)^(5/6)*Sin[c + d*x])/((a^2 - b^2)*d*Cos[c
 + d*x]^(5/3))
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)(a+b\cos(c+dx))}\,dx = a\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\left(a^2-b^2\cos^2(c+dx)\right)}\,dx - b\int \frac{\sqrt[3]{\cos(c+dx)}}{a^2-b^2\cos^2(c+dx)}\,dx$$

$$= -\frac{\left(b\sqrt[3]{\cos^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x^2}\left(a^2-b^2+b^2x^2\right)}\,dx, x, \sin(c+dx)\right)}{d\cos^{\frac{2}{3}}(c+dx)} + \frac{\left(a\cos^2(c \cdots}{\cdots}$$

$$= -\frac{bF_1\left(\frac{1}{2};\frac{1}{3},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\sqrt[3]{\cos^2(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\cos^{\frac{2}{3}}(c+dx)} + \frac{aF_1 \cdots}{\cdots}$$

Mathematica [B] time = 21.3517, size = 4608, normalized size = 26.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Cos[c + d*x]^(2/3)*(a + b*Cos[c + d*x])),x]
```

```
[Out] (9*(a^2 - b^2)*Sin[c + d*x]*((a*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)
*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 -
 b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[
c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c
+ d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*Appell
F1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])
/(-9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
 d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2,
 -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2))
)/(d*Cos[c + d*x]^(5/3)*(a + b*Cos[c + d*x])*(Sec[c + d*x]^2)^(1/6)*(-b^2 +
 a^2*Sec[c + d*x]^2)*((9*(a^2 - b^2)*(Sec[c + d*x]^2)^(5/6)*((a*AppellF1[1/
2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt
[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^
2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*Ap
pellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2
))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^
2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/
2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^
2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2) - (18*a^2*(a^
2 - b^2)*(Sec[c + d*x]^2)^(5/6)*Tan[c + d*x]^2*((a*AppellF1[1/2, -1/3, 1, 3
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]
^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c +
d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2
/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c +
d*x]^2) + (b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/
2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*Appe
llF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
])*Tan[c + d*x]^2)))/(-b^2 + a^2*Sec[c + d*x]^2)^2 - (3*(a^2 - b^2)*Tan[c +
```

d*x]^2*((a*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))]*Sqrt[Sec[c + d*x]^2])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3,
1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*A
ppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b
^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan
[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (b*AppellF1[1/2, 1/6, 1, 3/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])/(-9*(a^2 - b^2)*App
ellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2)
)] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)))/((Sec[c + d*x]^2)^
(1/6)*(-b^2 + a^2*Sec[c + d*x]^2)) + (9*(a^2 - b^2)*Tan[c + d*x]*((a*Appell
F1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
*Sqrt[Sec[c + d*x]^2]*Tan[c + d*x])/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3
/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*Appell
F1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]
+ (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) + (a*Sqrt[Sec[c + d*x]^2]*((-2*a^2*A
ppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b
^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 - b^2)) + (2*AppellF1[3/2, 2/3,
1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^
2*Tan[c + d*x])/9))/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2,
5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*
AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b
^2))])*Tan[c + d*x]^2) + (b*((-2*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*
x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*
(a^2 - b^2)) - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c +
d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9))/(-9*(a^2 - b^2)*Appe
llF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))
] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^
2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2) - (a*AppellF1[1/2, -1
/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sqrt[Sec[
c + d*x]^2]*(-4*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2, -((a^2*
Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3, 1, 5/2, -Ta
n[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c +
d*x] + 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2 -
b^2)) + (2*AppellF1[3/2, 2/3, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) - 2*Tan[c + d*x]^2*(3*a^2
*((-12*a^2*AppellF1[5/2, -1/3, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) + (2*AppellF
1[5/2, 2/3, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*S
ec[c + d*x]^2*Tan[c + d*x])/5) + (-a^2 + b^2)*((-6*a^2*AppellF1[5/2, 2/3, 2
, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2
*Tan[c + d*x])/(5*(a^2 - b^2)) - (4*AppellF1[5/2, 5/3, 1, 7/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5)))
)/(9*(a^2 - b^2)*AppellF1[1/2, -1/3, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c
+ d*x]^2)/(a^2 - b^2))] - 2*(3*a^2*AppellF1[3/2, -1/3, 2, 5/2, -Tan[c + d*x
]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (-a^2 + b^2)*AppellF1[3/2, 2/3,
1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x
]^2)^2 - (b*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]
^2)/(a^2 - b^2))]*(2*(6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Sec[c + d*x]^2*Tan[c
+ d*x] - 9*(a^2 - b^2)*((-2*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2
, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(3*(a^2
- b^2)) - (AppellF1[3/2, 7/6, 1, 5/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]

```
^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/9) + Tan[c + d*x]^2*(6*a^2*(
(-12*a^2*AppellF1[5/2, 1/6, 3, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)
/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/(5*(a^2 - b^2)) - (AppellF1[5/2
, 7/6, 2, 7/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c
+ d*x]^2*Tan[c + d*x])/5) + (a^2 - b^2)*((-6*a^2*AppellF1[5/2, 7/6, 2, 7/2,
 -Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c
 + d*x])/(5*(a^2 - b^2)) - (7*AppellF1[5/2, 13/6, 1, 7/2, -Tan[c + d*x]^2,
-((a^2*Tan[c + d*x]^2)/(a^2 - b^2))]*Sec[c + d*x]^2*Tan[c + d*x])/5))))/(-9
*(a^2 - b^2)*AppellF1[1/2, 1/6, 1, 3/2, -Tan[c + d*x]^2, -((a^2*Tan[c + d*x
]^2)/(a^2 - b^2))] + (6*a^2*AppellF1[3/2, 1/6, 2, 5/2, -Tan[c + d*x]^2, -((
a^2*Tan[c + d*x]^2)/(a^2 - b^2))] + (a^2 - b^2)*AppellF1[3/2, 7/6, 1, 5/2,
-Tan[c + d*x]^2, -((a^2*Tan[c + d*x]^2)/(a^2 - b^2))])*Tan[c + d*x]^2)^2))/
((Sec[c + d*x]^2)^(1/6)*(-b^2 + a^2*Sec[c + d*x]^2))))
```

Maple [F] time = 0.188, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\cos(dx+c)}(\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] int(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(2/3)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\cos(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

3.680 $\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi [A] time = 0.0535747, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 31.3721, size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{7}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(7/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Maple [A] time = 0.365, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{7}{3}}\frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

[Out] int(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{7}{3}}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/3)/(a+b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{7}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(7/3)/sqrt(b*cos(d*x + c) + a), x)

3.681 $\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi [A] time = 0.0543584, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 78.1857, size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{5}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(5/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Maple [A] time = 0.378, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{5}{3}}\frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{5}{3}}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{5}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.682 $\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi [A] time = 0.0545425, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 18.3268, size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{4}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(4/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Maple [A] time = 0.333, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{4}{3}}\frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{4}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{4}{3}}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(4/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{4}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(4/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.683 $\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi [A] time = 0.0549703, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 8.51073, size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(2/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Maple [A] time = 0.32, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{2}{3}}\frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{2}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{2}{3}}}{\sqrt{b\cos(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^{\frac{2}{3}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(2/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(2/3)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{2}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(2/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.684 $\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi [A] time = 0.0537361, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Defer[Int][Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Rubi steps

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 2.37486, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] Integrate[Cos[c + d*x]^(1/3)/Sqrt[a + b*Cos[c + d*x]], x]
```

Maple [A] time = 0.372, size = 0, normalized size = 0.

$$\int \sqrt[3]{\cos(dx+c)}\frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{1}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^{\frac{1}{3}}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**(1/3)/sqrt(a + b*cos(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^{\frac{1}{3}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(1/3)/sqrt(b*cos(d*x + c) + a), x)
```

3.685 $\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi [A] time = 0.0530685, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 1.74168, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Integrate[1/(Cos[c + d*x]^(1/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Maple [A] time = 0.315, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\cos(dx+c)}}\frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{2}{3}}}{b\cos(dx+c)^2+a\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^2 + a*
cos(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt[3]{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(1/3)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)), x)
```

3.686 $\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi [A] time = 0.0530306, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 0.508689, size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{2}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Integrate[1/(Cos[c + d*x]^(2/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Maple [A] time = 0.372, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{-\frac{2}{3}}\frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{1}{3}}}{b\cos(dx+c)^2+a\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)/(b*cos(d*x + c)^2 + a*
cos(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\cos^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(2/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*cos(c + d*x)**(2/3)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(2/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)), x)
```

3.687 $\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi [A] time = 0.0533637, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 79.1618, size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{4}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Integrate[1/(Cos[c + d*x]^(4/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Maple [A] time = 0.32, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{-\frac{4}{3}} \frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(4/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{2}{3}}}{b\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^3 + a*
cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(4/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\cos\left(dx+c\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(4/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(4/3)), x)
```

3.688 $\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi [A] time = 0.0535666, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx = \int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 26.7068, size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{5}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Integrate[1/(Cos[c + d*x]^(5/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Maple [A] time = 0.335, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{-\frac{5}{3}} \frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{1}{3}}}{b\cos(dx+c)^{3}+a\cos(dx+c)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(1/3)/(b*cos(d*x + c)^3 + a*
cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(5/3)), x)
```

3.689 $\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}},x\right)$$

```
[Out] Unintegrable[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi [A] time = 0.0535432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Defer[Int][1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx=\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Mathematica [A] time = 82.6583, size = 0, normalized size = 0.

$$\int \frac{1}{\cos^{\frac{7}{3}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]),x]
```

```
[Out] Integrate[1/(Cos[c + d*x]^(7/3)*Sqrt[a + b*Cos[c + d*x]]), x]
```

Maple [A] time = 0.395, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{-\frac{7}{3}}\frac{1}{\sqrt{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] int(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/3)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{2}{3}}}{b\cos(dx+c)^4+a\cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(2/3)/(b*cos(d*x + c)^4 + a*
cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/3)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\cos(dx+c)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/3)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*cos(d*x + c)^(7/3)), x)
```

3.690 $\int (A + B\cos(c + dx)) \sec^{\frac{7}{2}}(c + dx)\,dx$

Optimal. Leaf size=151

$$\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{6A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin}{}$$

```
[Out] (-6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*B*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.112296, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3768, 3771, 2641, 2639}

$$\frac{2A\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{6A\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{6A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d
) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
3*d) + (6*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*B*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*d) + (2*A*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (A+B\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)\,dx &= \int \sec^{\frac{5}{2}}(c+dx)(B+A\sec(c+dx))\,dx\\ &= A\int \sec^{\frac{7}{2}}(c+dx)\,dx+B\int \sec^{\frac{5}{2}}(c+dx)\,dx\\ &= \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2A\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{1}{5}(3A)\int \sec^{\frac{3}{2}}(c-\\ &= \frac{6A\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2A\sec^{\frac{5}{2}}(c+dx)\,\mathrm{s}}{5d}\\ &= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{6A\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\\ &= -\frac{6A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right.}{3d}\end{aligned}$$

Mathematica [A] time = 0.32215, size = 97, normalized size = 0.64

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(21A\sin(c+dx)+9A\sin(3(c+dx))-36A\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10B\sin(2(c+dx))+20B\cos^{\frac{5}{2}}(\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-36*A*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 2
0*B*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*A*Sin[c + d*x] + 10*B
*Sin[2*(c + d*x)] + 9*A*Sin[3*(c + d*x)]))/(30*d)
```

Maple [B] time = 6.905, size = 502, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*A/(8*sin(1
/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d
*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c
)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/
```

```
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(7/2), x)
```

3.691 $\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c}}{}$$

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rubi [A] time = 0.0962124, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3768, 3771, 2639, 2641}

$$\frac{2A\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}-\frac{2B\sqrt{\cos(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*A*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (A+B\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\,dx &= \int \sec^{\frac{3}{2}}(c+dx)(B+A\sec(c+dx))\,dx\\ &= A\int \sec^{\frac{5}{2}}(c+dx)\,dx + B\int \sec^{\frac{3}{2}}(c+dx)\,dx\\ &= \frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}A\int \sqrt{\sec(c\\ &= \frac{2B\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{2A\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(A\sqrt{\cos(c+\right.\\ &= -\frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}\end{aligned}$$

Mathematica [A] time = 0.21675, size = 85, normalized size = 0.69

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(2\sin(c+dx)(A+3B\cos(c+dx)) + 2A\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right) - 6B\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-6*B*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 2*
A*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(A + 3*B*Cos[c + d*x])*S
in[c + d*x]))/(3*d)
```

Maple [B] time = 6.014, size = 397, normalized size = 3.2

$$\frac{2}{3\,d}\sqrt{-\left(-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(2\,A\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}-1}\mathrm{EllipticF}\left(\cos\left(1/2\,dx+c/2\right),\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*A*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
```

```
1/2))+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+6*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(5/2), x)
```

3.692 $\int (A + B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d} - \frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.0843604, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3771, 2641, 3768, 2639}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d} - \frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
(2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (A+B\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\,dx &= \int \sqrt{\sec(c+dx)}(B+A\sec(c+dx))\,dx\\ &= A\int \sec^{\frac{3}{2}}(c+dx)\,dx + B\int \sqrt{\sec(c+dx)}\,dx\\ &= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \Big(B\sqrt{\cos(c+dx)}\sqrt{\sec(c\\ &= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d} -\\ &= -\frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx\right.}{d}\end{aligned}$$

Mathematica [A] time = 0.103447, size = 71, normalized size = 0.73

$$\frac{2\sqrt{\sec(c+dx)}\left(A\sin(c+dx)-A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) +
B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/d
```

Maple [A] time = 2.783, size = 148, normalized size = 1.5

$$-2\,\frac{A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)-2\,A\cos(1/2\,dx+c/2)(\sin(1/}{\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^(3/2), x)
```

3.693 $\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rubi [A] time = 0.0757031, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3238, 3787, 3771, 2639, 2641}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx = \int \frac{B+A\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$= A\int \sqrt{\sec(c+dx)}\,dx + B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx$$
$$= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx + \left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$
$$= \frac{2B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{d}$$

Mathematica [A] time = 0.0654656, size = 52, normalized size = 0.69

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2])*Sqrt[Sec[c + d*x]])/d
```

Maple [A] time = 2.369, size = 152, normalized size = 2.

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(A\mathrm{EllipticF}\right.}{\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\mathrm{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maximа [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(A+B\cos(c+dx))\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(B\cos(dx+c)+A)\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(sec(d*x + c)), x)
```

3.694 $\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.0867673, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2641, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (
2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
+ (2*B*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx &= \int \frac{B+A\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx+B\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}B\int \sqrt{\sec(c+dx)}\,dx+\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,d\\ &= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2B\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{1}{3}\left(B\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.\\ &= \frac{2A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{3d}\end{aligned}$$

Mathematica [A] time = 0.127173, size = 76, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(6A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(\sin(2(c+dx))+2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + B*(
2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d)
```

Maple [A] time = 2.537, size = 229, normalized size = 2.3

$$\frac{2}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+3\,A\sqrt{\left(\sin(1/2\,dx+c/2)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))+2*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
```

```
^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(sec(d*x + c)), x)
```

3.695 $\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*A*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.103304, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2639, 2641}

$$\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*d) + (2*B*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*A*Sin[c + d*x])/(3*d
*Sqrt[Sec[c + d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{B+A\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\ &= A\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + B\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx\\ &= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}A\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}(3B)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2B\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx\\ &= \frac{6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{3d}\end{aligned}$$

Mathematica [A] time = 0.300945, size = 88, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(5A+3B\cos(c+dx))+10A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+18B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+\right.\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(18*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10
*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 3*B*Cos[c + d*x])*
Sin[2*(c + d*x)]))/(15*d)
```

Maple [A] time = 2.684, size = 262, normalized size = 2.1

$$-\frac{2}{15\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-24\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(20\,A+24\,B\right)\left(\mathrm{s}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x
```

```
+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(3/2), x)
```

3.696 $\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10B\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10B\sqrt{\cos(}}{}$$

```
[Out] (6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*A*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*B*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.1135, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3787, 3769, 3771, 2641, 2639}

$$\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{6A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{10B\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{10B\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```


```
[Out] (6*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d)
 + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
21*d) + (2*B*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*A*Sin[c + d*x])/(5
*d*Sec[c + d*x]^(3/2)) + (10*B*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{B+A\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx \\
&= A\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + B\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3A)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{7}(5B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx \\
&= \frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10B\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(5B)\int \sqrt{\sec(c+dx)}\,dx + \frac{1}{5}\left(3A\right. \\
&= \frac{6A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2B\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{10B\sin}{21d\sqrt{}} \\
&= \frac{6A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{10B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{21d}
\end{aligned}$$

Mathematica [A] time = 0.523397, size = 99, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 1
00*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*
x] + 15*B*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 2.556, size = 290, normalized size = 1.9

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-168\,A-360\,B\right)\right(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*cos(
1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42*A
-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sec(d*x + c)^(5/2), x)
```

3.697 $\int (a+b\cos(c+dx))^2 \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=200

$$\frac{2\left(5a^2+7b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+dx)\mathrm{se}}{7d}$$

```
[Out] (-12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (12*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (
2*(5*a^2 + 7*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a*b*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7
*d)
```

Rubi [A] time = 0.166321, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(5a^2+7b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2\sin(c+dx)\mathrm{se}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
5*d) + (2*(5*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (12*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (
2*(5*a^2 + 7*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (4*a*b*Sec[c +
d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a^2*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7
*d)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^{\frac{9}{2}}(c+dx)\,dx = \int \sec^{\frac{5}{2}}(c+dx)(b+a\sec(c+dx))^2\,dx$$
$$= (2ab)\int \sec^{\frac{7}{2}}(c+dx)\,dx + \int \sec^{\frac{5}{2}}(c+dx)\left(b^2+a^2\sec^2(c+dx)\right)dx$$
$$= \frac{4ab\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{5}(6ab)\int \sec^{\frac{3}{2}}(c$$
$$= \frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2\left(5a^2+7b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{4ab\,\mathrm{se}}{}$$
$$= \frac{12ab\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{2\left(5a^2+7b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{4ab\,\mathrm{se}}{}$$
$$= -\frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(5a^2+7b^2\right)\sqrt{\cos(c+dx)}}{2}$$

Mathematica [A] time = 0.781179, size = 139, normalized size = 0.7

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(20\left(5a^2+7b^2\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\left(5\left(5a^2+7b^2\right)\cos(2(c+dx))+55a^2+273ab\,\mathrm{c}\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-504*a*b*Cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2]
+ 20*(5*a^2 + 7*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(55*a
^2 + 35*b^2 + 273*a*b*Cos[c + d*x] + 5*(5*a^2 + 7*b^2)*Cos[2*(c + d*x)] + 6
3*a*b*Cos[3*(c + d*x)])*Sin[c + d*x]))/(210*d)
```

Maple [B] time = 8.171, size = 689, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/5*a*b/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+2*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*
(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{9}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{9}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(9/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(9/2), x)
```

3.698 $\int (a+b\cos(c+dx))^2 \sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=175

$$\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\,\mathrm{s}}{5d}$$

```
[Out] (-2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*(3*a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d) + (4*a*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.15712, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2641, 4046, 2639}

$$\frac{2\left(3a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2\sin(c+dx)\,\mathrm{s}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

```
[Out] (-2*(3*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (2*(3*a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d) + (4*a*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a^2*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+b\cos(c+dx))^2\sec^{\frac{7}{2}}(c+dx)\,dx &= \int \sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^2\,dx\\
&= (2ab)\int \sec^{\frac{5}{2}}(c+dx)\,dx + \int \sec^{\frac{3}{2}}(c+dx)\left(b^2+a^2\sec^2(c+dx)\right)\,dx\\
&= \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{3}(2ab)\int \sqrt{\sec(\cdots}\\
&= \frac{2\left(3a^2+5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{4ab\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2a^2\,\mathrm{se}\cdots}{\cdots}\\
&= \frac{4ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{2\left(3a^2+5b^2\right)\sqrt{\sec(c+dx)}\sin\cdots}{5d}\\
&= -\frac{2\left(3a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}F\cdots}{\cdots}
\end{aligned}$$

Mathematica [A] time = 1.2167, size = 126, normalized size = 0.72

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12\left(3a^2+5b^2\right)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\left(3\left(3a^2+5b^2\right)\cos(2(c+dx))+15\left(a^2+b^2\right)+\cdots\right.\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(3*a^2 + 5*b^2)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 40*a*b*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(a
^2 + b^2) + 20*a*b*Cos[c + d*x] + 3*(3*a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c
 + d*x]))/(30*d)
```

Maple [B] time = 7.854, size = 660, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^2*(-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/5*a^2/(8
*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin
(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(
1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)+4*a*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(7/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(7/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(7/2), x)
```

3.699 $\int (a+b\cos(c+dx))^2 \sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] (-4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (4*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.136263, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3768, 3771, 2639, 4046, 2641}

$$\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{4ab\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
+ (2*(a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(3*d) + (4*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a^2*Sec[c +
 d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\,dx &= \int \sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2\,dx\\
&= (2ab)\int \sec^{\frac{3}{2}}(c+dx)\,dx+\int \sqrt{\sec(c+dx)}\left(b^2+a^2\sec^2(c+dx)\right)dx\\
&= \frac{4ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}-(2ab)\int \frac{1}{\sqrt{\sec(c}\cdots}\\
&= \frac{4ab\sqrt{\sec(c+dx)}\sin(c+dx)}{d}+\frac{2a^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}-\left(2ab\sqrt{\cos(c+dx}\cdots\right.\\
&= -\frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2\left(a^2+3b^2\right)\sqrt{\cos(c+dx)}F\left(\cdots\right.}{3\cdots}\end{aligned}$$

Mathematica [A] time = 0.311636, size = 93, normalized size = 0.69

$$\frac{2\sec^{\frac{3}{2}}(c+dx)\left(\left(a^2+3b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-6ab\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\sin(c+dx)(a+6b\cos(c\cdots\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*Sec[c + d*x]^(3/2)*(-6*a*b*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
+ (a^2 + 3*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + a*(a + 6*b*C
os[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] time = 7.66, size = 514, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*a^2*sin(1/2*d*x+1/2*c)^2+6*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^2*sin(1/2*
d*x+1/2*c)^2+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b*sin(1/2*d*x+1/2*c)^2-24*a*b*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*b^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+2*a^2*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^2+12*a*b*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2 + 2\,ab\cos(dx+c) + a^2\right)\sec(dx+c)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2), x)
```

3.700 $\int (a+b\cos(c+dx))^2 \sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=108

$$-\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}$$

```
[Out] (-2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.129556, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3788, 3771, 2641, 4046, 2639}

$$-\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+d}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*(a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/d + (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
```

```
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\,dx &= \int \frac{(b+a\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx\\ &= (2ab)\int \sqrt{\sec(c+dx)}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx\\ &= \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(-a^2+b^2\right)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \left(2ab\sqrt{\cos(c+\right.\\ &= \frac{4ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{d}\\ &= -\frac{2\left(a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4ab\sqrt{\cos(c+dx)}F\left(\right.}{}\end{aligned}$$

Mathematica [A] time = 0.186231, size = 83, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left(a\left(a\sin(c+dx)+2b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)-\left(a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2]) + a*(2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*Sin[c + d
*x])))/d
```

Maple [A] time = 3.987, size = 202, normalized size = 1.9

$$-2\,\frac{\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\left(\cos(1/2\,dx+c/2)\,,\sqrt{2}\right)a^2-\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\mathrm{si}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x)
```

```
[Out] -2*((sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+2*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*a*b-2*a^2*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c)+a)^2\sec (dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos (dx+c)^2+2\,ab\cos (dx+c)+a^2\right)\sec (dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^(3/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c)+a)^2\sec (dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2), x)
```

3.701 $\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=112

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+d}{3d\sqrt{\sec(c+d}}$$

```
[Out] (4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.1325, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3238, 3788, 3771, 2639, 4045, 2641}

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2\sin(c+d}{3d\sqrt{\sec(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (4*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d +
 (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= (2ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(-3a^2-b^2\right)\int \sqrt{\sec(c+dx)}\,dx + \left(2ab\sqrt{\cos(c+dx)}\sqrt{\mathrm{se}\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2b^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}\left(\left(-3a\right.\right.$$
$$= \frac{4ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\ldots}{}$$

Mathematica [A] time = 0.16982, size = 87, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+b^2\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b^2*Sin[2*(c
 + d*x)]))/(3*d)
```

Maple [A] time = 2.734, size = 283, normalized size = 2.5

$$-\frac{2}{3d}\sqrt{\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(4\,b^2\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^4+3\sqrt{\left(\sin(1/2\,dx+c/\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2-6*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
```

```
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-2*b^2*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(sec(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)
```

3.702 $\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}$$

```
[Out] (2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.14436, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{4ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```



```
[Out] (2*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*d) + (2*b^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (4*a*
b*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx &= \int \frac{(b+a\sec(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= (2ab)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx \\
&= \frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(2ab)\int \sqrt{\sec(c+dx)}\,dx - \frac{1}{5}\left(-5a^2-3b^2\right)\int \frac{}{\sqrt{\sec(}} \\
&= \frac{2b^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}\left(2ab\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx \\
&= \frac{2\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{4ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}
\end{aligned}$$

Mathematica [A] time = 0.419755, size = 100, normalized size = 0.71

$$\frac{\sqrt{\sec(c+dx)}\left(6\left(5a^2+3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+b\sin(2(c+dx))(10a+3b\cos(c+dx))+20ab\sqrt{\cos(c+dx)}F\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(5*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*(10*a +
3*b*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [A] time = 3.155, size = 321, normalized size = 2.3

$$-\frac{2}{15\,d}\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,b^2\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^6+\left(40\,ab+24\,b^2\right)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*b^2*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(40*a*b+24*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-20*a*b-6*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a*b-15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-9*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/sqrt(sec(d*x + c))
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\cos(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sqrt(sec(d*x + c)), x)
```

3.703 $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{12ab\sqrt{\cos(}}{}$$

```
[Out] (12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(21*d) + (2*b^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*a^2 + 5*b^2)*Sin[c + d*x
])/(21*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.164179, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2639, 4045, 2641}

$$\frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{12ab\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```


```
[Out] (12*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5
*d) + (2*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(21*d) + (2*b^2*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (4*
a*b*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(7*a^2 + 5*b^2)*Sin[c + d*x
])/(21*d*Sqrt[Sec[c + d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= (2ab)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(6ab)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx - \frac{1}{7}\left(-7a^2-5b^2\right)\int \frac{}{\sec^{\frac{3}{2}}(}$$
$$= \frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2\left(7a^2+5b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} - \frac{1}{21}\left(-7a^2-5b^2\right)\int \sqrt{\sec(}$$
$$= \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(}{}$$
$$= \frac{12ab\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

Mathematica [A] time = 0.635744, size = 120, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(70a^2+84ab\cos(c+dx)+15b^2\cos(2(c+dx))+65b^2\right)+20\left(7a^2+5b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(504*a*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] +
 20*(7*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (70*a^2
+ 65*b^2 + 84*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])
)/(210*d)
```

Maple [A] time = 2.925, size = 362, normalized size = 2.1

$$-\frac{2}{105d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,b^{2}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-336\,ab-360\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^2*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-336*a*b-360*b^2)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(140*a^2+336*a*b+280*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2
*d*x+1/2*c)+(-70*a^2-84*a*b-80*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
+35*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))*a^2+25*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-126*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\cos\left(dx+c\right)+a\right)^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}}{\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/sec(d*x + c)^(3/2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b\cos\left(c+dx\right)\right)^{2}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(3/2), x)
```

3.704 $\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{2(9a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{20ab\sin(c}{21d\sqrt{\sec(c}}$$

```
[Out] (2*(9*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (2*b^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (4
*a*b*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(9*a^2 + 7*b^2)*Sin[c + d*
x])/(45*d*Sec[c + d*x]^(3/2)) + (20*a*b*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]])
```

Rubi [A] time = 0.174146, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3788, 3769, 3771, 2641, 4045, 2639}

$$\frac{2(9a^2+7b^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9a^2+7b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{20ab\sin(c}{21d\sqrt{\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```


```
[Out] (2*(9*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (20*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(21*d) + (2*b^2*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (4
*a*b*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(9*a^2 + 7*b^2)*Sin[c + d*
x])/(45*d*Sec[c + d*x]^(3/2)) + (20*a*b*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\cos(c+dx))^2}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \int \frac{(b+a\sec(c+dx))^2}{\sec^{\frac{9}{2}}(c+dx)}\,dx \\
&= (2ab)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \int \frac{b^2+a^2\sec^2(c+dx)}{\sec^{\frac{9}{2}}(c+dx)}\,dx \\
&= \frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}(10ab)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx - \frac{1}{9}\left(-9a^2-7b^2\right)\int \frac{}{\sec^{\frac{5}{2}}(}\\
&= \frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(9a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{20ab\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}(10\\
&= \frac{2b^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{4ab\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2\left(9a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{20ab\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{1}{21}\left(1\right.\\
&= \frac{2\left(9a^2+7b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{20ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{21d}
\end{aligned}$$

Mathematica [A] time = 1.02458, size = 135, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(7\left(36a^2+43b^2\right)\cos(c+dx)+5b(36a\cos(2(c+dx))+156a+7b\cos(3(c+dx)))\right)+168\left(9\right.\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(9*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
d*x)/2, 2] + 1200*a*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36
*a^2 + 43*b^2)*Cos[c + d*x] + 5*b*(156*a + 36*a*Cos[2*(c + d*x)] + 7*b*Cos[
3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [A] time = 3.279, size = 398, normalized size = 2.

$$-\frac{2}{315d}\sqrt{\left(2\left(\cos\left(1/2dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-1120b^{2}\left(\sin\left(1/2dx+c/2\right)\right)^{10}\cos\left(1/2dx+c/2\right)+\left(1440ab+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^2*s
in(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(1440*a*b+2240*b^2)*sin(1/2*d*x+1/2
*c)^8*cos(1/2*d*x+1/2*c)+(-504*a^2-2160*a*b-2072*b^2)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(504*a^2+1680*a*b+952*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+(-126*a^2-480*a*b-168*b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*
c)+150*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-147*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2\cos(dx+c)^2+2ab\cos(dx+c)+a^2}{\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)/sec(d*x + c)^(5/2)
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^2}{\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2/sec(d*x + c)^(5/2), x)
```

3.705 $\int (a+b\cos(c+dx))^3 \sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=234

$$\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2b\left(9a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\sqrt{}}{21d}$$

```
[Out] (-2*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*(9*a^2 + 5*b^2)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (32*a^2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a^2
*Sec[c + d*x]^(5/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.268907, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3842, 4047, 3768, 3771, 2641, 4046, 2639}

$$\frac{2a\left(5a^2+21b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2b\left(9a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}\sqrt{}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```


```
[Out] (-2*b*(9*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*b*(9*a^2 + 5*b^2)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(5*d) + (2*a*(5*a^2 + 21*b^2)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (32*a^2*b*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a^2
*Sec[c + d*x]^(5/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
```

```
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^{\frac{9}{2}}(c+dx)\,dx=\int \sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))^3\,dx$$

$$=\frac{2a^2\sec^{\frac{5}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{7d}+\frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}b\left(3a^2+\right.\right.$$

$$=\frac{2a^2\sec^{\frac{5}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{7d}+\frac{2}{7}\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}b\left(3a^2+\right.\right.$$

$$=\frac{2a\left(5a^2+21b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{32a^2b\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}+\frac{2a}{}$$

$$=\frac{2b\left(9a^2+5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d}+\frac{2a\left(5a^2+21b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+}{21d}$$

$$=\frac{2a\left(5a^2+21b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{21d}+\frac{2b\left(9a^2+5b^2\right)\sqrt{}}{}$$

$$=-\frac{2b\left(9a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a\left(5a^2+21b^2\right)}{}$$

Mathematica [A] time = 1.079, size = 191, normalized size = 0.82

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(10a\left(5a^2+21b^2\right)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-42b\left(9a^2+5b^2\right)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+63a^2b\sin(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(9/2),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-42*b*(9*a^2 + 5*b^2)*Cos[c + d*x]^(7/2)*EllipticE[(c
+ d*x)/2, 2] + 10*a*(5*a^2 + 21*b^2)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)
/2, 2] + 30*a^3*Sin[c + d*x] + 50*a^3*Cos[c + d*x]^2*Sin[c + d*x] + 210*a*b
^2*Cos[c + d*x]^2*Sin[c + d*x] + 378*a^2*b*Cos[c + d*x]^3*Sin[c + d*x] + 21
0*b^3*Cos[c + d*x]^3*Sin[c + d*x] + 63*a^2*b*Sin[2*(c + d*x)]))/(105*d)
```

Maple [B] time = 10.832, size = 847, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(-(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1
/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2*a^3*(-1/
56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-6/5*a
^2*b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2
-1)/sin(1/2*d*x+1/2*c)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-
24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(
1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+6*a*b^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}\right)\sec\left(dx+c\right)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(9/2), x)
```

3.706 $\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=189

$$\frac{6a\left(a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6a\left(a^2+5b^2\right)\sqrt{\mathrm{co}}}{}$$

```
[Out] (-6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (6*a*(a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(5*d) + (8*a^2*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Sec[
c + d*x]^(3/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.237839, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3842, 4047, 3768, 3771, 2639, 4046, 2641}

$$\frac{6a\left(a^2+5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2b\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{6a\left(a^2+5b^2\right)\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```


```
[Out] (-6*a*(a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*b*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/d + (6*a*(a^2 + 5*b^2)*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(5*d) + (8*a^2*b*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a^2*Sec[
c + d*x]^(3/2)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^{\frac{7}{2}}(c+dx)\,dx = \int \sqrt{\sec(c+dx)}(b+a\sec(c+dx))^3\,dx$$
$$= \frac{2a^2\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}b\left(a^2+5\right.\right.$$
$$= \frac{2a^2\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\sec(c+dx)}\left(\frac{1}{2}b\left(a^2+5\right.\right.$$
$$= \frac{6a\left(a^2+5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{8a^2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2 s}{}$$
$$= \frac{6a\left(a^2+5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{5d} + \frac{8a^2b\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a^2 s}{}$$
$$= -\frac{6a\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\left(a^2+b^2\right)\sqrt{\cos}}{}$$

Mathematica [A] time = 1.51934, size = 134, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(5b\left(a^2+b^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)-3a\left(a^2+5b^2\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{a\sin(c+dx)\left(3\left(a^2+5b^2\right)\cos(2(c+dx))\right.}{2\cos^{\frac{5}{2}}(c+}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*a*(a^2 + 5*b^2)*EllipticE[(c +
 d*x)/2, 2] + 5*b*(a^2 + b^2)*EllipticF[(c + d*x)/2, 2] + (a*(5*(a^2 + 3*b^
2) + 10*a*b*Cos[c + d*x] + 3*(a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/
(2*Cos[c + d*x]^(5/2))))/(5*d)
```

Maple [B] time = 9.477, size = 738, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*a*b
^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1
)+6*a^2*b*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2/5*a^3/(8*sin(1/2*d*x+1
/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c
)^2*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)
^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*s
in(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2
*d*x+1/2*c)^2cos(1/2*d*x+1/2*c))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3\cos(dx+c)^3 + 3\,ab^2\cos(dx+c)^2 + 3\,a^2b\cos(dx+c) + a^3\right)\sec(dx+c)^{\frac{7}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(7/2), x)
```

3.707 $\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$

```
[Out] (-2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a^2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
3*d) + (2*a^2*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.23123, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3842, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```


```
[Out] (-2*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*a*(a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (16*a^2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
3*d) + (2*a^2*Sqrt[Sec[c + d*x]]*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\,dx = \int \frac{(b+a\sec(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$$

$$= \frac{2a^2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{1}{2}b\left(a^2-3b^2\right)+\frac{1}{2}a\left(a^2\right.}{}$$

$$= \frac{2a^2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int \frac{-\frac{1}{2}b\left(a^2-3b^2\right)+4a^2b\,\mathrm{se}}{\sqrt{\sec(c+dx)}}$$

$$= \frac{16a^2b\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} + \frac{2a^2\sqrt{\sec(c+dx)}(b+a\sec(c+dx))\sin(c+dx}{3d}$$

$$= \frac{2a\left(a^2+9b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d} + \frac{16a^2b\sqrt{\sec(c+dx)}}{3d}$$

$$= -\frac{2b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{2a\left(a^2+9b^2\right)\sqrt{\mathrm{co}}}{}$$

Mathematica [A] time = 0.44409, size = 106, normalized size = 0.66

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(6b\left(b^2-3a^2\right)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+a\left(2\left(a^2+9b^2\right)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2a\sin(c+dx)(\right.\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(6*b*(-3*a^2 + b^2)*Cos[c + d*x]^(3/2)*EllipticE[(c + d
*x)/2, 2] + a*(2*(a^2 + 9*b^2)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2]
 + 2*a*(a + 9*b*Cos[c + d*x])*Sin[c + d*x])))/(3*d)
```

Maple [B] time = 6.875, size = 631, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a^3*sin(1/2*d*x+1/2*c)^2+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2*sin(1
/2*d*x+1/2*c)^2+18*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b*sin(1/2*d*x+1/2*c)^2-6*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))*b^3*sin(1/2*d*x+1/2*c)^2-36*a^2*b*cos(1/2*d*x+1/2*c)
*sin(1/2*d*x+1/2*c)^4-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*a*b^2-9*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3+2*
a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+18*a^2*b*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos{(dx+c)}+a)^3 \sec{(dx+c)}^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2), x)
```

3.708 $\int (a+b\cos(c+dx))^3 \sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=166

$$\frac{2a\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2-3b^2\right)\sqrt{\mathrm{co}}}{}$$

```
[Out] (-2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*d) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
 d*x]])
```

Rubi [A] time = 0.228518, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3841, 4047, 3771, 2641, 4046, 2639}

$$\frac{2a\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a\left(a^2-3b^2\right)\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```


```
[Out] (-2*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*(3*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(3*d) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c +
 d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)\,dx=\int \frac{(b+a\sec(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$=\frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\int \frac{4ab^2+\frac{1}{2}b\left(9a^2+b^2\right)\sec(c+dx)+\frac{1}{2}a}{\sqrt{\sec(c+dx)}}$$
$$=\frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2}{3}\int \frac{4ab^2+\frac{1}{2}a\left(3a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}}\,dx$$
$$=\frac{2a\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d}+\frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-($$
$$=\frac{2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}+\frac{2a\left(3a^2-b^2\right)\sqrt{\sec(}}{3}$$
$$=-\frac{2a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2b\left(9a^2+b^2\right)\sqrt{\cos}}{}$$

Mathematica [A] time = 0.551762, size = 108, normalized size = 0.65

$$\frac{\sqrt{\sec(c+dx)}\left(2\sin(c+dx)\left(3a^3+b^3\cos(c+dx)\right)+2b\left(9a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-6a\left(a^2-3b^2\right)\sqrt{\cos(c+}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-6*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, 2] + 2*b*(9*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
+ 2*(3*a^3 + b^3*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [A] time = 3.428, size = 303, normalized size = 1.8

$$-\frac{2}{3d}\left(4b^3\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^4+9a^2b\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\text{EllipticF}\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(4*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*a^2*b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-9*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b^2-6*a^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2
*b^3*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2
*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3ab^2\cos(dx+c)^2+3a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (dx+c)+a)^3 \sec (dx+c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2), x)
```

3.709 $\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{2a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+$$

```
[Out] (6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d + (8*a*b^2*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
 + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rubi [A] time = 0.22292, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3841, 4047, 3771, 2639, 4045, 2641}

$$\frac{2a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/d + (8*a*b^2*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]])
 + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^3}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$
$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{6ab^2+\frac{3}{2}b\left(5a^2+b^2\right)\sec(c+dx)+\frac{1}{2}a}{\sec^{\frac{3}{2}}(c+dx)}$$
$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{6ab^2+\frac{1}{2}a\left(5a^2+b^2\right)\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{8ab^2\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \left(a\left(a^2+b^2\right)\right)\int \sqrt{\sec(c}$$
$$= \frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{8ab^2\sin(c+dx)}{5d\sqrt{\sec(c+dx)}} +$$
$$= \frac{6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2a\left(a^2+b^2\right)\sqrt{\cos(c}}{}$$

Mathematica [A] time = 0.43062, size = 106, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(10a\left(a^2+b^2\right)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+6b\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+b^2\sin(2(c+dx\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*b*(5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*a*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
b^2*(5*a + b*Cos[c + d*x])*Sin[2*(c + d*x)]))/(5*d)
```

Maple [A] time = 3.22, size = 376, normalized size = 2.4

$$-\frac{2}{5d}\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-8\,b^{3}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{6}+\left(20\,ab^{2}+8\,b^{3}\right)\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x)
```

```
[Out] -2/5*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*b^3*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*a*b^2+8*b^3)*sin(1/2*d*x+1/2*c)^4*cos
(1/2*d*x+1/2*c)+(-10*a*b^2-2*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*a^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-15*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-
1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\cos\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}\right)\sqrt{\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)
```

3.710 $\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=199

$$\frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5a^2+9b^2)\sqrt{\cos(c+\ldots}}{}$$

```
[Out] (2*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (32*a*b^2*Sin[c + d*x])/(35*d*Sec[c
 + d*x]^(3/2)) + (2*b*(21*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rubi [A] time = 0.247532, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3841, 4047, 3769, 3771, 2641, 4045, 2639}

$$\frac{2b(21a^2+5b^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2b(21a^2+5b^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5a^2+9b^2)\sqrt{\cos(c+\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```


```
[Out] (2*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(5*d) + (2*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (32*a*b^2*Sin[c + d*x])/(35*d*Sec[c
 + d*x]^(3/2)) + (2*b*(21*a^2 + 5*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x
]]) + (2*b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\sqrt{\sec(c+dx)}}\,dx = \int \frac{(b+a\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8ab^2+\frac{1}{2}b\left(21a^2+5b^2\right)\sec(c+dx)+\frac{1}{2}a\left(7a^2+3b\right.}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{8ab^2+\frac{1}{2}a\left(7a^2+3b^2\right)\sec^2(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{7}\left(b\left(21a\right.\right.$$

$$= \frac{32ab^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{21}\left(\right.$$

$$= \frac{32ab^2\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b\left(21a^2+5b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{21}\left(\right.$$

$$= \frac{2a\left(5a^2+9b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d} + \frac{2b\left(21a^2+5b^2\right)\sqrt{\cos(c+dx)}F}{21d}$$

Mathematica [A] time = 0.875641, size = 132, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(b\sin(2(c+dx))\left(210a^2+126ab\cos(c+dx)+15b^2\cos(2(c+dx))+65b^2\right)+20b\left(21a^2+5b^2\right)\sqrt{\cos(c+dx)}\right.}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*a*(5*a^2 + 9*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c +
 d*x)/2, 2] + 20*b*(21*a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2] + b*(210*a^2 + 65*b^2 + 126*a*b*Cos[c + d*x] + 15*b^2*Cos[2*(c + d*x)
])*Sin[2*(c + d*x)]))/(210*d)
```

Maple [A] time = 3.052, size = 421, normalized size = 2.1

$$-\frac{2}{105\,d}\sqrt{\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(240\,b^{3}\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{8}+\left(-504\,ab^{2}-36\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*b^3*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-504*a*b^2-360*b^3)*sin(1/2*d*x+1/2*c
)^6*cos(1/2*d*x+1/2*c)+(420*a^2*b+504*a*b^2+280*b^3)*sin(1/2*d*x+1/2*c)^4*c
os(1/2*d*x+1/2*c)+(-210*a^2*b-126*a*b^2-80*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/
2*d*x+1/2*c)+105*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+25*b^3*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-189*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*c
os(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b\cos\left(dx+c\right)+a\right)^{3}}{\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}}{\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\cos(c + dx))^3}{\sqrt{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx + c) + a)^3}{\sqrt{\sec(dx + c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sqrt(sec(d*x + c)), x)
```

3.711 $\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{2b\left(27a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\left(7a^2+15b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a\left(7a^2+15b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

```
[Out] (2*b*(27*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (2*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (40*a*b^2*Sin[c + d*x])/(63*d*Sec
[c + d*x]^(5/2)) + (2*b*(27*a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(
3/2)) + (2*a*(7*a^2 + 15*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*
b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rubi [A] time = 0.276859, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3841, 4047, 3769, 3771, 2639, 4045, 2641}

$$\frac{2b\left(27a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a\left(7a^2+15b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a\left(7a^2+15b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\right|}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*b*(27*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (2*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (40*a*b^2*Sin[c + d*x])/(63*d*Sec
[c + d*x]^(5/2)) + (2*b*(27*a^2 + 7*b^2)*Sin[c + d*x])/(45*d*Sec[c + d*x]^(
3/2)) + (2*a*(7*a^2 + 15*b^2)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*
b^2*(b + a*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3841

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)
*(d*Csc[e + f*x])^n)/(f*n), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m
- 3)*(d*Csc[e + f*x])^(n + 1)*Simp[a^2*b*(m - 2*n - 2) - a*(3*b^2*n + a^2*(
n + 1))*Csc[e + f*x] - b*(b^2*n + a^2*(m + n - 1))*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 2] && ((Inte
gerQ[m] && LtQ[n, -1]) || (IntegersQ[m + 1/2, 2*n] && LeQ[n, -1]))
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
```

```
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 4045

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^3}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\sec(c+dx))^3}{\sec^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{10ab^2+\frac{1}{2}b\left(27a^2+7b^2\right)\sec(c+dx)+\frac{1}{2}a\left(9a^2+5\right.}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{10ab^2+\frac{1}{2}a\left(9a^2+5b^2\right)\sec^2(c+dx)}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \frac{1}{9}\left(b\left(27\right.\right.$$

$$= \frac{40ab^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2b^2(b+a\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{1}{15}\left(\right.$$

$$= \frac{40ab^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27a^2+7b^2\right)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a\left(7a^2+15b^2\right)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} + \frac{2b^2(b+a}{9}$$

$$= \frac{2b\left(27a^2+7b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{40ab^2\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2b\left(27a^2\right.}{45d}$$

$$= \frac{2b\left(27a^2+7b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d} + \frac{2a\left(7a^2+15b^2\right)\sqrt{\cos(c+dx)}}{21}$$

Mathematica [A] time = 1.12106, size = 159, normalized size = 0.68

$$\frac{\sqrt{\sec(c+dx)}\left(\sin(2(c+dx))\left(7b\left(108a^2+43b^2\right)\cos(c+dx)+5\left(84a^3+54ab^2\cos(2(c+dx))+234ab^2+7b^3\cos(3(c\right.\right.\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*b*(27*a^2 + 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2] + 120*a*(7*a^2 + 15*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] + (7*b*(108*a^2 + 43*b^2)*Cos[c + d*x] + 5*(84*a^3 + 234*a*b^2 + 5
4*a*b^2*Cos[2*(c + d*x)] + 7*b^3*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(126
0*d)
```

Maple [A] time = 2.982, size = 470, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*b^3*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2160*a*b^2+2240*b^3)*sin(1/2*d*x+1
/2*c)^8*cos(1/2*d*x+1/2*c)+(-1512*a^2*b-3240*a*b^2-2072*b^3)*sin(1/2*d*x+1/
2*c)^6*cos(1/2*d*x+1/2*c)+(420*a^3+1512*a^2*b+2520*a*b^2+952*b^3)*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-210*a^3-378*a^2*b-720*a*b^2-168*b^3)*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+105*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3+225*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*a*b^2-567*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-147*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^3}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b\cos{(c + dx)})^3}{\sec^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*cos(c + d*x))**3/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos{(dx + c)} + a)^3}{\sec{(dx + c)}^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3/sec(d*x + c)^(3/2), x)
```

3.712 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=188

$$\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{a^2d}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a*d) + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^2*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.546839, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3851, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2b^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```


```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d) + (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3
*a*d) + (2*b^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^2*d) + (2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3851

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(
n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n
 - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e
 + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n
, 3]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
```

```
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx$$

$$= \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{b}{2}+\frac{1}{2}a\sec(c+dx)-\frac{3}{2}b\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{3a}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{4\int \frac{\frac{3b^2}{4}+ab\sec(c+dx)+\frac{1}{4}\left(a^2+3b^2\right)\sec}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{4\int \frac{\frac{3b^3}{4}+\frac{1}{4}ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{3a^2b^2} + \frac{b^2\int}{}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\int\sqrt{\sec(c+dx)}\,dx}{3a} + \frac{b\int \frac{}{\sqrt{\sec}}}{a}$$

$$= \frac{2b^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a+b)d} - \frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2d} + \frac{2\sec^{\frac{3}{2}}}{}$$

$$= \frac{2b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{3ad}$$

Mathematica [A] time = 2.79578, size = 167, normalized size = 0.89

$$-\frac{\cot(c+dx)\left(-2\left(a^2+3ab+3b^2\right)\sqrt{-\tan^2(c+dx)}F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-a^2\sec^{\frac{5}{2}}(c+dx)+a^2\cos(2(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]*(-(a^2*Sec[c + d*x]^(5/2)) + a^2*Cos[2*(c + d*x)]*Sec[c + d*
x]^(5/2) + 6*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*
x]^2] - 2*(a^2 + 3*a*b + 3*b^2)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
qrt[-Tan[c + d*x]^2] - 6*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[-Tan[c + d*x]^2]))/(3*a^3*d)
```

Maple [A] time = 7.937, size = 452, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/a^2*b^3/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-2/a^2*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*
```

```
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a), x)
```

3.713 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{ad}$$

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(a*(a + b)*d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rubi [A] time = 0.208898, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3238, 3850, 3768, 3771, 2639, 3849, 2805}

$$-\frac{2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}+\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
- (2*b*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(a*(a + b)*d) + (2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3850

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(5/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d/b, Int[(d*Csc[e + f*x])^(3/2), x], x] - Dist[(a*
d)/b, Int[(d*Csc[e + f*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx$$
$$= \frac{\int \sec^{\frac{3}{2}}(c+dx)\,dx}{a} - \frac{b\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{a}$$
$$= \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{\left(b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos}$$
$$= -\frac{2b\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a+b)d} + \frac{2\sqrt{\sec(c+dx)}\sin(c+dx)}{ad} - \frac{\left(\sqrt{\cos(c+\right.}}{}$$
$$= -\frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2b\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{a(a+b)d}$$

Mathematica [A] time = 4.25602, size = 86, normalized size = 0.74

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(-(a+b)F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-b\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+aE\left(\sin^{-1}\right.\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(a*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1] - (a + b)*Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - b*EllipticPi[-(a/b), -ArcSin[Sqrt[S
ec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(a^2*d)
```

Maple [B] time = 3.911, size = 354, normalized size = 3.

$$-2\frac{1}{(a-b)a\sqrt{-2(\sin(1/2dx+c/2))^4+(\sin(1/2dx+c/2))^2}\sin(1/2dx+c/2)\sqrt{2(\cos(1/2dx+c/2))^2-1}d}\left(-2\sqrt{-2(\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*(-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(a-b)*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/a/(a-b)/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a), x)
```

3.714 $\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/((a + b)*d)
```

Rubi [A] time = 0.130593, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3238, 3849, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/((a + b)*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))}\,dx$$
$$= \frac{2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a+b)d}$$

Mathematica [A] time = 0.291963, size = 63, normalized size = 1.29

$$\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\left(\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a
/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2])/(a*d)
```

Maple [B] time = 2.789, size = 150, normalized size = 3.1

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{(a-b)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\mathrm{EllipticPi}\left(\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),-2*b/(a-b),2^(1/2))/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c)),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a), x)
```

3.715 $\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d) -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(b*(a + b)*d)
```

Rubi [A] time = 0.187072, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3238, 3848, 2803, 2641, 2805}

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d) -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(b*(a + b)*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3848

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[(Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]])/d, Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 2803

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{b+a\sec(c+dx)}\,dx$$
$$= \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{a+b\cos(c+dx)}\,dx$$
$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)}{b}$$
$$= \frac{2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+\right.}{b(a+b)}$$

Mathematica [A] time = 0.219348, size = 49, normalized size = 0.53

$$-\frac{2\sqrt{-\tan^2(c+dx)}\cot(c+dx)\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*Cot[c + d*x]*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-
Tan[c + d*x]^2])/(b*d)
```

Maple [A] time = 2.5, size = 188, normalized size = 2.

$$-2\,\frac{\sqrt{\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}}{b\,(a-b)\sqrt{-2\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2}\sin(1/2\,dx+c/2)\sqrt{2\,(\cos(1/2\,dx+c/2))^2-1}d}\left(\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*a-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b-a*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2)))/b/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b\cos{(c + dx)})\sqrt{\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos{(dx + c)} + a)\sqrt{\sec{(dx + c)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.716 $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sqrt{\cos(c+dx)}\cdots}{\cdots}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d) -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2
*d) + (2*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rubi [A] time = 0.235854, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3238, 3852, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}-\frac{2a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2\sqrt{\cos(c+dx)}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d) -
 (2*a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2
*d) + (2*a^2*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a + b)*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3852

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_))), x_Symbol] :> Dist[b^2/(a^2*d^2), Int[(d*Csc[e + f*x])^(3/2)/(a + b
*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a - b*Csc[e + f*x])/Sqrt[d*Csc[e
+ f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}\,dx$$
$$= \frac{\int \frac{b-a\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{b^2} + \frac{a^2\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{b^2}$$
$$= -\frac{a\int \sqrt{\sec(c+dx)}\,dx}{b^2} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} + \frac{\left(a^2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\mathrm{co}}}}{b^2}$$
$$= \frac{2a^2\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{b^2}$$
$$= \frac{2\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd} - \frac{2a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}$$

Mathematica [A] time = 5.79362, size = 178, normalized size = 1.32

$$\frac{\cot(c+dx)\left(2a\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+b\sec^{\frac{7}{2}}(c+dx)-b\sec^{\frac{3}{2}}(c+dx)+b\cos(2(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cot[c + d*x]*(-(b*Sec[c + d*x]^(3/2)) - b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3
/2) + b*Sec[c + d*x]^(7/2) + b*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2) - 2*b*El
lipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*Ellipti
cF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2*a*EllipticPi[-
```

```
(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(b^2*d)
```

Maple [A] time = 3.253, size = 227, normalized size = 1.7

$$2\frac{\sqrt{\left(2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}}{(a-b)\,b^{2}\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{2\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1}d}\Bigg(\mathrm{EllipticF}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2-EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*a*b-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-a^2*Ellipt
icPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^2/(a-b)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\cos\left(c+dx\right)\right)\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))*sec(c + d*x)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.717 $\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2a\sqrt{c}}{}$$

```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2
*d) + (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*b^3*d) - (2*a^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a + b)*d) + (2*Sin[c + d*x])/(3
*b*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.391052, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3853, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d}-\frac{2a^3\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2a\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (-2*a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2
*d) + (2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*b^3*d) - (2*a^3*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a + b)*d) + (2*Sin[c + d*x])/(3
*b*d*Sqrt[Sec[c + d*x]])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3853

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x
] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(c+dx)(b+a\sec(c+dx))}\,dx$$

$$= \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}\,dx}{3b}$$

$$= \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} + \frac{2\int \frac{-\frac{3ab}{2}-\left(-\frac{3a^2}{2}-\frac{b^2}{2}\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}\,dx}{3b^3} - \frac{a^3\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}\,dx}{b^3}$$

$$= \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b^2} + \frac{\left(3a^2+b^2\right)\int \sqrt{\sec(c+dx)}\,dx}{3b^3} - \frac{\left(a^3\sqrt{\cos\ldots\right.}{}$$

$$= -\frac{2a^3\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d} + \frac{2\sin(c+dx)}{3bd\sqrt{\sec(c+dx)}} - \frac{(a\ldots}{}$$

$$= -\frac{2a\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,F\left(\ldots\right.}{3b^3\ldots}$$

Mathematica [A] time = 6.12514, size = 198, normalized size = 1.15

$$-\frac{\cot(c+dx)\left(12a^2\sqrt{-\tan^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+6ab\sec^{\frac{3}{2}}(c+dx)-6ab\cos(2(c+dx))\sec^{\frac{3}{2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -(Cot[c + d*x]*(-(b^2*Sqrt[Sec[c + d*x]]) + 6*a*b*Sec[c + d*x]^(3/2) - 6*a*
b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + b^2*Cos[3*(c + d*x)]*Sec[c + d*x]^(
3/2) - 12*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^
2] + 4*(3*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c +
d*x]^2] + 12*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-
Tan[c + d*x]^2]))/(6*b^3*d)
```

Maple [B] time = 3.504, size = 516, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((4*a*b^2-4*b^
3)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+(-2*a*b^2+2*b^3)*sin(1/2*d*x+1/2
*c)^2*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-3*a^2*b*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-b^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*a^3*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/b^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.718 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=341

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^3d\left(a^2-b^2\right)}+\frac{(2a}{}$$

```
[Out] (b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d) + (b^2*(7*a^2 -
 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) - (b*(4*a^2 - 5*b^2)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.965983, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)}-\frac{b\left(4a^2-5b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^3d\left(a^2-b^2\right)}+\frac{(2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (b*(4*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d) + (b^2*(7*a^2 -
 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(a^3*(a - b)*(a + b)^2*d) - (b*(4*a^2 - 5*b^2)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((2*a^2 - 5*b^2)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3b^2}{2}-ab\sec(c+dx)+\frac{1}{2}\left(2a^2-5b^2\right)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{4}\right.}{}}{}$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\,\mathrm{s}}{a\left(a^2-\right.}$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\,\mathrm{s}}{a\left(a^2-\right.}$$

$$= -\frac{b\left(4a^2-5b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b^2\,\mathrm{s}}{a\left(a^2-\right.}$$

$$= \frac{b^2\left(7a^2-5b^2\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a-b)(a+b)^2d} - \frac{b\left(4a^2-5b^2\right)\sqrt{\sec(c+}}{a^3\left(a^2-b^2\right.}$$

$$= \frac{b\left(4a^2-5b^2\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2-5b^2\right)\sqrt{\cos(c+dx)}\,F\left(\right.}{3a^2\left(a^2-\right.}$$

Mathematica [A] time = 6.71211, size = 661, normalized size = 1.94

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{b\left(4a^2-5b^2\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)}-\frac{b^3\sin(c+dx)}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))}+\frac{2\tan(c+dx)}{3a^2}\right)}{d} + \frac{-\frac{2\left(40ab^3-28a^3b\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\,\mathrm{s}}{b\left(1-\cos^2(c+dx)\right)(a+b}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((-2*(-28*a^3*b + 40*a*b^3)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[
Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d
*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-4*a^4 - 44*a^2*b^
2 + 45*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ell
ipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt
[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)) + ((-12*a^2*b^2 + 15*b^4)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a
*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]
*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4
*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^
2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c +
 d*x]^2)))/(12*a^3*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((b*(4*a^2 -
 5*b^2)*Sin[c + d*x])/(a^3*(a^2 - b^2))) - (b^3*Sin[c + d*x])/(a^2*(a^2 - b
^2)*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*a^2)))/d
```

Maple [B] time = 14.516, size = 1008, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8/a^3*b^3/(-2*
a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+
1/2*c),-2*b/(a-b),2^(1/2))-4/a^3*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/
2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+2/a^2*b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d
*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^
2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2)))+2/a^2*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx + c)^{\frac{5}{2}}}{(b\cos (dx + c) + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^2, x)
```

3.719 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=277

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-$$

```
[Out] -(((2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^2*(a^2 - b^2)*d)) + (b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - (b*(5*a^2 - 3*b^2)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
(a - b)*(a + b)^2*d) + ((2*a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*
(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.7112, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3845, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(2a^2-3b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d\left(a^2-b^2\right)}+\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -(((2*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a^2*(a^2 - b^2)*d)) + (b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) - (b*(5*a^2 - 3*b^2)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
(a - b)*(a + b)^2*d) + ((2*a^2 - 3*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*
(b + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{b^2}{2}-ab\sec(c+dx)+\frac{1}{2}\left(2a^2-3b^2\right)\sec^2(c+dx)\right)}{b+a\sec(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$= \frac{\left(2a^2-3b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{2\int -\frac{1}{4}b\left(2a^2-3b^2\right)-\frac{1}{2}}{}$$

$$= \frac{\left(2a^2-3b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{2\int -\frac{1}{4}b^2\left(2a^2-3b^2\right)-(}{}$$

$$= \frac{\left(2a^2-3b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{a^2\left(a^2-b^2\right)d} + \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{b\int \sqrt{\sec(c+dx)}}{2a\left(a^2-b^2\right)}$$

$$= -\frac{b\left(5a^2-3b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a-b)(a+b)^2d} + \frac{\left(2a^2-3b^2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d}$$

$$= -\frac{\left(2a^2-3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2\left(a^2-b^2\right)d} + \frac{b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d}$$

Mathematica [A] time = 4.26419, size = 355, normalized size = 1.28

$$\frac{2a\sin(c+dx)\left(2a\left(a^2-b^2\right)\sec(c+dx)+2a^2b-3b^3\right)}{\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))} + \frac{\cot(c+dx)\left(-2\left(4a^2b+2a^3-3ab^2-3b^3\right)\sqrt{-\tan^2(c+dx)}F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+2a\left(2a^2-3b^2\right)\sqrt{-\tan^2(c+dx)}E\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] ((2*a*(2*a^2*b - 3*b^3 + 2*a*(a^2 - b^2)*Sec[c + d*x])*Sin[c + d*x])/((a^2
- b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]) + (Cot[c + d*x]*(-2*a^3*Sec
[c + d*x]^(3/2) + 3*a*b^2*Sec[c + d*x]^(3/2) + 2*a^3*Cos[2*(c + d*x)]*Sec[c
 + d*x]^(3/2) - 3*a*b^2*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*a*(2*a^2 -
3*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*
(2*a^3 + 4*a^2*b - 3*a*b^2 - 3*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[-Tan[c + d*x]^2] - 10*a^2*b*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*b^3*EllipticPi[-(a/b), -ArcSin[Sqrt[
Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*(a + b)))/(2*a^3*d)
```

Maple [B] time = 9.095, size = 874, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/a^2*b^2/(-2*a
*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
```

```
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))+2/a^2*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1/2*c
)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/a*b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*
c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(
a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b
/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^2, x)
```

3.720 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$

Optimal. Leaf size=217

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{ad\left(a^2-b^2\right)}$$

```
[Out] -((b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a
^2 - b^2)*d)) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/((a^2 - b^2)*d) + ((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*(a + b)^2*d) + (b^
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.43544, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3845, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\left(a^2-b^2\right)}-\frac{b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+d\right.}{ad\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] -((b*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a
^2 - b^2)*d)) - (Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/((a^2 - b^2)*d) + ((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)
/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a - b)*(a + b)^2*d) + (b^
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
```

```
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{b^2}{2}-ab\sec(c+dx)+\frac{1}{2}\left(2a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{b^3}{2}-\frac{1}{2}ab^2\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{ab^2\left(a^2-b^2\right)} + \frac{\left(3a^2-b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \sqrt{\sec(c+dx)}\,dx}{2\left(a^2-b^2\right)} - \frac{b\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2a\left(a^2-b^2\right)} + \frac{\left(\left(3a^2-b^2\right)\sqrt{\right.}{}$$

$$= \frac{\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a-b)(a+b)^2d} + \frac{b^2\sqrt{\sec(c+dx)}\sin(c+}{a\left(a^2-b^2\right)d(b+a\sec(c+}$$

$$= -\frac{b\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d} - \frac{\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{\left(a^2-b^2\right)d}$$

Mathematica [B] time = 6.60425, size = 590, normalized size = 2.72

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{b\sin(c+dx)}{a\left(a^2-b^2\right)}+\frac{b\sin(c+dx)}{\left(b^2-a^2\right)(a+b\cos(c+dx))}\right)}{d} + \frac{2\left(3b^2-4a^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c+dx)}(a\sec(c+dx)+b)\left(\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+\right.\right.\right.}}{a\left(1-\cos^2(c+dx)\right)(a+b\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((b*Sin[c + d*x])/(a*(a^2 - b^2)) + (b*Sin[c + d*x])/((
-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d + ((-8*a*Cos[c + d*x]^2*EllipticPi[-(
a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c
+ d*x]^2]*Sin[c + d*x])/((a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-
4*a^2 + 3*b^2)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] +
EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*S
qrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c +
d*x]^2)) + (Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d
*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/
b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[
c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*a*(-a + b)
*(a + b)*d)
```

Maple [B] time = 5.45, size = 612, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a*b^2/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-b/(a^2-b^2)/a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+b/(a^2-b^2)
/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-6*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2/a/(a^2-b^2)/(-2*a*b
+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec (dx + c)}}{(b \cos (dx + c) + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^2, x)
```

3.721 $\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\left(a^2-b^2\right)}$$

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a^2 - b
^2)*d) + (a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b*(a^2 - b^2)*d) - ((a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b*(a + b)^2*d) - (b*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.419767, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3844, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\right.}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a^2 - b
^2)*d) + (a*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b*(a^2 - b^2)*d) - ((a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b*(a + b)^2*d) - (b*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b+a\sec(c+dx))^2}\,dx$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{-\frac{b}{2}-a\sec(c+dx)+\frac{1}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a^2-b^2}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\int \frac{-\frac{b^2}{2}-\frac{1}{2}ab\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^2\left(a^2-b^2\right)} - \frac{\left(a^2+b^2\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}}{2b\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2\left(a^2-b^2\right)} + \frac{a\int \sqrt{\sec(c+dx)}\,dx}{2b\left(a^2-b^2\right)} - \frac{((a^2}{}$$

$$= -\frac{\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b(a+b)^2d} - \frac{b\sqrt{\sec(c+dx)}}{\left(a^2-b^2\right)d(b+a}$$

$$= \frac{\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{\left(a^2-b^2\right)d} + \frac{a\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d}$$

Mathematica [B] time = 6.59814, size = 580, normalized size = 2.79

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{a\sin(c+dx)}{\left(a^2-b^2\right)(a+b\cos(c+dx))}-\frac{\sin(c+dx)}{a^2-b^2}\right)}{d} + \frac{\sin(c+dx)\cos(2(c+dx))(a\sec(c+dx)+b)\left(4a^2\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\sec(c+}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(-(Sin[c + d*x]/(a^2 - b^2)) + (a*Sin[c + d*x])/((a^2 -
 b^2)*(a + b*Cos[c + d*x]))))/d + ((-8*a*Cos[c + d*x]^2*EllipticPi[-(a/b),
-ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]
^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) - (2*b*Cos[
c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b),
-ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (Cos[2*(
c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*Ellip
ticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c
+ d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Se
c[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*Ellip
ticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
Sec[c + d*x]^2])*Sin[c + d*x])/(a*b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*(a - b)*(a + b)*d)
```

Maple [B] time = 7.491, size = 713, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/(-2*a*b+2*b^
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-
2*b/(a-b),2^(1/2))-2*a/b*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/
2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2
*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\cos(c+dx))^2\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*cos(c + d*x))**2*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^2\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sqrt(sec(d*x + c))), x)
```

3.722 $\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=223

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{bd\left(a^2-b^2\right)}$$

```
[Out] -((a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a
^2 - b^2)*d)) + ((a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - (a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*
b^2*(a + b)^2*d) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b +
a*Sec[c + d*x]))
```

Rubi [A] time = 0.411831, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3843, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a\sec(c+dx)+b)}+\frac{\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\left(a^2-b^2\right)}-\frac{a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E}{bd\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```


```
[Out] -((a*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a
^2 - b^2)*d)) + ((a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) - (a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*
x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*
b^2*(a + b)^2*d) + (a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(b +
a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
```

```
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(b+a\sec(c+dx))^2}\,dx$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{a}{2}-b\sec(c+dx)+\frac{1}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{a^2-b^2}$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))} + \frac{\left(a\left(3-\frac{a^2}{b^2}\right)\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{b+a\sec(c+dx)}dx}{2(a^2-b^2)} + \frac{\int \frac{-\frac{ab}{2}-\left(-\frac{a^2}{2}\right.}{\sqrt{\text{se}}}}{b^2(}$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d(b+a\sec(c+dx))} - \frac{a\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{2b(a^2-b^2)} + \frac{(a^2-2b^2)\int \sqrt{\sec(c+d}}{2b^2(a^2-b^2)}$$

$$= -\frac{a(a^2-3b^2)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^2(a+b)^2d} + \frac{a\sqrt{\sec(c}}{(a^2-b^2)\,a}$$

$$= -\frac{a\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b(a^2-b^2)\,d} + \frac{(a^2-2b^2)\sqrt{\cos(c+dx)}F(}{b^2(a^2}$$

Mathematica [A] time = 5.14587, size = 251, normalized size = 1.13

$$\frac{\cos(2(c+dx))\csc(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(\left(a^2-3b^2\right)\sqrt{-\tan^2(c+dx)}\sqrt{\sec(c+dx)}(a+b\cos(c+dx))\Pi\left(-\frac{a}{b};-\sin^{-1}\left(\sqrt{\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Cos[2*(c + d*x)]*Csc[c + d*x]*Sec[c + d*x]^(3/2)*(-(b*(-a + b)*(a + b*Cos[
c + d*x])*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[-Tan[c + d*x]^2]) + (a^2 - 3*b^2)*(a + b*Cos[c + d*x])*EllipticPi[-(a/b),
-ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2] +
 a*b*(a*Tan[c + d*x]^2 - (a + b*Cos[c + d*x])*EllipticE[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[-Tan[c + d*x]^2])))/((a - b)*b^2*(a +
b)*d*(b + a*Sec[c + d*x])*(-2 + Sec[c + d*x]^2))
```

Maple [B] time = 6.832, size = 794, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8/b*a
/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),-2*b/(a-b),2^(1/2))+2*a^2/b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1
```

```
/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x
+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-
3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)
*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{2}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(3/2)), x)
```

3.723 $\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\cdots}{b^2d\left(\cdots\right.}$$

```
[Out] ((3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(b^2*(a^2 - b^2)*d) - (a*(3*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) + (a^2*(3*a^2 - 5
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - (a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.473508, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3238, 3847, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$-\frac{a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a\sec(c+dx)+b)}-\frac{a\left(3a^2-4b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\left(a^2-b^2\right)}+\frac{\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}\cdots}{b^2d\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```


```
[Out] ((3*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(b^2*(a^2 - b^2)*d) - (a*(3*a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Elliptic
F[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d) + (a^2*(3*a^2 - 5
*b^2)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/((a - b)*b^3*(a + b)^2*d) - (a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(b*(a^2 - b^2)*d*(b + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3847

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2
 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a^2*(m + 1)
- b^2*(m + n + 1) - a*b*(m + 1)*Csc[e + f*x] + b^2*(m + n + 2)*Csc[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && LtQ[m,
-1] && IntegersQ[2*m, 2*n]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
```

```
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{1}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}\,dx$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{\frac{3a^2}{2}-b^2+ab\sec(c+dx)-\frac{1}{2}a^2\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))}dx}{b\left(a^2-b^2\right)}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} + \frac{\int \frac{b\left(\frac{3a^2}{2}-b^2\right)-\left(-ab^2+a\left(\frac{3a^2}{2}-b^2\right)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{b^3\left(a^2-b^2\right)} + \frac{(a^2}{}$$

$$= -\frac{a^2\sqrt{\sec(c+dx)}\sin(c+dx)}{b\left(a^2-b^2\right)d(b+a\sec(c+dx))} - \frac{\left(a\left(3a^2-4b^2\right)\right)\int\sqrt{\sec(c+dx)}\,dx}{2b^3\left(a^2-b^2\right)} + \frac{(3a^2-}{}$$

$$= \frac{a^2\left(3a^2-5b^2\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{(a-b)b^3(a+b)^2d} - \frac{a^2\sqrt{\sec(c+}}{b\left(a^2-b^2\right)d(}$$

$$= \frac{\left(3a^2-2b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2\left(a^2-b^2\right)d} - \frac{a\left(3a^2-4b^2\right)\sqrt{\cos(c}}{}$$

Mathematica [A] time = 6.63468, size = 323, normalized size = 1.32

$$\frac{4a^2\sin(c+dx)}{b\left(b^2-a^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))} - \frac{2\cot(c+dx)\left(2b\left(-3a^2+ab+2b^2\right)\sqrt{-\tan^2(c+dx)}F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+2b\left(3a^2-2b^2\right)\sqrt{-\tan^2(c+dx)}E\left(\sin^{-1}\left(\sqrt{\sec(c}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((4*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x
]]) - (2*Cot[c + d*x]*(-3*a^2*b*Sec[c + d*x]^(3/2) + 2*b^3*Sec[c + d*x]^(3/
2) + 3*a^2*b*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) - 2*b^3*Cos[2*(c + d*x)]*S
ec[c + d*x]^(3/2) + 2*b*(3*a^2 - 2*b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[-Tan[c + d*x]^2] + 2*b*(-3*a^2 + a*b + 2*b^2)*EllipticF[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*a^3*EllipticPi[-(a/b), -
ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 10*a*b^2*EllipticPi
[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)*
b^3*(a + b)))/(4*d)
```

Maple [B] time = 9.097, size = 815, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/b^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)-12/b^2*a^2/(-2*a*b+2*b^2)*(sin(1/2
```

```
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2))-2/b^3*a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+
b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*
b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/s
in(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**2/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^2 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^2/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^2*sec(d*x + c)^(5/2)), x)
```

3.724 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=455

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-61a^2b^2+8a^4+35b^4\right)\sin(c+dx)\,\text{se}}{12a^3d\left(a^2-b^2\right)^2}$$

```
[Out] (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35
*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*
a^3*(a^2 - b^2)^2*d) + (b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a
- b)^2*(a + b)^3*d) - (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]^(
7/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (b^2*(13*a^
2 - 7*b^2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*S
ec[c + d*x]))
```

Rubi [A] time = 1.45148, antiderivative size = 455, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3238, 3845, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b^2\left(13a^2-7b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-61a^2b^2+8a^4+35b^4\right)\sin(c+dx)\,\text{se}}{12a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35
*b^4)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*
a^3*(a^2 - b^2)^2*d) + (b^2*(63*a^4 - 86*a^2*b^2 + 35*b^4)*Sqrt[Cos[c + d*x
]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a
- b)^2*(a + b)^3*d) - (b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*d) + ((8*a^4 - 61*a^2*b^2 + 35*b^4)*Sec[
c + d*x]^(3/2)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]^(
7/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (b^2*(13*a^
2 - 7*b^2)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*S
ec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
```

```
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{11}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5b^2}{2}-2ab\sec(c+dx)+\frac{1}{2}\left(4a^2-7b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b^2\left(13a^2-7b^2\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} + \int \frac{\sec^{\frac{3}{2}}(c+ \cdots$$

$$= \frac{\left(8a^4-61a^2b^2+35b^4\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2 d} + \frac{b^2\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b^2 \cdots$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sec^{\frac{3}{2}}(c+d \cdots}{12a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sec^{\frac{3}{2}}(c+d \cdots}{12a^3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+35b^4\right)\sec^{\frac{3}{2}}(c+d \cdots}{12a^3\left(a^2-b^2\right)^2 d}$$

$$= \frac{b^2\left(63a^4-86a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4(a-b)^2(a+b)^3 d} - \frac{b\left(24a^4-6 \cdots\right.}{\cdots}$$

$$= \frac{b\left(24a^4-65a^2b^2+35b^4\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4\left(a^2-b^2\right)^2 d} + \frac{\left(8a^4-61a^2b^2+ \cdots\right.}{\cdots}$$

Mathematica [A] time = 6.77974, size = 753, normalized size = 1.65

$$\frac{\sqrt{\sec(c+dx)}\left(-\frac{b\left(-65a^2b^2+24a^4+35b^4\right)\sin(c+dx)}{4a^4\left(a^2-b^2\right)^2} - \frac{b^3\sin(c+dx)}{2a^2\left(a^2-b^2\right)(a+b\cos(c+dx))^2} - \frac{3\left(5a^2b^3\sin(c+dx)-3b^5\sin(c+dx)\right)}{4a^3\left(a^2-b^2\right)^2(a+b\cos(c+dx))} + \frac{2\tan(c+dx)}{3a^3}\right)}{d} + \frac{-\frac{2(- \cdots}{\cdots}}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(160*a^5*b - 512*a^3*b^3 + 280*a*b^5)*Cos[c + d*x]^2*EllipticPi[-(a/b)
, -ArcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*
x]^2]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*
a^6 + 328*a^4*b^2 - 641*a^2*b^4 + 315*b^6)*Cos[c + d*x]^2*(EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]],
 -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b
*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((72*a^4*b^2 - 195*a^2*b^4 + 105*b^6
)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*
a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - S
ec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sq
rt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*
b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Co
s[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*a^4*(a - b)^2*(
a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-(b*(24*a^4 - 65*a^2*b^2 + 35*b^4)*Sin[c
 + d*x])/(4*a^4*(a^2 - b^2)^2) - (b^3*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*(a +
 b*Cos[c + d*x])^2) - (3*(5*a^2*b^3*Sin[c + d*x] - 3*b^5*Sin[c + d*x]))/(4*
a^3*(a^2 - b^2)^2*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 20.158, size = 2128, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-12*b^3/a^4/(-2
*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),-2*b/(a-b),2^(1/2))-6/a^4*b*(-(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/sin(1/2*d*x+1
/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)+4*b^2/a^3*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*
x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*
d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b
^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2)))+2/a^2*b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*
c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)
```

```
-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^
2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(
a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a
^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2/a^3*(-1/6*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^
2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^3, x)
```

3.725 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=388

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-29a^2b^2+8a^4+15b^4\right)\sin(c+dx)\sqrt{s}}{4a^3d\left(a^2-b^2\right)^2}$$

```
[Out] -((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) + (b*(11*a^2 - 5*b^2)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2
)^2*d) - (b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3
*d) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^
3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*(b + a*Sec[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 1.03233, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3238, 3845, 4098, 4102, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{b^2\left(11a^2-5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{\left(-29a^2b^2+8a^4+15b^4\right)\sin(c+dx)\sqrt{s}}{4a^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) + (b*(11*a^2 - 5*b^2)*Sqrt[Co
s[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2
)^2*d) - (b*(35*a^4 - 38*a^2*b^2 + 15*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2
*b)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)^2*(a + b)^3
*d) + ((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^
3*(a^2 - b^2)^2*d) + (b^2*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)
*d*(b + a*Sec[c + d*x])^2) + (b^2*(11*a^2 - 5*b^2)*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
```

```
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4102

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(C*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(d*Csc[e + f*x])^(n - 1))/(b*f*(m + n + 1)), x] + Dist[d/(b*(m + n + 1)),
Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A*b
*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[e
+ f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 -
b^2, 0] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
```

```
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{9}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3b^2}{2}-2ab\sec(c+dx)+\frac{1}{2}\left(4a^2-5b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b^2\left(11a^2-5b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2d(b+a\sec(c+dx))} + \int \frac{\sqrt{\sec(c+}}{}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b}{}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b}{}$$

$$= \frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2d} + \frac{b^2\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{b}{}$$

$$= -\frac{b\left(35a^4-38a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3(a-b)^2(a+b)^3d} + \frac{\left(8a^4-29a\right.}{}$$

$$= -\frac{\left(8a^4-29a^2b^2+15b^4\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3\left(a^2-b^2\right)^2d} + \frac{b\left(11a^2-5b^2\right)\sqrt{\mathrm{co}}}{}$$

Mathematica [A] time = 6.82133, size = 729, normalized size = 1.88

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(-29a^2b^2+8a^4+15b^4\right)\sin(c+dx)}{4a^3\left(a^2-b^2\right)^2}+\frac{b^2\sin(c+dx)}{2a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{11a^2b^2\sin(c+dx)-5b^4\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d} - \frac{-\frac{2\left(-80a^3b^2+16a^5+40ab^4\right)\mathrm{si}}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] -((-2*(16*a^5 - 80*a^3*b^2 + 40*a*b^4)*Cos[c + d*x]^2*EllipticPi[-(a/b), -A
rcSin[Sqrt[Sec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2
```

```
]*Sin[c + d*x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(56*a^4*
b - 95*a^2*b^3 + 45*b^5)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c
 + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1
- Cos[c + d*x]^2)) + ((8*a^4*b - 29*a^2*b^3 + 15*b^5)*Cos[2*(c + d*x)]*(b +
 a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sq
rt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a
 - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b),
-ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[
c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec
[c + d*x]]*(((8*a^4 - 29*a^2*b^2 + 15*b^4)*Sin[c + d*x])/(4*a^3*(a^2 - b^2)
^2) + (b^2*Sin[c + d*x])/(2*a*(a^2 - b^2)*(a + b*Cos[c + d*x])^2) + (11*a^2
*b^2*Sin[c + d*x] - 5*b^4*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*(a + b*Cos[c +
 d*x]))))/d
```

Maple [B] time = 14.863, size = 1992, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*b^2/a^3/(-2*a
*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),-2*b/(a-b),2^(1/2))-2/a*b*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^
2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/
8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b
),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-
b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
```

```
Pi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+2/a^3*(-(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2)/
sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)-2/a^2*b*(-1/a*b^2/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a
^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/
(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*
c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**3,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec\left(dx+c\right)^{\frac{3}{2}}}{\left(b\cos\left(dx+c\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^3, x)
```

3.726 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$

Optimal. Leaf size=321

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{4ad\left(a^2-b^2\right)^2}$$

```
[Out] (-3*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a^2 - b^2)^2*d) + (3*(5*a^4
- 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*(a + b)^3*d) + (b^2*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (3*b^2*(3*
a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*S
ec[c + d*x]))
```

Rubi [A] time = 0.770272, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3845, 4098, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{3b^2\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F}{4ad\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```


```
[Out] (-3*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*a^2*(a^2 - b^2)^2*d) - ((7*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a^2 - b^2)^2*d) + (3*(5*a^4
- 2*a^2*b^2 + b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*(a + b)^3*d) + (b^2*Sec[c + d*x]^
(3/2)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (3*b^2*(3*
a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*S
ec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4098

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> -Simp[(d*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b
*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1))/(b*f*(a^2 - b^2)*(m + 1)),
 x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*C
sc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A -
b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n + b
^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C},
x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx = \int \frac{\sec^{\frac{7}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{b^2}{2}-2ab\sec(c+dx)+\frac{1}{2}\left(4a^2-3b^2\right)\sec^2(c+dx)\right)}{(b+a\sec(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{3}{4}b^2(3a^2}{}}{}$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} + \frac{\int \frac{-\frac{3}{4}b^3(3a^2}{}}{}$$

$$= \frac{b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3b^2\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} - \frac{\left(3b\left(3a^2-\right.\right.}{8}$$

$$= \frac{3\left(5a^4-2a^2b^2+b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a-b)^2(a+b)^3d} + \frac{b^2\sec^{\frac{3}{2}}(c+d}{2a\left(a^2-b^2\right)d(b}$$

$$= -\frac{3b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2\left(a^2-b^2\right)^2 d} - \frac{\left(7a^2-b^2\right)\sqrt{\cos(c+dx)}F\left(\right.}{4a\left(a^2-\right.}$$

Mathematica [B] time = 6.66215, size = 700, normalized size = 2.18

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{3b\left(3a^2-b^2\right)\sin(c+dx)}{4a^2\left(a^2-b^2\right)^2}-\frac{b\sin(c+dx)}{2\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{b^3\sin(c+dx)-7a^2b\sin(c+dx)}{4a\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d} + \frac{-\frac{2\left(8ab^3-32a^3b\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-s}}{b(1-\cos}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^3,x]
```

```
[Out] ((-2*(-32*a^3*b + 8*a*b^3)*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[S
ec[c + d*x]]], -1]*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*
x])/(b*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(16*a^4 - 19*a^2*b^2
 + 9*b^4)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + Ellip
ticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1
 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^
2)) + ((-9*a^2*b^2 + 3*b^4)*Cos[2*(c + d*x)]*(b + a*Sec[c + d*x])*(-4*a*b +
 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*(2*a - b)*b*EllipticF[ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2
*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a*b^2*(a
 + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x
]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((3*b*(3*a^2 -
b^2)*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2) - (b*Sin[c + d*x])/(2*(a^2 - b^2)*
(a + b*Cos[c + d*x])^2) + (-7*a^2*b*Sin[c + d*x] + b^3*Sin[c + d*x])/(4*a*(
a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 7.448, size = 1176, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/a*b^2/(a^2-b
^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/2*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1
/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos
(1/2*d*x+1/2*c)^2+a-b)-7/4/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/(a+b)/(a^2-b^2)/a*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+
3/4/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/4*b^3/a^2/(a^2-b^2)
^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+9/4*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))-3/4*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/2*a^2/(a^2-b^2)
^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b)
,2^(1/2))-3/2/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**3,x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^3, x)
```

3.727 $\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=317

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{4bd\left(a^2-b^2\right)^2}$$

```
[Out] ((5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(4*a*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b*(a^2 - b^2)^2*d) - ((3*a^4 + 10*a^2*
b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*(a - b)^2*b*(a + b)^3*d) + (b^2*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) - (b*(7*a^2 - b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.751586, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3845, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}-\frac{b\left(7a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{3\left(a^2+b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c\right.}{4bd\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] ((5*a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(4*a*(a^2 - b^2)^2*d) + (3*(a^2 + b^2)*Sqrt[Cos[c + d*x]]*EllipticF[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b*(a^2 - b^2)^2*d) - ((3*a^4 + 10*a^2*
b^2 - b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*(a - b)^2*b*(a + b)^3*d) + (b^2*Sqrt[Sec[c + d*x]]*Si
n[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) - (b*(7*a^2 - b^2)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3845

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(a^2*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m
 + 1)*(d*Csc[e + f*x])^(n - 3))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[d^3/(b
*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n
 - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m +
 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n, 2
]))
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3\sqrt{\sec(c+dx)}}\,dx = \int \frac{\sec^{\frac{5}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{b^2}{2}-2ab\sec(c+dx)+\frac{1}{2}\left(4a^2-b^2\right)\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} + \int$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} + \int$$

$$= \frac{b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{b\left(7a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4a\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} + (3$$

$$= -\frac{\left(3a^4+10a^2b^2-b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a(a-b)^2b(a+b)^3d} + \frac{b^2}{2a\left(a^2\right.}$$

$$= \frac{\left(5a^2+b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a\left(a^2-b^2\right)^2 d} + \frac{3\left(a^2+b^2\right)\sqrt{\cos(c+}}{4}$$

Mathematica [A] time = 6.61226, size = 401, normalized size = 1.26

$$-\frac{\frac{4b\sin(c+dx)\left(b\left(5a^2+b^2\right)\cos(c+dx)+7a^3-ab^2\right)}{a\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2} + \frac{2\cot(c+dx)\left(-2b\left(-7a^2b+5a^3+ab^2+b^3\right)\sqrt{-\tan^2(c+dx)}F\left(\sin^{-1}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+2ab\left(5a^2+b^2\right)\sqrt{-\tan^2(c+d}\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] -((4*b*(7*a^3 - a*b^2 + b*(5*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x])/(a*(a^2
 - b^2)^2*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) + (2*Cot[c + d*x]*(-5*
a^3*b*Sec[c + d*x]^(3/2) - a*b^3*Sec[c + d*x]^(3/2) + 5*a^3*b*Cos[2*(c + d*
x)]*Sec[c + d*x]^(3/2) + a*b^3*Cos[2*(c + d*x)]*Sec[c + d*x]^(3/2) + 2*a*b*
(5*a^2 + b^2)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^
2] - 2*b*(5*a^3 - 7*a^2*b + a*b^2 + b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]
]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^4*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 20*a^2*b^2*EllipticPi[-(a/b), -ArcSi
n[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*b^4*EllipticPi[-(a/b),
 -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/(a^2*(a - b)^2*b*
(a + b)^2))/(16*d)
```

Maple [B] time = 11.668, size = 1736, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b*(-1/a*b^2/(
a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a
^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))-2*a/b*(-1/2/a*b^2/(a^2-b^2)*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*c
os(1/2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+
1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*
x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+
b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*
a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1
/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1
/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^3\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{3}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sqrt(sec(d*x + c))), x)
```

3.728 $\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=302

$$\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{a\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{4b^2d\left(a^2-b^2\right)^2}$$

```
[Out] -((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(4*b*(a^2 - b^2)^2*d) + (a*(a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) - ((a^4 - 10*a
^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^2*(a + b)^3*d) - (b*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.696477, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3844, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{3\left(a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}-\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{a\left(a^2-7b^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}\right.}{4b^2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```



```
[Out] -((a^2 + 5*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(4*b*(a^2 - b^2)^2*d) + (a*(a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) - ((a^4 - 10*a
^2*b^2 - 3*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a + b), (c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^2*(a + b)^3*d) - (b*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2) + (3*(a^2 + b^2)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(4*(a^2 - b^2)^2*d*(b + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3844

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(a*d^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m +
1)*(d*Csc[e + f*x])^(n - 2))/(f*(m + 1)*(a^2 - b^2)), x] - Dist[d^2/((m + 1
)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(
a*(n - 2) + b*(m + 1)*Csc[e + f*x] - a*(m + n)*Csc[e + f*x]^2), x], x] /; F
reeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[1, n, 2]
 && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{\sec^{\frac{3}{2}}(c+dx)}{(b+a\sec(c+dx))^3}\,dx$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} - \frac{\int \frac{-\frac{b}{2}-2a\sec(c+dx)+\frac{3}{2}b\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} -$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} -$$

$$= -\frac{b\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{3\left(a^2+b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4\left(a^2-b^2\right)^2 d(b+a\sec(c+dx))} +$$

$$= -\frac{\left(a^4-10a^2b^2-3b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^2(a+b)^3d} - \frac{}{2(}$$

$$= -\frac{\left(a^2+5b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b\left(a^2-b^2\right)^2 d} + \frac{a\left(a^2-7b^2\right)\sqrt{\mathrm{co}}}{}$$

Mathematica [B] time = 6.63965, size = 671, normalized size = 2.22

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(a^2+5b^2\right)\sin(c+dx)}{4b\left(b^2-a^2\right)^2} + \frac{a^2\sin(c+dx)}{2b\left(b^2-a^2\right)(a+b\cos(c+dx))^2} + \frac{a^3\sin(c+dx)-7ab^2\sin(c+dx)}{4b\left(b^2-a^2\right)^2(a+b\cos(c+dx))}\right)}{d} - \frac{2\left(-5a^2-b^2\right)\sin(c+dx)\cos^2(c+dx)\sqrt{1-\sec^2(c\cdots}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(3/2)),x]
```

```
[Out] -((-48*a*Cos[c + d*x]^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]
*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/((a + b*Cos[c
+ d*x])*(1 - Cos[c + d*x]^2)) + (2*(-5*a^2 - b^2)*Cos[c + d*x]^2*(EllipticF
[ArcSin[Sqrt[Sec[c + d*x]]], -1] + EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c +
d*x]]], -1])*(b + a*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a
*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((a^2 + 5*b^2)*Cos[2*(c + d*x
)]*(b + a*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
 2*(2*a - b)*b*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] + 4*a^2*EllipticPi[-(a/b), -ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*b^2*EllipticPi[-
(a/b), -ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]^2])*Sin[c + d*x])/(a*b^2*(a + b*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sq
rt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(16*(a - b)^2*(a + b)^2*d) + (Sqrt[
Sec[c + d*x]]*(((a^2 + 5*b^2)*Sin[c + d*x])/(4*b*(-a^2 + b^2)^2) + (a^2*Sin
[c + d*x])/(2*b*(-a^2 + b^2)*(a + b*Cos[c + d*x])^2) + (a^3*Sin[c + d*x] -
7*a*b^2*Sin[c + d*x])/(4*b*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d
```

Maple [B] time = 11.98, size = 1836, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4/b/(-2*a*b+2*
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c)
,-2*b/(a-b),2^(1/2))-4*a/b^2*(-1/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b
)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)
/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1
/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2
^(1/2)))+2*a^2/b^2*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)^2-3/4*
b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-7/8/(a+b)/(a^2-
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a^2-b^2)/a^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2-9/8*b/
(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*b^3/a
^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+3/
2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-3/4/a^2/(a^2-b^2)^2/(-2*a*
b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1
)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^3\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(3/2)), x)
```

3.729 $\int \frac{1}{(a+b\cos(c+dx))^3 \sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=319

$$\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(-5a^2b^2+3a^4+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{4b^3d\left(a^2-b^2\right)^2}$$

```
[Out] (-3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4 - 5*a^2*b^2 + 8*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2
*d) - (3*a*(a^4 - 2*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^3*(a + b)^3*d) +
(a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2
) + (a*(a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*
(b + a*Sec[c + d*x]))
```

Rubi [A] time = 0.685024, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3238, 3843, 4100, 4106, 3849, 2805, 3787, 3771, 2639, 2641}

$$\frac{a\left(a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\sec(c+dx)+b)}+\frac{a\sin(c+dx)\sqrt{\sec(c+dx)}}{2d\left(a^2-b^2\right)(a\sec(c+dx)+b)^2}+\frac{\left(-5a^2b^2+3a^4+8b^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+}}{4b^3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (-3*a*(a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4 - 5*a^2*b^2 + 8*b^4)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2
*d) - (3*a*(a^4 - 2*a^2*b^2 + 5*b^4)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*b)/(a
 + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^3*(a + b)^3*d) +
(a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Sec[c + d*x])^2
) + (a*(a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*
(b + a*Sec[c + d*x]))
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3843

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*(d*Csc[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(
a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp
[b*d*(n - 1) + a*d*(m + 1)*Csc[e + f*x] - b*d*(m + n + 1)*Csc[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &
& LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 4100

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*(m + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4106

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3849

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}\,dx = \int \frac{\sqrt{\sec(c+dx)}}{(b+a\sec(c+dx))^3}\,dx$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{a}{2}-2b\sec(c+dx)+\frac{3}{2}a\sec^2(c+dx)}{\sqrt{\sec(c+dx)}(b+a\sec(c+dx))^2}\,dx}{2\left(a^2-b^2\right)}$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2d(b+a\sec(c+dx))} + \frac{\int -$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2d(b+a\sec(c+dx))} + \frac{\int -$$

$$= \frac{a\sqrt{\sec(c+dx)}\sin(c+dx)}{2\left(a^2-b^2\right)d(b+a\sec(c+dx))^2} + \frac{a\left(a^2-7b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{4b\left(a^2-b^2\right)^2d(b+a\sec(c+dx))} - \frac{(3a$$

$$= -\frac{3a\left(a^4-2a^2b^2+5b^4\right)\sqrt{\cos(c+dx)}\Pi\left(\frac{2b}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4(a-b)^2b^3(a+b)^3d} + \frac{a}{2(a$$

$$= -\frac{3a\left(a^2-3b^2\right)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2d} + \frac{\left(3a^4-5a^2b^2+8b\right.}{}$$

Mathematica [A] time = 6.30227, size = 282, normalized size = 0.88

$$\frac{\frac{2ab^2\sin(c+dx)\left(3b\left(a^2-3b^2\right)\cos(c+dx)+a^3-7ab^2\right)}{\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^2} - \frac{2\cot(c+dx)\left(3ab\left(a^2-3b^2\right)\sin^2(c+dx)\sec^{\frac{3}{2}}(c+dx)+b\left(-a^2b+3a^3-9ab^2+7b^3\right)\sqrt{-\tan^2(c+dx)}F\left(\sin^{-1}\left(\sqrt{\sec(c\cdot\right.\right.\right.}{}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)),x]
```

```
[Out] ((2*a*b^2*(a^3 - 7*a*b^2 + 3*b*(a^2 - 3*b^2)*Cos[c + d*x])*Sin[c + d*x])/((
a^2 - b^2)^2*(a + b*Cos[c + d*x])^2*Sqrt[Sec[c + d*x]]) - (2*Cot[c + d*x]*(
3*a*b*(a^2 - 3*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x]^2 - 3*a*b*(a^2 - 3*b^2)
*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + b*(3*a^3
 - a^2*b - 9*a*b^2 + 7*b^3)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
-Tan[c + d*x]^2] + 3*(a^4 - 2*a^2*b^2 + 5*b^4)*EllipticPi[-(a/b), -ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*b
^3*d)
```

Maple [B] time = 12.25, size = 1914, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/b^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
```

```
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12/b^
2*a/(-2*a*b+2*b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-2/b^3*a^3*(-1/2/a*b^2/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/
2*d*x+1/2*c)^2+a-b)^2-3/4*b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*b*cos(1/2*d*x+1/2
*c)^2+a-b)-7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/4/(a+b)/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+3/8/(a+b)/(a
^2-b^2)/a^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*b^2-9/8*b/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*b
/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-3/8*b^3/a^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-15/4*a^2/(a^2-b^2)^2/(-2*a*b+2
*b^2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),-2*b/(a-b),2^(1/2))+3/2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))-
3/4/a^2/(a^2-b^2)^2/(-2*a*b+2*b^2)*b^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2)))+6/b^3*a^2*(-1/a*b^2
/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(2*b*cos(1/2*d*x+1/2*c)^2+a-b)-1/2/a/(a+b)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2*b/(a^2-b^2)/a*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+1/2*b/(a^2-b^2)/a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))-3*a/(a^2-b^2)/(-2*a*b+2*b^2)*b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))+1/a/
(a^2-b^2)/(-2*a*b+2*b^2)*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),-2*b/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{3}\sec\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**3/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^3 \sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^3/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^3*sec(d*x + c)^(5/2)), x)
```

3.730 $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=369

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d\sqrt{\sec(c+dx)}}-$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(9
*a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*S
qrt[Sec[c + d*x]]) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(15*a*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(5*d)
```

Rubi [A] time = 0.752761, antiderivative size = 369, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2796, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{15a^3d\sqrt{\sec(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(15*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(9
*a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(15*a^2*d*S
qrt[Sec[c + d*x]]) + (2*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c
 + d*x])/(15*a*d) + (2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
d*x])/(5*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} + \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c}{5d}$$

$$= \frac{2b\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15ad} + \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c}{5d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2-2b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{15a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 12.0213, size = 353, normalized size = 0.96

$$2\left(\sqrt{\sec(c+dx)}(a+b\cos(c+dx))\left(\left(9a^2-2b^2\right)\sin(c+dx)+a\tan(c+dx)(3a\sec(c+dx)+b)\right)+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(9*a^3 + 9*a^2*b - 2*a*b^2 -
 2*b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec
[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] +
2*a*(9*a^2 + 7*a*b - 2*b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1
+ Sec[c + d*x]))] - (9*a^2 - 2*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2] + (a + b*Cos[c +
d*x])*Sqrt[Sec[c + d*x]]*((9*a^2 - 2*b^2)*Sin[c + d*x] + a*(b + 3*a*Sec[c +
 d*x])*Tan[c + d*x])))/(15*a^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.586, size = 1563, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15/d/a^2*(-3*a^3-9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3+a*b^2*cos(d*x+c)^2+9*cos(d*x+c)^4*
a^2*b+cos(d*x+c)^4*a*b^2-5*cos(d*x+c)^3*a^2*b-2*cos(d*x+c)^3*a*b^2-4*cos(d*
x+c)*a^2*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^2*sin(d*x+c)*b^3+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
```

```
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3-2*cos(d*x+c)^4*b^3+9*c
os(d*x+c)^3*a^3+2*cos(d*x+c)^3*b^3-6*cos(d*x+c)^2*a^3-9*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*
b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)^3*sin(d*x+c)*a*b^2+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b-2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2-9*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^
2*sin(d*x+c)*a^2*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)*a*b^2+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^3*sin(d*x+c)*b^3+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3-9*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3
)*cos(d*x+c)*(1/cos(d*x+c))^(7/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(7/2), x)
```

3.731 $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=311

$$\frac{2b(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{2\sin(c+dx)\,\mathrm{s}}{}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a
 + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.50191, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2796, 2998, 2816, 2994}

$$\frac{2b(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}}+\frac{2\sin(c+dx)\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a
 + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+a\right.$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left((a-b)\sqrt{\cos(c+dx)}\sqrt{\sec\right.$$

$$= \frac{2(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{}}{3a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 10.4975, size = 301, normalized size = 0.97

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2b\sin(c+dx)}{3a}+\frac{2}{3}\tan(c+dx)\right)}{d} - \frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(b\cos(c+dx)\tan\left(\frac{1}{2}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*(a + b)*Sqrt[Cos
[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
```

```
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*d*Sq
rt[a + b*Cos[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*
b*Sin[c + d*x])/(3*a) + (2*Tan[c + d*x])/3))/d
```

Maple [B] time = 0.626, size = 888, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b-(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^2+cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^2+a*b*cos(d*x+c)^3+cos(d*x+c)^3*b^2+cos(d*x+c)^2*a^2+cos(d*x+c)^2*a*b-b^2
*cos(d*x+c)^2-2*cos(d*x+c)*a*b-a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*co
s(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos\left(dx + c\right) + a}\sec\left(dx + c\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\cos\left(dx + c\right) + a}\sec\left(dx + c\right)^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

3.732 $\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=269

$$\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2(a-b)\sqrt{a+b}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.359825, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4222, 2795, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}-\frac{2(a-b)\sqrt{a+b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
```

```
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1+\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + \left((-a$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{a}}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 5.68529, size = 215, normalized size = 0.8

$$\frac{2\left(\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\cos(c+dx)) + \frac{-\tan\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx)) - \frac{(a+b)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{b-a}{a+b}\right)-F\left(\sin\right.\right.}{\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}}}{\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}}\right.}{d\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*((a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (-(((a + b)*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*(EllipticE[ArcSin[Tan[(
c + d*x)/2]], (-a + b)/(a + b)] - EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a +
 b)/(a + b)]))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]) - (a + b*Cos[c + d*x]
)*Tan[(c + d*x)/2])/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]])))/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.546, size = 797, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+
c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(
d*x+c)+b*cos(d*x+c)^2+cos(d*x+c)*a-b*cos(d*x+c)-a)*cos(d*x+c)*(1/cos(d*x+c)
)^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

3.733 $\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)}\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right)|\right.}{d\sqrt{a+b}}$$

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sqrt[(a*(1 - Cos[c + d*x]))/(a + b*Cos[c + d*x])]*Sq
rt[(a*(1 + Cos[c + d*x]))/(a + b*Cos[c + d*x])]*(a + b*Cos[c + d*x])*Csc[c
+ d*x]*EllipticPi[b/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[Cos[c + d*x]])/Sqrt[a
 + b*Cos[c + d*x]]], -((a - b)/(a + b))]*Sqrt[Sec[c + d*x]])/(Sqrt[a + b]*d
)
```

Rubi [A] time = 0.143433, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {4222, 2811}

$$-\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\sec(c+dx)}\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\right)|\right.}{d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Sqrt[(a*(1 - Cos[c + d*x]))/(a + b*Cos[c + d*x])]*Sq
rt[(a*(1 + Cos[c + d*x]))/(a + b*Cos[c + d*x])]*(a + b*Cos[c + d*x])*Csc[c
+ d*x]*EllipticPi[b/(a + b), ArcSin[(Sqrt[a + b]*Sqrt[Cos[c + d*x]])/Sqrt[a
 + b*Cos[c + d*x]]], -((a - b)/(a + b))]*Sqrt[Sec[c + d*x]])/(Sqrt[a + b]*d
)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2811

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Simp[(2*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d
)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(
1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticPi[(b*(c + d))/
(d*(a + b)), ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[
a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))])/(d*f*Rt[(a + b)
/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(a + b)/(c + d)]
```

Rubi steps

$$\int \sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$$
$$= -\frac{2\sqrt{\cos(c+dx)}\sqrt{\frac{a(1-\cos(c+dx))}{a+b\cos(c+dx)}}\sqrt{\frac{a(1+\cos(c+dx))}{a+b\cos(c+dx)}}(a+b\cos(c+dx))\csc(c+dx)}{\sqrt{a+b}d}$$

Mathematica [A] time = 1.31723, size = 148, normalized size = 0.95

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a+b\cos(c+dx)}\left((a-b)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)-2b\Pi\left(-1;-\right.\right.}{d(a+b)\sqrt{\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*((a - b)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] - 2*b*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)])*Sqrt[Cos[c + d*x]*Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]])/((a + b)*
d*Sqrt[((a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)])
```

Maple [A] time = 0.616, size = 199, normalized size = 1.3

$$2\frac{\sqrt{(\cos(dx+c))^{-1}}(\sin(dx+c))^2}{d\sqrt{a+b\cos(dx+c)}(-1+\cos(dx+c))}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\left(a\mathrm{EllipticF}\left(\frac{-1+\cos(dx+}{\sin(dx+c)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*(a*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))-El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b+2*b*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)))/(a+b*cos(d*x+c))^(1/2)*
(1/cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\cos(c + dx)}\sqrt{\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\cos(dx + c) + a}\sqrt{\sec(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

3.734 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=431

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}+\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{d\sqrt{\sec(c+dx)}}$$

```
[Out] -(((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (a*Sqrt[a + b]*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]
]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 0.649834, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2821, 3054, 2809, 12, 2801, 2816, 2994}

$$\frac{\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}+\frac{\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\right.\right.}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] -(((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (a*Sqrt[a + b]*Sqrt[Cos[c +
d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(S
qrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]
]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
```

```
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3054

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :
> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[1/b^2, Int[(A*b^2 - a^2*C - 2*a*b*C*Sin[e + f*x])/((a + b*Sin[e + f
*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A,
 C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-\frac{ab}{2}+\frac{1}{2}ab\,c}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}}{b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \frac{1}{2}\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\text{co}}}{\sqrt{a+b}}$$

$$= -\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.5347, size = 403, normalized size = 0.94

$$\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(-4a\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2(a+b)\sqrt{\frac{\text{c}}{\text{co}}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 - 4*a*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
 + d*x)/2]^2 - 4*a*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2 + a*Tan[(c + d*x)/2] + b*Tan[(c +
 d*x)/2] - 2*b*Tan[(c + d*x)/2]^3 - a*Tan[(c + d*x)/2]^5 + b*Tan[(c + d*x)/
2]^5))/(2*Sqrt[2]*d*((1 + Cos[c + d*x])^(-1))^(3/2)*Sqrt[a + b*Cos[c + d*x]
])
```

Maple [B] time = 0.753, size = 806, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
```

```
s(d*x+c)*sin(d*x+c)*a-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x
+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))
*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*b*sin(d*x+c)-b*cos(d*x+c)^3-a*cos(d*x+c)^2+b*cos(d*x+c)^2+cos(d*x+
c)*a)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

3.735 $\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=498

$$\frac{\sqrt{a+b}\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}+\frac{a\sin(c}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(
a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(2*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.991045, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2821, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^2d\sqrt{\sec(c+dx)}}+\frac{a\sin(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
 + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(
a*(1 + Sec[c + d*x]))/(a - b)])/(4*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(
a^2 - 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/
(2*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(4*b*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}\,dx$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{ab}{2}+b^2\cos(c+dx)+\frac{1}{2}ab\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,d}{2b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4bd} + \frac{\left(\sqrt{\cos(c+\right.}{}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4bd} + \frac{\left(\sqrt{\cos(c+\right.}{}$$

$$= \frac{\sqrt{a+b}\left(a^2-4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+}{a+b}}}{4b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1}{}}}{4bd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 18.0463, size = 1113, normalized size = 2.23

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4d} + \frac{a^2\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right) - ab\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right) + 2ab\sqrt{\frac{a-b}{a+b}}\tan^3\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*Cos[c + d*x]]/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (-(a
^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]) - a*b*Sqrt[(a - b)/(a + b)]*Tan[
(c + d*x)/2] + 2*a*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + a^2*Sqrt[(a
 - b)/(a + b)]*Tan[(c + d*x)/2]^5 - a*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]^5 - (2*I)*a^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b
)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*b^2
*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/
2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(
c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*EllipticPi[(a +
b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a
```

```
- b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*b^2*EllipticPi[(a
+ b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/
(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*T
an[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*a*(a - b)*EllipticE[
I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a^2 + a*b - 2*b^2)*Elli
pticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))
]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan
[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(4*b*Sqrt[(a - b)/(a + b)
]*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + Tan[
(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.546, size = 1241, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/4/d/b*(2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*sin(d*x+c)*a*b-4*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+cos(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^2+8*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+2*b^2*cos(d*x+c)^4+2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*
x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^
2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a^2*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+8*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)+3*a*b*cos(d*x+c)^3+cos(d*x
+c)^2*a^2-cos(d*x+c)^2*a*b-2*b^2*cos(d*x+c)^2-a^2*cos(d*x+c)-2*cos(d*x+c)*a
*b)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+b\cos(c+dx)}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*cos(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

3.736 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=427

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\sqrt{\cos(c+dx)}\csc(c-}{105a}$$

```
[Out] (4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b
]*(25*a^2 - 57*a*b - 6*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(105*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[
c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (16*b*Sqrt[a + b*Cos
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*Sqrt[a + b*Cos[c
+ d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.04999, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2799, 3055, 2998, 2816, 2994}

$$\frac{2\left(25a^2+3b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{105ad}+\frac{2(a-b)\sqrt{a+b}\left(25a^2-57ab-6b^2\right)\sqrt{\cos(c+dx)}\csc(c-}{105a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```


```
[Out] (4*(a - b)*b*Sqrt[a + b]*(41*a^2 - 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(105*a^3*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b
]*(25*a^2 - 57*a*b - 6*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(105*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(25*a^2 + 3*b^2)*Sqrt[a + b*Cos[
c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(105*a*d) + (16*b*Sqrt[a + b*Cos
[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*a*Sqrt[a + b*Cos[c
+ d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
```

```
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{16b\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2a\sqrt{a+b\cos(c+dx)}\,\mathrm{sec}}{7d}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105ad} + \frac{16b\sqrt{a+b\,\mathrm{co}}}{}$$

$$= \frac{2\left(25a^2+3b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105ad} + \frac{16b\sqrt{a+b\,\mathrm{co}}}{}$$

$$= \frac{4(a-b)b\sqrt{a+b}\left(41a^2-3b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c-}}\right.\right.}{105a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.6448, size = 441, normalized size = 1.03

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{4b(3b^2-41a^2)\sin(c+dx)}{105a^2} + \frac{2\sec(c+dx)(25a^2\sin(c+dx)+3b^2\sin(c+dx))}{105a} + \frac{2}{7}a\tan(c+dx)\sec^2(c+\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-41*a^3 - 41*a^2*b + 3*a*b^2
 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + a*(25*a^3 + 82*a^2*b + 51*a*b^2 - 6*b^3)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-41*a^2 + 3*b^2)*Cos[
c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*a^
2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]*((-4*b*(-41*a^2 + 3*b^2)*Sin[c + d*x])/(105*a^2)
 + (2*Sec[c + d*x]*(25*a^2*Sin[c + d*x] + 3*b^2*Sin[c + d*x]))/(105*a) + (1
6*b*Sec[c + d*x]*Tan[c + d*x])/35 + (2*a*Sec[c + d*x]^2*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.522, size = 1835, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/105/d/a^2*(6*b^4*cos(d*x+c)^4-27*a^2*b^2*cos(d*x+c)^2-15*a^4+25*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(
```

```
d*x+c)*a^4+25*cos(d*x+c)^4*a^4-10*cos(d*x+c)^2*a^4-6*cos(d*x+c)^5*b^4+25*co
s(d*x+c)^5*a^3*b+82*cos(d*x+c)^5*a^2*b^2+3*cos(d*x+c)^5*a*b^3+82*cos(d*x+c)
^4*a^3*b-55*cos(d*x+c)^4*a^2*b^2-6*cos(d*x+c)^4*a*b^3-68*cos(d*x+c)^3*a^3*b
+3*cos(d*x+c)^3*a*b^3-39*cos(d*x+c)*a^3*b+82*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+51*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*si
n(d*x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-82*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-82*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(
d*x+c)*a^2*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3+82*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+51*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*
x+c)*a^2*b^2-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3-82*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-82*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+
c)*a^2*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^3*sin(d*x+c)*a*b^3+6*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+25*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^
4+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)^3*sin(d*x+c)*b^4)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9
/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(9/2), x)
```

3.737 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=365

$$\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{5a^2d\sqrt{\sec(c+dx)}}+\frac{2a\sin}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(5*a^2*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*(3*a - b)*Sqrt[a
+ b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a*d*Sqrt[Sec[
c + d*x]]) + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(5*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*
d)
```

Rubi [A] time = 0.759106, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2799, 3055, 2998, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{5a^2d\sqrt{\sec(c+dx)}}+\frac{2a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
 + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d
*x]))/(a - b)])/(5*a^2*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*(3*a - b)*Sqrt[a
+ b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(5*a*d*Sqrt[Sec[
c + d*x]]) + (4*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])
/(5*d) + (2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*
d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
```

```
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{4b\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c}{5d}$$

$$= \frac{4b\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c}{5d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{5a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 11.4895, size = 345, normalized size = 0.95

$$\frac{2\left(\sqrt{\sec(c+dx)}(a+b\cos(c+dx))\left(\left(3a^2+b^2\right)\sin(c+dx)+a\tan(c+dx)(a\sec(c+dx)+2b)\right)+\frac{\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(-\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*((Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a^3 + 3*a^2*b + a*b^2 + b
^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c +
 d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + 2*a*
(3*a^2 + 4*a*b + b^2)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[
c + d*x]))] - (3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/Sqrt[Sec[(c + d*x)/2]^2] + (a + b*Cos[c + d*x])*Sq
rt[Sec[c + d*x]]*((3*a^2 + b^2)*Sin[c + d*x] + a*(2*b + a*Sec[c + d*x])*Tan
[c + d*x])))/(5*a*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.503, size = 1547, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/5/d/a*(3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*cos(d*x+c)^3*sin(d*x+c)*a^3+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b+(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2-
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
```

```
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*sin(d*x+c)*a^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2-(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*
x+c)*b^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*a^3+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*a^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x
+c)*b^3+3*cos(d*x+c)^4*a^2*b+2*cos(d*x+c)^4*a*b^2+cos(d*x+c)^4*b^3+3*cos(d*
x+c)^3*a^3+cos(d*x+c)^3*a*b^2-cos(d*x+c)^3*b^3-2*cos(d*x+c)^2*a^3-3*a*b^2*c
os(d*x+c)^2-3*cos(d*x+c)*a^2*b-a^3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/co
s(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(7/2), x)
```

3.738 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=317

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{2(a-3b)(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{3ad\sqrt{\sec(c+dx)}}$$

```
[Out] (8*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*(a - 3*b)*(a - b)*Sqrt[a + b]*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (
2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.52605, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2799, 2998, 2816, 2994}

$$\frac{2a\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3d}+\frac{2(a-3b)(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec}{}}}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```


```
[Out] (8*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (2*(a - 3*b)*(a - b)*Sqrt[a + b]*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*d*Sqrt[Sec[c + d*x]]) + (
2*a*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.$$

$$= \frac{2a\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left((a-3b)(a-b)\sqrt{\cos(c+d\right.$$

$$= \frac{8(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a}{}}}{3ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 7.1539, size = 291, normalized size = 0.92

$$\frac{\sqrt{\sec(c+dx)}\left(4\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\left(a^2+4ab+3b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+b\left(s\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(4*Cos[(c + d*x)/2]^2*(-4*b*(a + b)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a^2 + 4*a*b + 3*b^
```

```
2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
 + b*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(c
+ d*x))/2])) + 2*(a + b*Cos[c + d*x])*(a + 4*b*Cos[c + d*x])*Tan[c + d*x]))
/(3*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.615, size = 1083, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a^2+4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+3*cos(d*x+c)^2*sin(d*x+c)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b^2-4*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2+4*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+3*c
os(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*sin(d*x+c)*b^2-4*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+a*b*cos(d*x+c)^3+4*co
s(d*x+c)^3*b^2+cos(d*x+c)^2*a^2+4*cos(d*x+c)^2*a*b-4*b^2*cos(d*x+c)^2-5*cos
(d*x+c)*a*b-a^2)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin
(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}} \sec(dx+c)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2), x)
```

3.739 $\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=397

$$-\frac{2(a-2b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}+\frac{2(a-b)\sqrt{}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(d*Sqrt[Sec[c + d*x]]) - (2*(a - 2*b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec
[c + d*x]])
```

Rubi [A] time = 0.555988, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2798, 2809, 2998, 2816, 2994}

$$-\frac{2(a-2b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}+\frac{2(a-b)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(d*Sqrt[Sec[c + d*x]]) - (2*(a - 2*b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c
+ d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a
*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[a + b]*Sq
rt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec
[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2798

```
Int[((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_.) + (b_.)*sin[(e_.) +
 (f_.)*(x_)])^(3/2), x_Symbol] :> Dist[d^2/b^2, Int[Sqrt[a + b*Sin[e + f*x]
]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(b*c - a*d)/b^2, Int[Simp[b*c + a
*d + 2*b*d*Sin[e + f*x], x]/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e +
```

```
f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{a+2b\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx + \left(b^2\sqrt{\mathrm{co}\right.$$

$$= -\frac{2b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{d\sqrt{\sec(c+dx)}}$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.4493, size = 643, normalized size = 1.62

$$2\left(-\left(a^2+2ab-b^2\right)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*(a^2*
Tan[(c + d*x)/2] + a*b*Tan[(c + d*x)/2] - 2*a*b*Tan[(c + d*x)/2]^3 - a^2*Ta
n[(c + d*x)/2]^5 + a*b*Tan[(c + d*x)/2]^5 + 2*b^2*EllipticPi[-1, -ArcSin[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*b^2*EllipticPi
[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)
/2]^2)/(a + b)] + a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (a^2 + 2*a*b - b^2)
*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(a + b)]))/(d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + T
an[(c + d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.581, size = 1191, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -2/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^2+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2
-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
))*sin(d*x+c)*a*b+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c)
,-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
```

```
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+2*
b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*s
in(d*x+c)+cos(d*x+c)^2*a*b+a^2*cos(d*x+c)-cos(d*x+c)*a*b-a^2)*cos(d*x+c)/(a
+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2), x)
```

3.740 $\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=435

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}+\frac{\sqrt{a+b}(2a+b)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] -(((a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*a + b)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b]
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[
Sec[c + d*x]]) + (b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/d
```

Rubi [A] time = 0.715459, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2821, 3053, 2809, 2998, 2816, 2994}

$$\frac{b\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}+\frac{\sqrt{a+b}(2a+b)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```



```
[Out] -(((a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*(2*a + b)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b]
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Co
s[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[
Sec[c + d*x]]) + (b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/d
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
```

```
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= \frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right.$$

$$= -\frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 12.2385, size = 324, normalized size = 0.74

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(b\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos(c+dx))+4a(a-2b)\sqrt{\frac{\cos(c}{\cos(c+}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*a*(a - 2*b)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 12*a*b*Sq
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
] + b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
))/(d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.632, size = 1005, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^2+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+
c),-1,(-(a-b)/(a+b))^(1/2))*a*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-
```

```
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-4*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)
+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(
d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*a*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a^2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+cos(d*x+c)^3*b^2+cos(d*x+c)^2*a*b-b^2*co
s(d*x+c)^2-cos(d*x+c)*a*b)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)), x)
```

3.741 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=493

$$-\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd\sqrt{\sec(c+dx)}}+\sin(c$$

```
[Out] (-5*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a + 2*b)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(
3*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(4*b*d*Sqrt[Sec[c + d*x]]) + (3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(4*d) + ((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 1.2702, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4222, 2821, 3047, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4bd\sqrt{\sec(c+dx)}}+\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-5*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(4*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a + 2*b)*Sqrt[Cos[c + d*x]]*C
sc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos
[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(
3*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin
[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
 b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
 b)])/(4*b*d*Sqrt[Sec[c + d*x]]) + (3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(4*d) + ((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x
]]*Sin[c + d*x])/(2*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}\,dx$$

$$= \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+}}{}}{2b}$$

$$= -\frac{a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{2d} + \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c}{2d}$$

$$= \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d} + \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c}{2d}$$

$$= \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{4d} + \frac{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}\sin(c}{2d}$$

$$= -\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{4bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{5(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{4d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.729, size = 853, normalized size = 1.73

$$\frac{b\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4d} - \frac{-5a^2\tan^5\left(\frac{1}{2}(c+dx)\right)+5ab\tan^5\left(\frac{1}{2}(c+dx)\right)-10ab\tan^3\left(\frac{1}{2}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) - (5
*a^2*Tan[(c + d*x)/2] + 5*a*b*Tan[(c + d*x)/2] - 10*a*b*Tan[(c + d*x)/2]^3
- 5*a^2*Tan[(c + d*x)/2]^5 + 5*a*b*Tan[(c + d*x)/2]^5 - 6*a^2*EllipticPi[-1
, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^
2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/
(a + b)] - 6*a^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 8*b^2*EllipticPi[-1, -ArcSin[T
an[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)
] + 5*a*(a + b)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d
*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(4*a^2 - a*b + 2*b^2)*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d
*x)/2]^2)/(a + b)])/(4*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c +
 d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.557, size = 1423, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^2-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-5*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x
+c)*a^2-5*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*sin(d*x+c)*a*b-6*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-8*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+8*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d
*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*b^2*sin(d*x+c)-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
```

```
/(a+b))^(1/2))*a^2*sin(d*x+c)-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-8*b^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)-2*b^2*cos(d*x
+c)^4-7*a*b*cos(d*x+c)^3-5*cos(d*x+c)^2*a^2+5*cos(d*x+c)^2*a*b+2*b^2*cos(d*
x+c)^2+5*a^2*cos(d*x+c)+2*cos(d*x+c)*a*b)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(
a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)), x)
```

3.742 $\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=568

$$\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24bd}-\frac{(a-b)\sqrt{a+b}\left(3a^2+16b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(}{a+}}}{24abd\sqrt{\sec(c}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a + 2*b)*(3
*a + 8*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqr
t[Sec[c + d*x]]) + (a*Sqrt[a + b]*(a^2 - 12*b^2)*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (a*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + ((a + b*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((3*a^2 + 16*b^2
)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b*d)
```

Rubi [A] time = 1.35566, antiderivative size = 568, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4222, 2821, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(3a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24bd}-\frac{(a-b)\sqrt{a+b}\left(3a^2+16b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(}{a+}}}{24abd\sqrt{\sec(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(3*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elli
pticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -(
(a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c +
 d*x]))/(a - b)])/(24*a*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(a + 2*b)*(3
*a + 8*b)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c
 + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*b*d*Sqr
t[Sec[c + d*x]]) + (a*Sqrt[a + b]*(a^2 - 12*b^2)*Sqrt[Cos[c + d*x]]*Csc[c +
 d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b^2*d*Sqrt[Sec[c + d*x]]) + (a*S
qrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Sec[c + d*x]]) + ((a + b*Co
s[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]) + ((3*a^2 + 16*b^2
)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*b*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2821

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[1/(d*(m + n)), Int[
(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n - 1)*Simp[a^2*c*d*(m +
 n) + b*d*(b*c*(m - 1) + a*d*n) + (a*d*(2*b*c + a*d)*(m + n) - b*d*(a*c - b
*d*(m + n - 1)))*Sin[e + f*x] + b*d*(b*c*n + a*d*(2*m + n - 1))*Sin[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[0, m, 2] && LtQ[-1, n, 2] && NeQ
[m + n, 0] && (IntegerQ[m] || IntegersQ[2*m, 2*n])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
```

```
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{3/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}\,dx$$

$$= \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+dx)}\left(\frac{ab}{2}+2b^2\cos(c\right.}{\sqrt{\cos(c+c}}}{3b}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec\right.}{}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(3a^2+16b^2\right)\sqrt{a+}}{}$$

$$= \frac{a\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sqrt{\sec(c+dx)}} + \frac{(a+b\cos(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(3a^2+16b^2\right)\sqrt{a+}}{}$$

$$= \frac{a\sqrt{a+b}\left(a^2-12b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-s}{}}}{8b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(3a^2+16b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a}{}}}{24abd\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.5915, size = 969, normalized size = 1.71

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{12}b\sin(c+dx)+\frac{7}{24}a\sin(2(c+dx))+\frac{1}{12}b\sin(3(c+dx))\right)}{d} + \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(-3a^3\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(3/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b*Sin[c + d*x])/12 + (7*a*Si
n[2*(c + d*x)])/24 + (b*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[(c + d*x)
/2]^2)^(-1)]*(3*a^3*Tan[(c + d*x)/2] + 3*a^2*b*Tan[(c + d*x)/2] + 16*a*b^2*
Tan[(c + d*x)/2] + 16*b^3*Tan[(c + d*x)/2] - 6*a^2*b*Tan[(c + d*x)/2]^3 - 3
2*b^3*Tan[(c + d*x)/2]^3 - 3*a^3*Tan[(c + d*x)/2]^5 + 3*a^2*b*Tan[(c + d*x)
/2]^5 - 16*a*b^2*Tan[(c + d*x)/2]^5 + 16*b^3*Tan[(c + d*x)/2]^5 + 6*a^3*Ell
ipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
b)] - 72*a*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c
+ d*x)/2]^2)/(a + b)] + 6*a^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 72*a*b^2*Elliptic
Pi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*
x)/2]^2)/(a + b)] + (3*a^3 + 3*a^2*b + 16*a*b^2 + 16*b^3)*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[
(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/
(a + b)] + 2*a*(7*a - 26*b)*b*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*b*d*(1 + Tan[
(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/
2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.488, size = 1691, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/24/d/b*(-52*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))
/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*cos(d*x+c)^2*a^2*b-16
*cos(d*x+c)*a*b^2+72*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b
)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+16*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-6*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b*sin(d*x+c)+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
```

```
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-6*a*b^2*cos(d*x+c)^2+22
*cos(d*x+c)^4*a*b^2+17*cos(d*x+c)^3*a^2*b-14*cos(d*x+c)*a^2*b+3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*a^2*b+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*a*b^2+14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-52*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+7
2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)*sin(d*x+c)*a*b^2-3*a^3*cos(d*x+c)-16*cos(d*x+c)^2*b^3+8*cos(d*x+c)^
5*b^3+8*cos(d*x+c)^3*b^3+3*cos(d*x+c)^2*a^3)*cos(d*x+c)*(1/cos(d*x+c))^(3/2
)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{3}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2), x)
```

3.743 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx$

Optimal. Leaf size=494

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+a}}{315ad}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a -
 b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b + 165*a*b^2 + 10*b^3)*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*b
*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(315*a*d) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5
/2)*Sin[c + d*x])/(315*d) + (38*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(
7/2)*Sin[c + d*x])/(63*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(9
/2)*Sin[c + d*x])/(9*d)
```

Rubi [A] time = 1.5176, antiderivative size = 494, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(49a^2+75b^2\right)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{315d}+\frac{2b\left(163a^2+5b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+a}}{315ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sqrt[Cos[c + d*x]]*
Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Co
s[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^3*d*Sqrt[Sec[c + d*x]]) - (2*(a -
 b)*Sqrt[a + b]*(147*a^3 - 114*a^2*b + 165*a*b^2 + 10*b^3)*Sqrt[Cos[c + d*x
]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt
[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*
Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(315*a^2*d*Sqrt[Sec[c + d*x]]) + (2*b
*(163*a^2 + 5*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(315*a*d) + (2*(49*a^2 + 75*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5
/2)*Sin[c + d*x])/(315*d) + (38*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(
7/2)*Sin[c + d*x])/(63*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(9
/2)*Sin[c + d*x])/(9*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{11}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{9d} + \frac{1}{9}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{38ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec}{9d}$$

$$= \frac{2\left(49a^2+75b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{315d} + \frac{38ab\sqrt{a+b\cos}}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315ad} + \frac{2\left(49a^2+75b\right.}{}$$

$$= \frac{2b\left(163a^2+5b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315ad} + \frac{2\left(49a^2+75b\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(147a^4+279a^2b^2-10b^4\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\right.\right.}{315a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 16.3621, size = 521, normalized size = 1.05

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\left(279a^2b^2+147a^4-10b^4\right)\sin(c+dx)}{315a^2} + \frac{2}{315}\sec^2(c+dx)\left(49a^2\sin(c+dx)+75b^2\sin(c+dx)\right) + \right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(147*a^5 + 147*a^4*b + 279*a^3
*b^2 + 279*a^2*b^3 - 10*a*b^4 - 10*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 2*a*(147*a^4 + 261*a^3*b + 279*a^2*
b^2 + 155*a*b^3 - 10*b^4)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)] - (147*a^4 + 279*a^2*b^2 - 10*b^4)*Cos[c + d*x]*(a
+ b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*a^2*d*Sqrt[a +
 b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]*((2*(147*a^4 + 279*a^2*b^2 - 10*b^4)*Sin[c + d*x])/(315*a^2)
 + (2*Sec[c + d*x]^2*(49*a^2*Sin[c + d*x] + 75*b^2*Sin[c + d*x]))/315 + (2*
Sec[c + d*x]*(163*a^2*b*Sin[c + d*x] + 5*b^3*Sin[c + d*x]))/(315*a) + (38*a
*b*Sec[c + d*x]^2*Tan[c + d*x])/63 + (2*a^2*Sec[c + d*x]^3*Tan[c + d*x])/9)
)/d
```

Maple [B] time = 0.662, size = 2512, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x)

[Out] -2/315/d/a^2*(-82*cos(d*x+c)^3*a^4*b-80*cos(d*x+c)^3*a^2*b^3-170*cos(d*x+c)
^2*a^3*b^2-130*cos(d*x+c)*a^4*b+147*cos(d*x+c)^6*a^4*b+163*cos(d*x+c)^6*a^3
*b^2+279*cos(d*x+c)^6*a^2*b^3+5*cos(d*x+c)^6*a*b^4+65*cos(d*x+c)^5*a^4*b+27
9*cos(d*x+c)^5*a^3*b^2-199*cos(d*x+c)^5*a^2*b^3-10*cos(d*x+c)^5*a*b^4-272*c
os(d*x+c)^4*a^3*b^2+5*cos(d*x+c)^4*a*b^4-147*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5-35*a^5-147*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c
)*cos(d*x+c)^5*a^4*b-279*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^3*b^2-279*sin(d*x+c)*cos(d*x+c)^5
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3+
10*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a*b^4+261*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+279*sin(d*x+c)*cos(d*x+c)^
5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2
+155*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^2*b^3-10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a*b^4-147*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*
b-279*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin
(d*x+c)*cos(d*x+c)^4*a^3*b^2-279*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3+10*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a
*b^4+261*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^4*b+279*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-10*cos(d*x+c)^6*b^
5+147*cos(d*x+c)^5*a^5+10*cos(d*x+c)^5*b^5-98*cos(d*x+c)^4*a^5-14*cos(d*x+c
)^2*a^5+10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*sin(d*x+c)*cos(d*x+c)^5*b^5+147*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^5*a^5-147*sin(d*x+c)*cos(d*x
+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5
+10*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*b^5+147*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5+155*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^4*a^2*b^3-10
*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b

```
)/(a+b))^(1/2))*a*b^4)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(11
/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{11}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{11}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c)^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.744 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx$

Optimal. Leaf size=427

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}+\frac{2(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\sqrt{\cos(c+dx)}\csc(c+\ldots}{21ad\ldots}$$

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(21*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]
*(5*a^2 - 24*a*b + 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(21*a*d*Sqrt[Sec[c + d*x]]) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a*b*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 1.16731, antiderivative size = 427, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2792, 3055, 2998, 2816, 2994}

$$\frac{2\left(5a^2+9b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{21d}+\frac{2(a-b)\sqrt{a+b}\left(5a^2-24ab+3b^2\right)\sqrt{\cos(c+dx)}\csc(c+\ldots}{21ad\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a - b)*b*Sqrt[a + b]*(29*a^2 + 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*E
llipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(21*a^2*d*Sqrt[Sec[c + d*x]]) + (2*(a - b)*Sqrt[a + b]
*(5*a^2 - 24*a*b + 3*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(21*a*d*Sqrt[Sec[c + d*x]]) + (2*(5*a^2 + 9*b^2)*Sqrt[a + b*Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (6*a*b*Sqrt[a + b*Cos[c + d*x
]]*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(7*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(7*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
```

```
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{9}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{1}{7}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c$$

$$= \frac{6ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2a^2\sqrt{a+b\cos(c+dx)}\,\mathrm{se}}{7a}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{6ab\sqrt{a+b\cos}}{}$$

$$= \frac{2\left(5a^2+9b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d} + \frac{6ab\sqrt{a+b\cos}}{}$$

$$= \frac{2(a-b)b\sqrt{a+b}\left(29a^2+3b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{21a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.9729, size = 443, normalized size = 1.04

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2b(29a^2+3b^2)\sin(c+dx)}{21a} + \frac{2}{21}\sec(c+dx)\left(5a^2\sin(c+dx)+9b^2\sin(c+dx)\right) + \frac{2}{7}a^2\tan(c\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*b*(29*a^3 + 29*a^2*b + 3*a*b^2
 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(
a + b)] + 2*a*(5*a^3 + 29*a^2*b + 27*a*b^2 + 3*b^3)*Sqrt[Cos[c + d*x]/(1 +
Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elli
pticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - b*(29*a^2 + 3*b^2)*Cos[
c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(21*a*d
*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]*((2*b*(29*a^2 + 3*b^2)*Sin[c + d*x])/(21*a) + (2*Se
c[c + d*x]*(5*a^2*Sin[c + d*x] + 9*b^2*Sin[c + d*x]))/21 + (6*a*b*Sec[c + d
*x]*Tan[c + d*x])/7 + (2*a^2*Sec[c + d*x]^2*Tan[c + d*x])/7))/d
```

Maple [B] time = 0.569, size = 1835, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x)
```

```
[Out] -2/21/d/a*(-3*b^4*cos(d*x+c)^4-18*a^2*b^2*cos(d*x+c)^2-3*a^4+5*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
```

```
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+
c)*a^4+5*cos(d*x+c)^4*a^4-2*cos(d*x+c)^2*a^4+3*cos(d*x+c)^5*b^4+5*cos(d*x+c
)^5*a^3*b+29*cos(d*x+c)^5*a^2*b^2+9*cos(d*x+c)^5*a*b^3+29*cos(d*x+c)^4*a^3*
b-11*cos(d*x+c)^4*a^2*b^2+3*cos(d*x+c)^4*a*b^3-22*cos(d*x+c)^3*a^3*b-12*cos
(d*x+c)^3*a*b^3-12*cos(d*x+c)*a^3*b+29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b+27*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+
c)*a^2*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)^4*sin(d*x+c)*a*b^3-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)*a^3*b-29*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^4*sin(d*x+c)
*a^2*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^4*sin(d*x+c)*a*b^3+29*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b+27*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a
^2*b^2+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)^3*sin(d*x+c)*a*b^3-29*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b-29*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2
*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^3*sin(d*x+c)*a*b^3-3*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+5*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4-3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3
*sin(d*x+c)*b^4)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)/sin
(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{9}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.745 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx$

Optimal. Leaf size=378

$$-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{15ad\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(9*
a^2 - 8*a*b + 15*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(15*a*d*Sqrt[Sec[c + d*x]]) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(15*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.865177, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2792, 3055, 2998, 2816, 2994}

$$-\frac{2(a-b)\sqrt{a+b}\left(9a^2-8ab+15b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+}{a-}\right.}{15ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```


```
[Out] (2*(a - b)*Sqrt[a + b]*(9*a^2 + 23*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(15*a*d*Sqrt[Sec[c + d*x]]) - (2*(a - b)*Sqrt[a + b]*(9*
a^2 - 8*a*b + 15*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(15*a*d*Sqrt[Sec[c + d*x]]) + (22*a*b*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x
]^(3/2)*Sin[c + d*x])/(15*d) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(5*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
```

```
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{7}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{1}{5}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.$$

$$= \frac{22ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec}{5d}$$

$$= \frac{22ab\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec}{5d}$$

$$= \frac{2(a-b)\sqrt{a+b}\left(9a^2+23b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right.\right.}{15ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 15.3729, size = 376, normalized size = 0.99

$$\frac{2\sqrt{\sec(c+dx)}(a+b\cos(c+dx))\left(\left(9a^2+23b^2\right)\sin(c+dx)+a\tan(c+dx)(3a\sec(c+dx)+11b)\right)+\frac{2\left(\left(9a^2+23b^2\right)\cos(c+dx)\,t\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(7/2),x]
```

```
[Out] ((2*(2*(9*a^3 + 9*a^2*b + 23*a*b^2 + 23*b^3)*EllipticE[ArcSin[Tan[(c + d*x)
/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)]*Sqrt[(b + a*Sec[c + d
*x])/((a + b)*(1 + Sec[c + d*x]))] - 2*(9*a^3 + 17*a^2*b + 23*a*b^2 + 15*b^
3)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c +
d*x])^(-1)]*Sqrt[(b + a*Sec[c + d*x])/((a + b)*(1 + Sec[c + d*x]))] + (9*a^
2 + 23*b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2]))/(Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-
1 + Tan[(c + d*x)/2]^2)) + 2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*((9*a^
2 + 23*b^2)*Sin[c + d*x] + a*(11*b + 3*a*Sec[c + d*x])*Tan[c + d*x]))/(15*d
*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.533, size = 1758, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x)
```

```
[Out] -2/15/d*(-3*a^3-9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3-34*a*b^2*cos(d*x+c)^2+9*cos(d*x+c)^4*a
^2*b+11*cos(d*x+c)^4*a*b^2+5*cos(d*x+c)^3*a^2*b+23*cos(d*x+c)^3*a*b^2-14*co
s(d*x+c)*a^2*b-23*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
```

```
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3+15*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*b^3+23*cos(d*x+c)^4*b^3+9*cos(d*x+c)^3*a^3-23*cos(d*x+c)^3*b^3-6*cos(d*x+c
)^2*a^3+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)^2*sin(d*x+c)*b^3-9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b-23*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^
2+17*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)^3*sin(d*x+c)*a^2*b+23*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^2-9*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-23
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^2*sin(d*x+c)*a*b^2+17*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+23*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-23*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^
3*sin(d*x+c)*b^3+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3-9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3)*cos(d*x+c)/(a+b
*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.746 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx$

Optimal. Leaf size=452

$$\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d\sqrt{\sec(c+dx)}}+2a$$

```
[Out] (14*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(a^2 - 7*a*b + 9*b^2)*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d*Sqrt[Sec[c + d*x]]) -
(2*b^2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.823625, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2792, 3053, 2809, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}\left(a^2-7ab+9b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d\sqrt{\sec(c+dx)}}+2a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] (14*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(a^2 - 7*a*b + 9*b^2)*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*d*Sqrt[Sec[c + d*x]]) -
(2*b^2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sec[c +
d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c\right.$$

$$= -\frac{2b^2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{d\sqrt{\sec(c+dx)}}$$

$$= \frac{14(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 11.8948, size = 401, normalized size = 0.89

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2}{3}a^2\tan(c+dx)+\frac{14}{3}ab\sin(c+dx)\right)}{d} - \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-4\left(7a^2b+a^3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2),x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]]*(28*a*b*(a + b)*Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*
EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 4*(a^3 + 7*a^2*b +
9*a*b^2 - 3*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)] + 24*b^3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c +
d*x)/2]], (-a + b)/(a + b)] + 14*a*b*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*d*Sqrt[a + b*Cos[c + d*x]]) + (Sqrt[a
+ b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((14*a*b*Sin[c + d*x])/3 + (2*a^2*Tan[
c + d*x])/3))/d
```

Maple [B] time = 0.606, size = 1493, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*a^3+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+9*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-3*
```

```
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)^2*sin(d*x+c)*b^3-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2+6*cos(d*x+
c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+
b))^(1/2))*b^3+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)*sin(d*x+c)*a^3+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+9*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)*sin(d*x+c)*b^3-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+6*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x
+c)*b^3+cos(d*x+c)^3*a^2*b+7*cos(d*x+c)^3*a*b^2+cos(d*x+c)^2*a^3+7*cos(d*x+
c)^2*a^2*b-7*a*b^2*cos(d*x+c)^2-8*cos(d*x+c)*a^2*b-a^3)*cos(d*x+c)/(a+b*cos
(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.747 $\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=505

$$-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}\left(2a^2-6ab-b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{d\sqrt{\sec(c+dx)}}$$

```
[Out] ((a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]
]) - (5*a*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/
b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rubi [A] time = 1.10704, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2792, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{d}-\frac{\sqrt{a+b}\left(2a^2-6ab-b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx)}{a+b}}}{d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(2*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
 b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x
]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a + b]*(2*a^2 - 6*a*b - b^2)
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]
]) - (5*a*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/
b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a
+ b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*
x]))/(a - b)])/(d*Sqrt[Sec[c + d*x]]) + (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} + \left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx}\right.$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \frac{\left(2a^2-b^2\right)\sqrt{a+b\cos(c+d}}{d}$$

$$= \frac{2a^2\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx)}{d} - \frac{\left(2a^2-b^2\right)\sqrt{a+b\cos(c+d}}{d}$$

$$= -\frac{5ab\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{d\sqrt{\sec(c+dx)}}$$

$$= \frac{(a-b)\sqrt{a+b}\left(2a^2-b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 16.5833, size = 740, normalized size = 1.47

$$\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(2a\left(a^2+3ab-3b^2\right)\sqrt{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)+a-b\tan^2\left(\frac{1}{2}(c+dx)\right)+b}{a+b}}F\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (Sqrt[
(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-2*a^3*Tan[(c + d*x)/2] - 2*a^2*b*Tan[(c +
d*x)/2] + a*b^2*Tan[(c + d*x)/2] + b^3*Tan[(c + d*x)/2] + 4*a^2*b*Tan[(c +
d*x)/2]^3 - 2*b^3*Tan[(c + d*x)/2]^3 + 2*a^3*Tan[(c + d*x)/2]^5 - 2*a^2*b*T
an[(c + d*x)/2]^5 - a*b^2*Tan[(c + d*x)/2]^5 + b^3*Tan[(c + d*x)/2]^5 - 10*
a*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]
^2)/(a + b)] - 10*a*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Ta
```

```
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*a^3 + 2*a^2*b - a*b^
2 - b^3)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*(a^2 + 3*a*b - 3*b^2)*EllipticF[Ar
cSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 +
 Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2
]^2)/(a + b)]))/(d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.415, size = 1631, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a*b^2-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-2*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*a*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+6*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-6*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a*b^2+10*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+6
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*si
n(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2*sin(d*x+c)+cos(d*x+c)^3*b^3+2*cos(d*x+c)^2*a^2*b+a*b^2*cos(d*x+c)^2
-cos(d*x+c)^2*b^3+2*a^3*cos(d*x+c)-2*cos(d*x+c)*a^2*b-cos(d*x+c)*a*b^2-2*a^
3)*cos(d*x+c)/(a+b*cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.748 $\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx$

Optimal. Leaf size=503

$$\frac{\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d\sqrt{\sec(c+dx)}} - \sqrt{\cdots}$$

```
[Out] (-9*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(4*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a^2 + 9*a*b + 2*b^2)*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) -
(Sqrt[a + b]*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a
 + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 1.10091, antiderivative size = 503, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2793, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\sqrt{a+b}\left(8a^2+9ab+2b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4d\sqrt{\sec(c+dx)}} - \sqrt{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-9*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(4*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(8*a^2 + 9*a*b + 2*b^2)*Sqrt[C
os[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) -
(Sqrt[a + b]*(15*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a
 + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
 -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[
c + d*x]))/(a - b)])/(4*d*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[a + b*Cos[c + d*x
]]*Sin[c + d*x])/(2*d*Sqrt[Sec[c + d*x]]) + (9*a*b*Sqrt[a + b*Cos[c + d*x]]
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int (a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}a(4}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{9ab\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{4d}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2d\sqrt{\sec(c+dx)}} + \frac{9ab\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin}{4d}$$

$$= -\frac{\sqrt{a+b}\left(15a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx}}{\sqrt{a+b}\sqrt{\cos(c+dx}}\right.\right.}{4d\sqrt{\sec(c+dx)}}$$

$$= -\frac{9(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\mid -\frac{a+b}{a-b}\right.}{4d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 14.4943, size = 423, normalized size = 0.84

$$\frac{-4\left(-12a^2b+4a^3+ab^2-2b^3\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+4b\left(15a^2+4b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\Pi\left(-1;-\sin^{-1}(\right.}{\left(\tan^2\left(\frac{1}{2}(c+dx)\right)-1\right)\sqrt{\sec^2\left(\frac{1}{2}(c\right.}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]],x]
```

```
[Out] (b^2*(a + b*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)] + (-18*a*b*(a
 + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a +
b)] - 4*(4*a^3 - 12*a^2*b + a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + 4*b*(15*a^2 + 4*b^2)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 9*a*b
*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(Sq
rt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-1 + Tan[(c +
 d*x)/2]^2)))/(4*d*Sqrt[a + b*Cos[c + d*x]])
```

Maple [B] time = 0.557, size = 1631, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x)
```

```
[Out] -1/4/d*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*(9*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+9*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a*b^2+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+8*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+8*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin
(d*x+c)*a^3-24*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(
1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-4*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+9
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*si
n(d*x+c)+9*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a*b^2*sin(d*x+c)+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-
b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a
+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+8*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-24*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-
4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin
(d*x+c)+2*cos(d*x+c)^4*b^3+11*cos(d*x+c)^3*a*b^2+9*cos(d*x+c)^2*a^2*b-9*a*b
^2*cos(d*x+c)^2-2*cos(d*x+c)^2*b^3-9*cos(d*x+c)*a^2*b-2*cos(d*x+c)*a*b^2)/s
in(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)*sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)*sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)*sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.749 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=566

$$\frac{\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(33a^2+26ab+16b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1\cdots}{}}}{24d\sqrt{\sec(\cdots}}$$

```
[Out] -((a - b)*Sqrt[a + b]*(33*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(33*a^2 + 26*
a*b + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*S
qrt[Sec[c + d*x]]) - (5*a*Sqrt[a + b]*(a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) + (b^
2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (13*a*b
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Sec[c + d*x]]) + ((33*a^
2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*d
)
```

Rubi [A] time = 1.45095, antiderivative size = 566, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4222, 2793, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{\left(33a^2+16b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{24d}+\frac{\sqrt{a+b}\left(33a^2+26ab+16b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1\cdots}{}}}{24d\sqrt{\sec(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```


```
[Out] -((a - b)*Sqrt[a + b]*(33*a^2 + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*Ell
ipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -
((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
+ d*x]))/(a - b)])/(24*a*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(33*a^2 + 26*
a*b + 16*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*C
os[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(24*d*S
qrt[Sec[c + d*x]]) - (5*a*Sqrt[a + b]*(a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[
c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]
*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(8*b*d*Sqrt[Sec[c + d*x]]) + (b^
2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)) + (13*a*b
*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])/(12*d*Sqrt[Sec[c + d*x]]) + ((33*a^
2 + 16*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(24*d
)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3}{2}a\left(2\right.\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d\sqrt{\sec(c+dx)}} + \frac{\left(33a^2+16b^2\right)}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)} + \frac{13ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{12d\sqrt{\sec(c+dx)}} + \frac{\left(33a^2+16b^2\right)}{}$$

$$= -\frac{5a\sqrt{a+b}\left(a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{8bd\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(33a^2+16b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{24ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.367, size = 978, normalized size = 1.73

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{1}{12}\sin(c+dx)b^2+\frac{1}{12}\sin(3(c+dx))b^2+\frac{13}{24}a\sin(2(c+dx))b\right)}{d}+\frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\left(-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((b^2*Sin[c + d*x])/12 + (13*a
*b*Sin[2*(c + d*x)])/24 + (b^2*Sin[3*(c + d*x)])/12))/d + (Sqrt[(1 - Tan[(c
 + d*x)/2]^2)^(-1)]*(33*a^3*Tan[(c + d*x)/2] + 33*a^2*b*Tan[(c + d*x)/2] +
16*a*b^2*Tan[(c + d*x)/2] + 16*b^3*Tan[(c + d*x)/2] - 66*a^2*b*Tan[(c + d*x
)/2]^3 - 32*b^3*Tan[(c + d*x)/2]^3 - 33*a^3*Tan[(c + d*x)/2]^5 + 33*a^2*b*T
an[(c + d*x)/2]^5 - 16*a*b^2*Tan[(c + d*x)/2]^5 + 16*b^3*Tan[(c + d*x)/2]^5
 - 30*a^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[
1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x
)/2]^2)/(a + b)] - 120*a*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a
+ b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]
^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^3*EllipticPi[-1, -ArcSin[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 12
0*a*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c
+ d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2
 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (33*a^3 + 33*a^2*b + 16*a*b^2 + 16*b^3)
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(a + b)] - 2*a*(24*a^2 - 13*a*b + 38*b^2)*EllipticF[ArcSi
n[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2
)/(a + b)]))/(24*d*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.638, size = 1868, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -1/24/d*(-76*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*a*b^2*sin(d*x+c)-48*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+30*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-33*
```

```
cos(d*x+c)^2*a^2*b-16*cos(d*x+c)*a*b^2+120*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+33*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+
16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*b^3+30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+33*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+16*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+26*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c
)-18*a*b^2*cos(d*x+c)^2+34*cos(d*x+c)^4*a*b^2+59*cos(d*x+c)^3*a^2*b-26*cos(
d*x+c)*a^2*b+33*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+26*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^
2*b-76*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)*sin(d*x+c)*a*b^2+120*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-33*a^3*cos(d*x+c)-16*c
os(d*x+c)^2*b^3+8*cos(d*x+c)^5*b^3+8*cos(d*x+c)^3*b^3+33*cos(d*x+c)^2*a^3-4
8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*sin
(d*x+c))*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\sqrt{\sec(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)), x)
```

3.750 $\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=638

$$\frac{(59a^2+36b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{96d\sqrt{\sec(c+dx)}}+\frac{a(15a^2+284b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{\sqrt{a+b}\,(1}{}$$

```
[Out] -((a - b)*Sqrt[a + b]*(15*a^2 + 284*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(15*a^3 + 1
18*a^2*b + 284*a*b^2 + 72*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a^4 - 120*a^2*b^2
- 48*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(64*b^2*d*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(4*d*Sec[c + d*x]^(5/2)) + (17*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(24*d*Sec[c + d*x]^(3/2)) + ((59*a^2 + 36*b^2)*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(96*d*Sqrt[Sec[c + d*x]]) + (a*(15*a^2 + 284*b^2)*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)
```

Rubi [A] time = 1.88262, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4222, 2793, 3049, 3061, 3053, 2809, 2998, 2816, 2994}

$$\frac{(59a^2+36b^2)\sin(c+dx)\sqrt{a+b\cos(c+dx)}}{96d\sqrt{\sec(c+dx)}}+\frac{a(15a^2+284b^2)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{192bd}+\frac{\sqrt{a+b}\,(1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] -((a - b)*Sqrt[a + b]*(15*a^2 + 284*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*El
lipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])],
-((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c
 + d*x]))/(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(15*a^3 + 1
18*a^2*b + 284*a*b^2 + 72*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/
(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/
(a - b)])/(192*b*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b]*(5*a^4 - 120*a^2*b^2
- 48*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(64*b^2*d*Sqrt[Sec[c + d*x]]) + (b^2*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x]
)/(4*d*Sec[c + d*x]^(5/2)) + (17*a*b*Sqrt[a + b*Cos[c + d*x]]*Sin[c + d*x])
/(24*d*Sec[c + d*x]^(3/2)) + ((59*a^2 + 36*b^2)*Sqrt[a + b*Cos[c + d*x]]*Si
n[c + d*x])/(96*d*Sqrt[Sec[c + d*x]]) + (a*(15*a^2 + 284*b^2)*Sqrt[a + b*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(192*b*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{(a+b\cos(c+dx))^{5/2}}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}\,dx$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(\frac{1}{2}a\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+d\right.}}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(59a^2+36b\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(59a^2+36b\right.}{}$$

$$= \frac{b^2\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)} + \frac{17ab\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{24d\sec^{\frac{3}{2}}(c+dx)} + \frac{\left(59a^2+36b\right.}{}$$

$$= \frac{\sqrt{a+b}\left(5a^4-120a^2b^2-48b^4\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right|}{64b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\left(15a^2+284b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\right| - \frac{a+}{a-}}{192bd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 16.754, size = 1642, normalized size = 2.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(5/2)/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((17*a*b*Sin[c + d*x])/96 + ((
59*a^2 + 48*b^2)*Sin[2*(c + d*x)])/192 + (17*a*b*Sin[3*(c + d*x)])/96 + (b^
2*Sin[4*(c + d*x)])/32))/d + (-15*a^4*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
] - 15*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - 284*a^2*b^2*Sqrt[(a -
 b)/(a + b)]*Tan[(c + d*x)/2] - 284*a*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*
x)/2] + 30*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 568*a*b^3*Sqrt[
(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 15*a^4*Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]^5 - 15*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + 284*a^2*b^
2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - 284*a*b^3*Sqrt[(a - b)/(a + b)
]*Tan[(c + d*x)/2]^5 - (30*I)*a^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqr
t[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a +
 b)] + (720*I)*a^2*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (288
*I)*b^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c
+ d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (30*I)*a^4*Elliptic
Pi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
```

```
 + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (720*I)*a^2*b^2*
EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (288*I)*
b^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*
x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*a*
(15*a^3 - 15*a^2*b + 284*a*b^2 - 284*b^3)*EllipticE[I*ArcSinh[Sqrt[(a - b)/
(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c +
d*x)/2]^2)/(a + b)] + (2*I)*(15*a^4 + 59*a^3*b - 38*a^2*b^2 + 36*a*b^3 - 72
*b^4)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b
)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
 b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)])/(192*b*Sqrt[(a
- b)/(a + b)]*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^
2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.665, size = 2327, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] -1/192/d/b*(24*b^4*cos(d*x+c)^4-72*b^4*cos(d*x+c)^2+30*a^2*b^2*cos(d*x+c)^2
-15*cos(d*x+c)^2*a^3*b-284*cos(d*x+c)^2*a*b^3-284*cos(d*x+c)*a^2*b^2-72*cos
(d*x+c)*a*b^3-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4-30*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+288*b^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)+15*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-1
44*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*b^4+15*cos(d*x+c)^2*a^4-144*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+184*cos(d*x+c)^5*a*b^3+25
4*cos(d*x+c)^4*a^2*b^2+133*cos(d*x+c)^3*a^3*b+172*cos(d*x+c)^3*a*b^3-118*co
s(d*x+c)*a^3*b+48*cos(d*x+c)^6*b^4-15*a^4*cos(d*x+c)+118*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b
-644*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^2*b^2+72*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+720*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+c
os(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^
2+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^3*b+284*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
```

```
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+284*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3+288
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*b^4+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4+118*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)-644*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+72*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x
+c)+720*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2
))*a^2*b^2*sin(d*x+c)+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+284*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+284*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c))*cos(d*x+c)*(1/c
os(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{b\cos(dx+c)+a}}{\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sqrt(b*cos(d*x + c
) + a)/sec(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(dx+c)+a)^{\frac{5}{2}}}{\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2), x)
```

3.751 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=314

$$\frac{2\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}} - 4b(a-b)\sqrt{\ldots}$$

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(a + 2*b)*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rubi [A] time = 0.484821, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2802, 2998, 2816, 2994}

$$\frac{2\sqrt{a+b}(a+2b)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2d\sqrt{\sec(c+dx)}} - 4b(a-b)\sqrt{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (-4*(a - b)*b*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a -
b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a -
b)])/(3*a^3*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[a + b]*(a + 2*b)*Sqrt[Cos[c + d
*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sq
rt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*d*Sqrt[Sec[c + d*x]]) + (2*S
qrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
```

```
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{-b+\frac{1}{2}a\cos}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}}{3a}$$

$$= \frac{2\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{\left(2b\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1+\cos}{\cos^{\frac{3}{2}}(c+dx)\sqrt{}}}{3a}$$

$$= -\frac{4(a-b)b\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{3a^3d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.299, size = 322, normalized size = 1.03

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\tan(c+dx)}{3a}-\frac{4b\sin(c+dx)}{3a^2}\right)}{d} + \frac{4\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(b\cos(c+dx)\tan\left(\frac{1}{2}(c+dx\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(a + b)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + a*(a - 2*b)*Sqrt[Cos[
c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*Cos[c +
 d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^2*d*S
qrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x
]]*Sqrt[Sec[c + d*x]]*((-4*b*Sin[c + d*x])/(3*a^2) + (2*Tan[c + d*x])/(3*a)
))/d
```

Maple [B] time = 0.551, size = 891, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/3/d/a^2*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),(-(a-b)/(a+b))^(1/2))*a^2-2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b+2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*cos(d*x+c)^2*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*b^2+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*b^2+a*b*cos(d*x+c)^3-2*cos(d*x+c)^3*b^2+cos(d*x+c)^2*a^2-2*co
s(d*x+c)^2*a*b+2*b^2*cos(d*x+c)^2+cos(d*x+c)*a*b-a^2)*cos(d*x+c)*(1/cos(d*x
+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.752 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=264

$$\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}\sqrt{\mathrm{co}}}{}$$

```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.309121, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {4222, 2801, 2816, 2994}

$$\frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{a^2d\sqrt{\sec(c+dx)}}-\frac{2\sqrt{a+b}\sqrt{\mathrm{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a^2*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]
])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 +
Sec[c + d*x]))/(a - b)])/(a*d*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
```

```
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\left(\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx\right) + \left(\sqrt{\cos(c+dx)}\sqrt{\se\right.$$

$$= \frac{2(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1}{}}}{a^2d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 12.1082, size = 296, normalized size = 1.12

$$\frac{2\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{ad} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\sec^2\left(\frac{1}{2}(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) - (2*Sqr
t[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[
ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - 2*a*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(a*d*Sqrt[a + b*Cos[c + d*x]
]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] time = 0.519, size = 620, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/d/a*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)-(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)+b*cos(d*x+
c)^2+cos(d*x+c)*a-b*cos(d*x+c)-a)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(a+b*cos(
d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{3}{2}}}{\sqrt{b\cos (dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sec (dx+c)^{\frac{3}{2}}}{\sqrt{b\cos (dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/sqrt(b*cos(d*x + c) + a), x)
```

3.753 $\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=129

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

```
[Out] (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*S
qrt[Sec[c + d*x]])
```

Rubi [A] time = 0.130848, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {4222, 2816}

$$\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*
Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(
a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*d*S
qrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+dx))}{a-b}}}{ad\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 0.77954, size = 103, normalized size = 0.8

$$\frac{2\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)}{d\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)])/(d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*
x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
```

Maple [A] time = 0.625, size = 125, normalized size = 1.

$$2\,\frac{\sqrt{(\cos(dx+c))^{-1}}(\sin(dx+c))^2}{d\sqrt{a+b\cos(dx+c)}(-1+\cos(dx+c))}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a+b\cos(dx+c)}{(a+b)(1+\cos(dx+c))}}\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/d*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))/(a+b*
cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*sin(d*x+c)^2/(-1+cos(d*x+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{\sqrt{b\cos(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/sqrt(b*cos(d*x + c) + a), x)
```

3.754 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=136

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rubi [A] time = 0.132036, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {4222, 2809}

$$-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{bd\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(b*d*Sqrt[Sec[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{bd\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 1.67505, size = 148, normalized size = 1.09

$$-\frac{2\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)+1}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-a}{a+b}\right)+2\Pi\left(-1;-\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{d\sqrt{\frac{1}{\cos(c+dx)+1}}\sqrt{a+b\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*(EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] + 2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)])*Sqrt[1
+ Sec[c + d*x]])/(d*Sqrt[(1 + Cos[c + d*x])^(-1)]*Sqrt[a + b*Cos[c + d*x]])
```

Maple [A] time = 0.648, size = 143, normalized size = 1.1

$$2\frac{1}{d\sqrt{a+b\cos(dx+c)}\sqrt{(\cos(dx+c))^{-1}}}\left(\mathrm{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\sqrt{-\frac{a-b}{a+b}}\right)-2\,\mathrm{EllipticPi}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d/(a+b*cos(d*x+c))^(1/2)*(EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))-2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2)
))*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)/(1/cos(d*x+c))^(1/2)/(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*cos(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))), x)
```

3.755 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=474

$$\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{a\sin(c+dx)\sqrt{\ }}{bd\sqrt{a+b\cos}}$$

```
[Out] -(((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c
+ d*x]]) + Sin[c + d*x]/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (
a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.796291, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {4222, 2820, 2809, 3003, 2993, 12, 2801, 2816, 2994}

$$\frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^2d\sqrt{\sec(c+dx)}}+\frac{a\sin(c+dx)\sqrt{\ }}{bd\sqrt{a+b\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```


```
[Out] -(((a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(a*b*d*Sqrt[Sec[c + d*x]])) + (Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]]
)], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + S
ec[c + d*x]))/(a - b)])/(b*d*Sqrt[Sec[c + d*x]]) + (a*Sqrt[a + b]*Sqrt[Cos[
c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c
+ d*x]]) + Sin[c + d*x]/(d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (
a*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2820

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f
_.)*(x_)]], x_Symbol] :> -Dist[(a*d)/(2*b), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a
```

```
 + b*Sin[e + f*x]], x], x] + Dist[d/(2*b), Int[(Sqrt[d*Sin[e + f*x]]*(a + 2
*b*Sin[e + f*x]))/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 3003

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-2*B*Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n)/(f*(2
*n + 3)), x] + Dist[1/(2*n + 3), Int[((c + d*Sin[e + f*x])^(n - 1)*Simp[a*A
*c*(2*n + 3) + B*(b*c + 2*a*d*n) + (B*(a*c + b*d)*(2*n + 1) + A*(b*c + a*d)
*(2*n + 3))*Sin[e + f*x] + (A*b*d*(2*n + 3) + B*(a*d + 2*b*c*n))*Sin[e + f*
x]^2, x])/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B
}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ
[n^2, 1/4]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2801

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin
[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[1/(a - b), Int[1/(Sqrt[a + b*Sin[
e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[b/(a - b), Int[(1 + Sin[
e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}(a+2b\cos(c+dx))}{\sqrt{a+b\cos(c+dx)}}\,dx}{2b} - \frac{\left(a\sqrt{\cos(c+dx)}\cdots\right.}{\cdots}$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1\cdots}{\cdots}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1\cdots}{\cdots}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1\cdots}{\cdots}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1\cdots}{\cdots}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\cdots}}{abd\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 23.8163, size = 5017, normalized size = 10.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)),x]
```

```
[Out] Result too large to show
```

Maple [A] time = 0.753, size = 630, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d/b*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
```

```
os(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(
d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c)+(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*sin(d*x+c
)+b*cos(d*x+c)^3+a*cos(d*x+c)^2-b*cos(d*x+c)^2-cos(d*x+c)*a)*cos(d*x+c)*(1/
cos(d*x+c))^(3/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos\left(dx+c\right)+a}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)), x)
```

3.756 $\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=505

$$-\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d\sqrt{\sec(c+dx)}}-\frac{3a\sin}{}$$

```
[Out] (3*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(4*b^2*d*Sqrt[Sec[c + d*x]]) - ((3*a - 2*b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a
 + b]*(3*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
 ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(4*b^3*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(2*b*d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(4*b^2*d)
```

Rubi [A] time = 0.9543, antiderivative size = 505, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2793, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{4b^3d\sqrt{\sec(c+dx)}}-\frac{3a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (3*(a - b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))
]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)]
)/(4*b^2*d*Sqrt[Sec[c + d*x]]) - ((3*a - 2*b)*Sqrt[a + b]*Sqrt[Cos[c + d*x]
]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[
Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(4*b^2*d*Sqrt[Sec[c + d*x]]) - (Sqrt[a
 + b]*(3*a^2 + 4*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b,
 ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(4*b^3*d*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Cos[c + d*x]]*Sin[c
 + d*x])/(2*b*d*Sqrt[Sec[c + d*x]]) - (3*a*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(4*b^2*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{a}{2}+b\cos(c+dx)-\frac{3}{2}}{\sqrt{\cos(c+dx)}\sqrt{a+b}}}{2b}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx}{4b^2d}$$

$$= \frac{\sqrt{a+b\cos(c+dx)}\sin(c+dx)}{2bd\sqrt{\sec(c+dx)}} - \frac{3a\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(c+dx}{4b^2d}$$

$$= -\frac{\sqrt{a+b}\left(3a^2+4b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\right.}{4b^3d\sqrt{\sec(c+dx)}}$$

$$= \frac{3(a-b)\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{4b^2d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 18.6174, size = 1153, normalized size = 2.28

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\sin(2(c+dx))}{4bd} - \frac{\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx)\right)+a+b}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}\left(-3a^2\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)+3ab\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*b*d) - (S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x
)/2]^2)]*(3*a^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] + 3*a*b*Sqrt[(a - b)
/(a + b)]*Tan[(c + d*x)/2] - 6*a*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3
 - 3*a^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*a*b*Sqrt[(a - b)/(a +
 b)]*Tan[(c + d*x)/2]^5 + (6*I)*a^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[S
qrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + (8*I)*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a +
```

```
b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*a^
2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*
b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*
x)/2]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (3*I
)*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -(
(a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*(3
*a^2 - a*b + 2*b^2)*EllipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)
/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2
]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/
(4*b^2*Sqrt[(a - b)/(a + b)]*d*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c +
 d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b*(-1 + Tan[(c + d*x)/2]^2) - a*(1 +
 Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.533, size = 1248, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/4/d/b^2*(2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*sin(d*x+c)*a*b-4*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-3*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2-3*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*a*b+6*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+8*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-4*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin
(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*b*sin(d*x+c)+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(
a+b))^(1/2))*a^2*sin(d*x+c)+8*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x
+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)+2*b^2*cos(d*x+c)^4-a*b*cos(d*x+c)^3
-3*cos(d*x+c)^2*a^2+3*cos(d*x+c)^2*a*b-2*b^2*cos(d*x+c)^2+3*a^2*cos(d*x+c)-
2*cos(d*x+c)*a*b)*cos(d*x+c)^2*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(a+b*cos(d*x
+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)), x)
```

3.757 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=397

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}-\frac{2b\left(5a^2-8b^2\right)\sqrt{\cos(c+}}{}$$

```
[Out] (-2*b*(5*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(a + 2*b)*(a + 4*b)*Sqrt[Cos
[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec
[c + d*x]]) + (2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.82327, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2802, 3055, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-4b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}{3a^2d\left(a^2-b^2\right)}-\frac{2b\left(5a^2-8b^2\right)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-2*b*(5*a^2 - 8*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt
[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]
*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])
/(3*a^4*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*(a + 2*b)*(a + 4*b)*Sqrt[Cos
[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a +
 b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^3*Sqrt[a + b]*d*Sqrt[Sec
[c + d*x]]) + (2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]) + (2*(a^2 - 4*b^2)*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d
*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
```

```
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}\left(a^2-4b^2\right)-\frac{1}{2}ab\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\cos}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3a^2\left(a^2-b^2\right)d}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^2-4b^2\right)\sqrt{a+b\cos(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3a^2\left(a^2-b^2\right)d}$$

$$= -\frac{2b\left(5a^2-8b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{3a^4\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 14.0058, size = 440, normalized size = 1.11

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2b\left(5a^2-8b^2\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)} - \frac{2b^3\sin(c+dx)}{a^2\left(a^2-b^2\right)(a+b\cos(c+dx))} + \frac{2\tan(c+dx)}{3a^2}\right)}{d} - \frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*b*(-5*a^3 - 5*a^2*b + 8*a*b^2
+ 8*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] - 2*a*(a^3 - 5*a^2*b + 2*a*b^2 + 8*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-5*a^2 + 8*b^2)*Cos[c + d
*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^3*(a^2
- b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*b*(5*a^2 - 8*b^2)*Sin[c + d*x])/(3*a^3
*(a^2 - b^2)) - (2*b^3*Sin[c + d*x])/(a^2*(a^2 - b^2)*(a + b*Cos[c + d*x]))
 + (2*Tan[c + d*x])/(3*a^2)))/d
```

Maple [B] time = 0.487, size = 1789, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3/d/(a+b)/(a-b)/a^3*(-8*b^4*cos(d*x+c)^2+a^2*b^2+4*a^2*b^2*cos(d*x+c)^2-
5*cos(d*x+c)^2*a^3*b+8*cos(d*x+c)^2*a*b^3-4*cos(d*x+c)*a*b^3+8*cos(d*x+c)^2
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2
```

```
))*a*b^3+5*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b+5*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-8*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a
*b^3-5*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^3*b+2*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2-a^4-8*cos(d*x+c)^2*sin
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b
^4+cos(d*x+c)^2*a^4+cos(d*x+c)^3*a^3*b-4*cos(d*x+c)^3*a*b^3+4*cos(d*x+c)*a^
3*b+8*cos(d*x+c)^3*b^4-5*cos(d*x+c)^3*a^2*b^2+cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4-8*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b^4+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*a^4-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+2*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+8*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*a*b^3+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b+5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-8*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{5}{2}}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{(b\cos (dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.758 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=325

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3
*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b^
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rubi [A] time = 0.548995, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2802, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\,\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+}}{\sqrt{a+b}\sqrt{\cos(c}}\right.\right.}{a^3d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2 - 2*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^3
*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*(a + 2*b)*Sqrt[Cos[c + d*x]]*Csc[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(
1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*b^
2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]
])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}\left(a^2-2b^2\right)-\frac{1}{2}ab\cos(c+dx}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+d}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} - \frac{\left((a-b)(a+2b)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(c+}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2\left(a^2-2b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{}}{a^3\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 11.0416, size = 369, normalized size = 1.14

$$2\left(\sin(c+dx)\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\left(\left(a^2-2b^2\right)(a+b\cos(c+dx))+ab^2\right)-\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*((a*b^2 + (a^2 - 2*b^2)*(a + b*Cos[c + d*x]))*Sqrt[Sec[(c + d*x)/2]^2]*S
qrt[Sec[c + d*x]]*Sin[c + d*x] - Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(
a^3 + a^2*b - 2*a*b^2 - 2*b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] - 2*a*(a^2 - a*b - 2*b^2)*Sqrt[Cos[c + d*x]/(1
 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*E
llipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + (a^2 - 2*b^2)*Cos[c
+ d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])))/(a^2*(a^
2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] time = 0.541, size = 1460, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/a^2/(a-b)/(a+b)*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-2*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*
b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d
*x+c)*sin(d*x+c)*a^3-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*b^3+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2*b*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*
x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a
+b))^(1/2))*a*b^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+cos(d*x+c)^2*a
^2*b+a*b^2*cos(d*x+c)^2-2*cos(d*x+c)^2*b^3+a^3*cos(d*x+c)-cos(d*x+c)*a^2*b-
2*cos(d*x+c)*a*b^2+2*cos(d*x+c)*b^3-a^3+a*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(3
/2)/(a+b*cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{3}{2}}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(3/2), x)
```

3.759 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=307

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{a^2d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]
*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.479046, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2800, 2998, 2816, 2994}

$$-\frac{2b\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2b\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a}{a}\right.}{a^2d\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a^2*Sqrt[a + b]
*d*Sqrt[Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*b*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2800

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[(2*b*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
 b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(b + a
*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{b+a\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\cos(c+dx)}}}{a^2-b^2}$$

$$= -\frac{2b\sqrt{\sec(c+dx)}\sin(c+dx)}{\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}} + \frac{\left((a-b)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{a+b\text{c}}}}{a^2-b^2}$$

$$= \frac{2b\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(1+\sec(c+}{a-b}}}{a^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 7.63532, size = 237, normalized size = 0.77

$$\frac{2\sqrt{\sec(c+dx)}\left(b(b-a)\cos(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)+2a(a+b)\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}F\left(\text{sin}\right.\right.}{ad\left(a^2-b^2\right)\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-2*b*(a + b)*Cos[(c + d*x)/2]^2*Sqrt[(a + b*Cos[c +
d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + 2*a*(a + b)*Cos[(c + d*x)/2]
^2*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[(1 + Sec[c + d*x])^(-1)] + b*(-a
```

```
 + b)*Cos[c + d*x]*Tan[(c + d*x)/2]))/(a*(a^2 - b^2)*d*Sqrt[a + b*Cos[c + d
*x]])
```

Maple [B] time = 0.636, size = 832, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/d/(a+b)/(a-b)/a*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(1/2)*((cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(
d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^2+cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2+(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d
*x+c)-cos(d*x+c)^2*a*b+b^2*cos(d*x+c)^2+cos(d*x+c)*a*b-cos(d*x+c)*b^2)/sin(
d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(b^2*cos(d*x + c)^2 +
2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(sec(c + d*x))/(a + b*cos(c + d*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(3/2), x)
```

3.760 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=306

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d*
Sqrt[Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.418758, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4222, 2794, 2795, 2816, 2994}

$$\frac{2a\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}+\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x
]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*Sqrt[a + b]*d*
Sqrt[Sec[c + d*x]]) + (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(a*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (2*a*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/((a^2 - b^2)*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
```

```
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\cos(c+}}{\cos^{\frac{3}{2}}(c+d}}{a^2-b^2}$$

$$= \frac{2a\sqrt{\sec(c+dx)}\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\cos(c+dx)}} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1+c}{\cos^{\frac{3}{2}}(c+dx}}{a^2-b^2}$$

$$= -\frac{2\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{a\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 3.64888, size = 235, normalized size = 0.77

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left((a-b)\sin(c+dx)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\frac{a+b\cos(c+dx)}{(a+b)(\cos(c+dx)+1)}}-(a+b\cos(c+dx))F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{b-}{a+}\right.\right.}{d\left(a^2-b^2\right)\sqrt{\frac{\cos(c+dx)}{\cos(c+dx)+1}}\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\sqrt{\frac{a+b\cos(}{(a+b)(\cos(}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*((a + b*Cos[c + d*x])*EllipticE[ArcSin[Tan[(c + d*x)/2]
], (-a + b)/(a + b)] - (a + b*Cos[c + d*x])*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)] + (a - b)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sin[c + d*x]))/((a^2 -
b^2)*d*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[a + b*Cos[c + d*x]]*Sqrt[
```

```
(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sqrt[Sec[c + d*x]])
```

Maple [B] time = 0.657, size = 811, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] 2/d/(a-b)/(a+b)*((cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a+EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*b-(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a-(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
b+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*sin(d
*x+c)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b*s
in(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))
*a*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*b*sin(d*x+c)-a*cos(d*x+c)^2+b*cos(d*x+c)^2+cos(d*x+c)*a-b*cos(d*x+c))*
(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sqrt{\sec(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
 + a^2)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c))), x)
```

3.761 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=447

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{co}}\right.\right.}{b^2d\sqrt{\sec(c+dx)}}$$

```
[Out] (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d*S
qrt[Sec[c + d*x]]) - (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c + d*x]])
- (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]])
```

Rubi [A] time = 0.595291, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {4222, 2797, 2809, 2794, 2795, 2816, 2994}

$$-\frac{2a^2\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\cos(c+dx)}}-\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos}}{\sqrt{a+b}\sqrt{co}}\right.\right.}{b^2d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```


```
[Out] (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b*Sqrt[a + b]*d*S
qrt[Sec[c + d*x]]) - (2*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sq
rt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b)
)]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)
])/(b*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]
*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a
 + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^2*d*Sqrt[Sec[c + d*x]])
- (2*a^2*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Cos[c
 + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2797

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(3/2), x_Symbol] :> Dist[d/b, Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*Sin
```

```
[e + f*x]], x], x] - Dist[(a*d)/b, Int[Sqrt[d*Sin[e + f*x]]/(a + b*Sin[e +
f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2794

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2), x_Symbol] :> Simp[(-2*a*d*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]]), x] - Dist[d^2/(a^2 - b^2), Int[Sqrt[
a + b*Sin[e + f*x]]/(d*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, d, e, f}
, x] && NeQ[a^2 - b^2, 0]
```

Rule 2795

```
Int[Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(3/2), x_Symbol] :> Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin
[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), In
t[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{a+b\cos(c+dx)}}\,dx}{b} - \frac{\left(a\sqrt{\cos(c+dx)}\sqrt{\sec(c+d\right.}{b}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{b^2d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}\sqrt{\frac{a}{}}}{b\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 17.7353, size = 1175, normalized size = 2.63

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(\frac{2\sin(c+dx)a^2}{b(b^2-a^2)(a+b\cos(c+dx))}+\frac{2\sin(c+dx)a}{b(a^2-b^2)}\right)}{d} - \frac{2\left(a^2\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+dx)\right)-ab\sqrt{\frac{a-b}{a+b}}\tan^5\left(\frac{1}{2}(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*Sin[c + d*x])/(b*(a^2 -
b^2)) + (2*a^2*Sin[c + d*x])/(b*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))/d - (2
*(-(a^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]) - a*b*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2] + 2*a*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + a^2*Sq
rt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - a*b*Sqrt[(a - b)/(a + b)]*Tan[(c +
 d*x)/2]^5 - (2*I)*a^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(
a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]
*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I
)*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c +
d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*
Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*EllipticPi[
(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a +
b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*b^2*EllipticP
i[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
+ b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - I*a*(a - b)*Ellip
ticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(2*a^2 - a*b - b^2)*Ell
ipticF[I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b)
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Ta
n[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b*Sqrt[(a - b)/(a + b)
]*(a^2 - b^2)*d*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^
2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Ta
n[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] time = 0.608, size = 1214, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 2/d/(a+b)/(a-b)/b*(-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(
a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*si
n(d*x+c)*a*b-2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+2*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*sin(d*x+c
)+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b*sin
(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+co
s(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1
/2))*a^2*sin(d*x+c)+2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*sin(d*x+c)+cos(d*x+c)^2*a^2-cos(d*x+c)^2*a*b-a^2*cos(d
*x+c)+cos(d*x+c)*a*b)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(a+b*cos(d
*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\left(b^{2}\cos\left(dx+c\right)^{2}+2\,ab\cos\left(dx+c\right)+a^{2}\right)\sec\left(dx+c\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
 + a^2)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2)), x)
```

3.762 $\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=525

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{\left(3a^2-b^2\right)\cdots}{\cdots}$$

```
[Out] -(((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b
^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])) + ((3*a + b)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (3*a
*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rubi [A] time = 1.09094, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2792, 3061, 3053, 2809, 2998, 2816, 2994}

$$-\frac{2a^2\sin(c+dx)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}+\frac{\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}}{b^2d\left(a^2-b^2\right)}-\frac{\left(3a^2-b^2\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] -(((3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt
[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(a*b
^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]])) + ((3*a + b)*Sqrt[Cos[c + d*x]]*Csc[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c +
 d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*
(1 + Sec[c + d*x]))/(a - b)])/(b^2*Sqrt[a + b]*d*Sqrt[Sec[c + d*x]]) + (3*a
*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticPi[(a + b)/b, ArcSin[S
qrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b
))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b
)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*a^2*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt
[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + ((3*a^2 - b^2)*Sqrt[a + b*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x
]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d
- C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*
c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e
+ f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/
Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B
- 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] &&
NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
```

```
0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{3/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\cos(c}}{b^2\,(}$$

$$= -\frac{2a^2\sin(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}} + \frac{\left(3a^2-b^2\right)\sqrt{a+b\cos(c}}{b^2\,(}$$

$$= \frac{3a\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)}{b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{}}{ab^2\sqrt{a+b}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 15.2836, size = 1033, normalized size = 1.97

$$\frac{\sqrt{a+b\cos(c+dx)}\sqrt{\sec(c+dx)}\left(-\frac{2\sin(c+dx)a^3}{b^2\left(b^2-a^2\right)(a+b\cos(c+dx))}-\frac{2\sin(c+dx)a^2}{b^2\left(a^2-b^2\right)}\right)}{d} - \frac{\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}}\sqrt{\frac{a\tan^2\left(\frac{1}{2}(c+dx)\right)-b\tan^2\left(\frac{1}{2}(c+dx\right.}{\tan^2\left(\frac{1}{2}(c+dx)\right)+1}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*a^2*Sin[c + d*x])/(b^2*(a
^2 - b^2)) - (2*a^3*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(a + b*Cos[c + d*x]))))
/d - (Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^
2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a^3*Tan[(c + d*x)/2
] - 3*a^2*b*Tan[(c + d*x)/2] + a*b^2*Tan[(c + d*x)/2] + b^3*Tan[(c + d*x)/2
] + 6*a^2*b*Tan[(c + d*x)/2]^3 - 2*b^3*Tan[(c + d*x)/2]^3 + 3*a^3*Tan[(c +
d*x)/2]^5 - 3*a^2*b*Tan[(c + d*x)/2]^5 - a*b^2*Tan[(c + d*x)/2]^5 + b^3*Tan
[(c + d*x)/2]^5 - 6*a^3*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/
```

```
(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 -
b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*a*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[
(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^3*EllipticPi[-1, -Arc
Sin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a
 + b)] + 6*a*b^2*EllipticPi[-1, -ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c +
d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (3*a^3 + 3*a^2*b - a*b^2 - b^3
)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*T
an[(c + d*x)/2]^2)/(a + b)] + 2*a*b*(a + b)*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2
)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(b^
2*(-a^2 + b^2)*d*Sqrt[1 + Tan[(c + d*x)/2]^2]*(b*(-1 + Tan[(c + d*x)/2]^2)
- a*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] time = 0.528, size = 1675, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -1/d/(a+b)/(a-b)/b^2*(-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-3*cos(d*x+c)^2*a
^2*b+cos(d*x+c)*a*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(
a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3-6*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^2*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-a*b^2*cos(d*x+c)^2+cos(d*x+
c)^3*a^2*b+2*cos(d*x+c)*a^2*b+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*s
```

```
in(d*x+c)*a^2*b-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-3*a^3*cos(d*x
+c)+cos(d*x+c)^2*b^3-cos(d*x+c)^3*b^3+3*cos(d*x+c)^2*a^3)*cos(d*x+c)^2*(1/c
os(d*x+c))^(5/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
 + a^2)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(3/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{3}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(3/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2)), x)
```

3.763 $\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=513

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-13a^2b^2+a^4+8b^4\right)\sin(c+dx)\,\mathrm{s}}{3a^3d\left(a^2-\right.}$$

```
[Out] (-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(3*a^5*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a^4 +
 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) +
(2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^(3/2)) + (4*b^2*(5*a^2 - 3*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^
2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^2 - b^
2)^2*d)
```

Rubi [A] time = 1.30173, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2802, 3055, 2998, 2816, 2994}

$$\frac{4b^2\left(5a^2-3b^2\right)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}+\frac{2\left(-13a^2b^2+a^4+8b^4\right)\sin(c+dx)\,\mathrm{s}}{3a^3d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a +
b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]
))/(a - b)])/(3*a^5*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a^4 +
 9*a^3*b + 16*a^2*b^2 - 12*a*b^3 - 16*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])]
, -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec
[c + d*x]))/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) +
(2*b^2*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d
*x])^(3/2)) + (4*b^2*(5*a^2 - 3*b^2)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^
2*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]) + (2*(a^4 - 13*a^2*b^2 + 8*b^4)
*Sqrt[a + b*Cos[c + d*x]]*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^3*(a^2 - b^
2)^2*d)
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{3}{2}\left(a^2-2b^2\right)-\frac{3}{2}ab\cos(c}{\cos^{\frac{5}{2}}(c+dx)(a+}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left(4\sqrt{c\right.}{}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^4\right.}{}$$

$$= \frac{2b^2\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{4b^2\left(5a^2-3b^2\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{2\left(a^4\right.}{}$$

$$= -\frac{8b\left(2a^4-7a^2b^2+4b^4\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{3a^5(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 17.2324, size = 546, normalized size = 1.06

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{8b\left(-7a^2b^2+2a^4+4b^4\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)^2} - \frac{2b^3\sin(c+dx)}{3a^2\left(a^2-b^2\right)(a+b\cos(c+dx))^2} - \frac{2\left(11a^2b^3\sin(c+dx)-7b^5\sin(c+dx)\right)}{3a^3\left(a^2-b^2\right)^2(a+b\cos(c+dx))} + \right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(5/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(4*b*(2*a^5 + 2*a^4*b - 7*a^3*b^2
- 7*a^2*b^3 + 4*a*b^4 + 4*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)] + a*(a^5 - 8*a^4*b + 7*a^3*b^2 + 28*a^2*b^3 -
4*a*b^4 - 16*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-
a + b)/(a + b)] + 2*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Cos[c + d*x]*(a + b*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*Sqrt[
a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[a + b*Cos[c + d*x]]*S
qrt[Sec[c + d*x]]*((-8*b*(2*a^4 - 7*a^2*b^2 + 4*b^4)*Sin[c + d*x])/(3*a^4*(
a^2 - b^2)^2) - (2*b^3*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*(a + b*Cos[c + d*x]
)^2) - (2*(11*a^2*b^3*Sin[c + d*x] - 7*b^5*Sin[c + d*x]))/(3*a^3*(a^2 - b^2
)^2*(a + b*Cos[c + d*x])) + (2*Tan[c + d*x])/(3*a^3)))/d
```

Maple [B] time = 0.725, size = 4197, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)^2/(a+b)^2/a^4*(cos(d*x+c)^2*a^7-16*cos(d*x+c)^4*b^7+16*cos(d*x
+c)^3*b^7-a^3*b^4+16*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^7-a^7+cos(d*x+c)^4*a^5*b^2+2*a^5*b^2-
8*cos(d*x+c)^4*a^4*b^3-13*cos(d*x+c)^4*a^3*b^4+28*cos(d*x+c)^4*a^2*b^5+8*co
s(d*x+c)^4*a*b^6+2*cos(d*x+c)^3*a^6*b-16*cos(d*x+c)^3*a^5*b^2-8*cos(d*x+c)^
3*a^4*b^3+56*cos(d*x+c)^3*a^3*b^4-18*cos(d*x+c)^3*a^2*b^5-32*cos(d*x+c)^3*a
*b^6-8*cos(d*x+c)^2*a^6*b+13*cos(d*x+c)^2*a^5*b^2+28*cos(d*x+c)^2*a^4*b^3-4
2*cos(d*x+c)^2*a^3*b^4-16*cos(d*x+c)^2*a^2*b^5+24*cos(d*x+c)^2*a*b^6+6*cos(
d*x+c)*a^6*b-12*cos(d*x+c)*a^4*b^3+6*cos(d*x+c)*a^2*b^5+cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^7-
12*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^5+32*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^6-8*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*b+7*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^5*b^2+28*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3-4*cos(d*x+c)*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^4-16*cos(d*
x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^
(1/2))*a^2*b^5+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)*sin(d*x+c)*a^6*b+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/
(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5*b^2-28*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^4*b^3-28*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*a^3*b^4+16*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^5+16*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^6+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^3*sin(d*x+c)*a^6*b-8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^5*b^2+7*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4*b^3+2
8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x
+c)^3*sin(d*x+c)*a^3*b^4-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^2*b^5-16*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^6+8
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
```

```
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+
c)^3*sin(d*x+c)*a^5*b^2+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^4*b^3-28*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a^3*b^4-
28*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^2*b^5+16*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^3*sin(d*x+c)*a*b^6-7*cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6*b-co
s(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^5*b^2+35*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3+24*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^4-20
*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^2*b^5-16*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^6+16*cos(d*x+c)^2*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^7+cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^7+16*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b^2-20*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^3-56*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)*a^3*b^4+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^6*b)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)/(a+b
*cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec (dx+c)^{\frac{5}{2}}}{(b\cos (dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sec(dx+c)^{\frac{5}{2}}}{b^3\cos(dx+c)^3+3ab^2\cos(dx+c)^2+3a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(5/2)/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(5/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.764 $\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=438

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(9a^2b+3a^3-6ab^2-8b^3\right)\sqrt{\cos(c+}}{}$$

```
[Out] (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*a^3 +
 9*a^2*b - 6*a*b^2 - 8*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (
8*b^2*(2*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d
*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.925147, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2802, 3055, 2998, 2816, 2994}

$$\frac{8b^2\left(2a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\cos(c+dx))^{3/2}}-\frac{2\left(9a^2b+3a^3-6ab^2-8b^3\right)\sqrt{\cos(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*(3*a^4 - 15*a^2*b^2 + 8*b^4)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)
/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))
/(a - b)])/(3*a^4*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*a^3 +
 9*a^2*b - 6*a*b^2 - 8*b^3)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSi
n[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a
- b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a
- b)])/(3*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sqrt[Sec
[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)) + (
8*b^2*(2*a^2 - b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d
*Sqrt[a + b*Cos[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
```

```
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}\left(3a^2-4b^2\right)-\frac{3}{2}ab\cos}{\cos^{\frac{3}{2}}(c+dx)(a+}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} + \frac{\left(4\sqrt{\text{co}}\right.}{}$$

$$= \frac{2b^2\sqrt{\sec(c+dx)}\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}} + \frac{8b^2\left(2a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}} - \frac{((a-b}{}$$

$$= \frac{2\left(3a^4-15a^2b^2+8b^4\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-}{}}}{3a^4(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 16.352, size = 525, normalized size = 1.2

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{2\left(-15a^2b^2+3a^4+8b^4\right)\sin(c+dx)}{3a^3\left(a^2-b^2\right)^2}+\frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{8\left(2a^2b^2\sin(c+dx)-b^4\sin(c+dx)\right)}{3a^2\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d} + \frac{2\sqrt{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*(3*a^4 - 15*a^2*b^2 + 8*b^
4)*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b
^2)*(a + b*Cos[c + d*x])^2) + (8*(2*a^2*b^2*Sin[c + d*x] - b^4*Sin[c + d*x]
))/(3*a^2*(a^2 - b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*Sqrt[Cos[(c + d*x)/2
]^2*Sec[c + d*x]]*(-2*(3*a^5 + 3*a^4*b - 15*a^3*b^2 - 15*a^2*b^3 + 8*a*b^4
+ 8*b^5)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a
 + b)] + 2*a*(3*a^4 - 6*a^3*b - 15*a^2*b^2 + 2*a*b^3 + 8*b^4)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^4 - 15*a
^2*b^2 + 8*b^4)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2]))/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c +
d*x)/2]^2])
```

Maple [B] time = 0.48, size = 3701, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/(a-b)^2/(a+b)^2/a^3*(-3*a^2*b^4-15*cos(d*x+c)^3*a^2*b^4+3*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
```

)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^6-3*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
a^6+6*a^4*b^2-8*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6-3*a^6-4*cos(d*x+c)^3*a*b^5+3*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*a^6+6*cos(d*x+c)^2*a^5*b-30*cos(d*x+c)^2*a^3*b^3+10*cos(d*x+c)^2*a^2*b^4+
16*cos(d*x+c)^2*a*b^5-15*cos(d*x+c)*a^4*b^2+22*cos(d*x+c)*a^3*b^3+8*cos(d*x
+c)*a^2*b^4-12*cos(d*x+c)*a*b^5+3*cos(d*x+c)^3*a^4*b^2+2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2
b^4+8(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)*a*b^5-3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b-3*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b^2
+15*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^3*b^3+15*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^4-8*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+
c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^5
-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*a^5*b-21*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2-13*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3+10*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*a^2*b^4+8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^5-6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b+12*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2)
)*a^4*b^2+30*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+
b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4-16*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*
b^5+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos
(d*x+c)^2*sin(d*x+c)*a^5*b-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b^2-15*(cos(d*x+c)/(1+cos(d
x+c)))^(1/2)(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b
^3+8*cos(d*x+c)^3*a^3*b^3+8*cos(d*x+c)^3*b^6-8*cos(d*x+c)^2*b^6+3*cos(d*x+c
)*a^6-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(

```
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*c
os(d*x+c)*sin(d*x+c)*a^6-8*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^6-6*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*b-15*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2+2*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*
b^3+8*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^2*b^4-3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^5*b+15*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b^2+15*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^3-8*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^4-8*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)-6
*cos(d*x+c)*a^5*b+6*cos(d*x+c)^2*a^4*b^2)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/(
a+b*cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\,\sec(dx+c)^{\frac{3}{2}}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sec(d*x + c)^(3/2)/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^(3/2)/(b*cos(d*x + c) + a)^(5/2), x)
```

3.765 $\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=421

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2-3ab-2b^2\right)\sqrt{\cos}}{}$$

```
[Out] (4*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(3*a^2 - 3*a*b - 2*b^
2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(a - b)*(a
+ b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (4*b*(3*a^2 - b^2)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.844057, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2802, 2993, 2998, 2816, 2994}

$$\frac{2b^2\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{4b\left(3a^2-b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2\left(3a^2-3ab-2b^2\right)\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (4*b*(3*a^2 - b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*a^3*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(3*a^2 - 3*a*b - 2*b^
2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]
]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a^2*(a - b)*(a
+ b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*b^2*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*
(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]) - (4*b*(3*a^2 - b^2)*Sqrt[Se
c[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2802

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)
), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n +
```

```
2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2*m
, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n]
&& LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\frac{1}{2}(}{\sqrt{co}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{4b\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+d}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{2b^2\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{4b\left(3a^2-b^2\right)\sqrt{\sec(c+dx)}\sin(c+d}{3a\left(a^2-b^2\right)^2 d\sqrt{a+b\cos(c+dx)}}$$

$$= \frac{4b\left(3a^2-b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx))}{a+b}}}{3a^3(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.9232, size = 471, normalized size = 1.12

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(\frac{4b\left(3a^2-b^2\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)^2} - \frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)(a+b\cos(c+dx))^2} - \frac{2\left(5a^2b\sin(c+dx)-b^3\sin(c+dx)\right)}{3a\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d} + \frac{4\sqrt{\cos^2\left(\frac{1}{2}(\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]/(a + b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((4*b*(3*a^2 - b^2)*Sin[c + d*
x])/(3*a^2*(a^2 - b^2)^2) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[c
+ d*x])^2) - (2*(5*a^2*b*Sin[c + d*x] - b^3*Sin[c + d*x]))/(3*a*(a^2 - b^2)
^2*(a + b*Cos[c + d*x]))))/d + (4*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*
b*(-3*a^3 - 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (-a + b)/(a + b)] + a*(3*a^3 + 6*a^2*b + a*b^2 - 2*b^3)*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] + b*(-
3*a^2 + b^2)*Cos[c + d*x]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2]))/(3*(a^3 - a*b^2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/
2]^2])
```

Maple [B] time = 0.667, size = 2743, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3/d/a^2/(a+b)^2/(a-b)^2*(1/cos(d*x+c))^(1/2)/(a+b*cos(d*x+c))^(3/2)*(-2*
cos(d*x+c)^3*b^5+2*cos(d*x+c)^2*b^5-4*cos(d*x+c)^2*a*b^4+6*cos(d*x+c)^3*a^2
```

```
*b^3+12*cos(d*x+c)^2*a^3*b^2+6*cos(d*x+c)*a^4*b+2*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)+6*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)+(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+
c)-2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*
b^3*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c)
)/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))
^(1/2))*a^4*b*sin(d*x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*
cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a
-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)+2*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^5+3*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*a^5-2*cos(d*x+c)*a^2*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4-6*cos(d*x+c)^2*a^
4*b-4*cos(d*x+c)^2*a^2*b^3-7*cos(d*x+c)*a^3*b^2+3*cos(d*x+c)*a*b^4-5*cos(d*
x+c)^3*a^3*b^2+cos(d*x+c)^3*a*b^4+(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3-2*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a
*b^4+9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^4*b+7*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3-2*cos
(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a*b^4-12*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-6*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)
*a^4*b-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
cos(d*x+c)*sin(d*x+c)*a^2*b^3-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*
(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2-6*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2
*b^3+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*co
s(d*x+c)^2*sin(d*x+c)*a^4*b+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2+2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^5+3*(
```

```
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5*sin(d*
x+c))/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}\sqrt{\sec(dx+c)}}{b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*sqrt(sec(d*x + c))/(b^3*cos(d*x + c)^3 +
3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)/(a+b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sec(dx+c)}}{(b\cos(dx+c)+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)/(a+b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sec(d*x + c))/(b*cos(d*x + c) + a)^(5/2), x)
```

3.766 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=399

$$\frac{2\left(3a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,\csc(c}{}$$

```
[Out] (-2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(3*a - b)*Sqrt[Cos[c +
 d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt
[Sec[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)*Sqrt[Sec[c + d*x]]) + (2*(3*a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.757563, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2796, 2993, 2998, 2816, 2994}

$$\frac{2\left(3a^2+b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2b\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\,\csc(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```


```
[Out] (-2*(3*a^2 + b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqr
t[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*
a^2*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(3*a - b)*Sqrt[Cos[c +
 d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*
Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt
[Sec[c + d*x]]) - (2*b*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^
(3/2)*Sqrt[Sec[c + d*x]]) + (2*(3*a^2 + b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x
])/(3*(a^2 - b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2796

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])
^(m + 1)*(c + d*Sin[e + f*x])^n)/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m +
 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n
- 1)*Simp[a*c*(m + 1) + b*d*n + (a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] -
b*d*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1
] && LtQ[0, n, 1] && IntegersQ[2*m, 2*n]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sqrt{\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\cos(c+dx)}}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{3}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(3a^2+b^2\right)\sqrt{\sec(c+dx)}}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2b\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{2\left(3a^2+b^2\right)\sqrt{\sec(c+dx)}}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\,\mathrm{co}}}$$

$$= -\frac{2\left(3a^2+b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{3a^2(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 13.2316, size = 455, normalized size = 1.14

$$\frac{\sqrt{\sec(c+dx)}\sqrt{a+b\cos(c+dx)}\left(-\frac{2\left(3a^2+b^2\right)\sin(c+dx)}{3a\left(a^2-b^2\right)^2}+\frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)(a+b\cos(c+dx))^2}+\frac{4\left(a^2\sin(c+dx)+b^2\sin(c+dx)\right)}{3\left(a^2-b^2\right)^2(a+b\cos(c+dx))}\right)}{d} - 2\sqrt{\cos^2\left(\frac{1}{2}(c+\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sqrt[Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((-2*(3*a^2 + b^2)*Sin[c + d*x
])/(3*a*(a^2 - b^2)^2) + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*(a + b*Cos[c + d
*x])^2) + (4*(a^2*Sin[c + d*x] + b^2*Sin[c + d*x]))/(3*(a^2 - b^2)^2*(a + b
*Cos[c + d*x]))))/d - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(-2*(3*a^3 +
 3*a^2*b + a*b^2 + b^3)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (-a + b)/(a + b)] + 2*a*(3*a^2 + 4*a*b + b^2)*Sqrt[Cos[c + d*x]/(1 + Co
s[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipt
icF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (3*a^2 + b^2)*Cos[c + d*x
]*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*a*(a^2 - b^
2)^2*d*Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
```

Maple [B] time = 0.645, size = 2419, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x)
```

```
[Out] -2/3/d/a/(a+b)^2/(a-b)^2*(b^4*cos(d*x+c)^2+4*a^2*b^2*cos(d*x+c)^2-6*cos(d*x
+c)^2*a^3*b-2*cos(d*x+c)^2*a*b^3-cos(d*x+c)*a^2*b^2-cos(d*x+c)^2*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3+3*
```

```
cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*a^3*b+3*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3-3*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*a^3*b-4*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2+3*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),(-(a-b)/(a+b))^(1/2))*a^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)^2*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^4+3*cos(d*
x+c)^2*a^4+2*cos(d*x+c)^3*a^3*b+2*cos(d*x+c)^3*a*b^3+4*cos(d*x+c)*a^3*b-cos
(d*x+c)^3*b^4-3*cos(d*x+c)^3*a^2*b^2-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1
/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-3*a^4*cos(d*x+c)+cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*
b^4-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b
*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(
a-b)/(a+b))^(1/2))*a^4-7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*co
s(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b
)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b-5*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2-(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a*b^3+6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a^3*b+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^2+2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^3
+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)*sin(d*x+c)*a^4-4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*a^3*b*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b*sin(d*x+c)+(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^2*sin(d*x+c)+(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^3*sin(d*x+c))*(1/cos
(d*x+c))^(1/2)/sin(d*x+c)/(a+b*cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\cos\left(dx+c\right)+a}}{\left(b^{3}\cos\left(dx+c\right)^{3}+3\,ab^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}b\cos\left(dx+c\right)+a^{3}\right)\sqrt{\sec\left(dx+c\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sqrt(sec(d*x + c))), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\cos\left(dx+c\right)+a\right)^{\frac{5}{2}}\sqrt{\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c))), x)
```

3.767 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=382

$$-\frac{8ab\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2(a-3b)\sqrt{\cos(c+dx)}\csc(c+$$

```
[Out] (8*b*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a
+ b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a - 3*b)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]])
 + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[
c + d*x]]) - (8*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqr
t[a + b*Cos[c + d*x]])
```

Rubi [A] time = 0.728897, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {4222, 2799, 2993, 2998, 2816, 2994}

$$-\frac{8ab\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}+\frac{2a\sin(c+dx)}{3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}+\frac{2(a-3b)\sqrt{\cos(c+dx)}\csc(c+$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```


```
[Out] (8*b*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Cos[c + d*
x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a
+ b)^(3/2)*d*Sqrt[Sec[c + d*x]]) + (2*(a - 3*b)*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*
x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1
+ Sec[c + d*x]))/(a - b)])/(3*a*(a - b)*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]])
 + (2*a*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[
c + d*x]]) - (8*a*b*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)^2*d*Sqr
t[a + b*Cos[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2799

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin
[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1))/(f*(m + 1)*(a^2 - b^2)), x
] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*S
in[e + f*x])^(n - 2)*Simp[c*(a*c - b*d)*(m + 1) + d*(b*c - a*d)*(n - 1) + (
```

```
d*(a*c - b*d)*(m + 1) - c*(b*c - a*d)*(m + 2))*Sin[e + f*x] - d*(b*c - a*d)
*(m + n + 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& LtQ[1, n, 2] && IntegersQ[2*m, 2*n]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{8ab\sqrt{\sec(c+dx)}\sin(c}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

$$= \frac{2a\sin(c+dx)}{3\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{8ab\sqrt{\sec(c+dx)}\sin(c}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\cos}}$$

$$= \frac{8b\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)\middle|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1-\sec(c+dx}{a+b}}}{3a(a-b)(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [A] time = 7.81494, size = 359, normalized size = 0.94

$$-\frac{2\sqrt{\sec(c+dx)}\left(a^2\left(a^2-b^2\right)\sin(c+dx)-a\left(a^2-5b^2\right)\sin(c+dx)(a+b\cos(c+dx))+2b\cos^2\left(\frac{1}{2}(c+dx)\right)(a+b\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[Sec[c + d*x]]*(a^2*(a^2 - b^2)*Sin[c + d*x] - a*(a^2 - 5*b^2)*(a +
 b*Cos[c + d*x])*Sin[c + d*x] - 4*b^2*(a + b*Cos[c + d*x])^2*Sin[c + d*x] +
 2*b*Cos[(c + d*x)/2]^2*(a + b*Cos[c + d*x])*(4*b*(a + b)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b)] - (a^2 + 4*a*b + 3*
b^2)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(a + b*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (-a + b)/(a + b
)] - b*(a + b*Cos[c + d*x])*Sec[(c + d*x)/2]^3*(Sin[(c + d*x)/2] - Sin[(3*(
c + d*x))/2]))))/(3*b*(a^2 - b^2)^2*d*(a + b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 0.61, size = 1793, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x)
```

```
[Out] 2/3/d/(a+b)^2/(a-b)^2*(-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-
b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3+4*cos(d*x+c)^2*a^2*b+3*c
os(d*x+c)*a*b^2+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
```

```
))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(
a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+4*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-4*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-8*a*b^2*co
s(d*x+c)^2+5*cos(d*x+c)^3*a*b^2-4*cos(d*x+c)*a^2*b+4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^3+4*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)
*sin(d*x+c)*a^2*b+8*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b-7*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a*b^2+a^3*cos(d*x+c)+4*cos(d*x+c)^2*b^3-cos(d*x+c)^3*a^3-4*cos(d*x+c)^3*b^3
-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*
x+c)^2*sin(d*x+c)*b^3+4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-4*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin
(d*x+c)*a*b^2-(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1
/2))*a^3*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d
*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(
a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^3)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)/sin
(d*x+c)/(a+b*cos(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(3/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2)), x)
```

3.768 $\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=557

$$-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+ab-6b^2\right)\sqrt{\cos(}}{}$$

```
[Out] (2*(3*a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*a^2 + a*b - 6*b^2)
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^2*(a +
b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*a^2*
Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*
x]]) - (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2
- b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```

Rubi [A] time = 1.21284, antiderivative size = 557, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {4222, 2792, 3051, 2809, 2993, 2998, 2816, 2994}

$$-\frac{2a^2\left(3a^2-7b^2\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\cos(c+dx)}}-\frac{2a^2\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}(a+b\cos(c+dx))^{3/2}}-\frac{2\left(3a^2+ab-6b^2\right)\sqrt{\cos(}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```


```
[Out] (2*(3*a^2 - 7*b^2)*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticE[ArcSin[Sqrt[a
+ b*Cos[c + d*x]]/(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sq
rt[(a*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3
*(a - b)*b^2*(a + b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*(3*a^2 + a*b - 6*b^2)
*Sqrt[Cos[c + d*x]]*Csc[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Cos[c + d*x]]/
(Sqrt[a + b]*Sqrt[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c +
 d*x]))/(a + b)]*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(3*(a - b)*b^2*(a +
b)^(3/2)*d*Sqrt[Sec[c + d*x]]) - (2*Sqrt[a + b]*Sqrt[Cos[c + d*x]]*Csc[c +
d*x]*EllipticPi[(a + b)/b, ArcSin[Sqrt[a + b*Cos[c + d*x]]/(Sqrt[a + b]*Sqr
t[Cos[c + d*x]])], -((a + b)/(a - b))]*Sqrt[(a*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[(a*(1 + Sec[c + d*x]))/(a - b)])/(b^3*d*Sqrt[Sec[c + d*x]]) - (2*a^2*
Sin[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*
x]]) - (2*a^2*(3*a^2 - 7*b^2)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2
- b^2)^2*d*Sqrt[a + b*Cos[c + d*x]])
```


Rule 4222

```
Int[(csc[(a_.) + (b_.)*(x_)]*(c_.))^(m_.)*(u_), x_Symbol] :> Dist[(c*Csc[a
+ b*x])^m*(c*Sin[a + b*x])^m, Int[ActivateTrig[u]/(c*Sin[a + b*x])^m, x], x
```

```
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 2792

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(
n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e
+ f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2 +
 a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 + b
^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 - d^
2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || Int
egersQ[2*m, 2*n])
```

Rule 3051

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)])^(3/2)), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[d*Sin[e + f*x]]/Sqrt[a + b*
Sin[e + f*x]], x], x] + Dist[1/b, Int[(A*b + (b*B - a*C)*Sin[e + f*x])/((a
+ b*Sin[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, d, e,
 f, A, B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2809

```
Int[Sqrt[(b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)
*(x_)]], x_Symbol] :> Simp[(2*b*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 +
Csc[e + f*x]))/(c - d)]*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticPi[(c
+ d)/d, ArcSin[Sqrt[c + d*Sin[e + f*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b,
 2])], -((c + d)/(c - d))])/(d*f), x] /; FreeQ[{b, c, d, e, f}, x] && NeQ[c
^2 - d^2, 0] && PosQ[(c + d)/b]
```

Rule 2993

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[(2*(
A*b - a*B)*Cos[e + f*x])/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin
[e + f*x]]), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 2816

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[(-2*Tan[e + f*x]*Rt[(a + b)/d, 2]*Sqrt[(a*(1
 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[A
rcSin[Sqrt[a + b*Sin[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*Rt[(a + b)/d, 2])], -(
```

```
(a + b)/(a - b))])/(a*f), x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2,
 0] && PosQ[(a + b)/d]
```

Rule 2994

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[(-2*A
*(c - d)*Tan[e + f*x]*Rt[(c + d)/b, 2]*Sqrt[(c*(1 + Csc[e + f*x]))/(c - d)]
*Sqrt[(c*(1 - Csc[e + f*x]))/(c + d)]*EllipticE[ArcSin[Sqrt[c + d*Sin[e + f
*x]]/(Sqrt[b*Sin[e + f*x]]*Rt[(c + d)/b, 2])], -((c + d)/(c - d))])/(f*b*c^
2), x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B]
 && PosQ[(c + d)/b]
```

Rubi steps

$$\int \frac{1}{(a+b\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(a+b\cos(c+dx))^{5/2}}\,dx$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{}$$

$$= -\frac{2a^2\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\cos(c+dx))^{3/2}\sqrt{\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+\right.}{b^2}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$= -\frac{2\sqrt{a+b}\sqrt{\cos(c+dx)}\csc(c+dx)\Pi\left(\frac{a+b}{b};\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(}{}}}{b^3d\sqrt{\sec(c+dx)}}$$

$$= \frac{2\left(3a^2-7b^2\right)\sqrt{\cos(c+dx)}\csc(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\cos(c+dx)}}{\sqrt{a+b}\sqrt{\cos(c+dx)}}\right)|-\frac{a+b}{a-b}\right)\sqrt{\frac{a(1}{}}}{3(a-b)b^2(a+b)^{3/2}d\sqrt{\sec(c+dx)}}$$

Mathematica [C] time = 13.9552, size = 1716, normalized size = 3.08

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)),x]
```

```
[Out] (Sqrt[a + b*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*((2*a*(3*a^2 - 7*b^2)*Sin[c +
d*x])/(3*b^2*(a^2 - b^2)^2) - (2*a^3*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)*(a +
 b*Cos[c + d*x])^2) - (8*(a^4*Sin[c + d*x] - 2*a^2*b^2*Sin[c + d*x]))/(3*b^
2*(-a^2 + b^2)^2*(a + b*Cos[c + d*x]))))/d + (2*(3*a^4*Sqrt[(a - b)/(a + b)
]*Tan[(c + d*x)/2] + 3*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - 7*a^2
*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2] - 7*a*b^3*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2] - 6*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 + 14*a
*b^3*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^3 - 3*a^4*Sqrt[(a - b)/(a + b)]
*Tan[(c + d*x)/2]^5 + 3*a^3*b*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 + 7*
a^2*b^2*Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]^5 - 7*a*b^3*Sqrt[(a - b)/(a
```

```
+ b)]*Tan[(c + d*x)/2]^5 + (6*I)*a^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[
Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(
a + b)] - (12*I)*a^2*b^2*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)
/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6
*I)*b^4*EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c
+ d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b +
a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*a^4*EllipticP
i[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a
+ b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
+ a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] - (12*I)*a^2*b^2*El
lipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]]
, -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[
(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + (6*I)*b^4*
EllipticPi[(a + b)/(a - b), I*ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2
]], -((a + b)/(a - b))]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)] + I*a*(3*a
^3 - 3*a^2*b - 7*a*b^2 + 7*b^3)*EllipticE[I*ArcSinh[Sqrt[(a - b)/(a + b)]*T
an[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan
[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)
/(a + b)] - I*(6*a^4 - 2*a^3*b - 13*a^2*b^2 + 6*a*b^3 + 3*b^4)*EllipticF[I*
ArcSinh[Sqrt[(a - b)/(a + b)]*Tan[(c + d*x)/2]], -((a + b)/(a - b))]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b + a*Tan[(c + d*
x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(a + b)]))/(3*b^2*Sqrt[(a - b)/(a + b)]*(a^
2 - b^2)^2*d*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*Sqrt[(
a + b + a*Tan[(c + d*x)/2]^2 - b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^
2)]*(-1 + Tan[(c + d*x)/2]^4))
```

Maple [B] time = 0.648, size = 3921, normalized size = 7.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x)
```

```
[Out] -2/3/d/b^2/(a+b)^2/(a-b)^2*(7*cos(d*x+c)^2*a*b^4-4*cos(d*x+c)^3*a^4*b+8*cos
(d*x+c)^3*a^2*b^3+4*cos(d*x+c)^2*a^3*b^2-2*cos(d*x+c)*a^4*b+2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)-(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*
x+c)-6*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b^3*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+
c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b
))^(1/2))*a^4*b*sin(d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+
b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-
(a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),(-(a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)+6*cos(d*x+c)^2*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*b^5
+6*cos(d*x+c)*a^2*b^3+7*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos
(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)
/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^4-3*(cos(d*x+c)/(1+cos(d*x+c)))^
```

```
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^4*b+3*a^5*c
os(d*x+c)+6*cos(d*x+c)^2*a^4*b-14*cos(d*x+c)^2*a^2*b^3-7*cos(d*x+c)*a^3*b^2
+3*cos(d*x+c)^3*a^3*b^2-7*cos(d*x+c)^3*a*b^4-(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^3-6*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*s
in(d*x+c)*a*b^4+2*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),(-(a-b)/(a+b))^(1/2))*a^4*b+cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2-7*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^
2*b^3-9*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a
+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
-(a-b)/(a+b))^(1/2))*a*b^4+4*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^3*b^2+7*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b^4-6*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)*
sin(d*x+c)*a^4*b+14*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^3-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2+7*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin
(d*x+c)*a^2*b^3+2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c
))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b^2+6*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*b-12*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x
+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)
/(a+b))^(1/2))*a^2*b^3+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^4*b-12*cos(d*x+c)*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2
-12*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*c
os(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(
-(a-b)/(a+b))^(1/2))*a^2*b^3+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4-3*cos(d*x+c)^2*sin(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*b^5-
3*cos(d*x+c)^2*a^5+6*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b)
)^(1/2))*a^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)
*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),(-(a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)+6*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^5+6*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+c
os(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(
```

```
1/2))*b^5-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b
)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),(-(a-b)/(a+b))^(1/2))*b^5-3*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a^5-12*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a^3*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+6*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,(-(a-b)/(a+b))^(1/2))*a*b^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)-3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(1/(a+b)*(a+b*cos(d*x+c))/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(-(a-b)/(a+b))^(1/2))*a*b
^4*sin(d*x+c))*cos(d*x+c)^2*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(a+b*cos(d*x+c)
)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\cos(dx+c)+a}}{\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)/((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x +
c)^2 + 3*a^2*b*cos(d*x + c) + a^3)*sec(d*x + c)^(5/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))**(5/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\cos(dx+c)+a)^{\frac{5}{2}}\sec(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cos(d*x+c))^(5/2)/sec(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2)), x)
```

3.769 $\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx$

Optimal. Leaf size=330

$$-\frac{\left(6a^2b^2\left(m^2+5m+4\right)+a^4\left(m^2+6m+8\right)+b^4\left(m^2+4m+3\right)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c\right.}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (b^2*(b^2*(3 + m) + a^2*(22 + 5*m))*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(
2 + m)*(4 + m)) + (2*a*b^3*(5 + m)*Cos[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(3
 + m)*(4 + m)) + (b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(d*(4 + m)) - ((b^4*(3 + 4*m + m^2) + 6*a^2*b^2*(4 + 5*m + m^2) + a^4*
(8 + 6*m + m^2))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c
 + d*x]^2]) - (4*a*b*(b^2*(2 + m) + a^2*(3 + m))*Cos[c + d*x]^(2 + m)*Hyper
geometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2
 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.674238, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2793, 3033, 3023, 2748, 2643}

$$-\frac{\left(6a^2b^2\left(m^2+5m+4\right)+a^4\left(m^2+6m+8\right)+b^4\left(m^2+4m+3\right)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c\right.}{d(m+1)(m+2)(m+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (b^2*(b^2*(3 + m) + a^2*(22 + 5*m))*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(
2 + m)*(4 + m)) + (2*a*b^3*(5 + m)*Cos[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(3
 + m)*(4 + m)) + (b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])^2*Sin[c + d
*x])/(d*(4 + m)) - ((b^4*(3 + 4*m + m^2) + 6*a^2*b^2*(4 + 5*m + m^2) + a^4*
(8 + 6*m + m^2))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*(4 + m)*Sqrt[Sin[c
 + d*x]^2]) - (4*a*b*(b^2*(2 + m) + a^2*(3 + m))*Cos[c + d*x]^(2 + m)*Hyper
geometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2
 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^4\,dx = \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))^2\sin(c+dx)}{d(4+m)} + \frac{\int \cos^m(c+dx)(a+b\cos(c \ldots}{}$$

$$= \frac{2ab^3(5+m)\cos^{2+m}(c+dx)\sin(c+dx)}{d(3+m)(4+m)} + \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))^{\ldots}}{d(4+m)}$$

$$= \frac{b^2\left(b^2(3+m)+a^2(22+5m)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(4+m)} + \frac{2ab^3(5+m)\cos^{2+m\ldots}}{d(3+m\ldots}$$

$$= \frac{b^2\left(b^2(3+m)+a^2(22+5m)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(4+m)} + \frac{2ab^3(5+m)\cos^{2+m\ldots}}{d(3+m\ldots}$$

$$= \frac{b^2\left(b^2(3+m)+a^2(22+5m)\right)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(4+m)} + \frac{2ab^3(5+m)\cos^{2+m\ldots}}{d(3+m\ldots}$$

Mathematica [A] time = 1.6425, size = 242, normalized size = 0.73

$$\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(b\cos(c+dx)\left(b\cos(c+dx)\left(b\cos(c+dx)\left(-\frac{4a\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(c+dx)\right)}{m+4}-\frac{b\cos(\ldots}{}\right.\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((a^4*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[c + d*x]^2])/(1 + m)) + b*Cos[c + d*x]*((-4*a^3*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m) + b*Cos[c + d*x
]*((-6*a^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])/(3
 + m) + b*Cos[c + d*x]*((-4*a*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2,
Cos[c + d*x]^2])/(4 + m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + m)/2
, (7 + m)/2, Cos[c + d*x]^2])/(5 + m)))))*Sqrt[Sin[c + d*x]^2])/d
```

Maple [F] time = 1.476, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (a+b\cos(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^4 \cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^4\cos(dx+c)^4+4\,ab^3\cos(dx+c)^3+6\,a^2b^2\cos(dx+c)^2+4\,a^3b\cos(dx+c)+a^4\right)\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] integral((b^4*cos(d*x + c)^4 + 4*a*b^3*cos(d*x + c)^3 + 6*a^2*b^2*cos(d*x +
 c)^2 + 4*a^3*b*cos(d*x + c) + a^4)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^4\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^4*cos(d*x + c)^m, x)
```

3.770 $\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx$

Optimal. Leaf size=250

$$-\frac{a\left(a^2(m+2)+3b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{b\left(3a^2(m+3)+b^2(m+2)\right)}{}$$

```
[Out] (a*b^2*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (
b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) - (
a*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2,
 (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqr
t[Sin[c + d*x]^2]) - (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[c + d*x]^(2 + m)*
Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/
(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.316659, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2793, 3023, 2748, 2643}

$$-\frac{a\left(a^2(m+2)+3b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{b\left(3a^2(m+3)+b^2(m+2)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (a*b^2*(7 + 2*m)*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (
b^2*Cos[c + d*x]^(1 + m)*(a + b*Cos[c + d*x])*Sin[c + d*x])/(d*(3 + m)) - (
a*(3*b^2*(1 + m) + a^2*(2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2,
 (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqr
t[Sin[c + d*x]^2]) - (b*(b^2*(2 + m) + 3*a^2*(3 + m))*Cos[c + d*x]^(2 + m)*
Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/
(d*(2 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2793

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
 n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*(m
 + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) - 3*
a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin[e
 + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m] |
| IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &&
NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^3\,dx = \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))\sin(c+dx)}{d(3+m)} + \frac{\int \cos^m(c+dx)\left(a\left(b^2(1+m\right.\right.}{}$$

$$= \frac{ab^2(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))}{d(3+m)}$$

$$= \frac{ab^2(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))}{d(3+m)}$$

$$= \frac{ab^2(7+2m)\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)(3+m)} + \frac{b^2\cos^{1+m}(c+dx)(a+b\cos(c+dx))}{d(3+m)}$$

Mathematica [A] time = 0.807916, size = 197, normalized size = 0.79

$$\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(b\cos(c+dx)\left(b\cos(c+dx)\left(-\frac{3a\,{}_2F_1\left(\frac{1}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(c+dx)\right)}{m+3}-\frac{b\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\right.}{m+4}\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(-((a^3*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Cos[c + d*x]^2])/(1 + m)) + b*Cos[c + d*x]*((-3*a^2*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m) + b*Cos[c + d*x
]*((-3*a*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])/(3 +
 m) - (b*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Cos[c +
d*x]^2])/(4 + m))))*Sqrt[Sin[c + d*x]^2])/d
```

Maple [F] time = 1.426, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m\,(a+b\cos(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\cos(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*cos(d*x + c)^m, x)
```

3.771 $\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx$

Optimal. Leaf size=179

$$-\frac{\left(a^2(m+2)+b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2ab\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F}{d(m+2)\sqrt{\sin}}$$

```
[Out] (b^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - ((b^2*(1 + m) + a^2*(
2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*
a*b*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.127536, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2789, 2643, 3014}

$$-\frac{\left(a^2(m+2)+b^2(m+1)\right)\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2ab\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F}{d(m+2)\sqrt{\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] (b^2*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(2 + m)) - ((b^2*(1 + m) + a^2*(
2 + m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*
a*b*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c
 + d*x]^2]*Sin[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2789

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Dist[(2*c*d)/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] +
 Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d, e
, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))^2\,dx = (2ab)\int \cos^{1+m}(c+dx)\,dx + \int \cos^m(c+dx)\left(a^2+b^2\cos^2(c+dx)\right)dx$$

$$= \frac{b^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} - \frac{2ab\cos^{2+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(c\right.}{d(2+m)\sqrt{\sin^2(c+dx)}}$$

$$= \frac{b^2\cos^{1+m}(c+dx)\sin(c+dx)}{d(2+m)} - \frac{\left(a^2+\frac{b^2(1+m)}{2+m}\right)\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\right.}{d(1+m)\sqrt{\sin^2(c+}}$$

Mathematica [A] time = 0.316214, size = 168, normalized size = 0.94

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(a^2\left(m^2+5m+6\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)+b(m+1)\cos(c+dx)\right.}{d(m+1)(m+2)(m}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(a^2*(6 + 5*m + m^2)*Hypergeometric2F1
[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2] + b*(1 + m)*Cos[c + d*x]*(2*a*(
3 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2] + b*(2
+ m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]
^2]))*Sqrt[Sin[c + d*x]^2])/(d*(1 + m)*(2 + m)*(3 + m)))
```

Maple [F] time = 1.218, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (a+b\cos(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\cos(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*cos(d*x + c)^m, x)
```

3.772 $\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx$

Optimal. Leaf size=131

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(c\right.}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (b*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0640834, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2748, 2643}

$$-\frac{a\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)}{d(m+1)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(c\right.}{d(m+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a + b*Cos[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 + m)*Sqrt[Sin[c + d*x]^2])) - (b*Cos[c + d*
x]^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2]*Sin
[c + d*x])/(d*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a+b\cos(c+dx))\,dx = a\int \cos^m(c+dx)\,dx + b\int \cos^{1+m}(c+dx)\,dx$$

$$= -\frac{a\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(1+m)\sqrt{\sin^2(c+dx)}}-\frac{b\cos^{2+m}(c+}{}$$

Mathematica [A] time = 0.152915, size = 112, normalized size = 0.85

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(a(m+2)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\cos^2(c+dx)\right)+b(m+1)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{}\right.\right.}{d(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a + b*Cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(a*(2 + m)*Hypergeometric2F1[1/2, (1 +
 m)/2, (3 + m)/2, Cos[c + d*x]^2] + b*(1 + m)*Cos[c + d*x]*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 +
 m)*(2 + m)))
```

Maple [F] time = 1.27, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (a+b\cos(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(a+b*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)\cos(dx+c)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\cos(dx+c)+a)\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\cos(c+dx))\cos^m(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a+b*cos(d*x+c)),x)
```

```
[Out] Integral((a + b*cos(c + d*x))*cos(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)\cos(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*cos(d*x + c)^m, x)
```

3.773 $\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=190

$$\frac{a\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\cos^m(c+dx)\cos^2}{}$$

```
[Out] (a*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/
(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)*Sin[c + d*
x])/((a^2 - b^2)*d) - (b*AppellF1[1/2, -m/2, 1, 3/2, Sin[c + d*x]^2, -((b^2
*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^m*Sin[c + d*x])/((a^2 - b^2)*d*
(Cos[c + d*x]^2)^(m/2))
```

Rubi [A] time = 0.232686, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2823, 3189, 429}

$$\frac{a\sin(c+dx)\cos^{m-1}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}F_1\left(\frac{1}{2};\frac{1-m}{2},1;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)}-\frac{b\sin(c+dx)\cos^m(c+dx)\cos^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a + b*Cos[c + d*x]),x]
```

```
[Out] (a*AppellF1[1/2, (1 - m)/2, 1, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/
(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d*x]^2)^((1 - m)/2)*Sin[c + d*
x])/((a^2 - b^2)*d) - (b*AppellF1[1/2, -m/2, 1, 3/2, Sin[c + d*x]^2, -((b^2
*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^m*Sin[c + d*x])/((a^2 - b^2)*d*
(Cos[c + d*x]^2)^(m/2))
```

Rule 2823

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[a, Int[(d*Sin[e + f*x])^n/(a^2 - b^2*Sin[e + f*x]^
2), x], x] - Dist[b/d, Int[(d*Sin[e + f*x])^(n + 1)/(a^2 - b^2*Sin[e + f*x]
^2), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{a+b\cos(c+dx)}\,dx = a\int \frac{\cos^m(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx - b\int \frac{\cos^{1+m}(c+dx)}{a^2-b^2\cos^2(c+dx)}\,dx$$

$$= \frac{\left(a\cos^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}(c+dx)\cos^2(c+dx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \frac{(1-x^2)^{\frac{1}{2}(-1+m)}}{a^2-b^2+b^2x^2}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\left(b\cos^{n}\right.}{}$$

$$= \frac{aF_1\left(\frac{1}{2}; \frac{1-m}{2}, 1; \frac{3}{2}; \sin^2(c+dx), -\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{-1+m}(c+dx)\cos^2(c+dx)^{\frac{1-m}{2}}\sin(c+dx)}{(a^2-b^2)\,d} -$$

Mathematica [B] time = 24.4947, size = 6703, normalized size = 35.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^m/(a + b*Cos[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.111, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m}{a+b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a+b*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m/(a+b*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^m}{b\cos(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{b\cos(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a), x)
```

3.774 $\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2} \, dx$

Optimal. Leaf size=294

$$\frac{b^2 \sin(c+dx)\cos^{m+1}(c+dx)\cos^2(c+dx)^{\frac{1}{2}(-m-1)}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)^2}+\frac{a^2\sin(c+dx)\cos}{}$$

```
[Out] (b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^
2)/(a^2 - b^2))]*Cos[c + d*x]^(1 + m)*(Cos[c + d*x]^2)^((-1 - m)/2)*Sin[c +
 d*x])/((a^2 - b^2)^2*d) + (a^2*AppellF1[1/2, (1 - m)/2, 2, 3/2, Sin[c + d*
x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d
*x]^2)^((1 - m)/2)*Sin[c + d*x])/((a^2 - b^2)^2*d) - (2*a*b*AppellF1[1/2, -
m/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d
*x]^m*Sin[c + d*x])/((a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2))
```

Rubi [A] time = 0.35351, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2824, 3189, 429}

$$\frac{b^2 \sin(c+dx)\cos^{m+1}(c+dx)\cos^2(c+dx)^{\frac{1}{2}(-m-1)}F_1\left(\frac{1}{2};\frac{1}{2}(-m-1),2;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)}{d\left(a^2-b^2\right)^2}+\frac{a^2\sin(c+dx)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a + b*Cos[c + d*x])^2,x]
```


```
[Out] (b^2*AppellF1[1/2, (-1 - m)/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^
2)/(a^2 - b^2))]*Cos[c + d*x]^(1 + m)*(Cos[c + d*x]^2)^((-1 - m)/2)*Sin[c +
 d*x])/((a^2 - b^2)^2*d) + (a^2*AppellF1[1/2, (1 - m)/2, 2, 3/2, Sin[c + d*
x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d*x]^(-1 + m)*(Cos[c + d
*x]^2)^((1 - m)/2)*Sin[c + d*x])/((a^2 - b^2)^2*d) - (2*a*b*AppellF1[1/2, -
m/2, 2, 3/2, Sin[c + d*x]^2, -((b^2*Sin[c + d*x]^2)/(a^2 - b^2))]*Cos[c + d
*x]^m*Sin[c + d*x])/((a^2 - b^2)^2*d*(Cos[c + d*x]^2)^(m/2))
```


Rule 2824

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^n/((a - b*sin[e +
 f*x])^m/(a^2 - b^2*sin[e + f*x]^2)^m), x], x] /; FreeQ[{a, b, d, e, f, n},
 x] && NeQ[a^2 - b^2, 0] && ILtQ[m, -1]
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{(a+b\cos(c+dx))^2}\,dx = \int \left(\frac{a^2\cos^m(c+dx)}{\left(a^2-b^2\cos^2(c+dx)\right)^2} - \frac{2ab\cos^{1+m}(c+dx)}{\left(a^2-b^2\cos^2(c+dx)\right)^2} + \frac{b^2\cos^{2+m}(c+dx)}{\left(-a^2+b^2\cos^2(c+dx)\right)^2}\right)dx$$

$$= a^2\int \frac{\cos^m(c+dx)}{\left(a^2-b^2\cos^2(c+dx)\right)^2}\,dx - (2ab)\int \frac{\cos^{1+m}(c+dx)}{\left(a^2-b^2\cos^2(c+dx)\right)^2}\,dx + b^2\int \frac{\cos^{2+m}(c}{\left(-a^2+b^2\cos\right.}$$

$$= \frac{\left(b^2\cos^{2\left(\frac{1}{2}+\frac{m}{2}\right)}(c+dx)\cos^2(c+dx)^{-\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int \frac{(1-x^2)^{\frac{1+m}{2}}}{\left(-a^2+b^2-b^2x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d} + \frac{\left(a^2\,\mathrm{c}\right.}{}$$

$$= \frac{b^2F_1\left(\frac{1}{2};\frac{1}{2}(-1-m),2;\frac{3}{2};\sin^2(c+dx),-\frac{b^2\sin^2(c+dx)}{a^2-b^2}\right)\cos^{1+m}(c+dx)\cos^2(c+dx)^{\frac{1}{2}(-1-m)}\sin(}{\left(a^2-b^2\right)^2 d}$$

Mathematica [B] time = 26.08, size = 7214, normalized size = 24.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^m/(a + b*Cos[c + d*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.606, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m}{(a+b\cos(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^m}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a+b*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(dx+c)^m}{(b\cos(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c) + a)^2, x)
```

3.775 $\int (a+b\cos(c+dx))^3 \sec^m(c+dx)\,dx$

Optimal. Leaf size=282

$$-\frac{b\left(3a^2(3-m)+b^2(2-m)\right)\sin(c+dx)\sec^{m-4}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(c+dx)\right)}{d(2-m)(4-m)\sqrt{\sin^2(c+dx)}}-\frac{a\left(a^2(2-m)+3b^2(1-m)\right)\sin(}{d(1}$$

```
[Out] -((a^2*b*(1 - 2*m)*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(d*(1 - m)*(2 - m)))
 - (a^2*Sec[c + d*x]^(-2 + m)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(d*(1 - m)
) - (b*(b^2*(2 - m) + 3*a^2*(3 - m))*Hypergeometric2F1[1/2, (4 - m)/2, (6 -
 m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-4 + m)*Sin[c + d*x])/(d*(2 - m)*(4 -
m)*Sqrt[Sin[c + d*x]^2]) - (a*(3*b^2*(1 - m) + a^2*(2 - m))*Hypergeometric2
F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-3 + m)*Sin[c +
 d*x])/(d*(1 - m)*(3 - m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.424042, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3238, 3842, 4047, 3772, 2643, 4046}

$$-\frac{b\left(3a^2(3-m)+b^2(2-m)\right)\sin(c+dx)\sec^{m-4}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4-m}{2};\frac{6-m}{2};\cos^2(c+dx)\right)}{d(2-m)(4-m)\sqrt{\sin^2(c+dx)}}-\frac{a\left(a^2(2-m)+3b^2(1-m)\right)\sin(}{d(1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^m,x]
```


```
[Out] -((a^2*b*(1 - 2*m)*Sec[c + d*x]^(-2 + m)*Sin[c + d*x])/(d*(1 - m)*(2 - m)))
 - (a^2*Sec[c + d*x]^(-2 + m)*(b + a*Sec[c + d*x])*Sin[c + d*x])/(d*(1 - m)
) - (b*(b^2*(2 - m) + 3*a^2*(3 - m))*Hypergeometric2F1[1/2, (4 - m)/2, (6 -
 m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-4 + m)*Sin[c + d*x])/(d*(2 - m)*(4 -
m)*Sqrt[Sin[c + d*x]^2]) - (a*(3*b^2*(1 - m) + a^2*(2 - m))*Hypergeometric2
F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-3 + m)*Sin[c +
 d*x])/(d*(1 - m)*(3 - m)*Sqrt[Sin[c + d*x]^2])
```


Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3842

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> -Simp[(b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2
)*(d*Csc[e + f*x])^n)/(f*(m + n - 1)), x] + Dist[1/(d*(m + n - 1)), Int[(a
+ b*Csc[e + f*x])^(m - 3)*(d*Csc[e + f*x])^n*Simp[a^3*d*(m + n - 1) + a*b^2
*d*n + b*(b^2*d*(m + n - 2) + 3*a^2*d*(m + n - 1))*Csc[e + f*x] + a*b^2*d*(
3*m + 2*n - 4)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, n}, x] &
& NeQ[a^2 - b^2, 0] && GtQ[m, 2] && (IntegerQ[m] || IntegersQ[2*m, 2*n]) &&
  !(IGtQ[n, 2] &&  !IntegerQ[m])
```

Rule 4047

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^3\sec^m(c+dx)\,dx=\int \sec^{-3+m}(c+dx)(b+a\sec(c+dx))^3\,dx$$

$$=-\frac{a^2\sec^{-2+m}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{d(1-m)}+\frac{\int \sec^{-3+m}(c+dx)\left(-b\right.}{}$$

$$=-\frac{a^2\sec^{-2+m}(c+dx)(b+a\sec(c+dx))\sin(c+dx)}{d(1-m)}+\left(a\left(3b^2+\frac{a^2(2-m)}{1-m}\right)\right)$$

$$=-\frac{a^2b(1-2m)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(1-m)(2-m)}-\frac{a^2\sec^{-2+m}(c+dx)(b+a\sec(c}{d(1-m)}$$

$$=-\frac{a^2b(1-2m)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(1-m)(2-m)}-\frac{a^2\sec^{-2+m}(c+dx)(b+a\sec(c}{d(1-m)}$$

$$=-\frac{a^2b(1-2m)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(1-m)(2-m)}-\frac{a^2\sec^{-2+m}(c+dx)(b+a\sec(c}{d(1-m)}$$

Mathematica [A] time = 0.792781, size = 222, normalized size = 0.79

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-4}(c+dx)\left(\frac{1}{2}a(m-3)\sec^3(c+dx)\left(2a(m-2)\left(a(m-1)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\sec^2(c+dx)\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^3*Sec[c + d*x]^m,x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]^(-4 + m)*(b^3*m*(2 - 3*m + m^2)*Hypergeometric2F
1[1/2, (-3 + m)/2, (-1 + m)/2, Sec[c + d*x]^2] + (a*(-3 + m)*(6*b^2*(-1 + m
```

```
)*m*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-2 + m)/2, m/2, Sec[c + d*x]^2]
+ 2*a*(-2 + m)*(3*b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 +
m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, S
ec[c + d*x]^2]))*Sec[c + d*x]^3)/2)*Sqrt[-Tan[c + d*x]^2])/(d*(-3 + m)*(-2
+ m)*(-1 + m)*m)
```

Maple [F] time = 1.984, size = 0, normalized size = 0.

$$\int (a+b\cos(dx+c))^3 (\sec(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x)
```

```
[Out] int((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3 \sec(dx+c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3\cos(dx+c)^3+3\,ab^2\cos(dx+c)^2+3\,a^2b\cos(dx+c)+a^3\right)\sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)^2 + 3*a^2*b*cos(d*x + c
) + a^3)*sec(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**3*sec(d*x+c)**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^3\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^3*sec(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^3*sec(d*x + c)^m, x)
```

3.776 $\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx$

Optimal. Leaf size=200

$$-\frac{\left(a^2(2-m)+b^2(1-m)\right)\sin(c+dx)\sec^{m-3}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(c+dx)\right)}{d(1-m)(3-m)\sqrt{\sin^2(c+dx)}}-\frac{a^2\sin(c+dx)\sec^{m-1}(c+dx)}{d(1-m)}-\frac{2a}{}$$

```
[Out] -((a^2*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m))) - ((b^2*(1 - m) + a
^2*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Se
c[c + d*x]^(-3 + m)*Sin[c + d*x])/(d*(1 - m)*(3 - m)*Sqrt[Sin[c + d*x]^2])
- (2*a*b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.188567, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3238, 3788, 3772, 2643, 4046}

$$-\frac{\left(a^2(2-m)+b^2(1-m)\right)\sin(c+dx)\sec^{m-3}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(c+dx)\right)}{d(1-m)(3-m)\sqrt{\sin^2(c+dx)}}-\frac{a^2\sin(c+dx)\sec^{m-1}(c+dx)}{d(1-m)}-\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^m,x]
```

```
[Out] -((a^2*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m))) - ((b^2*(1 - m) + a
^2*(2 - m))*Hypergeometric2F1[1/2, (3 - m)/2, (5 - m)/2, Cos[c + d*x]^2]*Se
c[c + d*x]^(-3 + m)*Sin[c + d*x])/(d*(1 - m)*(3 - m)*Sqrt[Sin[c + d*x]^2])
- (2*a*b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3788

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[(2*a*b)/d, Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1))
, x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; Fr
eeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx = \int \sec^{-2+m}(c+dx)(b+a\sec(c+dx))^2\,dx$$

$$= (2ab)\int \sec^{-1+m}(c+dx)\,dx + \int \sec^{-2+m}(c+dx)\left(b^2+a^2\sec^2(c+dx)\right)\,dx$$

$$= -\frac{a^2\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m)} + \left(b^2+\frac{a^2(2-m)}{1-m}\right)\int \sec^{-2+m}(c+dx)\,dx +$$

$$= -\frac{a^2\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m)} - \frac{2ab\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(c+dx)\right)\sec^{-2}}{d(2-m)\sqrt{\sin^2(c+d}}$$

$$= -\frac{a^2\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m)} - \frac{\left(b^2+\frac{a^2(2-m)}{1-m}\right)\,{}_2F_1\left(\frac{1}{2},\frac{3-m}{2};\frac{5-m}{2};\cos^2(c+\right.}{d(3-m)\sqrt{\sin^2(}}$$

Mathematica [A] time = 0.308575, size = 159, normalized size = 0.8

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-3}(c+dx)\left(a(m-2)\sec^2(c+dx)\left(a(m-1)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\sec^2(c+dx)\right)+2bm\cos(\right.\right.}{d(m-2)(m-1)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])^2*Sec[c + d*x]^m,x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]^(-3 + m)*(b^2*(-1 + m)*m*Hypergeometric2F1[1/2,
(-2 + m)/2, m/2, Sec[c + d*x]^2] + a*(-2 + m)*(2*b*m*Cos[c + d*x]*Hypergeom
etric2F1[1/2, (-1 + m)/2, (1 + m)/2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeom
etric2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2])*Sec[c + d*x]^2)*Sqrt[-Tan[c
+ d*x]^2])/(d*(-2 + m)*(-1 + m)*m)
```

Maple [F] time = 1.473, size = 0, normalized size = 0.

$$\int (a+b\cos(dx+c))^2(\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x)
```

```
[Out] int((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2\right)\sec(dx+c)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\cos(c+dx))^2\sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**2*sec(d*x+c)**m,x)
```

```
[Out] Integral((a + b*cos(c + d*x))**2*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)^2\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^2*sec(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

3.777 $\int (a + b\cos(c+dx))\sec^m(c+dx)\,dx$

Optimal. Leaf size=143

$$-\frac{a\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d(1-m)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c + d
*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])) - (a*Hypergeom
etric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*S
in[c + d*x])/(d*(1 - m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.112146, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3238, 3787, 3772, 2643}

$$-\frac{a\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d(1-m)\sqrt{\sin^2(c+dx)}}-\frac{b\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(c+dx)\right)}{d(2-m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])*Sec[c + d*x]^m,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (2 - m)/2, (4 - m)/2, Cos[c + d*x]^2]*Sec[c + d
*x]^(-2 + m)*Sin[c + d*x])/(d*(2 - m)*Sqrt[Sin[c + d*x]^2])) - (a*Hypergeom
etric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*S
in[c + d*x])/(d*(1 - m)*Sqrt[Sin[c + d*x]^2])
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)
*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] &&
 !IntegerQ[m] && IntegersQ[n, p]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\cos(c+dx))\sec^m(c+dx)\,dx = \int \sec^{-1+m}(c+dx)(b+a\sec(c+dx))\,dx$$
$$= a\int \sec^m(c+dx)\,dx + b\int \sec^{-1+m}(c+dx)\,dx$$
$$= (a\cos^m(c+dx)\sec^m(c+dx))\int \cos^{-m}(c+dx)\,dx + (b\cos^m(c+dx)\sec^m(c+dx$$
$$= -\frac{b\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{2};\frac{4-m}{2};\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{d(2-m)\sqrt{\sin^2(c+dx)}} - \frac{a\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3}{}\right.}{}$$

Mathematica [A] time = 0.182855, size = 107, normalized size = 0.75

$$\frac{\sqrt{-\tan^2(c+dx)}\csc(c+dx)\sec^{m-1}(c+dx)\left(a(m-1)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\sec^2(c+dx)\right)+bm\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\mathrm{se}\right.\right.}{d(m-1)m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])*Sec[c + d*x]^m,x]
```

```
[Out] (Csc[c + d*x]*(b*m*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/
2, Sec[c + d*x]^2] + a*(-1 + m)*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Sec[
c + d*x]^2])*Sec[c + d*x]^(-1 + m)*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + m)*m)
```

Maple [F] time = 1.573, size = 0, normalized size = 0.

$$\int (a+b\cos(dx+c))(\sec(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))*sec(d*x+c)^m,x)
```

```
[Out] int((a+b*cos(d*x+c))*sec(d*x+c)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx+c)+a)\sec(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\cos(dx+c)+a)\sec(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\cos(c + dx))\sec^m(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)**m,x)
```

```
[Out] Integral((a + b*cos(c + d*x))*sec(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(dx + c) + a)\sec(dx + c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))*sec(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c) + a)*sec(d*x + c)^m, x)
```

3.778 $\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx$

Optimal. Leaf size=26

$$-2\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)$$

```
[Out] -2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]
```

Rubi [A] time = 0.0776947, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2775, 204}

$$-2\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[x]]/Sqrt[a - Cos[x]],x]
```

```
[Out] -2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)\\ &= -2\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)\end{aligned}$$

Mathematica [C] time = 0.0674033, size = 47, normalized size = 1.81

$$i\sqrt{2-2\cos(x)}\csc\left(\frac{x}{2}\right)\log\left(\sqrt{a-\cos(x)}+i\sqrt{2}\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]]/Sqrt[a - Cos[x]],x]
```

```
[Out] I*Sqrt[2 - 2*Cos[x]]*Csc[x/2]*Log[I*Sqrt[2]*Cos[x/2] + Sqrt[a - Cos[x]]]
```

Maple [B] time = 0.346, size = 67, normalized size = 2.6

$$-\frac{1}{\sin(x)(-1+\cos(x))}(2-2\cos(x))^{\frac{3}{2}}\sqrt{a-\cos(x)}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\frac{-a+\cos(x)}{\cos(x)+1}}\right)\frac{1}{\sqrt{-2\frac{-a+\cos(x)}{\cos(x)+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x)
```

```
[Out] -(2-2*cos(x))^(3/2)*(a-cos(x))^(1/2)*arctan(1/2*2^(1/2)*(-2*(-a+cos(x))/(co
s(x)+1))^(1/2))/sin(x)/(-1+cos(x))/(-2*(-a+cos(x))/(cos(x)+1))^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.6862, size = 103, normalized size = 3.96

$$\arctan\left(\frac{(a-2\cos(x)-1)\sqrt{-\cos(x)+1}}{2\sqrt{a-\cos(x)}\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(1/2*(a - 2*cos(x) - 1)*sqrt(-cos(x) + 1)/(sqrt(a - cos(x))*sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))**(1/2)/(a-cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cos(x))/sqrt(a - cos(x)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2)/(a-cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.779 $\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}}$$

```
[Out] (-2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]*Sqrt[(1 - Cos[x])/(a
 - Cos[x])]*Sqrt[a - Cos[x]])/Sqrt[1 - Cos[x]]
```

Rubi [A] time = 0.0999325, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4400, 2775, 204}

$$-\frac{2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - Cos[x])/(a - Cos[x])],x]
```

```
[Out] (-2*ArcTan[Sin[x]/(Sqrt[1 - Cos[x]]*Sqrt[a - Cos[x]])]*Sqrt[(1 - Cos[x])/(a
 - Cos[x])]*Sqrt[a - Cos[x]])/Sqrt[1 - Cos[x]]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !I
nertTrigFreeQ[w])
```

Rule 2775

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b + d*x^2), x], x
, (b*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\,dx = \frac{\left(\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\right)\int \frac{\sqrt{1-\cos(x)}}{\sqrt{a-\cos(x)}}\,dx}{\sqrt{1-\cos(x)}}$$
$$= \frac{\left(2\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)}{\sqrt{1-\cos(x)}}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sin(x)}{\sqrt{1-\cos(x)}\sqrt{a-\cos(x)}}\right)\sqrt{\frac{1-\cos(x)}{a-\cos(x)}}\sqrt{a-\cos(x)}}{\sqrt{1-\cos(x)}}$$

Mathematica [A] time = 0.068058, size = 64, normalized size = 0.98

$$-\sqrt{2}\csc\left(\frac{x}{2}\right)\sqrt{\frac{\cos(x)-1}{\cos(x)-a}}\sqrt{\cos(x)-a}\log\left(\sqrt{\cos(x)-a}+\sqrt{2}\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - Cos[x])/(a - Cos[x])],x]
```

```
[Out] -(Sqrt[2]*Sqrt[(-1 + Cos[x])/(-a + Cos[x])]*Sqrt[-a + Cos[x]]*Csc[x/2]*Log[
Sqrt[2]*Cos[x/2] + Sqrt[-a + Cos[x]]])
```

Maple [A] time = 0.5, size = 67, normalized size = 1.

$$-\frac{\sqrt{2}\sin(x)}{-1+\cos(x)}\sqrt{\frac{-1+\cos(x)}{-a+\cos(x)}}\sqrt{-2\,\frac{-a+\cos(x)}{\cos(x)+1}}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-2\,\frac{-a+\cos(x)}{\cos(x)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-cos(x))/(a-cos(x)))^(1/2),x)
```

```
[Out] -2^(1/2)*((-1+cos(x))/(-a+cos(x)))^(1/2)*sin(x)*(-2*(-a+cos(x))/(cos(x)+1))
^(1/2)*arctan(1/2*2^(1/2)*(-2*(-a+cos(x))/(cos(x)+1))^(1/2))/(-1+cos(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55248, size = 100, normalized size = 1.54

$$-\arctan\left(-\frac{(a-2\cos(x)-1)\sqrt{-\frac{\cos(x)-1}{a-\cos(x)}}}{2\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*(a - 2*cos(x) - 1)*sqrt(-(cos(x) - 1)/(a - cos(x)))/sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-cos(x))/(a-cos(x)))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-cos(x))/(a-cos(x)))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.780 $\int (a+a\cos(c+dx)) \left(-\frac{B}{2}+B\cos(c+dx)\right) dx$

Optimal. Leaf size=37

$$\frac{aB\sin(c+dx)}{2d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

```
[Out] (a*B*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0201143, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2734}

$$\frac{aB\sin(c+dx)}{2d}+\frac{aB\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])*(-B/2 + B*Cos[c + d*x]),x]
```

```
[Out] (a*B*Sin[c + d*x])/(2*d) + (a*B*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx)) \left(-\frac{B}{2}+B\cos(c+dx)\right) dx=\frac{aB\sin(c+dx)}{2d}+\frac{aB\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.0505303, size = 29, normalized size = 0.78

$$\frac{aB(2\sin(c+dx)+\sin(2(c+dx))+2c)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])*(-B/2 + B*Cos[c + d*x]),x]
```

```
[Out] (a*B*(2*c + 2*Sin[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.044, size = 51, normalized size = 1.4

$$\frac{1}{2d}\left(2\,aB\left(1/2\,\cos\left(dx+c\right)\sin\left(dx+c\right)+1/2\,dx+c/2\right)+aB\sin\left(dx+c\right)-aB\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)*(-1/2*B+B*cos(d*x+c)),x)
```

```
[Out] 1/2/d*(2*a*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a*B*sin(d*x+c)-a*B*(
d*x+c))
```

Maxima [A] time = 1.05095, size = 61, normalized size = 1.65

$$\frac{(2\,dx+2\,c+\sin{(2\,dx+2\,c)})Ba-2\,(dx+c)Ba+2\,Ba\sin{(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 2*(d*x + c)*B*a + 2*B*a*sin(d*x
 + c))/d
```

Fricas [A] time = 1.38892, size = 61, normalized size = 1.65

$$\frac{(Ba\cos{(dx+c)}+Ba)\sin{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*a*cos(d*x + c) + B*a)*sin(d*x + c)/d
```

Sympy [A] time = 0.396081, size = 87, normalized size = 2.35

$$\begin{cases} \frac{Bax\sin^{2}{(c+dx)}}{2}+\frac{Bax\cos^{2}{(c+dx)}}{2}-\frac{Bax}{2}+\frac{Ba\sin{(c+dx)}\cos{(c+dx)}}{2d}+\frac{Ba\sin{(c+dx)}}{2d} & \text{for}\ d\neq 0 \\ x\left(B\cos{(c)}-\frac{B}{2}\right)(a\cos{(c)}+a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((B*a*x*sin(c + d*x)**2/2 + B*a*x*cos(c + d*x)**2/2 - B*a*x/2 + B*
a*sin(c + d*x)*cos(c + d*x)/(2*d) + B*a*sin(c + d*x)/(2*d), Ne(d, 0)), (x*(
B*cos(c) - B/2)*(a*cos(c) + a), True))
```

Giac [A] time = 1.74801, size = 41, normalized size = 1.11

$$\frac{Ba\sin{(2\,dx+2\,c)}}{4\,d}+\frac{Ba\sin{(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))*(-1/2*B+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*B*a*sin(2*d*x + 2*c)/d + 1/2*B*a*sin(d*x + c)/d
```

3.781 $\int (a + a\cos(c + dx))^4 \left(-\frac{4B}{5} + B\cos(c + dx)\right) dx$

Optimal. Leaf size=26

$$\frac{B\sin(c + dx)(a\cos(c + dx) + a)^4}{5d}$$

```
[Out] (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0313192, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2749}

$$\frac{B\sin(c + dx)(a\cos(c + dx) + a)^4}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^4*((-4*B)/5 + B*Cos[c + d*x]),x]
```

```
[Out] (B*(a + a*Cos[c + d*x])^4*Sin[c + d*x])/(5*d)
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int (a + a\cos(c + dx))^4 \left(-\frac{4B}{5} + B\cos(c + dx)\right) dx = \frac{B(a + a\cos(c + dx))^4 \sin(c + dx)}{5d}$$

Mathematica [A] time = 0.184874, size = 31, normalized size = 1.19

$$\frac{a^4 B \sin^9(c + dx) \csc^8\left(\frac{1}{2}(c + dx)\right)}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^4*((-4*B)/5 + B*Cos[c + d*x]),x]
```

```
[Out] (a^4*B*Csc[(c + d*x)/2]^8*Sin[c + d*x]^9)/(80*d)
```

Maple [B] time = 0.048, size = 150, normalized size = 5.8

$$\frac{1}{5d}\left(a^4B\left(\frac{8}{3} + (\cos(dx + c))^4 + \frac{4(\cos(dx + c))^2}{3}\right)\sin(dx + c) + 16a^4B\left(1/4\left((\cos(dx + c))^3 + 3/2\cos(dx + c)\right)\sin(dx + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^4*(-4/5*B+B*cos(d*x+c)),x)
```

```
[Out] 1/5/d*(a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+16*a^4*B*(1/4*(
cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+14/3*a^4*B*(2+cos(d*
x+c)^2)*sin(d*x+c)-4*a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-11*a^4
*B*sin(d*x+c)-4*a^4*B*(d*x+c))
```

Maxima [B] time = 1.0555, size = 194, normalized size = 7.46

$$\frac{2\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)Ba^{4}-28\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba^{4}+3\left(12\,dx+12\,c+\right.}{30\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(2*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 - 28
*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4
- 24*(d*x + c)*B*a^4 - 66*B*a^4*sin(d*x + c))/d
```

Fricas [B] time = 1.48511, size = 167, normalized size = 6.42

$$\frac{\left(Ba^{4}\cos\left(dx+c\right)^{4}+4\,Ba^{4}\cos\left(dx+c\right)^{3}+6\,Ba^{4}\cos\left(dx+c\right)^{2}+4\,Ba^{4}\cos\left(dx+c\right)+Ba^{4}\right)\sin\left(dx+c\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5*(B*a^4*cos(d*x + c)^4 + 4*B*a^4*cos(d*x + c)^3 + 6*B*a^4*cos(d*x + c)^2
 + 4*B*a^4*cos(d*x + c) + B*a^4)*sin(d*x + c)/d
```

Sympy [A] time = 3.54753, size = 333, normalized size = 12.81

$$\begin{cases}\frac{6Ba^{4}x\sin^{4}(c+dx)}{5}+\frac{12Ba^{4}x\sin^{2}(c+dx)\cos^{2}(c+dx)}{5}-\frac{2Ba^{4}x\sin^{2}(c+dx)}{5}+\frac{6Ba^{4}x\cos^{4}(c+dx)}{5}-\frac{2Ba^{4}x\cos^{2}(c+dx)}{5}-\frac{4Ba^{4}x}{5}+\frac{8Ba^{4}\sin^{5}(c+dx)}{15d} \\ x\left(B\cos\left(c\right)-\frac{4B}{5}\right)\left(a\cos\left(c\right)+a\right)^{4}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**4*(-4/5*B+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((6*B*a**4*x*sin(c + d*x)**4/5 + 12*B*a**4*x*sin(c + d*x)**2*cos(c
 + d*x)**2/5 - 2*B*a**4*x*sin(c + d*x)**2/5 + 6*B*a**4*x*cos(c + d*x)**4/5
- 2*B*a**4*x*cos(c + d*x)**2/5 - 4*B*a**4*x/5 + 8*B*a**4*sin(c + d*x)**5/(1
5*d) + 4*B*a**4*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + 6*B*a**4*sin(c + d*
x)**3*cos(c + d*x)/(5*d) + 28*B*a**4*sin(c + d*x)**3/(15*d) + B*a**4*sin(c
+ d*x)*cos(c + d*x)**4/d + 2*B*a**4*sin(c + d*x)*cos(c + d*x)**3/d + 14*B*a
```

```
**4*sin(c + d*x)*cos(c + d*x)**2/(5*d) - 2*B*a**4*sin(c + d*x)*cos(c + d*x)
/(5*d) - 11*B*a**4*sin(c + d*x)/(5*d), Ne(d, 0)), (x*(B*cos(c) - 4*B/5)*(a*
cos(c) + a)**4, True))
```

Giac [B] time = 1.82523, size = 119, normalized size = 4.58

$$\frac{Ba^4\sin(5\,dx+5\,c)}{80\,d}+\frac{Ba^4\sin(4\,dx+4\,c)}{10\,d}+\frac{27\,Ba^4\sin(3\,dx+3\,c)}{80\,d}+\frac{3\,Ba^4\sin(2\,dx+2\,c)}{5\,d}+\frac{21\,Ba^4\sin(dx+c)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^4*(-4/5*B+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/80*B*a^4*sin(5*d*x + 5*c)/d + 1/10*B*a^4*sin(4*d*x + 4*c)/d + 27/80*B*a^4
*sin(3*d*x + 3*c)/d + 3/5*B*a^4*sin(2*d*x + 2*c)/d + 21/40*B*a^4*sin(d*x +
c)/d
```

3.782 $\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n}+B\cos(c+dx)\right) dx$

Optimal. Leaf size=28

$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^n}{d(n+1)}$$

```
[Out] (B*(a + a*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n))
```

Rubi [A] time = 0.0447429, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2749}

$$\frac{B\sin(c+dx)(a\cos(c+dx)+a)^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^n*(-((B*n)/(1 + n)) + B*Cos[c + d*x]),x]
```

```
[Out] (B*(a + a*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n))
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^n \left(-\frac{Bn}{1+n}+B\cos(c+dx)\right) dx = \frac{B(a+a\cos(c+dx))^n\sin(c+dx)}{d(1+n)}$$

Mathematica [A] time = 0.166431, size = 28, normalized size = 1.

$$\frac{B\sin(c+dx)(a(\cos(c+dx)+1))^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^n*(-((B*n)/(1 + n)) + B*Cos[c + d*x]),x]
```

```
[Out] (B*(a*(1 + Cos[c + d*x]))^n*Sin[c + d*x])/(d*(1 + n))
```

Maple [B] time = 0.418, size = 74, normalized size = 2.6

$$2\,\frac{B\tan(1/2\,dx+c/2)}{d\,(1+n)\left(1+(\tan(1/2\,dx+c/2))^2\right)}e^{n\ln\left(a+\frac{a\left(1-(\tan(1/2\,dx+c/2))^2\right)}{1+(\tan(1/2\,dx+c/2))^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^n*(-B*n/(1+n)+B*cos(d*x+c)),x)
```

```
[Out] 2*B/d/(1+n)*tan(1/2*d*x+1/2*c)*exp(n*ln(a+a*(1-tan(1/2*d*x+1/2*c)^2)/(1+tan
(1/2*d*x+1/2*c)^2)))/(1+tan(1/2*d*x+1/2*c)^2)
```

Maxima [B] time = 2.17089, size = 193, normalized size = 6.89

$$-\frac{\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)^{n}Ba^{n}\sin\left(-(dx+c)(n+1)+2\,n\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)+\right.\right.}{2\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -1/2*((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*B*a^n*sin(-(
d*x + c)*(n + 1) + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1)) - (cos(d*x
+ c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)^n*B*a^n*sin(-(d*x + c)*(n - 1
) + 2*n*arctan2(sin(d*x + c), cos(d*x + c) + 1)))/(2^n*d*(n + 1))
```

Fricas [A] time = 1.31158, size = 66, normalized size = 2.36

$$\frac{\left(a\cos\left(dx+c\right)+a\right)^{n}B\sin\left(dx+c\right)}{dn+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] (a*cos(d*x + c) + a)^n*B*sin(d*x + c)/(d*n + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**n*(-B*n/(1+n)+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 46.1419, size = 1850, normalized size = 66.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^n*(-B*n/(1+n)+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(B*(sqrt(-tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^4 + 2*tan(d*x + c)^4*tan(1
/2*d*x + 1/2*c)^2 - tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^4 + 3*tan(d*x + c)^
4 + 6*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 7*tan(d*x + c)^2 + 4*tan(1/2*
d*x + 1/2*c)^2 + 4)*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x
 + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))^n*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x
+ 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*
d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)
*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/
2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*
x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/
2*c) - B*(sqrt(-tan(d*x + c)^4*tan(1/2*d*x + 1/2*c)^4 + 2*tan(d*x + c)^4*ta
n(1/2*d*x + 1/2*c)^2 - tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^4 + 3*tan(d*x +
c)^4 + 6*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 7*tan(d*x + c)^2 + 4*tan(1
/2*d*x + 1/2*c)^2 + 4)*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(
d*x + c)^2 + tan(1/2*d*x + 1/2*c)^2 + 1))^n*tan(1/2*d*x + 1/2*c))/(d*n*tan(
-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/
2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c))
- 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/
2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*
d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x
+ 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x
 + 1/2*c)))^2*tan(1/2*d*x + 1/2*c)^2 + d*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x
+ 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d
*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*
d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)
*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/
2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*
x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)))^2*tan(1/2*d*x + 1/
2*c)^2 + d*n*tan(-1/4*pi*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn
(tan(1/2*d*x + 1/2*c)) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1
/2*d*x + 1/2*c)) - 1/4*pi*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1
/4*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) -
 1/4*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)
*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*
n*sgn(tan(1/2*d*x + 1/2*c)))^2 + d*n*tan(1/2*d*x + 1/2*c)^2 + d*tan(-1/4*pi
*n*sgn(4*a*tan(1/2*d*x + 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) +
 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*p
i*n*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) + pi*n*floor(1/4*sgn(4*a*tan(1/2*d*x +
 1/2*c)^2 - 4*a)*sgn(a)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*sgn(tan(1/2*d*x + 1
/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1/4*sgn(a)*sgn(tan(1/2*d*x + 1/2*c
)) - 1/4*sgn(tan(1/2*d*x + 1/2*c)) + 1/2) + 1/4*pi*n*sgn(tan(1/2*d*x + 1/2*
c)))^2 + d*tan(1/2*d*x + 1/2*c)^2 + d*n + d)
```

3.783 $\int \frac{-\frac{3B}{2}+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx$

Optimal. Leaf size=26

$$-\frac{B\sin(c+dx)}{2d(a\cos(c+dx)+a)^3}$$

```
[Out] -(B*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.0328949, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2749}

$$-\frac{B\sin(c+dx)}{2d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3*B)/2 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(B*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^3)
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int \frac{-\frac{3B}{2}+B\cos(c+dx)}{(a+a\cos(c+dx))^3}\,dx=-\frac{B\sin(c+dx)}{2d(a+a\cos(c+dx))^3}$$

Mathematica [A] time = 0.103346, size = 27, normalized size = 1.04

$$-\frac{B\sin(c+dx)}{2a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3*B)/2 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^3,x]
```

```
[Out] -(B*Sin[c + d*x])/(2*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.06, size = 48, normalized size = 1.9

$$\frac{B}{8\,da^3}\left(-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-2\,(\tan(1/2\,dx+c/2))^3-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3/2*B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^3,x)
```

```
[Out] 1/8/d*B/a^3*(-tan(1/2*d*x+1/2*c)^5-2*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c
))
```

Maxima [B] time = 1.06946, size = 155, normalized size = 5.96

$$-\frac{\frac{B\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}-\frac{2B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/40*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 2*B*(5*sin(d*x +
c)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [B] time = 1.33963, size = 135, normalized size = 5.19

$$-\frac{B\sin\left(dx+c\right)}{2\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*B*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)
```

Sympy [A] time = 3.66162, size = 80, normalized size = 3.08

$$\begin{cases} -\frac{B\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^3d}-\frac{B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{4a^3d}-\frac{B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8a^3d} & \text{for } d \neq 0 \\ \frac{x\left(B\cos\left(c\right)-\frac{3B}{2}\right)}{\left(a\cos\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))**3,x)
```

```
[Out] Piecewise((-B*tan(c/2 + d*x/2)**5/(8*a**3*d) - B*tan(c/2 + d*x/2)**3/(4*a**
3*d) - B*tan(c/2 + d*x/2)/(8*a**3*d), Ne(d, 0)), (x*(B*cos(c) - 3*B/2)/(a*c
os(c) + a)**3, True))
```

Giac [A] time = 1.32672, size = 63, normalized size = 2.42

$$-\frac{B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{8\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3/2*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(B*tan(1/2*d*x + 1/2*c)^5 + 2*B*tan(1/2*d*x + 1/2*c)^3 + B*tan(1/2*d*x
 + 1/2*c))/(a^3*d)
```

3.784 $\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

```
[Out] (2*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.0424421, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2749}

$$\frac{2B\sin(c+dx)(a\cos(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(3/2)*((-3*B)/5 + B*Cos[c + d*x]),x]
```

```
[Out] (2*B*(a + a*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{3/2}\left(-\frac{3B}{5}+B\cos(c+dx)\right)\,dx=\frac{2B(a+a\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

Mathematica [A] time = 0.103002, size = 45, normalized size = 1.61

$$\frac{8aB\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(3/2)*((-3*B)/5 + B*Cos[c + d*x]),x]
```

```
[Out] (8*a*B*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Cos[c + d*x])]*Sin[(c + d*x)/2])/(5*d
)
```

Maple [A] time = 0.658, size = 48, normalized size = 1.7

$$\frac{8\,a^2B\sqrt{2}}{5\,d}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(3/2)*(-3/5*B+B*cos(d*x+c)),x)
```

```
[Out] 8/5*cos(1/2*d*x+1/2*c)^5*a^2*sin(1/2*d*x+1/2*c)*B*2^(1/2)/(cos(1/2*d*x+1/2*
c)^2*a)^(1/2)/d
```

Maxima [B] time = 1.90831, size = 124, normalized size = 4.43

$$\frac{\left(\sqrt{2}a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+20\,\sqrt{2}a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}-2\left(\sqrt{2}a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,c\right.\right.}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/10*((sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 5*sqrt(2)*a*sin(3/2*d*x + 3/2*c) +
20*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a) - 2*(sqrt(2)*a*sin(3/2*d*x + 3
/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [A] time = 1.37911, size = 95, normalized size = 3.39

$$\frac{2\left(Ba\cos\left(dx+c\right)+Ba\right)\sqrt{a\cos\left(dx+c\right)+a}\sin\left(dx+c\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 2/5*(B*a*cos(d*x + c) + B*a)*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(3/2)*(-3/5*B+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(3/2)*(-3/5*B+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.785 $\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=26

$$\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

```
[Out] (2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rubi [A] time = 0.0155015, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {21, 2646}

$$\frac{2B\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(B + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{B+B\cos(c+dx)}{\sqrt{a+a\cos(c+dx)}}\,dx &= \frac{B\int \sqrt{a+a\cos(c+dx)}\,dx}{a}\\ &= \frac{2B\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0427348, size = 33, normalized size = 1.27

$$\frac{2B\tan\left(\frac{1}{2}(c+dx)\right)\sqrt{a(\cos(c+dx)+1)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B + B*Cos[c + d*x])/Sqrt[a + a*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/(a*d)
```

Maple [A] time = 0.904, size = 43, normalized size = 1.7

$$2\,{\frac{B\cos \left(1/2\,dx+c/2 \right) \sin \left(1/2\,dx+c/2 \right) \sqrt{2}}{\sqrt{ \left(\cos \left(1/2\,dx+c/2 \right) \right) ^{2}a}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/2),x)
```

```
[Out] 2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(cos(1/2*d*x+1/2*c)^2*a)^
(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.3451, size = 92, normalized size = 3.54

$$\frac{2 \, \sqrt{a \cos \left(d x + c\right) + a} B \sin \left(d x + c\right)}{a d \cos \left(d x + c\right) + a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B \left(\int \frac{\cos{\left (c + d x \right)}}{\sqrt{a \cos{\left (c + d x \right)} + a}}\, dx + \int \frac{1}{\sqrt{a \cos{\left (c + d x \right)} + a}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/2),x)
```

```
[Out] B*(Integral(cos(c + d*x)/sqrt(a*cos(c + d*x) + a), x) + Integral(1/sqrt(a*c
os(c + d*x) + a), x))
```

Giac [A] time = 2.44693, size = 47, normalized size = 1.81

$$\frac{2\sqrt{2}B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*B*tan(1/2*d*x + 1/2*c)/(sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

3.786 $\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{2B\sin(c+dx)}{3d(a\cos(c+dx)+a)^{5/2}}$$

```
[Out] (-2*B*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^(5/2))
```

Rubi [A] time = 0.0423808, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2749}

$$-\frac{2B\sin(c+dx)}{3d(a\cos(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((-5*B)/3 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*B*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^(5/2))
```

Rule 2749

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*
(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + 1), 0]
```

Rubi steps

$$\int \frac{-\frac{5B}{3}+B\cos(c+dx)}{(a+a\cos(c+dx))^{5/2}}\,dx=-\frac{2B\sin(c+dx)}{3d(a+a\cos(c+dx))^{5/2}}$$

Mathematica [A] time = 0.0671509, size = 28, normalized size = 1.

$$-\frac{2B\sin(c+dx)}{3d(a(\cos(c+dx)+1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-5*B)/3 + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*B*Sin[c + d*x])/(3*d*(a*(1 + Cos[c + d*x]))^(5/2))
```

Maple [A] time = 0.828, size = 48, normalized size = 1.7

$$-\frac{B\sqrt{2}}{6\,a^2d}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}\frac{1}{\sqrt{\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5/3*B+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(5/2),x)
```

```
[Out] -1/6/cos(1/2*d*x+1/2*c)^3/a^2*sin(1/2*d*x+1/2*c)*B*2^(1/2)/(cos(1/2*d*x+1/2
*c)^2*a)^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="maxima
")
```

```
[Out] Timed out
```

Fricas [B] time = 1.3452, size = 169, normalized size = 6.04

$$-\frac{2\sqrt{a\cos\left(dx+c\right)+a}B\sin\left(dx+c\right)}{3\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="fricas
")
```

```
[Out] -2/3*sqrt(a*cos(d*x + c) + a)*B*sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*
d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.49017, size = 80, normalized size = 2.86

$$-\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{\sqrt{2}B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{3}}+\frac{\sqrt{2}B}{a^{3}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5/3*B+B*cos(d*x+c))/(a+a*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*B*tan(1/2*d*x + 1/2*c)^2/a
^3 + sqrt(2)*B/a^3)*tan(1/2*d*x + 1/2*c)/d
```

3.787 $\int (a+a\cos (c+dx))^{2/3}(A+B\cos (c+dx))\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt[6]{2}(5A+2B)\sin (c+dx)(a\cos (c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos (c+dx))\right)}{5d(\cos (c+dx)+1)^{7/6}}+\frac{3B\sin (c+dx)(a\cos (c+dx)+a)^{2/3}}{5d}$$

```
[Out] (3*B*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) + (2*2^(1/6)*(5*A + 2*B
)*(a + a*Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c +
 d*x])/2]*Sin[c + d*x])/(5*d*(1 + Cos[c + d*x])^(7/6))
```

Rubi [A] time = 0.0819791, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2652, 2651}

$$\frac{2\sqrt[6]{2}(5A+2B)\sin (c+dx)(a\cos (c+dx)+a)^{2/3}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos (c+dx))\right)}{5d(\cos (c+dx)+1)^{7/6}}+\frac{3B\sin (c+dx)(a\cos (c+dx)+a)^{2/3}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*B*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) + (2*2^(1/6)*(5*A + 2*B
)*(a + a*Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c +
 d*x])/2]*Sin[c + d*x])/(5*d*(1 + Cos[c + d*x])^(7/6))
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{3B(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{1}{5}(5A+2B)\int (a+a\cos(c+$$
$$= \frac{3B(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{\big((5A+2B)(a+a\cos(c+dx)\big)}{5(1+\cos(}$$
$$= \frac{3B(a+a\cos(c+dx))^{2/3}\sin(c+dx)}{5d} + \frac{2\sqrt[6]{2}(5A+2B)(a+a\cos(c+}{}$$

Mathematica [A] time = 0.591884, size = 164, normalized size = 1.58

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+1))^{2/3}\left(3\,2^{5/6}\sin(c+dx)(5A+2B\cos(c+dx)+4B)\sqrt[6]{1-\cos\left(dx-2\tan^{-1}\left(\cot\left(\frac{c}{2}\right.\right.\right.}}{20\,2^{5/6}d\sqrt[6]{1-\cos\left(dx-2\tan^{-1}\left(\mathrm{co}\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*(3*2^(5/6)*(5*A + 4*B + 2*
B*Cos[c + d*x])*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] - 2*
(5*A + 2*B)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/2]]
]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]]))/(20*2^(5/6)*d*(1 - Cos[d*x - 2*ArcTan[
Cot[c/2]]])^(1/6))
```

Maple [F] time = 0.246, size = 0, normalized size = 0.

$$\int (a+\cos(dx+c)\,a)^{\frac{2}{3}}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)
```

3.788 $\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{(4A+B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}d(\cos(c+dx)+1)^{5/6}}+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

```
[Out] (3*B*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) + ((4*A + B)*(a + a*Cos
[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin
[c + d*x])/(2*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))
```

Rubi [A] time = 0.0789982, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2652, 2651}

$$\frac{(4A+B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2\sqrt[6]{2}d(\cos(c+dx)+1)^{5/6}}+\frac{3B\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (3*B*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) + ((4*A + B)*(a + a*Cos
[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin
[c + d*x])/(2*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))
```


Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt[3]{a+a\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{3B\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{1}{4}(4A+B)\int \sqrt[3]{a+a\cos(c+dx)}\,dx$$
$$= \frac{3B\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left((4A+B)\sqrt[3]{a+a\cos(c+dx)}\right)\int \sqrt[3]{1}}{4\sqrt[3]{1+\cos(c+dx)}}$$
$$= \frac{3B\sqrt[3]{a+a\cos(c+dx)}\sin(c+dx)}{4d} + \frac{(4A+B)\sqrt[3]{a+a\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\right.}{2\sqrt[6]{2}d(1+\cos}$$

Mathematica [C] time = 3.16901, size = 213, normalized size = 2.09

$$\frac{3\sqrt[3]{a(\cos(c+dx)+1)}\left(\frac{2(4A+B)\csc\left(\frac{c}{4}\right)\sec\left(\frac{c}{4}\right)\sqrt[3]{i\sin(c)e^{idx}+\cos(c)e^{idx}+1}\left(2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-e^{idx}(\cos(c)+i\sin(c))\right)+e^{idx}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-e^{idx}(\cos(c)+i\sin(c))\right)\right)}{i\sin\left(\frac{c}{2}\right)\left(-1+e^{idx}\right)+\cos\left(\frac{c}{2}\right)\left(1+e^{idx}\right)}\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*(a*(1 + Cos[c + d*x]))^(1/3)*(-8*(4*A + B)*Cot[c/2] + 8*B*Cos[d*x]*Sin[c
] + (2*(4*A + B)*Csc[c/4]*(2*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^(I*d*x)*
(Cos[c] + I*Sin[c]))] + E^(I*d*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -(E^(I*d
*x)*(Cos[c] + I*Sin[c]))])*Sec[c/4]*(1 + E^(I*d*x)*Cos[c] + I*E^(I*d*x)*Sin
[c])^(1/3))/((1 + E^(I*d*x))*Cos[c/2] + I*(-1 + E^(I*d*x))*Sin[c/2]) + 8*B*
Cos[c]*Sin[d*x]))/(32*d)
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+\cos(dx+c)\,a}\,(A+B\cos(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)(a\cos(dx+c)+a)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a\left(\cos\left(c+dx\right)+1\right)}\left(A+B\cos\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((a*(cos(c + d*x) + 1))**(1/3)*(A + B*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos\left(dx+c\right)+A)(a\cos\left(dx+c\right)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)
```

3.789 $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{(2A-B)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}}+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

```
[Out] (3*B*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(1/3)) + ((2*A - B)*Hypergeome
tric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(5/6)*d*(1 +
Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rubi [A] time = 0.0771416, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2751, 2652, 2651}

$$\frac{(2A-B)\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{2^{5/6}d\sqrt[6]{\cos(c+dx)+1}\sqrt[3]{a\cos(c+dx)+a}}+\frac{3B\sin(c+dx)}{2d\sqrt[3]{a\cos(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*B*Sin[c + d*x])/(2*d*(a + a*Cos[c + d*x])^(1/3)) + ((2*A - B)*Hypergeome
tric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2^(5/6)*d*(1 +
Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))
```

Rule 2751

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+a\cos(c+dx)}}\,dx = \frac{3B\sin(c+dx)}{2d\sqrt[3]{a+a\cos(c+dx)}} + \frac{1}{2}(2A-B)\int \frac{1}{\sqrt[3]{a+a\cos(c+dx)}}\,dx$$
$$= \frac{3B\sin(c+dx)}{2d\sqrt[3]{a+a\cos(c+dx)}} + \frac{\left((2A-B)\sqrt[3]{1+\cos(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\cos(c+dx)}}\,dx}{2\sqrt[3]{a+a\cos(c+dx)}}$$
$$= \frac{3B\sin(c+dx)}{2d\sqrt[3]{a+a\cos(c+dx)}} + \frac{(2A-B)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)\sin(c+dx)}{2^{5/6}d\sqrt[6]{1+\cos(c+dx)}\sqrt[3]{a+a\cos(c+dx)}}$$

Mathematica [A] time = 0.2581, size = 133, normalized size = 1.32

$$\frac{3\ 2^{5/6}B\sin(c+dx)\sqrt[6]{1-\cos\left(dx-2\tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right)}-2(2A-B)\sin\left(dx-2\tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right){}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\cos^2\left(\frac{dx}{2}-\right.\right.}{4d\sqrt[3]{a(\cos(c+dx)+1)}\sqrt[6]{\sin^2\left(\frac{dx}{2}-\tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*2^(5/6)*B*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] - 2*(2*
A - B)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*
Sin[d*x - 2*ArcTan[Cot[c/2]]])/(4*d*(a*(1 + Cos[c + d*x]))^(1/3)*(Sin[(d*x)
/2 - ArcTan[Cot[c/2]]]^2)^(1/6))
```

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))\frac{1}{\sqrt[3]{a+\cos(dx+c)\,a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a(\cos(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a*(cos(c + d*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)
```

3.790 $\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=105

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{2^{5/6}(A-2B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{ad(\cos(c+dx)+1)^{5/6}}$$

```
[Out] (3*(A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) - (2^(5/6)*(A - 2*B
)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c +
d*x])/2]*Sin[c + d*x])/(a*d*(1 + Cos[c + d*x])^(5/6))
```

Rubi [A] time = 0.0894841, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {2750, 2652, 2651}

$$\frac{3(A-B)\sin(c+dx)}{d(a\cos(c+dx)+a)^{2/3}}-\frac{2^{5/6}(A-2B)\sin(c+dx)\sqrt[3]{a\cos(c+dx)+a}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right)}{ad(\cos(c+dx)+1)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*(A - B)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) - (2^(5/6)*(A - 2*B
)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c +
d*x])/2]*Sin[c + d*x])/(a*d*(1 + Cos[c + d*x])^(5/6))
```

Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+a\cos(c+dx))^{2/3}}\,dx = \frac{3(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} - \frac{(A-2B)\int \sqrt[3]{a+a\cos(c+dx)}\,dx}{a}$$
$$= \frac{3(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} - \frac{\left((A-2B)\sqrt[3]{a+a\cos(c+dx)}\right)\int \sqrt[3]{1+\cos(c+dx)}\,dx}{a\sqrt[3]{1+\cos(c+dx)}}$$
$$= \frac{3(A-B)\sin(c+dx)}{d(a+a\cos(c+dx))^{2/3}} - \frac{2^{5/6}(A-2B)\sqrt[3]{a+a\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx))\right) s}{ad(1+\cos(c+dx))^{5/6}}$$

Mathematica [C] time = 1.30371, size = 197, normalized size = 1.88

$$\frac{3\cos\left(\frac{1}{2}(c+dx)\right)\left(-4\csc\left(\frac{c}{2}\right)\left((3B-2A)\cos\left(\frac{dx}{2}\right)+B\cos\left(c+\frac{dx}{2}\right)\right)-(A-2B)\csc\left(\frac{c}{4}\right)\sec\left(\frac{c}{4}\right)e^{-\frac{1}{2}idx}\sqrt[3]{i\sin(c)e^{idx}+\cos(c}\right.}{4d(a(\cos(c+dx)+1))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + a*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*Cos[(c + d*x)/2]*(-4*((-2*A + 3*B)*Cos[(d*x)/2] + B*Cos[c + (d*x)/2])*Cs
c[c/2] - ((A - 2*B)*Csc[c/4]*(2*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^(I*d*
x)*(Cos[c] + I*Sin[c]))] + E^(I*d*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -(E^(
I*d*x)*(Cos[c] + I*Sin[c]))])*Sec[c/4]*(1 + E^(I*d*x)*Cos[c] + I*E^(I*d*x)*
Sin[c])^(1/3))/E^((I/2)*d*x)))/(4*d*(a*(1 + Cos[c + d*x]))^(2/3))
```

Maple [F] time = 0.219, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))(a+\cos(dx+c)a)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))/(a+cos(d*x+c)*a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(a\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)
```

3.791 $\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

```
[Out] (B*x)/b - (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a*b*d)
```

Rubi [A] time = 0.0931029, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2735, 2659, 205}

$$\frac{Bx}{b}-\frac{2B\sqrt{a-b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*x)/b - (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(a*b*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\frac{bB}{a}+B\cos(c+dx)}{a+b\cos(c+dx)}\,dx = \frac{Bx}{b} - \frac{\left(aB-\frac{b^2B}{a}\right)\int \frac{1}{a+b\cos(c+dx)}\,dx}{b}$$
$$= \frac{Bx}{b} - \frac{\left(2\left(a-\frac{b^2}{a}\right)B\right)\text{Subst}\left(\int \frac{1}{a+b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$
$$= \frac{Bx}{b} - \frac{2\sqrt{a-b}\sqrt{a+b}B\tan^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{abd}$$

Mathematica [A] time = 0.115834, size = 64, normalized size = 1.02

$$\frac{B\left(2\sqrt{b^2-a^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^2-a^2}}\right)+a(c+dx)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Cos[c + d*x])/(a + b*Cos[c + d*x]),x]
```

```
[Out] (B*(a*(c + d*x) + 2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[-a^2 + b^2]]))/(a*b*d)
```

Maple [B] time = 0.11, size = 117, normalized size = 1.9

$$2\,\frac{B\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{bd}-2\,\frac{aB}{bd\sqrt{(a-b)(a+b)}}\arctan\left(\frac{\tan\left(1/2\,dx+c/2\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)+2\,\frac{bB}{da\sqrt{(a-b)(a+b)}}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] 2/d*B/b*arctan(tan(1/2*d*x+1/2*c))-2/d*B*a/b/((a-b)*(a+b))^(1/2)*arctan(tan
(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d*B/a*b/((a-b)*(a+b))^(1/2)*ar
ctan(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52444, size = 437, normalized size = 6.94

$$\left[\frac{2\,Badx + \sqrt{-a^2+b^2}B\log\left(\frac{2\,ab\cos(dx+c)+\left(2\,a^2-b^2\right)\cos(dx+c)^2+2\,\sqrt{-a^2+b^2}(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\,abd}, \frac{Badx - \sqrt{a^2-b^2}B\,\mathrm{arctan}}{abd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*d*x + sqrt(-a^2 + b^2)*B*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2
)*cos(d*x + c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a
^2 + 2*b^2)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/(a*b*d), (B*a
*d*x - sqrt(a^2 - b^2)*B*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(
d*x + c))))/(a*b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19166, size = 146, normalized size = 2.32

$$\frac{\frac{(dx+c)B}{b} + \frac{2\left(Ba^2-Bb^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*cos(d*x+c))/(a+b*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*B/b + 2*(B*a^2 - B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2
*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(
a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b))/d
```

3.792 $\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx$

Optimal. Leaf size=22

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+b)}$$

```
[Out] Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.0311741, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2754, 8}

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])/(b + a*Cos[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cos(c+dx)}{(b+a\cos(c+dx))^2}\,dx &= \frac{\sin(c+dx)}{d(b+a\cos(c+dx))}+\frac{\int 0\,dx}{a^2-b^2}\\ &= \frac{\sin(c+dx)}{d(b+a\cos(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0918501, size = 22, normalized size = 1.

$$\frac{\sin(c+dx)}{d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cos[c + d*x])/(b + a*Cos[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]/(d*(b + a*Cos[c + d*x]))
```

Maple [B] time = 0.052, size = 51, normalized size = 2.3

$$-2\frac{\tan(1/2\,dx+c/2)}{d\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))/(b+cos(d*x+c)*a)^2,x)
```

```
[Out] -2/d*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36325, size = 53, normalized size = 2.41

$$\frac{\sin(dx+c)}{ad\cos(dx+c)+bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(a*d*cos(d*x + c) + b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2601, size = 68, normalized size = 3.09

$$-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))/(b+a*cos(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)
^2 - a - b)*d)
```

3.793 $\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx$

Optimal. Leaf size=47

$$\frac{10\tan^{-1}\left(\frac{\sin(c+dx)}{-\cos(c+dx)+\sqrt{3}+2}\right)}{\sqrt{3}d}+\frac{5x}{\sqrt{3}}-x$$

```
[Out] -x + (5*x)/Sqrt[3] + (10*ArcTan[Sin[c + d*x]/(2 + Sqrt[3] - Cos[c + d*x])])
/(Sqrt[3]*d)
```

Rubi [A] time = 0.0670049, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2735, 2657}

$$\frac{10\tan^{-1}\left(\frac{\sin(c+dx)}{-\cos(c+dx)+\sqrt{3}+2}\right)}{\sqrt{3}d}+\frac{5x}{\sqrt{3}}-x$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Cos[c + d*x])/(2 - Cos[c + d*x]),x]
```

```
[Out] -x + (5*x)/Sqrt[3] + (10*ArcTan[Sin[c + d*x]/(2 + Sqrt[3] - Cos[c + d*x])])
/(Sqrt[3]*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\begin{aligned}\int \frac{3+\cos(c+dx)}{2-\cos(c+dx)}\,dx &= -x+5\int \frac{1}{2-\cos(c+dx)}\,dx\\ &= -x+\frac{5x}{\sqrt{3}}+\frac{10\tan^{-1}\left(\frac{\sin(c+dx)}{2+\sqrt{3}-\cos(c+dx)}\right)}{\sqrt{3}d}\end{aligned}$$

Mathematica [A] time = 0.0538518, size = 31, normalized size = 0.66

$$\frac{10\tan^{-1}\left(\sqrt{3}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{3}d}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + Cos[c + d*x])/(2 - Cos[c + d*x]),x]
```

```
[Out] -x + (10*ArcTan[Sqrt[3]*Tan[(c + d*x)/2]])/(Sqrt[3]*d)
```

Maple [A] time = 0.093, size = 39, normalized size = 0.8

$$-2\,{\frac{\arctan \left(\tan \left(1/2\,dx+c/2 \right) \right) }{d}}+{\frac{10\,\sqrt{3}}{3\,d}\arctan \left(\tan \left({\frac{dx}{2}}+{\frac{c}{2}} \right) \sqrt{3} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+cos(d*x+c))/(2-cos(d*x+c)),x)
```

```
[Out] -2/d*arctan(tan(1/2*d*x+1/2*c))+10/3/d*3^(1/2)*arctan(tan(1/2*d*x+1/2*c)*3^
(1/2))
```

Maxima [A] time = 1.57573, size = 70, normalized size = 1.49

$$\frac{2 \left(5 \sqrt{3} \arctan\left(\frac{\sqrt{3} \sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right) - 3 \arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)\right)}{3 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(5*sqrt(3)*arctan(sqrt(3)*sin(d*x + c)/(cos(d*x + c) + 1)) - 3*arctan(s
in(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.39132, size = 119, normalized size = 2.53

$$-\frac{3 \, dx + 5 \, \sqrt{3} \arctan\left(\frac{2 \, \sqrt{3} \cos\left(dx+c\right) - \sqrt{3}}{3 \, \sin\left(dx+c\right)}\right)}{3 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*d*x + 5*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(d*x + c) - sqrt(3))/sin(d
*x + c)))/d
```

Sympy [A] time = 7.03786, size = 56, normalized size = 1.19

$$\begin{cases} -x + \frac{10 \sqrt{3} \left(\operatorname{atan}{\left (\sqrt{3} \tan{\left (\frac{c}{2} + \frac{dx}{2} \right)} \right)} + \pi \left\lfloor{\frac{\frac{c}{2} + \frac{dx}{2} - \frac{\pi}{2}}{\pi}}\right\rfloor\right)}{3d} & \text{for}\: d \neq 0 \\ \frac{x \left(\cos{\left (c \right)} + 3\right)}{2 - \cos{\left (c \right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x)
```

```
[Out] Piecewise((-x + 10*sqrt(3)*(atan(sqrt(3)*tan(c/2 + d*x/2)) + pi*floor((c/2
+ d*x/2 - pi/2)/pi))/(3*d), Ne(d, 0)), (x*(cos(c) + 3)/(2 - cos(c)), True))
```

Giac [A] time = 1.28236, size = 97, normalized size = 2.06

$$-\frac{3\,dx-5\,\sqrt{3}\left(dx+c+2\,\arctan\left(-\frac{\sqrt{3}\sin(dx+c)-3\,\sin(dx+c)}{\sqrt{3}\cos(dx+c)+\sqrt{3}-3\,\cos(dx+c)+3}\right)\right)+3\,c}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+cos(d*x+c))/(2-cos(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*d*x - 5*sqrt(3)*(d*x + c + 2*arctan(-(sqrt(3)*sin(d*x + c) - 3*sin(
d*x + c))/(sqrt(3)*cos(d*x + c) + sqrt(3) - 3*cos(d*x + c) + 3))) + 3*c)/d
```

3.794 $\int \frac{aB+bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])
```

Rubi [A] time = 0.0447661, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {21, 2655, 2653}

$$\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*B + b*B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2655

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
*Sin[c + d*x])/(a + b)], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2653

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*Sqrt[a
 + b]*EllipticE[(1*(c - Pi/2 + d*x))/2, (2*b)/(a + b)])/d, x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rubi steps

$$\int \frac{aB + bB\cos(c+dx)}{\sqrt{a+b\cos(c+dx)}}\,dx = B\int \sqrt{a+b\cos(c+dx)}\,dx$$
$$= \frac{\left(B\sqrt{a+b\cos(c+dx)}\right)\int \sqrt{\frac{a}{a+b}+\frac{b\cos(c+dx)}{a+b}}\,dx}{\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$
$$= \frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 0.0683982, size = 58, normalized size = 1.

$$\frac{2B\sqrt{a+b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cos(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*B + b*B*Cos[c + d*x])/Sqrt[a + b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[a + b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*b)/(a + b)])/(d*Sqr
t[(a + b*Cos[c + d*x])/(a + b)])
```

Maple [B] time = 2.642, size = 171, normalized size = 3.

$$-2\frac{\sqrt{\left(2b\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+a-b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}B\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\left(a-b\right)}{\sqrt{-2b\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+\left(a+b\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sin\left(1/2\,dx+c/2\right)\sqrt{-2\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}b+a+b}d}\sqrt{2b(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)*sin(1/2*d*x+1/2*c)^2)^(1/2)*B*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*((2*b*cos(1/2*d*x+1/2*c)^2+a-b)/(a-b))^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),(-2*b/(a-b))^(1/2))*(a-b)/(-2*b*sin(1/2*d*x+1/2*c)^4+(a+b
)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^2*b
+a+b)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bb\cos\left(dx+c\right)+Ba}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/sqrt(b*cos(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)+a}B,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*cos(d*x + c) + a)*B, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$B\int\sqrt{a+b\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/2),x)
```

```
[Out] B*Integral(sqrt(a + b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{Bb\cos\left(dx+c\right)+Ba}{\sqrt{b\cos\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B+b*B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*b*cos(d*x + c) + B*a)/sqrt(b*cos(d*x + c) + a), x)
```

3.795 $\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=229

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{\sqrt{2}B(a+b)\sin(c+dx)(}{}$$

```
[Out] (Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(
1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*S
qrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b
 - a*B)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rubi [A] time = 0.222851, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)(a+b\cos(c+dx))^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}+\frac{\sqrt{2}B(a+b)\sin(c+dx)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(
1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*S
qrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b
 - a*B)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```


Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a+b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{B\int (a+b\cos(c+dx))^{5/3}\,dx}{b}+\frac{(Ab-aB)\int (a+b\cos(c+dx))^{2/3}\,dx}{b}$$

$$=-\frac{(B\sin(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx,x,\cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}}-\frac{((Ab-aB)}{}$$

$$=\frac{\left((-a-b)B(a+b\cos(c+dx))^{2/3}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{5/3}}{\sqrt{1-x}\sqrt{1+x}}\right.}{bd\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)}$$

$$=\frac{\sqrt{2}(a+b)BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{5}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos}{bd\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}}$$

Mathematica [A] time = 1.87, size = 259, normalized size = 1.13

$$\frac{3(a+b\cos(c+dx))^{2/3}\left(5B\left(a^2-b^2\right)\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*(a + b*Cos[c + d*x])^(2/3)*(5*(a^2 - b^2)*B*AppellF1[2/3, 1/2, 1/2, 5/3,
 (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1
+ Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*Csc[c +
d*x] - (5*A*b + 2*a*B)*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a
 - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b)
)]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x])*Csc[c + d*x]
+ 5*b^2*B*Sin[c + d*x]))/(25*b^2*d)
```

Maple [F] time = 0.221, size = 0, normalized size = 0.

$$\int (a+b\cos(dx+c))^{\frac{2}{3}}(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

3.796 $\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=229

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}B(a+b)\sin(c+dx)}{}$$

```
[Out] (Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(
1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*S
qrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b
 - a*B)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rubi [A] time = 0.189073, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\sqrt[3]{a+b\cos(c+dx)}F_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}+\frac{\sqrt{2}B(a+b)\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[2]*(a + b)*B*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(
1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*S
qrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b
 - a*B)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c +
 d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Cos
[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sqrt[3]{a+b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{B\int (a+b\cos(c+dx))^{4/3}\,dx}{b} + \frac{(Ab-aB)\int \sqrt[3]{a+b\cos(c+dx)}\,dx}{b}$$

$$= -\frac{(B\sin(c+dx))\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} - \frac{((Ab-aB)\sin(}{bd}$$

$$= \frac{\left((-a-b)B\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{4/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x,\right.}{bd\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}}$$

$$= \frac{\sqrt{2}(a+b)BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c}}{bd\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}}$$

Mathematica [A] time = 1.74165, size = 253, normalized size = 1.1

$$-\frac{3\csc(c+dx)\sqrt[3]{a+b\cos(c+dx)}\left(4B\left(b^2-a^2\right)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)+\right.}{16}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*B*AppellF1[1/3,
 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]
*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a
- b))] + (4*A*b + a*B)*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a
 - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b)
)]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*B*Sin
[c + d*x]^2))/(16*b^2*d)
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+b\cos(dx+c)}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A+B\cos(c+dx))\sqrt[3]{a+b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*(a + b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c)+a)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```

3.797 $\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}+\frac{\sqrt{2}B\sin(c+dx)(a+b\cos(c+d}{bd\sqrt{\cos}}$$

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[
c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 +
Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b - a*B)*
AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(
a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Co
s[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rubi [A] time = 0.183003, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}F_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}\sqrt[3]{a+b\cos(c+dx)}}+\frac{\sqrt{2}B\sin(c+dx)(a+b\cos(c+d}{bd\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[
c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 +
Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + (Sqrt[2]*(A*b - a*B)*
AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(
a + b)]*((a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Co
s[c + d*x]]*(a + b*Cos[c + d*x])^(1/3))
```

Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx = \frac{B\int (a+b\cos(c+dx))^{2/3}\,dx}{b} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a+b\cos(c+dx)}}\,dx}{b}$$

$$= -\frac{(B\sin(c+dx))\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} - \frac{((Ab-aB)\sin(c+dx))\operatorname{Subst}\Big(}{bd\sqrt{1-\cos(c+}}$$

$$= -\frac{\left(B(a+b\cos(c+dx))^{2/3}\sin(c+dx)\right)\operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}-\frac{bx}{-a-b}\right)^{2/3}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\left(-\frac{a+b\cos(c+dx)}{-a-b}\right)^{2/3}} - \Big((Ab$$

$$= \frac{\sqrt{2}BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)(a+b\cos(c+dx))^{2/3}\sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}} +$$

Mathematica [A] time = 0.413311, size = 189, normalized size = 0.84

$$-\frac{3\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}(a+b\cos(c+dx))^{2/3}\left(5(Ab-aB)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)\right.}{10b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-
a + b)]*(a + b*Cos[c + d*x])^(2/3)*(5*(A*b - a*B)*AppellF1[2/3, 1/2, 1/2, 5
/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)] + 2*B*Appel
lF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/
(a + b)]*(a + b*Cos[c + d*x]))*Csc[c + d*x])/(10*b^2*d)
```

Maple [F] time = 0.404, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))\frac{1}{\sqrt[3]{a+b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+B\cos(c+dx)}{\sqrt[3]{a+b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(a + b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)
```

3.798 $\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=226

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}+\frac{\sqrt{2}B\sin(c+dx)\sqrt[3]{a+b\cos}}{b}$$

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[
c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 +
Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b - a*B)*
AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(
a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Co
s[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```

Rubi [A] time = 0.185021, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {2756, 2665, 139, 138}

$$\frac{\sqrt{2}(Ab-aB)\sin(c+dx)\left(\frac{a+b\cos(c+dx)}{a+b}\right)^{2/3}F_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)}{bd\sqrt{\cos(c+dx)+1}(a+b\cos(c+dx))^{2/3}}+\frac{\sqrt{2}B\sin(c+dx)\sqrt[3]{a+b\cos}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(2/3),x]
```


```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[
c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[1 +
Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(A*b - a*B)*
AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(
a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(b*d*Sqrt[1 + Co
s[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))
```


Rule 2756

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(a+b\cos(c+dx))^{2/3}}\,dx = \frac{B\int \sqrt[3]{a+b\cos(c+dx)}\,dx}{b} + \frac{(Ab-aB)\int \frac{1}{(a+b\cos(c+dx))^{2/3}}\,dx}{b}$$

$$= -\frac{(B\sin(c+dx))\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}} - \frac{((Ab-aB)\sin(c+dx))\,\text{Subst}\left(\int \right.}{bd\sqrt{1-\cos(c+}}$$

$$= -\frac{\left(B\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[3]{-\frac{a}{-a-b}-\frac{bx}{-a-b}}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x, \cos(c+dx)\right)}{bd\sqrt{1-\cos(c+dx)}\sqrt{1+\cos(c+dx)}\sqrt[3]{-\frac{a+b\cos(c+dx)}{-a-b}}} - \frac{\left((Ab-aB\right.}{}$$

$$= \frac{\sqrt{2}BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\cos(c+dx)),\frac{b(1-\cos(c+dx))}{a+b}\right)\sqrt[3]{a+b\cos(c+dx)}\sin(c+dx)}{bd\sqrt{1+\cos(c+dx)}\sqrt[3]{\frac{a+b\cos(c+dx)}{a+b}}} + \frac{\sqrt{2}}{}$$

Mathematica [A] time = 0.418399, size = 188, normalized size = 0.83

$$-\frac{3\csc(c+dx)\sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}}\sqrt{\frac{b(\cos(c+dx)+1)}{b-a}}\sqrt[3]{a+b\cos(c+dx)}\left(4(Ab-aB)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{a+b\cos(c+dx)}{a-b},\frac{a+b\cos(c+dx)}{a+b}\right)+B(\right.}{4b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x])/(a + b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-
a + b)]*(a + b*Cos[c + d*x])^(1/3)*(4*(A*b - a*B)*AppellF1[1/3, 1/2, 1/2, 4
/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)] + B*AppellF
1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a
 + b)]*(a + b*Cos[c + d*x]))*Csc[c + d*x])/(4*b^2*d)
```

Maple [F] time = 0.321, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))(a+b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c)+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)
```

3.799 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=168

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{10B\sin(c+d}{}$$

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (10*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos
[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*(b*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c
+ d*x])/(7*b^2*d)
```

Rubi [A] time = 0.142352, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{10B\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```


```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (10*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos
[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*(b*Cos
[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c
+ d*x])/(7*b^2*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx &= \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b^2}\\ &= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b^2}+\frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^3}\\ &= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+}{7b^2d}\\ &= \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}+\frac{2A(b\cos(c+dx))^{3/2}\sin(c+}{5bd}\\ &= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c}{21d}\\ &= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{10bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{21d\sqrt{b\cos(c+d}}\end{aligned}$$

Mathematica [A] time = 0.516783, size = 100, normalized size = 0.6

$$\frac{\sqrt{b\cos(c+dx)}\left(2\sin(c+dx)\sqrt{\cos(c+dx)}(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252AE\left(\frac{1}{2}(c+dx)\middle|2\right)+10\right.}{210d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[(c
 + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2
*(c + d*x)])*Sin[c + d*x]))/(210*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 3.447, size = 299, normalized size = 1.8

$$-\frac{2b}{105d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8+(-168\,A-36\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(240*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-
42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] Timed out
```

3.800 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=139

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6BE\left(\frac{1}{2}(c+dx)\right|}{5d\sqrt{\text{co}}}$$

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*(b*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rubi [A] time = 0.116791, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6BE\left(\frac{1}{2}(c+dx)\right|}{5d\sqrt{\text{co}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```


```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*(b*Cos[c +
 d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx &= \frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx}{b}\\ &= \frac{A\int (b\cos(c+dx))^{3/2}\,dx}{b}+\frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^2}\\ &= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+d}{5bd}\\ &= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+d}{5bd}\\ &= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.281795, size = 91, normalized size = 0.65

$$\frac{2(b\cos(c+dx))^{3/2}\left(\sin(c+dx)\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c
 + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/
(15*b*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 3.091, size = 271, normalized size = 2.

$$-\frac{2b}{15d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+(20\,A+24\,B)\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-24*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c
)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c
)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

3.801 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=108

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
 + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
 d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0835939, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
 + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
 d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = A\int \sqrt{b\cos(c+dx)}\,dx + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b}$$
$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}(bB)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx + \frac{\left(A\sqrt{b\cos(\right.}{}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{(bB}{}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} +$$

Mathematica [A] time = 0.103184, size = 75, normalized size = 0.69

$$\frac{2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c +
d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.964, size = 238, normalized size = 2.2

$$\frac{2b}{3d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,B\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^4+3\,A\sqrt{(\sin(1/2\,dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-4*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+2*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

3.802 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=80

$$\frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
 + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d
*x]])
```

Rubi [A] time = 0.0867523, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```


```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
 + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d
*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= (Ab)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx + B\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{\left(Ab\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c}$$
$$= \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+d\right.}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0759697, size = 55, normalized size = 0.69

$$\frac{2b\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x
)/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.906, size = 161, normalized size = 2.

$$-2\,\frac{\sqrt{b\left(2\,\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin(1/2\,dx+c/2)\right)^2}b\sqrt{\left(\sin(1/2\,dx+c/2)\right)^2}\sqrt{-2\,\left(\cos(1/2\,dx+c/2)\right)^2+1}\left(A\mathrm{EllipticF}\right.}{\sqrt{-b\left(2\,\left(\sin(1/2\,dx+c/2)\right)^4-\left(\sin(1/2\,dx+c/2)\right)^2\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\,(c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

3.803 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=105

$$\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c +
d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.117293, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c +
d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + (bB)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - A\int \sqrt{b\cos(c+dx)}\,dx + \frac{\left(bB\sqrt{\cos(c+dx}\right.}{\sqrt{b\cos}}$$
$$= \frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{b}\right.}{}$$
$$= -\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{d\sqrt{b\cos(c+d}}$$

Mathematica [A] time = 0.171938, size = 73, normalized size = 0.7

$$\frac{2\sqrt{b\cos(c+dx)}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c +
d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 3.575, size = 213, normalized size = 2.

$$-2\,\frac{b\sqrt{-2\,b\left(\sin\left(1/2\,dx+c/2\right)\right)^4+\left(\sin\left(1/2\,dx+c/2\right)\right)^2b}\left(A\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^2}\sqrt{2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^2-1}\mathrm{EllipticE}\right.}{\sqrt{-b\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^4\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -2*b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1
```

```
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/
2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

3.804 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=136

$$\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b*B*Sin
[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.131399, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*A*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b*B*Sin
[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx+\left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{1}{3}(Ab)\int \frac{1}{\sqrt{b\cos(c+}\\ &= \frac{2Ab^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{\left(Ab\sqrt{\cos(c+dx)}\right)\int}{3\sqrt{b\cos(c+}}\\ &= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.239538, size = 85, normalized size = 0.62

$$\frac{2b\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*Sq
rt[b*Cos[c + d*x]])
```

Maple [B] time = 4.171, size = 453, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -2/3*(12*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
```

```
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*b*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))*b/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

3.805 $\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=169

$$\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sqrt{\cos(c+}}{3d\sqrt{b}}$$

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*B
*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b*Sin[c + d*x])/(5*d*Sqr
t[b*Cos[c + d*x]])
```

Rubi [A] time = 0.16253, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2bB\sqrt{\cos(c+}}{3d\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*B
*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b*Sin[c + d*x])/(5*d*Sqr
t[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\sec^4(c+dx)\,dx = b^4\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \left(Ab^4\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \left(b^3B\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{5}\left(3Ab^2\right)\int \frac{}{(b\cos}$$
$$= \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6Ab\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$
$$= \frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^2}{3d(b}$$
$$= -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+\right.}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.447722, size = 107, normalized size = 0.63

$$\frac{2\sec^2(c+dx)\sqrt{b\cos(c+dx)}\left(\frac{9}{2}A\sin(2(c+dx))+3A\tan(c+dx)-9A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sin(c+dx)+\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(-9*A*Cos[c + d*x]^(3/2)*EllipticE[(
c + d*x)/2, 2] + 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*B*Sin
[c + d*x] + (9*A*Sin[2*(c + d*x)])/2 + 3*A*Tan[c + d*x]))/(15*d)
```

Maple [B] time = 7.365, size = 575, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*
c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*
x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)
^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)
)-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

3.806 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=169

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{10b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2B\,\mathrm{si}}{}$$

```
[Out] (6*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (10*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (10*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*
(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Si
n[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.138917, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{10b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2B\,\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (10*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (10*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*A*
(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Si
n[c + d*x])/(7*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx &= \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b}\\
&= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^2}\\
&= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+ \\
&= \frac{10bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2A(b\cos(c+dx))^{3/2}\sin(c+ \\
&= \frac{6Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10bB\sqrt{b\cos(c+dx)}\sin(\\
&= \frac{6Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(\right.}{21d\sqrt{b\cos(c+d}}\end{aligned}$$

Mathematica [A] time = 0.291485, size = 103, normalized size = 0.61

$$\frac{(b\cos(c+dx))^{5/2}\left(2\sin(c+dx)\sqrt{\cos(c+dx)}(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252AE\left(\frac{1}{2}(c+dx)\middle|2\right)+10\right.}{210bd\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[
(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos
[2*(c + d*x)])*Sin[c + d*x]))/(210*b*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 3.297, size = 301, normalized size = 1.8

$$-\frac{2b^2}{105d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8+(-168\,A-360\,B)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(240*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^3+Ab\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

3.807 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bBE\left(\frac{1}{2}(c\right.}{5}$$

```
[Out] (6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*C
os[c + d*x]]) + (2*A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.104018, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bBE\left(\frac{1}{2}(c\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*C
os[c + d*x]]) + (2*A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*B*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = A\int (b\cos(c+dx))^{3/2}\,dx + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b}$$
$$= \frac{2Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{3}\left(Ab^2\right.$$
$$= \frac{2Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(Ab^2\sqrt{}\right.}{}$$
$$= \frac{6bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} +$$

Mathematica [A] time = 0.127823, size = 88, normalized size = 0.63

$$\frac{2(b\cos(c+dx))^{3/2}\left(\sin(c+dx)\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c
 + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/
(15*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 2.857, size = 273, normalized size = 2.

$$-\frac{2b^2}{15d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+(20\,A+24\,B)\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-24*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.808 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

```
[Out] (2*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos
[c + d*x]]) + (2*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.103683, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos
[c + d*x]]) + (2*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec(c+dx)\,dx &= b\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx\\ &= (Ab)\int \sqrt{b\cos(c+dx)}\,dx + B\int (b\cos(c+dx))^{3/2}\,dx\\ &= \frac{2bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(b^2B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\\ &= \frac{2Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2bB\sqrt{b\cos(c+dx)}\sin}{3d}\\ &= \frac{2Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}F\left(\right.}{3d\sqrt{b\cos(c+}}\end{aligned}$$

Mathematica [A] time = 0.0396753, size = 76, normalized size = 0.68

$$\frac{2b\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c
+ d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 3.18, size = 240, normalized size = 2.1

$$\frac{2\,b^2}{3\,d}\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^4+3\,A\sqrt{\left(\sin\left(1/2\,dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-4*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
```

```
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+2*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

3.809 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=83

$$\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c
 + d*x]])
```

Rubi [A] time = 0.100645, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c
 + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx &= b^2\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \left(Ab^2\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx+(bB)\int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{\left(Ab^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}+\frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \sqrt{c}}{\sqrt{\cos(c+dx}} \\
&= \frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c\right.}{d\sqrt{b\cos(c+dx}}
\end{aligned}
$$

Mathematica [A] time = 0.063803, size = 57, normalized size = 0.69

$$
\frac{2b^2\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d
*x)/2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.713, size = 163, normalized size = 2.

$$
-2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}b^2\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(A\mathrm{Elliptic}\right.}{\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\,(c\right.}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos\left(dx+c\right)^{2}+Ab\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}\sec\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

3.810 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (-2*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.120401, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (-2*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\left(b^2B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-(Ab)\int \sqrt{b\cos(c+dx)}\,dx+\frac{\left(b^2B\sqrt{\cos(\right.}{\sqrt{}}\\ &= \frac{2b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{(A}{}\\ &= -\frac{2Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sqrt{\cos(c+dx)}F}{d\sqrt{b\cos(c}}\end{aligned}$$

Mathematica [A] time = 0.158263, size = 73, normalized size = 0.66

$$\frac{2(b\cos(c+dx))^{3/2}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c
+ d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 3.645, size = 215, normalized size = 2.

$$-2\,\frac{b^2\sqrt{-2\,b\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2\,b}\left(A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{Elliptic}\right.}{\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -2*b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

3.811 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^2
*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.141157, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (-2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^2
*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= b^4\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \left(Ab^4\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx+\left(b^3B\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{1}{3}\left(Ab^2\right)\int \frac{1}{\sqrt{b\cos(\ldots}}\\ &= \frac{2Ab^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{\left(Ab^2\sqrt{\cos(c+dx)}\right)}{3\sqrt{b\cos(\ldots}}\\ &= -\frac{2bB\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^2\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\ldots\right.}{3d\sqrt{b\cos(c+\ldots}}\end{aligned}$$

Mathematica [A] time = 0.142433, size = 87, normalized size = 0.62

$$\frac{2b^2\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 4.066, size = 455, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -2/3*(12*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
```

```
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*b*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))*b^2/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

3.812 $\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=174

$$\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sqrt{\cos(c}}{3d\sqrt{}}$$

```
[Out] (-6*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b
^3*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b^2*Sin[c + d*x])/(5
*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.157485, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AbE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2B\sqrt{\cos(c}}{3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```


```
[Out] (-6*A*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b
^3*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b^2*Sin[c + d*x])/(5
*d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = b^5\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$

$$= \left(Ab^5\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \left(b^4B\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$

$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{5}\left(3Ab^3\right)\int \frac{}{(b\,c}$$

$$= \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^3B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6Ab^2\sin(c+dx}{5d\sqrt{b\cos(c+dx}}$$

$$= \frac{2b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2}{3d}$$

$$= -\frac{6Ab\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}\right.}{3d\sqrt{b\cos(c+}}$$

Mathematica [A] time = 0.264613, size = 107, normalized size = 0.61

$$\frac{2\sec^3(c+dx)(b\cos(c+dx))^{3/2}\left(\frac{9}{2}A\sin(2(c+dx))+3A\tan(c+dx)-9A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5B\sin(c+dx)\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(-9*A*Cos[c + d*x]^(3/2)*EllipticE
[(c + d*x)/2, 2] + 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 5*B*S
in[c + d*x] + (9*A*Sin[2*(c + d*x)])/2 + 3*A*Tan[c + d*x]))/(15*d)
```

Maple [B] time = 7.467, size = 576, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d
*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

3.813 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=171

$$\frac{6Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{10b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b^3B\ldots}{\ldots}$$

```
[Out] (6*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (10*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (10*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*A*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*B*(b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(7*d)
```

Rubi [A] time = 0.119754, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{6Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2Ab\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{10b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b^3B\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (6*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (10*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (10*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (
2*A*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*B*(b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(7*d)
```


Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = A\int (b\cos(c+dx))^{5/2}\,dx + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b}$$
$$= \frac{2Ab(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{5}\big(3$$
$$= \frac{10b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2Ab(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + 2B($$
$$= \frac{6Ab^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} +$$
$$= \frac{6Ab^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{10b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0587152, size = 100, normalized size = 0.58

$$\frac{(b\cos(c+dx))^{5/2}\left(2\sin(c+dx)\sqrt{\cos(c+dx)}(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252AE\left(\frac{1}{2}(c+dx)\middle|2\right)+10\right.}{210d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(252*A*EllipticE[(c + d*x)/2, 2] + 100*B*EllipticF[
(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(65*B + 42*A*Cos[c + d*x] + 15*B*Cos
[2*(c + d*x)])*Sin[c + d*x]))/(210*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 3.313, size = 301, normalized size = 1.8

$$-\frac{2b^3}{105d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^8+(-168A-360B)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(240*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*
```

```
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.814 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=145

$$\frac{2Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2Ab^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{6b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(}{}$$

```
[Out] (6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*
B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rubi [A] time = 0.123482, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2Ab^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{6b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```


```
[Out] (6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
d*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*
B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$$
$$= (Ab)\int (b\cos(c+dx))^{3/2}\,dx + B\int (b\cos(c+dx))^{5/2}\,dx$$
$$= \frac{2Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2bB(b\cos(c+dx))^{3/2}\sin(}{5d}$$
$$= \frac{2Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2bB(b\cos(c+dx))^{3/2}\sin(}{5d}$$
$$= \frac{6b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sqrt{\cos(c+dx)}F\left(\right.}{3d\sqrt{b\cos(c+}}$$

Mathematica [A] time = 0.059507, size = 89, normalized size = 0.61

$$\frac{2b(b\cos(c+dx))^{3/2}\left(\sin(c+dx)\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (2*b*(b*Cos[c + d*x])^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[
(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x])
)/(15*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 3.227, size = 273, normalized size = 1.9

$$-\frac{2b^3}{15d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+(20\,A+24\,B)\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-24*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

3.815 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*C
os[c + d*x]]) + (2*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.11374, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*C
os[c + d*x]]) + (2*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= \left(Ab^2\right)\int\sqrt{b\cos(c+dx)}\,dx + (bB)\int(b\cos(c+dx))^{3/2}\,dx$$
$$= \frac{2b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3d} + \frac{1}{3}\left(b^3B\right)\int\frac{1}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{2Ab^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2B\sqrt{b\cos(c+dx)}}{3d}$$
$$= \frac{2Ab^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}F}{3d\sqrt{b\cos(c}}$$

Mathematica [A] time = 0.0413694, size = 78, normalized size = 0.67

$$\frac{2b^2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*Sqrt[Cos[c + d*x]]
)
```

Maple [A] time = 2.875, size = 240, normalized size = 2.1

$$\frac{2\,b^3}{3\,d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,B\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^4+3\,A\sqrt{(\sin(1/2\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-4*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+2*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

3.816 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=85

$$\frac{2Ab^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos
[c + d*x]])
```

Rubi [A] time = 0.0986461, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2Ab^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```


```
[Out] (2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*
x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos
[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^3(c+dx)\,dx &= b^3\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \left(Ab^3\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx+\left(b^2B\right)\int \sqrt{b\cos(c+dx)}\,dx\\ &= \frac{\left(Ab^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)}{\sqrt{\cos(c+}}\\ &= \frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2Ab^3\sqrt{\cos(c+dx)}F}{d\sqrt{b\cos(c+}}\end{aligned}$$

Mathematica [A] time = 0.0965407, size = 54, normalized size = 0.64

$$\frac{2(b\cos(c+dx))^{5/2}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d
*x)/2, 2]))/(d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 2.623, size = 163, normalized size = 1.9

$$-2\,\frac{\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}b^{3}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\,\left(A\mathrm{Ellip}\right.}{\sqrt{-b\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\right)}\sin\left(1/2\,dx+c/2\right)\sqrt{b\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos\left(dx+c\right)^3+Ab^2\cos\left(dx+c\right)^2\right)\sqrt{b\cos\left(dx+c\right)}\sec\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}\sec\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

3.817 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (-2*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.120302, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (-2*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Co
s[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^4(c+dx)\,dx &= b^4\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \left(Ab^4\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\left(b^3B\right)\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\left(Ab^2\right)\int \sqrt{b\cos(c+dx)}\,dx+\frac{\left(b^3B\sqrt{\cos(c\right.}{\sqrt{b}}\\ &= \frac{2b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\left(Ab^2\right.}{}\\ &= -\frac{2Ab^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^3B\sqrt{\cos(c+dx)}F\left(\right.}{d\sqrt{b\cos(c+}}\end{aligned}$$

Mathematica [A] time = 0.190638, size = 73, normalized size = 0.65

$$\frac{2(b\cos(c+dx))^{5/2}\left(-AE\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{A\sin(c+dx)}{\sqrt{\cos(c+dx)}}+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*(-(A*EllipticE[(c + d*x)/2, 2]) + B*EllipticF[(c
+ d*x)/2, 2] + (A*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 3.265, size = 215, normalized size = 1.9

$$-2\frac{b^3\sqrt{-2b(\sin(1/2dx+c/2))^4+(\sin(1/2dx+c/2))^2b}\left(A\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\text{EllipticE}\left(\right.\right.}{\sqrt{-b\left(2(\sin(1/2dx+c/2))^4-(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] -2*b^3*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

3.818 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b
^3*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.138212, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2Ab^3\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (-2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d
*x]]) + (2*A*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b
^3*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^5(c+dx)\,dx = b^5\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \left(Ab^5\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + \left(b^4B\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}$$
$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{1}{3}\left(Ab^3\right)\int \frac{}{\sqrt{b\,c}}$$
$$= \frac{2Ab^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2b^3B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(Ab^3\sqrt{\cos(c+d}\right.}{3\sqrt{b\cos}}$$
$$= -\frac{2b^2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(}}$$

Mathematica [A] time = 0.181833, size = 87, normalized size = 0.61

$$\frac{2b^3\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c +
d*x]]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.306, size = 455, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x)
```

```
[Out] -2/3*(12*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
```

```
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*b*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))*b^3/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^5,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

3.819 $\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx$

Optimal. Leaf size=176

$$\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\sqrt{\cos(c+}}{3d\sqrt{b}}$$

```
[Out] (-6*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2
*b^4*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b^3*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.162002, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6Ab^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3B\sqrt{\cos(c+}}{3d\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (-6*A*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c +
 d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2
*b^4*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*b^3*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\sec^6(c+dx)\,dx = b^6\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \left(Ab^6\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \left(b^5B\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}$$
$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{1}{5}\left(3Ab^4\right)\int \frac{}{(}$$
$$= \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2b^4B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{6Ab^3\sin(c+}{5d\sqrt{b\cos(c+}}$$
$$= \frac{2b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2Ab^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} +$$
$$= -\frac{6Ab^2\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(}}$$

Mathematica [A] time = 0.235969, size = 102, normalized size = 0.58

$$-\frac{2b^4\left(-\frac{9}{2}A\sin(2(c+dx)) - 3A\tan(c+dx) + 9A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right) - 5B\sin(c+dx) - 5B\cos^{\frac{3}{2}}(c+dx)F\left(\right.\right.}{15d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x])*Sec[c + d*x]^6,x]
```

```
[Out] (-2*b^4*(9*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 5*B*Cos[c + d*x
]^(3/2)*EllipticF[(c + d*x)/2, 2] - 5*B*Sin[c + d*x] - (9*A*Sin[2*(c + d*x)
])/2 - 3*A*Tan[c + d*x]))/(15*d*(b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 7.057, size = 578, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb^2\cos(dx+c)^3+Ab^2\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}\sec(dx+c)^6,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))
*sec(d*x + c)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}\sec(dx+c)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))*sec(d*x+c)^6,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

3.820 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=173

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{10B\sin(c+d}{}$$

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*
x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos
[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*A*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Si
n[c + d*x])/(7*b^3*d)
```

Rubi [A] time = 0.134904, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{10B\sin(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*
x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos
[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*A*(b*C
os[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Si
n[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b^3}$$
$$= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b^3} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^4}$$
$$= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{(3A)}{}$$
$$= \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{2B(b\,\mathrm{cc}}{}$$
$$= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21bd} + \frac{2A(b}{}$$
$$= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + 10$$

Mathematica [A] time = 0.340427, size = 101, normalized size = 0.58

$$\frac{\sin(2(c+dx))(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos(c+d}}{210d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c +
 d*x)])*Sin[2*(c + d*x)])/(210*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.193, size = 298, normalized size = 1.7

$$-\frac{2}{105\,d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,B\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^8+(-168\,A-36\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-42
*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.821 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=144

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right.}{5bd\sqrt{\cos}}$$

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d) + (2*B*(b*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rubi [A] time = 0.113366, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right.}{5bd\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```


```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
 + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d) + (2*B*(b*Cos[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}dx = \frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{3/2}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^3}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{1}{3}A\int$$
$$= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{\left(A\sqrt{\cos\right.}}{}$$
$$= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2A}{}$$

Mathematica [A] time = 0.145038, size = 88, normalized size = 0.61

$$\frac{2\sqrt{\cos(c+dx)}\left(\sin(c+dx)\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d
*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*
d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.006, size = 270, normalized size = 1.9

$$-\frac{2}{15d}\sqrt{b\left(2\left(\cos(1/2\,dx+c/2)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)\left(\sin(1/2\,dx+c/2)\right)^6+(20\,A+24\,B)\right($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*B*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^
4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^
2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{b\cos(dx+c)}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.822 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=113

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]
]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
 d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rubi [A] time = 0.0925369, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]
]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c +
 d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^2}$$
$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{1}{3}B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx + \frac{\left(A\sqrt{b\cos(c+dx\right.}}{b\sqrt{\mathrm{co}}}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3bd} + \frac{\left(B\sqrt{\mathrm{co}}\right.}{}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{}}{}$$

Mathematica [A] time = 0.0491222, size = 78, normalized size = 0.69

$$\frac{2\sqrt{b\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c +
d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*b*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 2.878, size = 237, normalized size = 2.1

$$\frac{2}{3\,d}\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}\left(-4\,B\cos\left(1/2\,dx+c/2\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}+3\,A\sqrt{\left(\sin\left(1/2\,dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-4*B*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*d*
```

```
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))+2*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d
*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d
*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.823 $\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]
]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d
*x]])
```

Rubi [A] time = 0.0677927, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2748, 2642, 2641, 2640, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]
]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d
*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx+\frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b}\\
&= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}}\\
&= \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 0.0474339, size = 54, normalized size = 0.66

$$
\frac{2\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.728, size = 160, normalized size = 2.

$$
-2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\,\left(A\mathrm{EllipticF}\,(\right.}{\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2\right)}\sin(1/2\,dx+c/2)\,\sqrt{b\left(2\,(c\right.}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+
1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)
```

3.824 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c +
d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.103445, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```


```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c +
d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= (Ab)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int}{b\sqrt{\cos(c+}}$$
$$= -\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}} + \frac{2}{d}$$

Mathematica [A] time = 0.091587, size = 73, normalized size = 0.69

$$\frac{2\left(A\sin(c+dx) - A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.559, size = 212, normalized size = 2.

$$-2\,\frac{\sqrt{-2\,b\,(\sin(1/2\,dx+c/2))^4 + (\sin(1/2\,dx+c/2))^2\,b}\left(A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\mathrm{EllipticE}\right.}{\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4 -\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2
```

```
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*
d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b*cos(d*x
+ c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

3.825 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c +
 d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.130198, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```


```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c
+ d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c +
 d*x])/(d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + (bB)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{1}{3}A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx - \frac{B\int \sqrt{b\,c}}{}$$
$$= \frac{2Ab\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} - \frac{(}{}$$
$$= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2A}{3d(b}$$

Mathematica [A] time = 0.146592, size = 84, normalized size = 0.62

$$\frac{2\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*d*Sqrt
[b*Cos[c + d*x]])
```

Maple [B] time = 6.625, size = 405, normalized size = 3.

$$\frac{2}{3\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(2\,A\mathrm{EllipticF}\left(\cos(1/2\,dx+c/2),\sqrt{2}\right)\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x
+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*
x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+6*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b*cos(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

3.826 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=168

$$\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6A\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+}}{3d\sqrt{b}}$$

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*B*S
in[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*d*Sqrt[b*
Cos[c + d*x]])
```

Rubi [A] time = 0.153572, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6A\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+}}{3d\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*B*S
in[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*d*Sqrt[b*
Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\
&= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx+\left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{5}(3Ab)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx+\\
&= \frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{6A\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{(3A)\int \sqrt{b\cos}}{5}\\
&= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2Ab^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2bB\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\\
&= -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2A}{5d(b}
\end{aligned}$$

Mathematica [A] time = 0.224769, size = 101, normalized size = 0.6

$$\frac{2\left(9A\sin(c+dx)-9A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+3A\tan(c+dx)\sec(c+dx)+5B\tan(c+dx)+5B\sqrt{\cos(c+dx)}F\right.}{15d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*S
ec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 7.589, size = 578, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*
c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^3}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b*cos(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

3.827 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{10B\sin(c$$

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b
*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*
A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d) + (2*B*(b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(7*b^4*d)
```

Rubi [A] time = 0.135144, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{10B\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b
*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*
A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d) + (2*B*(b*Cos[c + d*x])^(5
/2)*Sin[c + d*x])/(7*b^4*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b^4}\\ &= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b^4}+\frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^5}\\ &= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}+\frac{(3A)\int \sqrt{\ }}{}\\ &= \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}+\frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{2B(b\cos(c}{}\\ &= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d}+\frac{2A(b\,\mathrm{co}}{}\\ &= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{10B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{10B\sqrt{}}{}\end{aligned}$$

Mathematica [A] time = 0.142845, size = 104, normalized size = 0.59

$$\frac{\sin(2(c+dx))(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos(c+dx)}F}{210bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c +
 d*x)])*Sin[2*(c + d*x)])/(210*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.817, size = 301, normalized size = 1.7

$$-\frac{2}{105\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,B\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^8+(-168\,A-360\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(240*B*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^6*
cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-
42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

3.828 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6BE\left(\frac{1}{2}(c+dx\right.}{5b^2d\sqrt{}}$$

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*C
os[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*B*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rubi [A] time = 0.1129, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6BE\left(\frac{1}{2}(c+dx\right.}{5b^2d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*C
os[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*B*(b
*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx}{b^3}\\ &= \frac{A\int (b\cos(c+dx))^{3/2}\,dx}{b^3}+\frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^4}\\ &= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{A\int \frac{1}{\sqrt{b\cos(c}}}{3b}\\ &= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}+\frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{\left(A\sqrt{\cos(c}\right.}{3b\sqrt{}}\\ &= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2A\sqrt{b}}{}\end{aligned}$$

Mathematica [A] time = 0.136124, size = 88, normalized size = 0.6

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\left(\sin(c+dx)\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d
*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*
d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 2.552, size = 273, normalized size = 1.9

$$-\frac{2}{15\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^6+(20\,A+24\,B)\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-24*B*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c
)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c
)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{b\cos(dx+c)}}{b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

3.829 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos
[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rubi [A] time = 0.0940435, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos
[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^3}$$
$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos}}{b^2\sqrt{\cos(c+dx}}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{\left(B\sqrt{\cos(}\right.}{3b}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2B\sqrt{b}}{}$$

Mathematica [A] time = 0.0915344, size = 75, normalized size = 0.65

$$\frac{2\cos^{\frac{3}{2}}(c+dx)\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*
x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 2.651, size = 240, normalized size = 2.1

$$\frac{2}{3\,bd}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,B\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+3\,A\sqrt{(\sin(1/2\,dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-4*B*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/2*
```

```
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+2*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2
*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2
*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.830 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c
 + d*x]])
```

Rubi [A] time = 0.0772422, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c
 + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b}\\ &= \frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b}+\frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b^2}\\ &= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}\\ &= \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0468542, size = 57, normalized size = 0.67

$$\frac{2\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2]))/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.652, size = 163, normalized size = 1.9

$$-2\,\frac{\sqrt{b\left(2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}-1\right)\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}}\sqrt{-2\,\left(\cos\left(1/2\,dx+c/2\right)\right)^{2}+1}\left(A\mathrm{Elliptic}\right.}{b\sqrt{-b\left(2\,\left(\sin\left(1/2\,dx+c/2\right)\right)^{4}-\left(\sin\left(1/2\,dx+c/2\right)\right)^{2}\right)}\sin\left(1/2\,dx+c/2\right)\sqrt{b\left(2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/b/(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^2\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.831 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=112

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0885878, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2636, 2640, 2639, 2642, 2641}

$$-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[
c + d*x]]) + (2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = A\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b}$$
$$= \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b^2} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}$$
$$= -\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0664098, size = 76, normalized size = 0.68

$$\frac{2\left(A\sin(c+dx) - A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.102, size = 215, normalized size = 1.9

$$-2\frac{\sqrt{-2b(\sin(1/2dx+c/2))^4+(\sin(1/2dx+c/2))^2b}\left(A\sqrt{(\sin(1/2dx+c/2))^2}\sqrt{2(\sin(1/2dx+c/2))^2-1}\mathrm{EllipticE}\left(\mathrm{co}\right.\right.}{b\sqrt{-b\left(2(\sin(1/2dx+c/2))^4-(\mathrm{s}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/
2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)
```

3.832 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c
 + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.127498, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d
*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c
 + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx\\ &= (Ab)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + B\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b} - \frac{B\int \sqrt{b\cos(}}{b^2}\\ &= \frac{2A\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}}\\ &= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{}{3d}\end{aligned}$$

Mathematica [A] time = 0.0650773, size = 87, normalized size = 0.62

$$\frac{2\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*b*d*Sq
rt[b*Cos[c + d*x]])
```

Maple [B] time = 3.389, size = 455, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -2/3*(12*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
```

```
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*b*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
3*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/b/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1
/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b^2*cos(d*
x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

3.833 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{6A\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+dx)}\ldots}{3bd\sqrt{b}\ldots}$$

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*S
in[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b*d*Sqrt[
b*Cos[c + d*x]])
```

Rubi [A] time = 0.160071, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{6A\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+dx)}\ldots}{3bd\sqrt{b}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```


```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*S
in[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b*d*Sqrt[
b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx\\ &= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx+(bB)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{5}(3A)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{6A\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}-\frac{(3A)\int}{}\\ &= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3d(b\cos(c+dx))}\\ &= -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\end{aligned}$$

Mathematica [A] time = 0.105126, size = 104, normalized size = 0.61

$$\frac{2\left(9A\sin(c+dx)-9A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+3A\tan(c+dx)\sec(c+dx)+5B\tan(c+dx)+5B\sqrt{\cos(c+dx)}\right.}{15bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*S
ec[c + d*x]*Tan[c + d*x]))/(15*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 7.915, size = 578, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)^2}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b^2*cos(
d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

3.834 $\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10B\sin(c+d}{2}$$

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt
[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (
2*A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d) + (2*B*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rubi [A] time = 0.133719, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10B\sin(c+d}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt
[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (
2*A*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d) + (2*B*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx}{b^5} \\
&= \frac{A\int (b\cos(c+dx))^{5/2}\,dx}{b^5} + \frac{B\int (b\cos(c+dx))^{7/2}\,dx}{b^6} \\
&= \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2B(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{(3A)}{\cdots} \\
&= \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2A(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2B(b\cos}{\cdots} \\
&= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2A(}{\cdots} \\
&= \frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{10B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + 10\cdots
\end{aligned}$$

Mathematica [A] time = 0.0883427, size = 104, normalized size = 0.59

$$\frac{\sin(2(c+dx))(42A\cos(c+dx)+15B\cos(2(c+dx))+65B)+252A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+100B\sqrt{\cos(c+d}\cdots}{210b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (252*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 100*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (65*B + 42*A*Cos[c + d*x] + 15*B*Cos[2*(c +
 d*x)])*Sin[2*(c + d*x)])/(210*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.22, size = 301, normalized size = 1.7

$$-\frac{2}{105\,b^2d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(240\,B\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^8+(-168\,A-3\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(240*B
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-168*A-360*B)*sin(1/2*d*x+1/2*c)^
6*cos(1/2*d*x+1/2*c)+(168*A+280*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
(-42*A-80*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-63*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)\sqrt{b\cos(dx+c)}}{b^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^5}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

3.835 $\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right.}{5b^3d\sqrt{\cos}}$$

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d) + (2*B*
(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rubi [A] time = 0.114353, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2A\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2B\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right.}{5b^3d\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b
*Cos[c + d*x]]) + (2*A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d) + (2*B*
(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx}{b^4}$$
$$= \frac{A\int (b\cos(c+dx))^{3/2}\,dx}{b^4} + \frac{B\int (b\cos(c+dx))^{5/2}\,dx}{b^5}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{A\int \frac{}{\sqrt{b\,c}}}{3}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2B(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{\left(A\sqrt{\cos}\right.}{3}$$
$$= \frac{6B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A}{}$$

Mathematica [A] time = 0.0774615, size = 91, normalized size = 0.62

$$\frac{2\sqrt{\cos(c+dx)}\left(\sin(c+dx)\sqrt{\cos(c+dx)}(5A+3B\cos(c+dx))+5AF\left(\frac{1}{2}(c+dx)\middle|2\right)+9BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(9*B*EllipticE[(c + d*x)/2, 2] + 5*A*EllipticF[(c + d
*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A + 3*B*Cos[c + d*x])*Sin[c + d*x]))/(15*
b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.027, size = 273, normalized size = 1.9

$$-\frac{2}{15\,b^2d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-24\,B\cos(1/2\,dx+c/2)\,(\sin(1/2\,dx+c/2))^6+(20\,A+24\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-24*B*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(20*A+24*B)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-10*A-6*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2
*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(B\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{b\cos(dx+c)}}{b^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^4}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

3.836 $\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*
x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*C
os[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rubi [A] time = 0.0910747, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {16, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*
x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*C
os[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx}{b^3}$$
$$= \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b^3} + \frac{B\int (b\cos(c+dx))^{3/2}\,dx}{b^4}$$
$$= \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sqrt{}}{b^3\sqrt{\cos(c+}}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{\left(B\sqrt{c}\right.}{}$$
$$= \frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2B}{}$$

Mathematica [A] time = 0.0766783, size = 78, normalized size = 0.67

$$\frac{2\sqrt{\cos(c+dx)}\left(3AE\left(\frac{1}{2}(c+dx)\middle|2\right)+B\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}\right)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(3*A*EllipticE[(c + d*x)/2, 2] + B*(EllipticF[(c + d*
x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin[c + d*x])))/(3*b^2*d*Sqrt[b*Cos[c + d*x]]
)
```

Maple [A] time = 3.504, size = 240, normalized size = 2.1

$$\frac{2}{3\,b^2d}\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-4\,B\cos(1/2\,dx+c/2)(\sin(1/2\,dx+c/2))^4+3\,A\sqrt{(\sin(1/2\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-4*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
```

```
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+2*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1
/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1
/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^3}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

3.837 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Rubi [A] time = 0.0746879, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {16, 2748, 2642, 2641, 2640, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*
x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int \frac{A+B\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}\\ &= \frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}+\frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b^3}\\ &= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}}\\ &= \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0455693, size = 57, normalized size = 0.67

$$\frac{2\sqrt{\cos(c+dx)}\left(AF\left(\frac{1}{2}(c+dx)\middle|2\right)+BE\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(B*EllipticE[(c + d*x)/2, 2] + A*EllipticF[(c + d*x)/
2, 2]))/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 2.763, size = 163, normalized size = 1.9

$$-2\,\frac{\sqrt{b\left(2\,(\cos(1/2\,dx+c/2))^2-1\right)(\sin(1/2\,dx+c/2))^2}\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{-2\,(\cos(1/2\,dx+c/2))^2+1}\left(A\mathrm{EllipticF}\left(\right.\right.}{b^2\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4-(\sin(1/2\,dx+c/2))^2\right)}\sin(1/2\,dx+c/2)\sqrt{b\left(2\,(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/b^2/(-b*(2*sin(1/2*
d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

3.838 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=112

$$\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0965816, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-2*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d
*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Co
s[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx}{b}$$
$$= \frac{A\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{b} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}$$
$$= \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{A\int \sqrt{b\cos(c+dx)}\,dx}{b^3} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}$$
$$= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{b\cos(c+dx)}\right)}{b^3\sqrt{\cos(}}$$
$$= -\frac{2A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{2}{b^2}$$

Mathematica [A] time = 0.060793, size = 76, normalized size = 0.68

$$\frac{2\left(A\sin(c+dx) - A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-(A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.316, size = 215, normalized size = 1.9

$$-2\,\frac{\sqrt{-2\,b\,(\sin(1/2\,dx+c/2))^4+(\sin(1/2\,dx+c/2))^2\,b}\left(A\sqrt{(\sin(1/2\,dx+c/2))^2}\sqrt{2\,(\sin(1/2\,dx+c/2))^2-1}\text{EllipticE}\right.}{b^2\sqrt{-b\left(2\,(\sin(1/2\,dx+c/2))^4\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
```

```
+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.839 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d
*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (2*B*S
in[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.10176, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}+\frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d
*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*
Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (2*B*S
in[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = A\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx + \frac{B\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{b}$$
$$= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2} - \frac{B\int \sqrt{b\cos(c+dx)}\,dx}{b^3}$$
$$= \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{b\cos(c\right.}{b}$$
$$= -\frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{3bd(b\cos(c+dx))^{3}}$$

Mathematica [A] time = 0.0572776, size = 87, normalized size = 0.61

$$\frac{2\left(A\tan(c+dx)+A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)+3B\sin(c+dx)-3B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + 3*B*Sin[c + d*x] + A*Tan[c + d*x]))/(3*b^2*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 4.189, size = 455, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -2/3*(12*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2*b)^(1/2)*(A+3*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*b*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2+A*(-2*b*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
```

```
3*B*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/b^2/(2*cos(1/2*d*x+1/2*c)^2-1)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(
1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^
(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)
```

3.840 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=173

$$\frac{6A\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{}{3b}$$

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*S
in[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b^2*d*S
qrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.143779, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {16, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{6A\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```


```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*S
in[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b^2*d*S
qrt[b*Cos[c + d*x]])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= (Ab)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + B\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{(3A)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b} + \frac{B\int}{}$$
$$= \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{6A\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{(3A)\int}{}$$
$$= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3bd(b\cos(c+dx))}$$
$$= -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{}{5d}$$

Mathematica [A] time = 0.0683959, size = 104, normalized size = 0.6

$$\frac{2\left(9A\sin(c+dx) - 9A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 3A\tan(c+dx)\sec(c+dx) + 5B\tan(c+dx) + 5B\sqrt{\cos(c+dx}\right.}{15b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*S
ec[c + d*x]*Tan[c + d*x]))/(15*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 8.306, size = 578, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}\sec(dx+c)}{b^3\cos(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b^3*cos(d*
x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

3.841 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=176

$$\frac{6A\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c}}{3b^3d}$$

```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (2*B
*Sin[c + d*x])/(3*b^2*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b^3
*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.11914, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{6A\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}-\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c}}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(7/2),x]
```


```
[Out] (-6*A*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c +
 d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^3*d*Sqrt[
b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (2*B
*Sin[c + d*x])/(3*b^2*d*(b*Cos[c + d*x])^(3/2)) + (6*A*Sin[c + d*x])/(5*b^3
*d*Sqrt[b*Cos[c + d*x]])
```


Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/2}}\,dx = A\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx + \frac{B\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx}{b}$$
$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{(3A)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2} + \frac{B\int \frac{1}{\sqrt{b\cos(c+dx)}}}{3b^3}$$
$$= \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{6A\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{(3A)\int \sqrt{b\cos(c+}}{5b^4}$$
$$= \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^{3/2}} + \frac{6A}{5b^3d}$$
$$= -\frac{6A\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^3d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+d}{5bd(b\cos(c+d}$$

Mathematica [A] time = 0.0650335, size = 104, normalized size = 0.59

$$\frac{2\left(9A\sin(c+dx) - 9A\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + 3A\tan(c+dx)\sec(c+dx) + 5B\tan(c+dx) + 5B\sqrt{\cos(c+d}\right.}{15b^3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (2*(-9*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*S
ec[c + d*x]*Tan[c + d*x]))/(15*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 7.52, size = 578, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
```

```
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^4-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/
2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+9*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{b^4\cos(dx+c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^4*cos(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)
```

3.842 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=172

$$-\frac{A\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos}}{4d}$$

```
[Out] (3*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x
]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]
]*Sin[c + d*x])/(4*d) - (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[C
os[c + d*x]])
```

Rubi [A] time = 0.0685106, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2633, 2635, 8}

$$-\frac{A\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (3*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x
]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]
]*Sin[c + d*x])/(4*d) - (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[C
os[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt{b\cos(c+dx)}\int \cos^3(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int}{\sqrt{\cos(c+}}$$
$$= \frac{B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3B\sqrt{b\cos(c+d}\right.}{4\sqrt{c}}$$
$$= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3B\sqrt{\cos(c+dx)}\sqrt{b\cos(c+d}}{8d}$$
$$= \frac{3Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3B\sqrt{co}}{}$$

Mathematica [A] time = 0.159078, size = 81, normalized size = 0.47

$$\frac{\sqrt{b\cos(c+dx)}(72A\sin(c+dx)+8A\sin(3(c+dx))+24B\sin(2(c+dx))+3B\sin(4(c+dx))+36Bc+36Bdx)}{96d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2*(
c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Sqrt[Cos[c
+ d*x]])
```

Maple [A] time = 0.548, size = 91, normalized size = 0.5

$$\frac{6\,B\sin(dx+c)(\cos(dx+c))^3+8\,A\sin(dx+c)(\cos(dx+c))^2+9\,B\sin(dx+c)\cos(dx+c)+16\,A\sin(dx+c)+9}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(1/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c)*cos
(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)
^(1/2)
```

Maxima [A] time = 2.17928, size = 126, normalized size = 0.73

$$\frac{3\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)\right)B\sqrt{b}+8\,A\sqrt{b}\left(\sin(3\,dx+3\,c)\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4
*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 8*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(
1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

Fricas [A] time = 1.80887, size = 699, normalized size = 4.06

$$\left[\frac{9\,B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2\left(6\,B\cos\left(dx+c\right)^{3}+\right.}{48\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/48*(9*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*cos(d*x + c)^3
 + 8*A*cos(d*x + c)^2 + 9*B*cos(d*x + c) + 16*A)*sqrt(b*cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*sqrt(b)*arctan(sqrt
(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) +
(6*B*cos(d*x + c)^3 + 8*A*cos(d*x + c)^2 + 9*B*cos(d*x + c) + 16*A)*sqrt(b*
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.843 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=136

$$\frac{Ax\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}-\frac{B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(2*d) - (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*
Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0550547, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ax\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}-\frac{B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(2*d) - (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*
Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \cos}{\sqrt{\cos(c+dx}}$$
$$= \frac{A\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)}{2\sqrt{\cos(c+dx}}$$
$$= \frac{Ax\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{\cos(c+}}{}$$

Mathematica [A] time = 0.113154, size = 69, normalized size = 0.51

$$\frac{\sqrt{b\cos(c+dx)}(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c +
d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.457, size = 74, normalized size = 0.5

$$\frac{2B\sin(dx+c)(\cos(dx+c))^2+3A\cos(dx+c)\sin(dx+c)+3A(dx+c)+4B\sin(dx+c)}{6d}\sqrt{b\cos(dx+c)}\frac{1}{\sqrt{\cos(dx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(
d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [A] time = 2.13139, size = 92, normalized size = 0.68

$$\frac{3(2dx+2c+\sin(2dx+2c))A\sqrt{b}+B\sqrt{b}\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + B*sqrt(b)*(sin(3*d*x +
 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d
```

Fricas [A] time = 1.7367, size = 639, normalized size = 4.7

$$\left[\frac{3\,A\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2\left(2\,B\cos\left(dx+c\right.\right.}{12\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
+ c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*cos(d*x + c)^2
 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^2 + 3
*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.844 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=98

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0222809, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{B\sqrt{\cos(c+}}{}\end{aligned}$$

Mathematica [A] time = 0.111821, size = 57, normalized size = 0.58

$$\frac{\sqrt{b\cos(c+dx)}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]
)))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.439, size = 55, normalized size = 0.6

$$\frac{B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c)}{2d}\sqrt{b\cos(dx+c)}\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c
))/cos(d*x+c)^(1/2)
```

Maxima [A] time = 2.14998, size = 54, normalized size = 0.55

$$\frac{(2dx+2c+\sin(2dx+2c))B\sqrt{b}+4A\sqrt{b}\sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B*sqrt(b) + 4*A*sqrt(b)*sin(d*x + c))
/d
```

Fricas [A] time = 1.66595, size = 570, normalized size = 5.82

$$\left[\frac{B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2(B\cos(dx+c)+2}{4d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c
))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*cos(d*x + c) + 2*A)
*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/
2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)
^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [A] time = 66.1476, size = 99, normalized size = 1.01

$$\begin{cases} \frac{A\sqrt{b}\sin(c+dx)}{d} + \frac{B\sqrt{b}x\sin^2(c+dx)}{2} + \frac{B\sqrt{b}x\cos^2(c+dx)}{2} + \frac{B\sqrt{b}\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x\sqrt{b\cos(c)}\,(A + B\cos(c))\sqrt{\cos(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Piecewise((A*sqrt(b)*sin(c + d*x)/d + B*sqrt(b)*x*sin(c + d*x)**2/2 + B*sqr
t(b)*x*cos(c + d*x)**2/2 + B*sqrt(b)*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*sqrt(b*cos(c))*(A + B*cos(c))*sqrt(cos(c)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.845 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=59

$$\frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (B*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.012328, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (B*Sqrt[b*Cos[c + d*x]]*Sin
[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(B\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Ax\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0469873, size = 42, normalized size = 0.71

$$\frac{\sqrt{b\cos(c+dx)}(A(c+dx)+B\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]]
)
```

Maple [A] time = 0.455, size = 39, normalized size = 0.7

$$\frac{A\left(dx+c\right)+B\sin\left(dx+c\right)}{d}\sqrt{b\cos\left(dx+c\right)}\frac{1}{\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [A] time = 1.99071, size = 54, normalized size = 0.92

$$\frac{2\,A\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+B\sqrt{b}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*sqrt(b)*sin(d*x +
c))/d
```

Fricas [A] time = 1.61787, size = 508, normalized size = 8.61

$$\left[\frac{A\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2\,\sqrt{b\cos\left(dx+c\right)}B\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c
))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))*B
*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*sqrt(b)*arctan(sqrt(
b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + s
qrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [A] time = 18.2536, size = 46, normalized size = 0.78

$$\begin{cases} A\sqrt{b}x + \frac{B\sqrt{b}\sin(c+dx)}{d} & \text{for } d \neq 0 \\ \frac{x\sqrt{b\cos(c)}(A+B\cos(c))}{\sqrt{\cos(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Piecewise((A*sqrt(b)*x + B*sqrt(b)*sin(c + d*x)/d, Ne(d, 0)), (x*sqrt(b*cos
(c))*(A + B*cos(c))/sqrt(cos(c)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

3.846 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{A\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0258115, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{A\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(A\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0317632, size = 40, normalized size = 0.67

$$\frac{\sqrt{b\cos(c+dx)}\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*
x]])
```

Maple [A] time = 0.384, size = 54, normalized size = 0.9

$$-\frac{1}{d}\sqrt{b\cos(dx+c)}\left(2\,A\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)\right)\frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] -1/d*(b*cos(d*x+c))^(1/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c
))/cos(d*x+c)^(1/2)
```

Maxima [A] time = 2.04823, size = 124, normalized size = 2.07

$$\frac{A\sqrt{b}\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*B*sqrt(b)*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.98703, size = 589, normalized size = 9.82

$$\left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)-B\sqrt{-b}\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) - B*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*sqrt(b)*arc
tan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A*sqr
t(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

3.847 $\int \frac{\sqrt{b \cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=68

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.0405146, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 8, 3770}

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int(A+B\cos(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{\left(A\sqrt{b\cos(c+dx)}\right)\mathrm{Subst}(\int 1\,dx, x, -}{d\sqrt{\cos(c+dx)}}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.0501688, size = 50, normalized size = 0.74

$$\frac{\sqrt{b\cos(c+dx)}\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x
]))/(d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.4, size = 59, normalized size = 0.9

$$\frac{1}{d}\left(-2B\cos(dx+c)\,\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)\sqrt{b\cos(dx+c)}\,(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*c
os(d*x+c))^(1/2)/cos(d*x+c)^(3/2)
```

Maxima [A] time = 2.26447, size = 162, normalized size = 2.38

$$\frac{B\sqrt{b}\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(B*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
 log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*A*sqrt(b)*s
in(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1))/d
```

Fricas [A] time = 1.55529, size = 564, normalized size = 8.29

$$\left[\frac{B\sqrt{b}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin}{2d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

3.848 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{A\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (B*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.0541124, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {17, 2748, 3768, 3770, 3767, 8}

$$\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{A\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (B*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int(A+B\cos(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}$$
$$= \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}} - \frac{(B}{}$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{B}{}$$

Mathematica [A] time = 0.103548, size = 65, normalized size = 0.61

$$\frac{\sqrt{b\cos(c+dx)}\left(\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Co
s[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.412, size = 120, normalized size = 1.1

$$\frac{1}{2\,d}\left(-A\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2+A\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 1/2/d*(-A*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+A*ln((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+2*B*sin(d*x+c)*cos(d*x+c)+A*
sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Maxima [B] time = 2.09577, size = 967, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] -1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
 + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*
d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c)
+ 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1) - 8*B*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.81333, size = 626, normalized size = 5.85

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx+c\right.}}{4\,d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))
*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*cos(d*
x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

3.849 $\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=145

$$\frac{A\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\tanh^{-1}(\ldots}{2d\sqrt{\cos(c+d\ldots}}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.0613371, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 3768, 3770}

$$\frac{A\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{A\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{B\sqrt{b\cos(c+dx)}\tanh^{-1}(\ldots}{2d\sqrt{\cos(c+d\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*Sqrt[b*C
os[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int(A+B\cos(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{b\cos(c+dx)}\right)\int\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sec^3(c+dx)$$
$$= \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(B\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}} - \frac{(A$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{A$$

Mathematica [A] time = 0.283179, size = 76, normalized size = 0.52

$$\frac{\sqrt{b\cos(c+dx)}\left(2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c
 + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(5/2)
)
```

Maple [A] time = 0.446, size = 139, normalized size = 1.

$$\frac{1}{6d}\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3+3B\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c
)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+
c)^(7/2)
```

Maxima [B] time = 2.1911, size = 1292, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*
d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*si
n(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c
)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c
) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4
*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)
+ 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(
2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.65394, size = 695, normalized size = 4.79

$$\left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,A\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right.\right.}{12\,d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)
*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*
cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

3.850 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=177

$$-\frac{Ab\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{}}{4d}$$

```
[Out] (3*b*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c +
 d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*b*B*Sqrt[Cos[c + d*x]]*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) - (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(
3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0694481, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2633, 2635, 8}

$$-\frac{Ab\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (3*b*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c +
 d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*b*B*Sqrt[Cos[c + d*x]]*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[
c + d*x]]*Sin[c + d*x])/(4*d) - (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(
3*d*Sqrt[Cos[c + d*x]])
```


Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx}\right.}{\sqrt{\cos}}$$
$$= \frac{bB\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3bB\sqrt{b\cos(\right.}{4}$$
$$= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3bB\sqrt{\cos(c+dx)}\sqrt{b\cos(c}}{8d}$$
$$= \frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3b}{}$$

Mathematica [A] time = 0.151681, size = 81, normalized size = 0.46

$$\frac{(b\cos(c+dx))^{3/2}(72A\sin(c+dx)+8A\sin(3(c+dx))+24B\sin(2(c+dx))+3B\sin(4(c+dx))+36Bc+36Bdx)}{96d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2
*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Cos[c + d
*x]^(3/2))
```

Maple [A] time = 0.367, size = 91, normalized size = 0.5

$$\frac{6B\sin(dx+c)(\cos(dx+c))^3+8A\sin(dx+c)(\cos(dx+c))^2+9B\sin(dx+c)\cos(dx+c)+16A\sin(dx+c)+9}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(3/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c)*cos
(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)
^(3/2)
```

Maxima [A] time = 2.07591, size = 135, normalized size = 0.76

$$\frac{8\left(b\sin(3dx+3c)+9b\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)A\sqrt{b}+3\left(12(dx+c)b+b\sin(4dx+4c)\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/96*(8*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d
*x + 3*c))))*A*sqrt(b) + 3*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1
/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b))/d
```

Fricas [A] time = 1.68214, size = 724, normalized size = 4.09

$$\left[\frac{9\,B\sqrt{-b}b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2\left(6\,Bb\cos\left(dx+c\right)^{3}\right.}{48\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/48*(9*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*b*cos(d*x +
c)^3 + 8*A*b*cos(d*x + c)^2 + 9*B*b*cos(d*x + c) + 16*A*b)*sqrt(b*cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B*b^(3/2)*a
rctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d
*x + c) + (6*B*b*cos(d*x + c)^3 + 8*A*b*cos(d*x + c)^2 + 9*B*b*cos(d*x + c)
 + 16*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.851 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{Abx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}-\frac{bB\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[Cos[c + d*x]]*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) - (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^
3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0562753, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Abx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}-\frac{bB\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[Cos[c + d*x]]*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d) - (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^
3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)}{\sqrt{\cos(c}}$$
$$= \frac{Ab\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\left(Ab\sqrt{b\cos(c+}\right.}{2\sqrt{\cos(c}}$$
$$= \frac{Abx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{c}}{}$$

Mathematica [A] time = 0.112481, size = 69, normalized size = 0.49

$$\frac{(b\cos(c+dx))^{3/2}(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c
+ d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.394, size = 74, normalized size = 0.5

$$\frac{2B\sin(dx+c)(\cos(dx+c))^2+3A\cos(dx+c)\sin(dx+c)+3A(dx+c)+4B\sin(dx+c)}{6d}(b\cos(dx+c))^{\frac{3}{2}}(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(3/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(
d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [A] time = 2.05249, size = 100, normalized size = 0.71

$$\frac{3(2(dx+c)b+b\sin(2dx+2c))A\sqrt{b}+\left(b\sin(3dx+3c)+9b\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)B\sqrt{b}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + (b*sin(3*d*x + 3*c
) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b))/d
```

Fricas [A] time = 1.80399, size = 657, normalized size = 4.69

$$\left[\frac{3\,A\sqrt{-b}b\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2\left(2\,Bb\cos\left(dx+\right.\right.}{12\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*A*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*
x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*b*cos(d*x +
c)^2 + 3*A*b*cos(d*x + c) + 4*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*A*b^(3/2)*arctan(sqrt(b*cos(d*x + c)
)*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*b*cos(d*x
+ c)^2 + 3*A*b*cos(d*x + c) + 4*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.852 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0229203, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{bBx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{\cos(c+dx)}\sqrt{b}}{}\end{aligned}$$

Mathematica [A] time = 0.039797, size = 58, normalized size = 0.57

$$\frac{b\sqrt{b\cos(c+dx)}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x
)])))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.444, size = 55, normalized size = 0.5

$$\frac{B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c)}{2d}(b\cos(dx+c))^{\frac{3}{2}}(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(3/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c
))/cos(d*x+c)^(3/2)
```

Maxima [A] time = 2.08354, size = 58, normalized size = 0.57

$$\frac{4Ab^{\frac{3}{2}}\sin(dx+c)+(2(dx+c)b+b\sin(2dx+2c))B\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(4*A*b^(3/2)*sin(d*x + c) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*B*sqrt
(b))/d
```

Fricas [A] time = 1.64022, size = 583, normalized size = 5.77

$$\left[\frac{B\sqrt{-b}b\cos(dx+c)\log\left(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2(Bb\cos(dx+c)}{4d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*b*cos(d*x + c) +
2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
)), 1/2*(B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2)))*cos(d*x + c) + (B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)), x
)
```

3.853 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*B*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0129245, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*B*Sqrt[b*Cos[c + d*x]]
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Abx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0629073, size = 42, normalized size = 0.69

$$\frac{(b\cos(c+dx))^{3/2}(A(c+dx)+B\sin(c+dx))}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/
2))
```

Maple [A] time = 0.237, size = 39, normalized size = 0.6

$$\frac{A(dx+c)+B\sin(dx+c)}{d}(b\cos(dx+c))^{\frac{3}{2}}(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(3/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.91375, size = 54, normalized size = 0.89

$$\frac{2\,Ab^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+Bb^{\frac{3}{2}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*b^(3/2)*sin(d*x +
c))/d
```

Fricas [A] time = 1.70696, size = 516, normalized size = 8.46

$$\left[\frac{A\sqrt{-b}b\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\,\sqrt{b\cos(dx+c)}Bb\sqrt{}}{2\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(A*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))
*B*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*b^(3/2)*arctan(s
qrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
 + sqrt(b*cos(d*x + c))*B*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x
)
```

3.854 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{Ab\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*ArcTanh[Sin[c + d*x]
]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0264272, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{Ab\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*ArcTanh[Sin[c + d*x]
]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{bBx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0415903, size = 40, normalized size = 0.65

$$\frac{(b\cos(c+dx))^{3/2}\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*(b*Cos[c + d*x])^(3/2))/(d*Cos[c + d*x]^
(3/2))
```

Maple [A] time = 0.237, size = 54, normalized size = 0.9

$$-\frac{1}{d}(b\cos(dx+c))^{\frac{3}{2}}\left(2\,A\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)\right)(\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] -1/d*(b*cos(d*x+c))^(3/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c
))/cos(d*x+c)^(3/2)
```

Maxima [A] time = 1.89732, size = 128, normalized size = 2.06

$$\frac{4\,Bb^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(b\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-b\log\left(\cos(dx+c)^2+\sin(dx\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*B*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b*log(cos(d*x +
 c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d
*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d
```

Fricas [A] time = 1.96146, size = 594, normalized size = 9.58

$$\left[-\frac{2\,A\sqrt{-b}b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)-B\sqrt{-b}b\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*
sqrt(cos(d*x + c)))) - B*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*b^(3/2)
*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A
*b^(3/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d
*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2), x
)
```

3.855 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (
A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.0405489, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 8, 3770}

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (
A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int(A+B\cos(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int\sec(c+dx)}{\sqrt{\cos(c+dx)}}$$
$$= \frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\text{Subst}(\int 1\,d}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.0519817, size = 50, normalized size = 0.71

$$\frac{(b\cos(c+dx))^{3/2}\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d
*x]))/(d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.207, size = 59, normalized size = 0.8

$$\frac{1}{d}\left(-2B\cos(dx+c)\,\text{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)(b\cos(dx+c))^{\frac{3}{2}}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*c
os(d*x+c))^(3/2)/cos(d*x+c)^(5/2)
```

Maxima [A] time = 2.05552, size = 166, normalized size = 2.37

$$\frac{\left(b\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-b\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)B\sqrt{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*((b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(c
os(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b) + 4*A*b^(3/
2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
```

```
+ 2*c) + 1))/d
```

Fricas [A] time = 1.62737, size = 572, normalized size = 8.17

$$\left[\frac{Bb^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}Ab\sqrt{\cos\left(dx+c\right)}\,\mathrm{si}}{2\,d\cos\left(dx+c\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*b^(3/2)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2), -(B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*b*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos\left(dx+c\right)+A)\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}{\cos\left(dx+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2), x
)
```

3.856 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*B*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.0551959, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {17, 2748, 3768, 3770, 3767, 8}

$$\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*B*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}+\frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+\\ &= \frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}-\\ &= \frac{Ab\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{Ab\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.0891647, size = 65, normalized size = 0.59

$$\frac{(b\cos(c+dx))^{3/2}\left(\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*
Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.217, size = 121, normalized size = 1.1

$$-\frac{1}{2d}\left(A\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2-A\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] -1/2/d*(A*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-A*ln((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-2*B*sin(d*x+c)*cos(d*x+c)-A*
sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 2.09507, size = 1008, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*B*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - (4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c
) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2
 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*
cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*si
n(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x +
2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*
c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x
 + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*co
s(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
 1))/d
```

Fricas [A] time = 1.63206, size = 640, normalized size = 5.82

$$\left[\frac{Ab^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2\,Bb\cos\left(dx+c\right)+Ab\right)\sqrt{b\cos\left(dx+\right.}}{4\,d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x
 + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*
b*cos(d*x + c) + A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2), x
)
```

3.857 $\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{Ab\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\tan}{2d\sqrt{\cos(c}}$$

```
[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*b*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.0625064, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 3768, 3770}

$$\frac{Ab\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{Ab\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sqrt{b\cos(c+dx)}\tan}{2d\sqrt{\cos(c}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*b*Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*b*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab\sqrt{b\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+$$
$$= \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(bB\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}} -$$
$$= \frac{bB\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{bB\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] time = 0.0371528, size = 77, normalized size = 0.52

$$\frac{b\sqrt{b\cos(c+dx)}\left(2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2)
,x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin
[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(5/
2))
```

Maple [A] time = 0.28, size = 139, normalized size = 0.9

$$\frac{1}{6d}\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3+3B\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c
)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+
c)^(9/2)
```

Maxima [B] time = 2.19075, size = 1339, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] -1/12*(16*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*
d*x + 2*c) + b)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 +
6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)
^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*
d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(4*(b*sin(4*d*x + 4*c) + 2*b*sin
(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*
sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin
(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2
*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c
) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x
+ 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c)
+ 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*
cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos
(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.71092, size = 714, normalized size = 4.79

$$\left[\frac{3\,Bb^{\frac{3}{2}}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,Ab\cos\left(dx+c\right)^{2}+3\,Bb\cos\left(dx+\right.\right.}{12\,d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] [1/12*(3*B*b^(3/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*B*sq
rt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x
+ c))))*cos(d*x + c)^4 - (4*A*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2),
x)
```

3.858 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=187

$$-\frac{Ab^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\cos^{\frac{5}{2}}(c+d}{4d}$$

```
[Out] (3*b^2*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*b^2*B*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*B*Cos[c + d*x]^(5/2)*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c +
 d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0719922, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2633, 2635, 8}

$$-\frac{Ab^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\cos^{\frac{5}{2}}(c+d}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (3*b^2*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (3*b^2*B*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*B*Cos[c + d*x]^(5/2)*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c +
 d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```


Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+\right.}{\sqrt{\mathrm{co}}}$$
$$= \frac{b^2B\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3b^2B\sqrt{b\,\mathrm{co}}\right.}{}$$
$$= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{3b^2B\sqrt{\cos(c+dx)}\sqrt{b\cos}}{8d}$$
$$= \frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + $$

Mathematica [A] time = 0.177683, size = 81, normalized size = 0.43

$$\frac{(b\cos(c+dx))^{5/2}(72A\sin(c+dx)+8A\sin(3(c+dx))+24B\sin(2(c+dx))+3B\sin(4(c+dx))+36Bc+36Bdx)}{96d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(36*B*c + 36*B*d*x + 72*A*Sin[c + d*x] + 24*B*Sin[2
*(c + d*x)] + 8*A*Sin[3*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/(96*d*Cos[c + d
*x]^(5/2))
```

Maple [A] time = 0.427, size = 91, normalized size = 0.5

$$\frac{6B\sin(dx+c)(\cos(dx+c))^3+8A\sin(dx+c)(\cos(dx+c))^2+9B\sin(dx+c)\cos(dx+c)+16A\sin(dx+c)+9}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] 1/24/d*(b*cos(d*x+c))^(5/2)*(6*B*sin(d*x+c)*cos(d*x+c)^3+8*A*sin(d*x+c)*cos
(d*x+c)^2+9*B*sin(d*x+c)*cos(d*x+c)+16*A*sin(d*x+c)+9*B*(d*x+c))/cos(d*x+c)
^(5/2)
```

Maxima [A] time = 2.50568, size = 149, normalized size = 0.8

$$\frac{8\left(b^2\sin(3dx+3c)+9b^2\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)A\sqrt{b}+3\left(12(dx+c)b^2+b^2\sin(4dx+\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorit
hm="maxima")
```

```
[Out] 1/96*(8*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))))*A*sqrt(b) + 3*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8
*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b))/d
```

Fricas [A] time = 1.80656, size = 748, normalized size = 4.

$$\left[\frac{9 B\sqrt{-b}b^2 \cos(dx+c)\log\left(2 b\cos(dx+c)^2 - 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c) - b\right) + 2\left(6 Bb^2\cos(dx+\right.}{48 d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorit
hm="fricas")
```

```
[Out] [1/48*(9*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*B*b^2*cos(d*
x + c)^3 + 8*A*b^2*cos(d*x + c)^2 + 9*B*b^2*cos(d*x + c) + 16*A*b^2)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(9*B
*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/
2)))*cos(d*x + c) + (6*B*b^2*cos(d*x + c)^3 + 8*A*b^2*cos(d*x + c)^2 + 9*B*
b^2*cos(d*x + c) + 16*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)),x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.859 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=148

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}-\frac{b^2B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+}{d\sqrt{c}}$$

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) - (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c
 + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0563016, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}-\frac{b^2B\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+}{d\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) - (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c
 + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+$$
$$= \frac{Ab^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}$$
$$= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{\cos(c+dx)}}{}$$

Mathematica [A] time = 0.142225, size = 69, normalized size = 0.47

$$\frac{(b\cos(c+dx))^{5/2}(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c
+ d*x)] + B*Sin[3*(c + d*x)]))/(12*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.419, size = 74, normalized size = 0.5

$$\frac{2\,B\sin(dx+c)(\cos(dx+c))^2+3\,A\cos(dx+c)\sin(dx+c)+3\,A(dx+c)+4\,B\sin(dx+c)}{6\,d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] 1/6/d*(b*cos(d*x+c))^(5/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(
d*x+c)+3*A*(d*x+c)+4*B*sin(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [A] time = 2.05939, size = 111, normalized size = 0.75

$$\frac{3\left(2(dx+c)b^2+b^2\sin(2\,dx+2\,c)\right)A\sqrt{b}+\left(b^2\sin(3\,dx+3\,c)+9\,b^2\sin\left(\frac{1}{3}\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)B}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + (b^2*sin(3*d*x
 + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqr
t(b))/d
```

Fricas [A] time = 1.76967, size = 676, normalized size = 4.57

$$\left[\frac{3\,A\sqrt{-b}b^2\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)+2\left(2\,Bb^2\cos\left(dx\right.\right.}{12\,d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*A*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(
d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*B*b^2*cos(d*
x + c)^2 + 3*A*b^2*cos(d*x + c) + 4*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*A*b^(5/2)*arctan(sqrt(b*cos(
d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*b^
2*cos(d*x + c)^2 + 3*A*b^2*cos(d*x + c) + 4*B*b^2)*sqrt(b*cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}{\sqrt{\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)), x
)
```

3.860 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0239772, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c
 + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{\cos(c+dx)}}{}\end{aligned}$$

Mathematica [A] time = 0.123914, size = 57, normalized size = 0.53

$$\frac{(b\cos(c+dx))^{5/2}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x
)])))/(4*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.246, size = 55, normalized size = 0.5

$$\frac{B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c)}{2d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(5/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c
))/cos(d*x+c)^(5/2)
```

Maxima [A] time = 2.03302, size = 63, normalized size = 0.59

$$\frac{4\,Ab^{\frac{5}{2}}\sin(dx+c)+\left(2\,(dx+c)b^2+b^2\sin(2\,dx+2\,c)\right)B\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(4*A*b^(5/2)*sin(d*x + c) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*B*
sqrt(b))/d
```

Fricas [A] time = 1.7651, size = 597, normalized size = 5.58

$$\left[\frac{B\sqrt{-b}b^2\cos(dx+c)\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\left(Bb^2\cos(dx+c\right.}{4\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(B*b^2*cos(d*x + c
) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)), 1/2*(B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
cos(d*x + c)^(3/2)))*cos(d*x + c) + (B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*c
```

```
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2), x
)
```

3.861 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*B*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0141022, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*B*Sqrt[b*Cos[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{Ab^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.076974, size = 42, normalized size = 0.65

$$\frac{(b\cos(c+dx))^{5/2}(A(c+dx)+B\sin(c+dx))}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Cos[c + d*x]^(5/
2))
```

Maple [A] time = 0.223, size = 39, normalized size = 0.6

$$\frac{A(dx+c)+B\sin(dx+c)}{d}(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] 1/d*(b*cos(d*x+c))^(5/2)*(A*(d*x+c)+B*sin(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [A] time = 1.904, size = 54, normalized size = 0.83

$$\frac{2\,Ab^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+Bb^{\frac{5}{2}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + B*b^(5/2)*sin(d*x +
c))/d
```

Fricas [A] time = 1.70098, size = 524, normalized size = 8.06

$$\left[\frac{A\sqrt{-b}b^{2}\cos(dx+c)\log\left(2\,b\cos(dx+c)^{2}-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\sqrt{b\cos(dx+c)}Bb^{2}}{2\,d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(A*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c
))*B*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), (A*b^(5/2)*arct
an(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x
+ c) + sqrt(b*cos(d*x + c))*B*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2), x
)
```

3.862 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=66

$$\frac{Ab^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*ArcTanh[Sin[c +
d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0263802, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{Ab^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}+\frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*ArcTanh[Sin[c +
d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= \frac{b^2Bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}+\frac{Ab^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.060486, size = 40, normalized size = 0.61

$$\frac{(b\cos(c+dx))^{5/2}\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*(b*Cos[c + d*x])^(5/2))/(d*Cos[c + d*x]^
(5/2))
```

Maple [A] time = 0.185, size = 54, normalized size = 0.8

$$-\frac{1}{d}(b\cos(dx+c))^{\frac{5}{2}}\left(2\,A\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)\right)(\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] -1/d*(b*cos(d*x+c))^(5/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c
))/cos(d*x+c)^(5/2)
```

Maxima [A] time = 1.82097, size = 134, normalized size = 2.03

$$\frac{4\,Bb^{\frac{5}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)+\left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-b^2\log\left(\cos(dx+c)^2+\sin(\ldots\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*B*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (b^2*log(cos(d*x
 + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + s
in(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d
```

Fricas [A] time = 2.08131, size = 599, normalized size = 9.08

$$\left[-\frac{2\,A\sqrt{-b}b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)-B\sqrt{-b}b^2\log\left(2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin\ldots\right.}{2\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c)))) - B*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*c
os(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(2*B*b^(
5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))
 + A*b^(5/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2), x
)
```

3.863 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) +
 (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.0426741, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 8, 3770}

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) +
 (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \sec(c+d}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\text{Subst}(\int 1}{d\sqrt{\cos(c+d}}$$
$$= \frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A] time = 0.0784006, size = 50, normalized size = 0.68

$$\frac{(b\cos(c+dx))^{5/2}\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),
x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d
*x]))/(d*Cos[c + d*x]^(7/2))
```

Maple [A] time = 0.224, size = 59, normalized size = 0.8

$$\frac{1}{d}\left(-2\,B\cos(dx+c)\,\text{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)(b\cos(dx+c))^{\frac{5}{2}}(\cos(dx+c))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*c
os(d*x+c))^(5/2)/cos(d*x+c)^(7/2)
```

Maxima [A] time = 1.9862, size = 171, normalized size = 2.31

$$\frac{\frac{4\,Ab^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1}+\left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-b^2\log\left(\cos(dx+c)\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*A*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*si
n(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c)
```

```
+ 1))*B*sqrt(b))/d
```

Fricas [A] time = 1.62102, size = 581, normalized size = 7.85

$$\left[\frac{Bb^{\frac{5}{2}}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}Ab^2\sqrt{\cos(dx+c)}\,s}{2d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*b^(5/2)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos
(d*x + c)^2), -(B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x
 + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*b^2*s
qrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2), x
)
```

3.864 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]])
 + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^
2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rubi [A] time = 0.0569699, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {17, 2748, 3768, 3770, 3767, 8}

$$\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{Ab^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2),x]
```

```
[Out] (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]])
 + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^
2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{11}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \sec^2(c\\ &= \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}\\ &= \frac{Ab^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2\sqrt{b\cos(c+dx)}\sin(c+d}{2d\cos^{\frac{5}{2}}(c+dx)}\end{aligned}$$

Mathematica [A] time = 0.145808, size = 65, normalized size = 0.56

$$\frac{(b\cos(c+dx))^{5/2}\left(\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(11/2)
,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*
Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.246, size = 121, normalized size = 1.

$$-\frac{1}{2d}\left(A\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2 - A\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x)
```

```
[Out] -1/2/d*(A*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-A*ln((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-2*B*sin(d*x+c)*cos(d*x+c)-A*
sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2)
```

Maxima [B] time = 2.02774, size = 1084, normalized size = 9.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="maxima")
```

```
[Out] 1/4*(8*B*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1) - (4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*
c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x
 + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(
4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x
+ 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*
cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4
*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 +
 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*
cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*A*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c
)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.71817, size = 653, normalized size = 5.63

$$\left[\frac{Ab^{\frac{5}{2}}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2Bb^2\cos(dx+c)+Ab^2\right)\sqrt{b\cos(dx}}{4d\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="fricas")
```

```
[Out] [1/4*(A*b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*b^2*arctan(sqrt(b*c
os(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 -
 (2*B*b^2*cos(d*x + c) + A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(11/2),x, algori
thm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2),
x)
```

3.865 $\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{Ab^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos}}$$

```
[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]])
 + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*
b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*b^2*Sqrt
[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.0635576, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 3768, 3770}

$$\frac{Ab^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{Ab^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2B\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2),x]
```

```
[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]])
 + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (A*
b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (A*b^2*Sqrt
[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}(A+B\cos(c+dx))}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int (A+B\cos(c+dx))\sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\left(Ab^2\sqrt{b\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \sec^3(c$$
$$= \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2B\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2B\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{b^2B\sqrt{b\cos(c+dx)}\sin(c+d}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A] time = 0.179833, size = 76, normalized size = 0.48

$$\frac{(b\cos(c+dx))^{5/2}\left(2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{6d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(13/2)
,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin
[c + d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(9/
2))
```

Maple [A] time = 0.257, size = 139, normalized size = 0.9

$$\frac{1}{6d}\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3+3B\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c
)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+
c)^(11/2)
```

Maxima [B] time = 2.07803, size = 1431, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algori
thm="maxima")
```

```
[Out] -1/12*(16*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)
*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*
b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
 + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*(4*(b^2*sin(4*d*x +
4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*co
s(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b
^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
(b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2
 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b
^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*
c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x
 + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b^
2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d
*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
+ 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
+ 2*c) + 1))/d
```

Fricas [A] time = 1.9629, size = 733, normalized size = 4.67

$$\left[\frac{3\, Bb^{\frac{5}{2}} \cos\left(dx+c\right)^{4} \log\left(-\frac{b\cos(dx+c)^{3} - 2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c) - 2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right) + 2\left(4\, Ab^{2}\cos\left(dx+c\right)^{2} + 3\, Bb^{2}\cos\left(dx\right.\right.}{12\, d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algori
thm="fricas")
```

```
[Out] [1/12*(3*B*b^(5/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(
b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(
3*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(
cos(d*x + c))))*cos(d*x + c)^4 - (4*A*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x
+ c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c))/cos(d*x+c)^(13/2),x, algori
thm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2),
x)
```

3.866 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=136

$$\frac{Ax\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sq
rt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0556648, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ax\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sq
rt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right)\text{S}}{}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx}{2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0983586, size = 69, normalized size = 0.51

$$\frac{\sqrt{\cos(c+dx)}(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*
x)] + B*Sin[3*(c + d*x)]))/(12*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.325, size = 74, normalized size = 0.5

$$\frac{2B\sin(dx+c)(\cos(dx+c))^2+3A\cos(dx+c)\sin(dx+c)+3A(dx+c)+4B\sin(dx+c)}{6d}\sqrt{\cos(dx+c)}\frac{1}{\sqrt{b\cos(d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*cos(d*x+c)^(1/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(d*x+
c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 2.04084, size = 92, normalized size = 0.68

$$\frac{\frac{3(2dx+2c+\sin(2dx+2c))A}{\sqrt{b}} + \frac{B\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)}{\sqrt{b}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + B*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b))/d
```

Fricas [A] time = 1.68327, size = 645, normalized size = 4.74

$$\left[-\frac{3\,A\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\left(2\,B\cos\left(dx+c\right)^{2}\right.}{12\,bd\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin
(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^2
+ 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c))/(b*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{5}{2}}}{\sqrt{b\cos\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)
```

3.867 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0238959, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```


```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(2*d*Sqrt[b*Cos[c + d*x]])
```


Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0794154, size = 57, normalized size = 0.58

$$\frac{\sqrt{\cos(c+dx)}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.33, size = 55, normalized size = 0.6

$$\frac{B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c)}{2d}\sqrt{\cos(dx+c)}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*cos(d*x+c)^(1/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c))/(
b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 2.03885, size = 54, normalized size = 0.55

$$\frac{\frac{(2dx+2c+\sin(2dx+2c))B}{\sqrt{b}}+\frac{4A\sin(dx+c)}{\sqrt{b}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/sqrt(b) + 4*A*sin(d*x + c)/sqrt(b))
/d
```

Fricas [A] time = 1.72982, size = 576, normalized size = 5.88

$$\left[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2(B\cos(dx+c)+2\ldots}{4bd\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)),
 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x +
 c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)
```

3.868 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=59

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (B*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0131365, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (B*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0417325, size = 42, normalized size = 0.71

$$\frac{\sqrt{\cos(c+dx)}(A(c+dx)+B\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]]
)
```

Maple [A] time = 0.366, size = 39, normalized size = 0.7

$$\frac{A\,(dx+c)+B\sin\,(dx+c)}{d}\sqrt{\cos\,(dx+c)}\frac{1}{\sqrt{b\cos\,(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)*(A*(d*x+c)+B*sin(d*x+c))/(b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.89028, size = 54, normalized size = 0.92

$$\frac{\frac{2\,A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}+\frac{B\sin(dx+c)}{\sqrt{b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + B*sin(d*x + c)/sqrt(
b))/d
```

Fricas [A] time = 1.62229, size = 514, normalized size = 8.71

$$\left[-\frac{A\sqrt{-b}\cos\,(dx+c)\log\left(2\,b\cos\,(dx+c)^2+2\,\sqrt{b\cos\,(dx+c)}\sqrt{-b}\sqrt{\cos\,(dx+c)}\sin\,(dx+c)-b\right)-2\,\sqrt{b\cos\,(dx+c)}\ldots}{2\,bd\cos\,(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*
B*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)), (A*sqrt(b)*arctan(sq
rt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)
+ sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)
)]
```

Sympy [A] time = 19.3705, size = 46, normalized size = 0.78

$$\begin{cases} \frac{Ax}{\sqrt{b}} + \frac{B\sin(c+dx)}{\sqrt{b}d} & \text{for } d \neq 0 \\ \frac{x(A+B\cos(c))\sqrt{\cos(c)}}{\sqrt{b\cos(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((A*x/sqrt(b) + B*sin(c + d*x)/(sqrt(b)*d), Ne(d, 0)), (x*(A + B*c
os(c))*sqrt(cos(c))/sqrt(b*cos(c)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)
```

3.869 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0264577, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {18, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sq
rt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\
&= \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}+\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0337165, size = 40, normalized size = 0.67

$$\frac{\sqrt{\cos(c+dx)}\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A] time = 0.326, size = 54, normalized size = 0.9

$$-\frac{1}{d}\left(2\,A\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\,(dx+c)\right)\sqrt{\cos(dx+c)}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c))*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 1.80266, size = 124, normalized size = 2.07

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/sqrt(b))/d
```

Fricas [B] time = 2.03033, size = 599, normalized size = 9.98

$$\left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)+B\sqrt{-b}\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+\right.}{2\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/2*(2*B*sqrt(b)
*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + A
*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d
*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + B\cos(c + dx)}{\sqrt{b\cos(c + dx)}\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*cos(c + d*x))/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx + c) + A}{\sqrt{b\cos(dx + c)}\sqrt{\cos(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))), x
)
```

3.870 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=68

$$\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (A*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0418939, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 8, 3770}

$$\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (A*
Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c}{d\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0415201, size = 50, normalized size = 0.74

$$\frac{A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.315, size = 59, normalized size = 0.9

$$\frac{1}{d}\left(-2B\cos(dx+c)\,\text{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)\frac{1}{\sqrt{\cos(dx+c)}}\frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))/cos(
d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.08948, size = 169, normalized size = 2.49

$$\frac{\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{\sqrt{b}}+\frac{4A\sqrt{b}\sin(2dx+2c)}{b\cos(2dx+2c)^2+b\sin(2dx+2c)^2+2b\cos(2dx+2c)+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*A*sqrt(b)*s
in(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*
x + 2*c) + b))/d
```

Fricas [A] time = 1.57537, size = 570, normalized size = 8.38

$$\left[\frac{B\sqrt{b}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin(d}{2bd\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d
*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*
x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)), x
)
```

3.871 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (
A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c +
d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0545298, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {18, 2748, 3768, 3770, 3767, 8}

$$\frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (
A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c +
d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+\right.}{}$$

$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\,}{d\sqrt{\cos(c+}}$$

Mathematica [A] time = 0.0643787, size = 65, normalized size = 0.61

$$\frac{\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*
x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.338, size = 120, normalized size = 1.1

$$\frac{1}{2\,d}\left(-A\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2 + A\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2 + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*(-A*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+A*ln((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+2*B*sin(d*x+c)*cos(d*x+c)+A*
sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.02027, size = 975, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c
)^2 + 2*b*cos(2*d*x + 2*c) + b) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c)
)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c
) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*c
os(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4
*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)
+ 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((2*(2*cos(2*d
*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Fricas [A] time = 1.6147, size = 632, normalized size = 5.91

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx\right.}}{4\,bd\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c
))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*cos(
d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*co
s(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)), x
)
```

3.872 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=145

$$\frac{A\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\mathrm{t}}{2d\sqrt{b\,\mathrm{c}}}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (
B*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c +
d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3)/(3*d
*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0649014, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 3768, 3770}

$$\frac{A\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\mathrm{t}}{2d\sqrt{b\,\mathrm{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (
B*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c +
d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3)/(3*d
*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{\cos(c+\right.}{}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{A\,\mathrm{s}}{d\sqrt{\cos(c+}}$$

Mathematica [A] time = 0.0805181, size = 76, normalized size = 0.52

$$\frac{2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{6d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos
[2*(c + d*x)])*Tan[c + d*x])/(6*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.378, size = 139, normalized size = 1.

$$\frac{1}{6d}\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3+3B\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c
)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c
))^(1/2)
```

Maxima [B] time = 2.06454, size = 1292, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1
)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4
*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x
 + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6
*cos(2*d*x + 2*c) + 1)*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*
c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4
*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4
*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d
```

Fricas [A] time = 1.68687, size = 701, normalized size = 4.83

$$\left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,A\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx\right.\right.}{12\,bd\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-
b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)))
)*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)), x
)
```

3.873 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{Ax\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(2*b*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(
3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0556814, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ax\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(2*b*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(
3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2b\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right)\text{Sub}}{bd}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} -$$

Mathematica [A] time = 0.0736401, size = 69, normalized size = 0.47

$$\frac{\cos^{\frac{3}{2}}(c+dx)(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*
x)] + B*Sin[3*(c + d*x)]))/(12*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.246, size = 74, normalized size = 0.5

$$\frac{2B\sin(dx+c)(\cos(dx+c))^2+3A\cos(dx+c)\sin(dx+c)+3A(dx+c)+4B\sin(dx+c)}{6d}(\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/6/d*cos(d*x+c)^(3/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(d*x+
c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 2.09645, size = 92, normalized size = 0.62

$$\frac{\frac{3(2dx+2c+\sin(2dx+2c))A}{b^{\frac{3}{2}}}+\frac{B\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)}{b^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + B*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(3/2))/d
```

Fricas [A] time = 1.64115, size = 651, normalized size = 4.4

$$\left[-\frac{3\,A\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\left(2\,B\cos\left(dx+\right.\right.}{12\,b^{2}d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b^2*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*s
in(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{7}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2), x
)
```

3.874 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0232636, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0864058, size = 57, normalized size = 0.53

$$\frac{\cos^{\frac{3}{2}}(c+dx)(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.262, size = 55, normalized size = 0.5

$$\frac{B\sin(dx+c)\cos(dx+c)+2A\sin(dx+c)+B(dx+c)}{2d}(\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*cos(d*x+c)^(3/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c))/(
b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.96502, size = 54, normalized size = 0.5

$$\frac{\frac{(2dx+2c+\sin(2dx+2c))B}{b^{\frac{3}{2}}}+\frac{4A\sin(dx+c)}{b^{\frac{3}{2}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(3/2) + 4*A*sin(d*x + c)/b^(3/2))
/d
```

Fricas [A] time = 1.63725, size = 582, normalized size = 5.44

$$\left[-\frac{B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2(B\cos(dx+c)+}{4b^2d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)
), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x
 + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sq
```

```
rt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2), x
)
```

3.875 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0136778, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0453462, size = 42, normalized size = 0.65

$$\frac{\cos^{\frac{3}{2}}(c+dx)(A(c+dx)+B\sin(c+dx))}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(A*(c + d*x) + B*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/
2))
```

Maple [A] time = 0.207, size = 39, normalized size = 0.6

$$\frac{A(dx+c)+B\sin(dx+c)}{d}(\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(3/2)*(A*(d*x+c)+B*sin(d*x+c))/(b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.87198, size = 54, normalized size = 0.83

$$\frac{\frac{2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}+\frac{B\sin(dx+c)}{b^{\frac{3}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2) + B*sin(d*x + c)/b^(3/
2))/d
```

Fricas [A] time = 1.63778, size = 520, normalized size = 8.

$$\left[-\frac{A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2\sqrt{b\cos(dx+c)}B\sqrt{\cdot}}{2b^2d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*
B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)), (A*sqrt(b)*arctan(
sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c
) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x
+ c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2), x
)
```

3.876 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]
]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0266115, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]
]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}+\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0381425, size = 40, normalized size = 0.61

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(3/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Cos[c + d*x]^(3/2))/(d*(b*Cos[c + d*x])^
(3/2))
```

Maple [A] time = 0.309, size = 54, normalized size = 0.8

$$-\frac{1}{d}(\cos(dx+c))^{\frac{3}{2}}\left(2\,A\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B\,(dx+c)\right)(b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(3/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c))/(
b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 1.87296, size = 124, normalized size = 1.88

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{b^{\frac{3}{2}}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/b^(3/2))/d
```

Fricas [A] time = 1.89476, size = 605, normalized size = 9.17

$$\left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)+B\sqrt{-b}\log\left(2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(d\right.}{2\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(2*B*sqrt(
b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) +
 A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos
(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2), x
)
```

3.877 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (
A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0415016, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 8, 3770}

$$\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (
A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c}{bd\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0534019, size = 50, normalized size = 0.68

$$\frac{\sqrt{\cos(c+dx)}\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])
)/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.315, size = 59, normalized size = 0.8

$$\frac{1}{d}\left(-2\,B\cos(dx+c)\,\text{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)\sqrt{\cos(dx+c)}\,(b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*cos(
d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.04411, size = 180, normalized size = 2.43

$$\frac{\frac{4\,A\sqrt{b}\sin(2\,dx+2\,c)}{b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2}+\frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\,\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\,\sin(dx+c)+1\right)\right)}{b^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + B*(log(cos(d*x + c)^2 + sin(d*x +
 c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
```

```
*x + c) + 1))/b^(3/2))/d
```

Fricas [A] time = 1.7188, size = 575, normalized size = 7.77

$$\left[\frac{B\sqrt{b}\cos(dx+c)^2\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\sqrt{b\cos(dx+c)}A\sqrt{\cos(dx+c)}\sin(dx+c)}{2b^2d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos
(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(
d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))),
 x)
```

3.878 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) +
 (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[
c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0561972, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {18, 2748, 3768, 3770, 3767, 8}

$$\frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) +
 (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[
c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}} - \left(B\sqrt{c\right.$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{}{bd\sqrt{}}$$

Mathematica [A] time = 0.0619756, size = 65, normalized size = 0.56

$$\frac{\sin(c+dx)(A+2B\cos(c+dx))+A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*
x])/(2*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.274, size = 120, normalized size = 1.

$$\frac{1}{2d}\left(-A\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2 + A\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(-A*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+A*ln((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2+2*B*sin(d*x+c)*cos(d*x+c)+A*
sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.04705, size = 998, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x +
 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b*co
s(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x +
 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b)))/d
```

Fricas [A] time = 1.71594, size = 637, normalized size = 5.49

$$\left[\frac{A\sqrt{b}\cos(dx+c)^3\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2(2B\cos(dx+c)+A)\sqrt{b\cos(dx+c}}{4b^2d\cos(dx+c)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b^2*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x +
 c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*co
s(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*
d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2)),
 x)
```

3.879 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=157

$$\frac{A\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\mathrm{t}}{2bd\sqrt{b\,\mathrm{c}}}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) +
 (B*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[
c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3
)/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0644936, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 3768, 3770}

$$\frac{A\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\,\mathrm{t}}{2bd\sqrt{b\,\mathrm{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) +
 (B*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*Sin[
c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x]^3
)/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A+B\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{c\right.}}{}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{}{bd\sqrt{}}$$

Mathematica [A] time = 0.079532, size = 76, normalized size = 0.48

$$\frac{2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))}{6d\sqrt{\cos(c+dx)}(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),
x]
```

```
[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c + d*x] + 2*A*(2 + Cos
[2*(c + d*x)])*Tan[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)
)
```

Maple [A] time = 0.262, size = 139, normalized size = 0.9

$$\frac{1}{6d}\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3+3B\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c
)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c
))^(3/2)
```

Maxima [B] time = 2.06332, size = 1327, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2
+ 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 +
18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*co
s(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*
d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x +
4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) - 3*(4*(sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c
) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
 + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x
+ 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))))*B/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x
 + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2
+ 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)
*sqrt(b)))/d
```

Fricas [A] time = 1.68517, size = 706, normalized size = 4.5

$$\left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^{4}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(4\,A\cos\left(dx+c\right)^{2}+3\,B\cos\left(dx+c\right.\right.}{12\,b^{2}d\cos\left(dx+c\right)^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4), -1/6*(3*B*sqrt
(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)),
 x)
```

3.880 $\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}} dx$

Optimal. Leaf size=148

$$\frac{Ax\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x
]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0569816, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 2635, 8, 2633}

$$\frac{Ax\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}-\frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x
]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)(A+B\cos(c+dx))\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int 1\,dx}{2b^2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos(c+dx)}\right)\text{S}}{}$$
$$= \frac{Ax\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\cos^{\frac{3}{2}}(c+dx)\sin(c+}{2b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0658698, size = 72, normalized size = 0.49

$$\frac{\sqrt{\cos(c+dx)}(3A\sin(2(c+dx))+6Ac+6Adx+9B\sin(c+dx)+B\sin(3(c+dx)))}{12b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A*c + 6*A*d*x + 9*B*Sin[c + d*x] + 3*A*Sin[2*(c + d*
x)] + B*Sin[3*(c + d*x)]))/(12*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.24, size = 74, normalized size = 0.5

$$\frac{2B\sin(dx+c)(\cos(dx+c))^2+3A\cos(dx+c)\sin(dx+c)+3A(dx+c)+4B\sin(dx+c)}{6d}(\cos(dx+c))^{\frac{5}{2}}(b\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/6/d*cos(d*x+c)^(5/2)*(2*B*sin(d*x+c)*cos(d*x+c)^2+3*A*cos(d*x+c)*sin(d*x+
c)+3*A*(d*x+c)+4*B*sin(d*x+c))/(b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 2.02031, size = 92, normalized size = 0.62

$$\frac{\frac{3(2dx+2c+\sin(2dx+2c))A}{b^{\frac{5}{2}}}+\frac{B\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)}{b^{\frac{5}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + B*(sin(3*d*x + 3*c) +
9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2))/d
```

Fricas [A] time = 1.90168, size = 651, normalized size = 4.4

$$\left[-\frac{3\,A\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^2+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\left(2\,B\cos\left(dx+c\right)^2\right.}{12\,b^3d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x
 + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b^3*d*cos(d*x + c)), 1/6*(3*A*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*s
in(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*B*cos(d*x + c)^
2 + 3*A*cos(d*x + c) + 4*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B\cos\left(dx+c\right)+A\right)\cos\left(dx+c\right)^{\frac{9}{2}}}{\left(b\cos\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x
)
```

3.881 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0240382, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2734}

$$\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (A*Sqrt[Cos[c + d*x
]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)(A+B\cos(c+dx))\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}+\frac{B\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0608085, size = 60, normalized size = 0.56

$$\frac{\sqrt{\cos(c+dx)}(4A\sin(c+dx)+B(2(c+dx)+\sin(2(c+dx))))}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(4*A*Sin[c + d*x] + B*(2*(c + d*x) + Sin[2*(c + d*x)]))
)/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.22, size = 55, normalized size = 0.5

$$\frac{B\sin\left(dx+c\right)\cos\left(dx+c\right)+2\,A\sin\left(dx+c\right)+B\left(dx+c\right)}{2\,d}\left(\cos\left(dx+c\right)\right)^{\frac{5}{2}}\left(b\cos\left(dx+c\right)\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/2/d*cos(d*x+c)^(5/2)*(B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c)+B*(d*x+c))/(
b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.97879, size = 54, normalized size = 0.5

$$\frac{\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))B}{b^{\frac{5}{2}}}+\frac{4\,A\sin(dx+c)}{b^{\frac{5}{2}}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(5/2) + 4*A*sin(d*x + c)/b^(5/2))
/d
```

Fricas [A] time = 1.97664, size = 582, normalized size = 5.44

$$\left[-\frac{B\sqrt{-b}\cos\left(dx+c\right)\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)-2\left(B\cos\left(dx+c\right)+2\right.}{4\,b^{3}d\cos\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*(B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(B*cos(d*x + c) + 2*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)
), 1/2*(B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x
 + c)^(3/2)))*cos(d*x + c) + (B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sq
```

```
rt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2), x
)
```

3.882 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0136397, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {17, 2637}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0433792, size = 45, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}(A(c+dx)+B\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*(c + d*x) + B*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d
*x]])
```

Maple [A] time = 0.23, size = 39, normalized size = 0.6

$$\frac{A(dx+c)+B\sin(dx+c)}{d}(\cos(dx+c))^{\frac{5}{2}}(b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/d*cos(d*x+c)^(5/2)*(A*(d*x+c)+B*sin(d*x+c))/(b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.86252, size = 54, normalized size = 0.83

$$\frac{\frac{2A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}+\frac{B\sin(dx+c)}{b^{\frac{5}{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] (2*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2) + B*sin(d*x + c)/b^(5/
2))/d
```

Fricas [A] time = 1.92784, size = 520, normalized size = 8.

$$\left[-\frac{A\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2\sqrt{b\cos(dx+c)}B}{2b^3d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(A*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*
B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), (A*sqrt(b)*arctan(
sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c
) + sqrt(b*cos(d*x + c))*B*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x
+ c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{5}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2), x
)
```

3.883 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=66

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*
x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0274445, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {17, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}+\frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*
x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}+\frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.0341598, size = 43, normalized size = 0.65

$$\frac{\sqrt{\cos(c+dx)}\left(A\tanh^{-1}(\sin(c+dx))+Bdx\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] ((B*d*x + A*ArcTanh[Sin[c + d*x]])*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c
+ d*x]])
```

Maple [A] time = 0.181, size = 54, normalized size = 0.8

$$-\frac{1}{d}(\cos(dx+c))^{\frac{5}{2}}\left(2\,A\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)-B(dx+c)\right)(b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/d*cos(d*x+c)^(5/2)*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c))/(
b*cos(d*x+c))^(5/2)
```

Maxima [A] time = 1.84922, size = 124, normalized size = 1.88

$$\frac{\frac{A\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)}{b^{\frac{5}{2}}}+\frac{4\,B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 4*B*arctan(si
n(d*x + c)/(cos(d*x + c) + 1))/b^(5/2))/d
```

Fricas [A] time = 2.51025, size = 605, normalized size = 9.17

$$\left[-\frac{2\,A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)+B\sqrt{-b}\log\left(2\,b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+\right.}{2\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c)))) + B*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x +
 c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/2*(2*B*sqrt(
b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) +
 A*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos
(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3))/(b^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^{\frac{3}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2), x
)
```

3.884 $\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0413233, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {17, 2748, 3767, 8, 3770}

$$\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) +
 (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx))}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int(A+B\cos(c+dx))\sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int\sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int\sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(A\sqrt{\cos(c+dx)}\right)\mathrm{Subst}(\int 1\,dx, x, -}{b^2d\sqrt{b\cos(c+dx)}}$$
$$= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0500516, size = 50, normalized size = 0.68

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(A\sin(c+dx)+B\cos(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(5/2),
x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])
)/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.331, size = 59, normalized size = 0.8

$$\frac{1}{d}\left(-2B\cos(dx+c)\,\mathrm{Artanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)+A\sin(dx+c)\right)(\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*cos(
d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [B] time = 2.02931, size = 180, normalized size = 2.43

$$\frac{\frac{4A\sqrt{b}\sin(2dx+2c)}{b^3\cos(2dx+2c)^2+b^3\sin(2dx+2c)^2+2b^3\cos(2dx+2c)+b^3} + \frac{B\left(\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+\right.\right.}{b^{\frac{5}{2}}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + B*(log(cos(d*x + c)^2 + sin(d*x +
 c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d
```

```
*x + c) + 1))/b^(5/2))/d
```

Fricas [A] time = 1.91438, size = 575, normalized size = 7.77

$$\left[\frac{B\sqrt{b}\cos\left(dx+c\right)^{2}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}A\sqrt{\cos\left(dx+c\right)}\sin\left(d\right.}{2b^{3}d\cos\left(dx+c\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/2*(B*sqrt(b)*cos(d*x + c)^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos
(d*x + c)^2), -(B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c))*A*sqrt(cos(
d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sqrt{\cos(dx+c)}}{(b\cos(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2), x
)
```

3.885 $\int \frac{A+B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
 + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*
Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0560446, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {18, 2748, 3768, 3770, 3767, 8}

$$\frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
 + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*
Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{A + B\cos(c+dx)}{\sqrt{\cos(c+dx)}(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}} - \frac{\left(B\sqrt{\cos\right.}{}$$
$$= \frac{A\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{}{b^2d\sqrt{}}$$

Mathematica [A] time = 0.0748962, size = 65, normalized size = 0.56

$$\frac{\sqrt{\cos(c+dx)}\left(\sin(c+dx)(A+2B\cos(c+dx)) + A\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{2d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[
c + d*x])*Sin[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.342, size = 121, normalized size = 1.

$$-\frac{1}{2d}\left(A\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2 - A\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^2 - 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] -1/2/d*(A*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-A*ln((1-c
os(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^2-2*B*sin(d*x+c)*cos(d*x+c)-A*
sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [B] time = 2.12923, size = 1022, normalized size = 8.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/4*(8*B*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x +
 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*
x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d
*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*
c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x
 + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*
cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*
b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*co
s(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*s
qrt(b)))/d
```

Fricas [A] time = 1.97045, size = 637, normalized size = 5.49

$$\left[\frac{A\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(2\,B\cos\left(dx+c\right)+A\right)\sqrt{b\cos\left(dx\right.}}{4\,b^{3}d\cos\left(dx+c\right)^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*(A*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c
)^3) + 2*(2*B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c))/(b^3*d*cos(d*x + c)^3), -1/2*(A*sqrt(-b)*arctan(sqrt(b*cos(d*x +
 c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - (2*B*co
s(d*x + c) + A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*
d*cos(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c))),
 x)
```

3.886 $\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=157

$$\frac{A\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+}}{2b}$$

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
 + (B*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*
Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c +
d*x]^3)/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0642757, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {18, 2748, 3767, 3768, 3770}

$$\frac{A\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{A\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{B\sqrt{\cos(c+}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
 + (B*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (A*
Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c +
d*x]^3)/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/
2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int (A+B\cos(c+dx))\sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\left(A\sqrt{\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2b^2\sqrt{b\cos(c+dx)}}-\frac{\left(A\sqrt{\cos\right.}{}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{B\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{}{b^2d\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0984708, size = 76, normalized size = 0.48

$$\frac{\sqrt{\cos(c+dx)}\left(2A(\cos(2(c+dx))+2)\tan(c+dx)+3B\sin(c+dx)+3B\cos^2(c+dx)\tanh^{-1}(\sin(c+dx))\right)}{6d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + 3*B*Sin[c +
 d*x] + 2*A*(2 + Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*(b*Cos[c + d*x])^(5/
2))
```

Maple [A] time = 0.286, size = 139, normalized size = 0.9

$$\frac{1}{6d}\left(-3B\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3+3B\ln\left(\frac{1-\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)(\cos(dx+c))^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
(1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c
)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c
))^(5/2)
```

Maxima [B] time = 2.2113, size = 1395, normalized size = 8.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c)
 + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x
+ 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c
)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x +
 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)
^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2
*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(
4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x +
6*c))*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*
c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d
*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(
4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((b^2*cos(4*d*x + 4*c)^2 + 4*
b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 +
2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)))/d
```

Fricas [A] time = 1.97355, size = 706, normalized size = 4.5

$$\left[\frac{3\,B\sqrt{b}\cos\left(dx+c\right)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(4\,A\cos\left(dx+c\right)^2+3\,B\cos\left(dx\right.\right.}{12\,b^3d\cos\left(dx+c\right)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x
 + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x
+ c)^3) + 2*(4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4), -1/6*(3*B*sqrt
(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c)
)))*cos(d*x + c)^4 - (4*A*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*c
os(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)),
 x)
```

3.887 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c\right.}{13b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]
^2]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*
b^4*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.07488, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c\right.}{13b^4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]
^2]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*
b^4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{7/3}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{7/3}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{10/3}\,dx}{b^3}$$
$$= -\frac{3A(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10b^3d\sqrt{\sin^2(c+dx)}} - \ldots$$

Mathematica [A] time = 0.17175, size = 94, normalized size = 0.79

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\left(13A\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)+10B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\ldots\right)\right)}{130d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(13*A*Hypergeometric
2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] + 10*B*Cos[c + d*x]*Hypergeometric2F1[1/
2, 13/6, 19/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(130*d)
```

Maple [F] time = 0.526, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2\sqrt[3]{b\cos(dx+c)}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)(b\cos(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.888 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1
0/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^3
*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0742312, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1
0/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^3
*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\cos(c+dx))^{4/3}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{7/3}\,dx}{b^2}$$
$$= -\frac{3A(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}} -$$

Mathematica [A] time = 0.132882, size = 89, normalized size = 0.75

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{4/3}\left(10A\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+\right.\right.}{70bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6, 1
3/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(70*b*d)
```

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int \cos(dx+c)\sqrt[3]{b\cos(dx+c)}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos(dx+c)^2+A\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.889 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)
*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^2*d*S
qrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0606157, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)
*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^2*d*S
qrt[Sin[c + d*x]^2])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx &= A\int \sqrt[3]{b\cos(c+dx)}\,dx+\frac{B\int (b\cos(c+dx))^{4/3}\,dx}{b}\\ &= -\frac{3A(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c\end{aligned}$$

Mathematica [A] time = 0.0753476, size = 86, normalized size = 0.72

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\left(7A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx\right.\right.}{28d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3, 5/
3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos
[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*d)
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos\left(dx+c\right)}\,\left(A+B\cos\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos\left(dx+c\right)+A\right)\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3), x)
```

3.890 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=114

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hyp
ergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin
[c + d*x]^2])
```

Rubi [A] time = 0.0789069, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hyp
ergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= (Ab)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx + B\int \sqrt[3]{b\cos(c+dx)}\,dx$$
$$= -\frac{3A\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}} - \text{3.}$$

Mathematica [A] time = 0.0937762, size = 86, normalized size = 0.75

$$-\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] +
B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.462, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos(dx+c)}\,(A+B\cos(dx+c))\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.891 $\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=112

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*A*b*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1/3)
*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin
[c + d*x]^2])
```

Rubi [A] time = 0.0897038, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (3*A*b*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1/3)
*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx + (bB)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3Ab\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3B\sqrt[3]{b\cos(c+dx)}}{}$$

Mathematica [A] time = 0.115637, size = 86, normalized size = 0.77

$$\frac{3b\sqrt{\sin^2(c+dx)}\csc(c+dx)\left(A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)-2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\right)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2*
B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.292, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos(dx+c)}\,(A+B\cos(dx+c))\,(\sec(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.892 $\int \sqrt[3]{b \cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[Out] (3*A*b^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5
*d*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[
-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*S
qrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0909964, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5
*d*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[
-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*S
qrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx$$
$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,d$$
$$= \frac{3Ab^2\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,{}_2F_1\left(-\frac{1}{3},\right.}{2d(b\cos}$$

Mathematica [A] time = 0.0885259, size = 94, normalized size = 0.8

$$\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\sec^2(c+dx)\sqrt[3]{b\cos(c+dx)}\left(2A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\right.\right.}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(2*A*Hypergeometric2F1[-5/6, 1/2, 1/
6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos
[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(10*d)
```

Maple [F] time = 0.344, size = 0, normalized size = 0.

$$\int \sqrt[3]{b\cos(dx+c)}\,(A+B\cos(dx+c))\,(\sec(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.893 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{16/3}\,{}_2F_1\left(\frac{1}{2},\frac{8}{3};\frac{11}{3};\cos^2(c\right)}{16b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*
x]^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])
^(16/3)*Hypergeometric2F1[1/2, 8/3, 11/3, Cos[c + d*x]^2]*Sin[c + d*x])/(16
*b^4*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0736791, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{16/3}\,{}_2F_1\left(\frac{1}{2},\frac{8}{3};\frac{11}{3};\cos^2(c\right)}{16b^4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*
x]^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])
^(16/3)*Hypergeometric2F1[1/2, 8/3, 11/3, Cos[c + d*x]^2]*Sin[c + d*x])/(16
*b^4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{10/3}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{10/3}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{13/3}\,dx}{b^3}$$
$$= -\frac{3A(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.201313, size = 94, normalized size = 0.79

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)(b\cos(c+dx))^{4/3}\left(16A\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)+13B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{8}{3}\right.\right.}{208d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(16*A*Hypergeometric
2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2] + 13*B*Cos[c + d*x]*Hypergeometric2F1[
1/2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(208*d)
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\cos(dx+c))^{\frac{4}{3}} (A+B\cos(dx+c))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^4 + Ab\cos(dx+c)^3\right)(b\cos(dx+c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^3)*(b*cos(d*x + c))^(1/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.894 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]
^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*
b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.074004, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-3*A*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]
^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(
13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*
b^3*d*Sqrt[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{7/3}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\cos(c+dx))^{7/3}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{10/3}\,dx}{b^2}$$
$$= -\frac{3A(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.178926, size = 89, normalized size = 0.75

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{7/3}\left(13A\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)+10B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\right)}{130bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 5/3, 8
/3, Cos[c + d*x]^2] + 10*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 13/6, 19/6,
Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(130*b*d)
```

Maple [F] time = 0.311, size = 0, normalized size = 0.

$$\int \cos(dx+c)(b\cos(dx+c))^{\frac{4}{3}}(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^3+Ab\cos(dx+c)^2\right)(b\cos(dx+c))^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\left(b\cos(dx+c)\right)^{\frac{4}{3}}\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

3.895 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+
}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(10/
3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d
*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0641907, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+
}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(10/
3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d
*Sqrt[Sin[c + d*x]^2])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = A\int (b\cos(c+dx))^{4/3}\,dx+\frac{B\int (b\cos(c+dx))^{7/3}\,dx}{b}$$
$$=-\frac{3A(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos
}{}$$

Mathematica [A] time = 0.0468766, size = 86, normalized size = 0.72

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{4/3}\left(10A\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+
}{70d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6, 1
3/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(70*d)
```

Maple [F] time = 0.204, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}} (A + B\cos(dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c) + A)(b\cos(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(Bb\cos(dx+c)^2 + Ab\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)
```

3.896 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*H
ypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.0801155, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```


```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*H
ypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[
Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$$
$$= (Ab)\int \sqrt[3]{b\cos(c+dx)}\,dx + B\int (b\cos(c+dx))^{4/3}\,dx$$
$$= -\frac{3A(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0118503, size = 87, normalized size = 0.75

$$-\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\left(7A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+d\right.\right.}{28d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3,
5/3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*d)
```

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}(A+B\cos(dx+c))\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.897 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=112

$$-\frac{3Ab\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*b*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x
]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*H
ypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin
[c + d*x]^2])
```

Rubi [A] time = 0.0956602, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3Ab\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (-3*A*b*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x
]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*H
ypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx + (bB)\int \sqrt[3]{b\cos(c+dx)}\,dx$$
$$= -\frac{3Ab\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}} - \text{-}$$

Mathematica [A] time = 0.0221056, size = 88, normalized size = 0.79

$$-\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
+ B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.355, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}(A+B\cos(dx+c))(\sec(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

3.898 $\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=115

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*A*b^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2
*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*(b*Cos[c + d*x])^(
1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0982533, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (3*A*b^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2
*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*(b*Cos[c + d*x])^(
1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt
[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{2/3}}$$
$$= \frac{3Ab^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3bB\sqrt[3]{b\cos(}}{}$$

Mathematica [A] time = 0.103253, size = 88, normalized size = 0.77

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\csc(c+dx)\left(A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)-2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\right)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] -
2*B*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin
[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^{\frac{4}{3}}(A+B\cos(dx+c))(\sec(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\left(b\cos(dx+c)\right)^{\frac{4}{3}}\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

3.899 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+\right.}{10b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1
0/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^4
*d*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.075725, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+\right.}{10b^4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1
0/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^4
*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\int (b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{4/3}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{7/3}\,dx}{b^3}$$
$$= -\frac{3A(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx}{}$$

Mathematica [A] time = 0.17494, size = 94, normalized size = 0.79

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)\left(10A\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\right)}{70d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*(10*A*Hypergeometric2F1[1/2, 7/6, 13/6, Cos
[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*
x]^2])*Sqrt[Sin[c + d*x]^2])/(70*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (A+B\cos(dx+c)) (b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(B\cos(dx+c)^2+A\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}}{b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.900 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/
3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^3*d
*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0700779, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```



```
[Out] (-3*A*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/
3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b^3*d
*Sqrt[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int \sqrt[3]{b\cos(c+dx)}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{4/3}\,dx}{b^2}$$
$$= -\frac{3A(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+a}{}$$

Mathematica [A] time = 0.0202145, size = 89, normalized size = 0.75

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\left(7A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx\right.\right.}{28bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 2/3, 5/
3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos
[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(28*b*d)
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int \cos(dx+c)(A+B\cos(dx+c))(b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.901 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=117

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*H
ypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b^2*d*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0672679, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2748, 2643}

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*H
ypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b^2*d*Sqrt
[Sin[c + d*x]^2])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= A\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx+\frac{B\int \sqrt[3]{b\cos(c+dx)}\,dx}{b}\\ &= -\frac{3A\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3}\right.}{4b^2d\sqrt{\sin}}\end{aligned}$$

Mathematica [A] time = 0.013814, size = 85, normalized size = 0.73

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(4A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)+B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\right)}{4d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + B*
Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c +
 d*x]^2])/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))\,(b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(2/3), x)
```

3.902 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1/3)*H
ypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin
[c + d*x]^2])
```

Rubi [A] time = 0.0850318, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(1/3)*H
ypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= (Ab)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx + B\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$$
$$= \frac{3A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} - \frac{3B\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos\right.}{bd\sqrt{\sin^2(c}$$

Mathematica [A] time = 0.0645689, size = 85, normalized size = 0.75

$$\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\left(A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)-2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\right)}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*Csc[c + d*x]*(A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] - 2*B*
Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sqrt[Sin[c +
 d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))\sec(dx+c)(b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(d*
x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B\cos(c + dx))\sec(c + dx)}{(b\cos(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx + c) + A)\sec(dx + c)}{(b\cos(dx + c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

3.903 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

```
[Out] (3*A*b*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d
*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3
, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.0973742, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*b*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d
*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3
, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{8/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx + (bB)\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx$$
$$= \frac{3Ab\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\text{s}}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c}}$$

Mathematica [A] time = 0.144967, size = 89, normalized size = 0.78

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(2A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\right)}{10d(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(2*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]
+ 5*B*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2])*Sqrt[
Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(8/3))
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))(\sec(dx+c))^2(b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^2}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*cos(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

3.904 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{8/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}$$

```
[Out] (3*A*b^2*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
8*d*(b*Cos[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1
[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)*
Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0997625, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{8/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*A*b^2*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
8*d*(b*Cos[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1
[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)*
Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{11/3}}\,dx$$
$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{11/3}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx$$
$$= \frac{3Ab^2\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+\right.}{5d(b\cos(c+dx))^{5/3}\sqrt{}}$$

Mathematica [A] time = 0.134134, size = 89, normalized size = 0.76

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\csc(c+dx)\left(5A\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)+8B\cos(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\right)}{40d(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(5*A*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]
 + 8*B*Cos[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(40*d*(b*Cos[c + d*x])^(8/3))
```

Maple [F] time = 0.297, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))(\sec(dx+c))^3(b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)^3}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

3.905 $\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+ \right.}{8b^4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8
/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^4*d
*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0734252, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+ \right.}{8b^4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8
/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^4*d
*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\int (b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{2/3}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{5/3}\,dx}{b^3}$$
$$= -\frac{3A(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+dx}{}$$

Mathematica [A] time = 0.166257, size = 94, normalized size = 0.79

$$-\frac{3\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)\left(8A\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)+5B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\right)}{40d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*(8*A*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[
c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(40*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F] time = 0.327, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (A+B\cos(dx+c)) (b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.906 $\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=119

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (-3*A*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/
3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d
*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0725229, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```


```
[Out] (-3*A*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^
2]*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/
3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d
*Sqrt[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+B\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}$$
$$= \frac{A\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{2/3}\,dx}{b^2}$$
$$= -\frac{3A(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B(b\cos(c+d}{}$$

Mathematica [A] time = 0.0819728, size = 89, normalized size = 0.75

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(5A\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)+2B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\right)}{10bd\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*
B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[Sin[
c + d*x]^2])/(10*b*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(A+B\cos(dx+c))\,(b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{2}{3}}}{b^2\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.907 $\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(2/3)*H
ypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0635625, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2748, 2643}

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(2/3)*H
ypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*Sqrt
[Sin[c + d*x]^2])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= A\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx+\frac{B\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}\\ &= \frac{3A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{3B(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx\right.}{2b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.116189, size = 85, normalized size = 0.73

$$-\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(B\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)-2A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\right)}{2d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*(-2*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] +
B*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2])*Sqrt[Sin[c
 + d*x]^2])/(2*d*(b*Cos[c + d*x])^(4/3))
```

Maple [F] time = 0.213, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))(b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{B\cos(dx+c)+A}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)
```

3.908 $\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

```
[Out] (3*A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Si
n[c + d*x]^2])
```

Rubi [A] time = 0.0839064, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{3A\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```


```
[Out] (3*A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Si
n[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = b\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{7/3}}\,dx$$
$$= (Ab)\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx + B\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$$
$$= \frac{3A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c}$$

Mathematica [A] time = 0.140284, size = 86, normalized size = 0.75

$$\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)+4B\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\right)}{4d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Cot[c + d*x]*(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*
B*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2])*Sqrt[Sin[
c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F] time = 0.244, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))\sec(dx+c)(b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(
d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

3.909 $\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

```
[Out] (3*A*b*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/
3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0971097, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}+\frac{3B\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*b*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/
3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt
[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = b^2\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{10/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\cos(c+dx))^{10/3}}\,dx + (bB)\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx$$
$$= \frac{3Ab\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2}}$$

Mathematica [A] time = 0.180191, size = 89, normalized size = 0.78

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(4A\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)+7B\cos(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\right)}{28d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]
 + 7*B*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])*Sqrt
[Sin[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))(\sec(dx+c))^2(b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^2}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*co
s(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^2}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

3.910 $\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{10/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}$$

```
[Out] (3*A*b^2*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
10*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2
F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/
3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0980307, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {16, 2748, 2643}

$$\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{10/3}}+\frac{3bB\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*b^2*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(
10*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2
F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/
3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A+B\cos(c+dx))\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = b^3\int \frac{A+B\cos(c+dx)}{(b\cos(c+dx))^{13/3}}\,dx$$
$$= \left(Ab^3\right)\int \frac{1}{(b\cos(c+dx))^{13/3}}\,dx + \left(b^2B\right)\int \frac{1}{(b\cos(c+dx))^{10/3}}\,dx$$
$$= \frac{3Ab^2\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}} + \frac{3bB\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c\right.}{7d(b\cos(c+dx))^{7/3}\sqrt{}}$$

Mathematica [A] time = 0.179286, size = 89, normalized size = 0.76

$$\frac{3b^2\sqrt{\sin^2(c+dx)}\csc(c+dx)\left(7A\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)+10B\cos(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\right)}{70d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x])*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(7*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]
 + 10*B*Cos[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2])*Sq
rt[Sin[c + d*x]^2])/(70*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F] time = 0.305, size = 0, normalized size = 0.

$$\int (A+B\cos(dx+c))(\sec(dx+c))^3(b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)^3}{b^2\cos(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*co
s(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\sec(dx+c)^3}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c))*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

3.911 $\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=157

$$-\frac{A\sin(c+dx)\cos^{m+1}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(c+dx)\right)}{d(m+n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)\cos^{m}}{}$$

```
[Out] -((A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + m
+ n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m + n)*Sqrt[Si
n[c + d*x]^2])) - (B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric
2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2
+ m + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0842588, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {20, 2748, 2643}

$$-\frac{A\sin(c+dx)\cos^{m+1}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(c+dx)\right)}{d(m+n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)\cos^{m}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```


```
[Out] -((A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + m
+ n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m + n)*Sqrt[Si
n[c + d*x]^2])) - (B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric
2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2
+ m + n)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{m+n}(c+dx)(A+B\cos(c-$$
$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{m+n}(c+dx)\,dx + (B\cos$$
$$= -\frac{A\cos^{1+m}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+\right.}{d(1+m+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.241586, size = 130, normalized size = 0.83

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)(b\cos(c+dx))^n\left(A(m+n+2)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(c+d\right.\right.}{d(m+n+1)(m+n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(2 + m + n)*Hype
rgeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2] + B*(1 + m
 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos
[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m + n)*(2 + m + n)))
```

Maple [F] time = 1.993, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (b\cos(dx+c))^n (A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos(c+dx))^n\,(A+B\cos(c+dx))\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))*cos(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^m, x)
```

3.912 $\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=141

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,{}_2F_1\left(\frac{1}{2},\frac{n+4}{2};\frac{n+6}{2};\cos^2(c+dx)\right)}{b^4d(n+4)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((A*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b
*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0991279, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+4}\,{}_2F_1\left(\frac{1}{2},\frac{n+4}{2};\frac{n+6}{2};\cos^2(c+dx)\right)}{b^4d(n+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b
*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{2+n}(A+B\cos(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\cos(c+dx))^{2+n}\,dx}{b^2} + \frac{B\int (b\cos(c+dx))^{3+n}\,dx}{b^3}$$
$$= -\frac{A(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\sin(c+}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.281681, size = 120, normalized size = 0.85

$$-\frac{\sqrt{\sin^2(c+dx)}\cos^2(c+dx)\cot(c+dx)(b\cos(c+dx))^n\left(A(n+4)\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)+B(n+3)\cos(c+\right.}{d(n+3)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(4 + n)*Hypergeometric
2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + B*(3 + n)*Cos[c + d*x]*Hyp
ergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(3 + n)*(4 + n)))
```

Maple [F] time = 1.819, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b\cos(dx+c))^n (A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n \cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^3 + A\cos(dx+c)^2\right)(b\cos(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

3.913 $\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=141

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\text{co}\right.}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((A*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b
*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0948405, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {16, 2748, 2643}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\text{co}\right.}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b
*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = \frac{\int (b\cos(c+dx))^{1+n}(A+B\cos(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\cos(c+dx))^{1+n}\,dx}{b} + \frac{B\int (b\cos(c+dx))^{2+n}\,dx}{b^2}$$
$$= -\frac{A(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^2 d(2+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.224801, size = 118, normalized size = 0.84

$$-\frac{\sqrt{\sin^2(c+dx)}\cos(c+dx)\cot(c+dx)(b\cos(c+dx))^n\left(A(n+3)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)+B(n+2)\cos(c+dx)\,{}_2\right.}{d(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + n)*Hypergeometric2F
1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + B*(2 + n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^
2])/(d*(2 + n)*(3 + n)))
```

Maple [F] time = 1.497, size = 0, normalized size = 0.

$$\int \cos(dx+c)\,(b\cos(dx+c))^n\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(B\cos(dx+c)^2 + A\cos(dx+c)\right)(b\cos(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas
")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\cos(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

3.914 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=141

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(\right.}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((A*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b*C
os[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d
*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0796793, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2748, 2643}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(\right.}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -((A*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])) - (B*(b*C
os[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d
*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n(A+B\cos(c+dx))\,dx &= A\int (b\cos(c+dx))^n\,dx+\frac{B\int (b\cos(c+dx))^{1+n}\,dx}{b}\\ &= -\frac{A(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}-\frac{B(b\cos(c}{}\end{aligned}$$

Mathematica [A] time = 0.157603, size = 112, normalized size = 0.79

$$-\frac{\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^n\left(A(n+2)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)+B(n+1)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\right.\right.}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(2 + n)*Hypergeometric2F1[1/2, (1 + n
)/2, (3 + n)/2, Cos[c + d*x]^2] + B*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[
1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + n
)*(2 + n)))
```

Maple [F] time = 1.506, size = 0, normalized size = 0.

$$\int (b\cos (dx+c))^n (A+B\cos (dx+c))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos (dx+c)+A)(b\cos (dx+c))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos (dx+c)+A)(b\cos (dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b\cos (c+dx))^n (A+B\cos (c+dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos (dx+c)+A) (b\cos (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

3.915 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx$

Optimal. Leaf size=132

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c\right.}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -((A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]
^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0937742, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {16, 2748, 2643}

$$-\frac{A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c\right.}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```


```
[Out] -((A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]
^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (B*(b*Cos[c + d*x])^(1 + n)
*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])
/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec(c+dx)\,dx = b\int (b\cos(c+dx))^{-1+n}(A+B\cos(c+dx))\,dx$$
$$= (Ab)\int (b\cos(c+dx))^{-1+n}\,dx + B\int (b\cos(c+dx))^n\,dx$$
$$= -\frac{A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}} - \frac{B(\ldots}{\ldots}$$

Mathematica [A] time = 0.180392, size = 109, normalized size = 0.83

$$-\frac{b\sqrt{\sin^2(c+dx)}\cot(c+dx)(b\cos(c+dx))^{n-1}\left(A(n+1)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)+Bn\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\ldots\right.\right.}{dn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x],x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(A*(1 + n)*Hypergeometric2F1[1/
2, n/2, (2 + n)/2, Cos[c + d*x]^2] + B*n*Cos[c + d*x]*Hypergeometric2F1[1/2
, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(1 + n)
))
```

Maple [F] time = 1.197, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n\,(A+B\cos(dx+c))\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="maxima
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="fricas
")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

3.916 $\int (b\cos(c+dx))^n (A+B\cos(c+dx))\sec^2(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{Ab\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+d\right.}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] (A*b*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Co
s[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.114124, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{Ab\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}-\frac{B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+d\right.}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] (A*b*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2
, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Co
s[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^2(c+dx)\,dx = b^2\int (b\cos(c+dx))^{-2+n}(A+B\cos(c+dx))\,dx$$
$$= \left(Ab^2\right)\int (b\cos(c+dx))^{-2+n}\,dx + (bB)\int (b\cos(c+dx))^{-1+n}\,d$$
$$= \frac{Ab(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)\mathrm{s}}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.174848, size = 109, normalized size = 0.83

$$-\frac{b\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^{n-1}\left(An\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)+B(n-1)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+}{2}\right.\right.}{d(n-1)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^2,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(A*n*Hypergeometric2F1[1/2, (-1
 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + B*(-1 + n)*Cos[c + d*x]*Hypergeometri
c2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n
)*n))
```

Maple [F] time = 1.366, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n\,(A+B\cos(dx+c))\,(\sec(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\sec(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.917 $\int (b\cos(c+dx))^n (A+B\cos(c+dx))\sec^3(c+dx)\,dx$

Optimal. Leaf size=139

$$\frac{Ab^2\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}+\frac{bB\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos\right.}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (A*b^2*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (b*B*(b*Cos[
c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.117374, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{Ab^2\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}+\frac{bB\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos\right.}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] (A*b^2*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (b*B*(b*Cos[
c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*
x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^3(c+dx)\,dx = b^3\int (b\cos(c+dx))^{-3+n}(A+B\cos(c+dx))\,dx$$
$$= \left(Ab^3\right)\int (b\cos(c+dx))^{-3+n}\,dx + \left(b^2B\right)\int (b\cos(c+dx))^{-2+n}\,dx$$
$$= \frac{Ab^2(b\cos(c+dx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+dx)\right)\sin(c}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.162323, size = 118, normalized size = 0.85

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\sec^2(c+dx)(b\cos(c+dx))^n\left(A(n-1)\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)+B(n-2)\cos(c+dx)\,{}_2F}{d(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^3,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + n)*Hypergeometric2F1[1/2, (-2 +
 n)/2, n/2, Cos[c + d*x]^2] + B*(-2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2
, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^
2])/(d*(-2 + n)*(-1 + n)))
```

Maple [F] time = 1.596, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (A+B\cos(dx+c))(\sec(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^3,x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

3.918 $\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{Ab^3\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}+\frac{b^2B\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(\right.}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (A*b^3*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2]) + (b^2*
B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.119445, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {16, 2748, 2643}

$$\frac{Ab^3\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}+\frac{b^2B\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(\right.}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```


```
[Out] (A*b^3*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n
)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2]) + (b^2*
B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```


Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^n(A+B\cos(c+dx))\sec^4(c+dx)\,dx = b^4\int (b\cos(c+dx))^{-4+n}(A+B\cos(c+dx))\,dx$$
$$= \left(Ab^4\right)\int (b\cos(c+dx))^{-4+n}\,dx + \left(b^3B\right)\int (b\cos(c+dx))^{-3+n}$$
$$= \frac{Ab^3(b\cos(c+dx))^{-3+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1+n);\cos^2(c+\right.}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.160283, size = 118, normalized size = 0.84

$$-\frac{\sqrt{\sin^2(c+dx)}\csc(c+dx)\sec^3(c+dx)(b\cos(c+dx))^n\left(A(n-2)\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)+B(n-3)\cos(c+\right.}{d(n-3)(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x])*Sec[c + d*x]^4,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-2 + n)*Hypergeometric2F1[1/2, (-3 +
 n)/2, (-1 + n)/2, Cos[c + d*x]^2] + B*(-3 + n)*Cos[c + d*x]*Hypergeometric
2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2])*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]
^2])/(d*(-3 + n)*(-2 + n)))
```

Maple [F] time = 1.182, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n(A+B\cos(dx+c))(\sec(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="maxi
ma")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sec(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*sec(d*x+c)^4,x, algorithm="giac
")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.919 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{9}{2}}(c+d$$

```
[Out] (-2*A*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n
)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c +
d*x]^2]) - (2*B*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2
, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqr
t[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0946512, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{2A\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{9}{2}}(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-2*A*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n
)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c +
d*x]^2]) - (2*B*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2
, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqr
t[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{5}{2}+n}(c+dx)(A+B\cos(c+$$
$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{5}{2}+n}(c+dx)\,dx + (B\cos$$
$$= -\frac{2A\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\right.}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.40253, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+9)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)+B(2r\right.}{d(2n+7)(2n+9)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(9 + 2*n)*Hyperge
ometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + B*(7 + 2*n)*Co
s[c + d*x]*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2
])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 2*n)*(9 + 2*n))
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{5}{2}}(b\cos(dx+c))^n(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^3+A\cos(dx+c)^2\right)(b\cos(dx+c))^n\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(
d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\cos(dx+c)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

3.920 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b$$

```
[Out] (-2*A*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n
)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0912102, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-2*A*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n
)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{3}{2}+n}(c+dx)(A+B\cos$$
$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{3}{2}+n}(c+dx)\,dx + (B$$
$$= -\frac{2A\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2\right.}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.321349, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+7)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)+B(\right.}{d(2n+5)(2n+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(7 + 2*n)*Hyperge
ometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + B*(5 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]
)*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(7 + 2*n))
```

Maple [F] time = 0.394, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{3}{2}}(b\cos(dx+c))^n(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(B\cos(dx+c)^2+A\cos(dx+c)\right)(b\cos(dx+c))^n\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*
x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^n\cos(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.921 $\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+d\text{…}}{}$$

```
[Out] (-2*A*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n
)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.0864296, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{5}{2}}(c+d\text{…}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-2*A*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n
)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}(b\cos(c+dx))^n(A+B\cos(c+dx))\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{1}{2}+n}(c+dx)(A+B\cos(c$$
$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{\frac{1}{2}+n}(c+dx)\,dx + (B\cos$$
$$= -\frac{2A\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\right.}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.236766, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+5)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)+B(2n\right.}{d(2n+3)(2n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(5 + 2*n)*Hyperge
ometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + B*(3 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2])
*Sqrt[Sin[c + d*x]^2])/(d*(3 + 2*n)*(5 + 2*n))
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (A+B\cos(dx+c))\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c))^n\sqrt{\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\left(b\cos(dx+c)\right)^n\sqrt{\cos(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

3.922 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b}{}$$

```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n
)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.0845629, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2B\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b}{}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```


```
[Out] (-2*A*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n
)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{1}{2}+n}(c+dx)(A+B\cos(c+dx))\,dx$$
$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{1}{2}+n}(c+dx)\,dx + (B\cos^{-n}(c+d$$
$$= -\frac{2A\sqrt{\cos(c+dx)}(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+\right.}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.214486, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+3)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)+B(\right.}{d(2n+1)(2n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(3 + 2*n)*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + B*(1 + 2*n)*Cos
[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])
*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(3 + 2*n))
```

Maple [F] time = 0.424, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n\,(A+B\cos(dx+c))\,\frac{1}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\cos(c+dx))^n\,(A+B\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*(A + B*cos(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

3.923 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}-\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx)}{d(2n+1)}$$

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d
*x]^2]) - (2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rubi [A] time = 0.0878826, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}-\frac{2B\sin(c+dx)\sqrt{\cos(c+dx)}(b\cos(c+dx)}{d(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d
*x]^2]) - (2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2,
 (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{3}{2}+n}(c+dx)(A+B\cos(c+dx))\,dx$$
$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx + (B\cos^{-n}(c+dx)($$
$$= \frac{2A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.246213, size = 133, normalized size = 0.82

$$-\frac{2\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\left(A(2n+1)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)+B(2n-1)\cos(c+d}{d\left(4n^2-1\right)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-1
 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + B*(-1 + 2*n)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x
]^2])/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

Maple [F] time = 0.39, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n(A+B\cos(dx+c))(\cos(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

3.924 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-\right.}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{c}}$$

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 +
 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt
[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0960241, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-\right.}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 +
 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{5}{2}+n}(c+dx)(A+B\cos(c+dx))\,dx$$

$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{5}{2}+n}(c+dx)\,dx + (B\cos^{-n}(c+d$$

$$= \frac{2A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)\sin(c+d}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.207713, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\left(A(2n-1)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)+B(2n-3)\cos(c}{d(2n-3)(2n-1)\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-
3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + B*(-3 + 2*n)*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

Maple [F] time = 0.379, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (A+B\cos(dx+c))(\cos(dx+c))^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\,(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

3.925 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2\right.}{d(3-2n)\sqrt{\sin^2(c+dx}}$$

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c +
d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1
+ 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqr
t[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0910064, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{5}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2\right.}{d(3-2n)\sqrt{\sin^2(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c +
d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1
+ 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{7}{2}+n}(c+dx)(A+B\cos(c+dx))\,dx$$

$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{7}{2}+n}(c+dx)\,dx + (B\cos^{-n}(c+dx)($$

$$= \frac{2A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.214401, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\left(A(2n-3)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)+B(2n-5)\cos(c+d\right.}{d(2n-5)(2n-3)\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(7/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-3 + 2*n)*Hypergeometric2F1[1/2, (-
5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + B*(-5 + 2*n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c +
d*x]^2])/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

Maple [F] time = 0.415, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (A+B\cos(dx+c))(\cos(dx+c))^{-\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

3.926 $\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-\right.}{d(5-2n)\sqrt{\sin^2(c+dx)}\,\mathrm{co}}$$

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Cos[c + d*x]^(7/2)*Sqrt[Sin[c +
d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1
 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sq
rt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0906357, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{2A\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)}+\frac{2B\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-\right.}{d(5-2n)\sqrt{\sin^2(c+dx)}\,\mathrm{co}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*A*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Cos[c + d*x]^(7/2)*Sqrt[Sin[c +
d*x]^2]) + (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1
 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sq
rt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n(A+B\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx = (\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{9}{2}+n}(c+dx)(A+B\cos(c+dx))\,dx$$
$$= (A\cos^{-n}(c+dx)(b\cos(c+dx))^n)\int \cos^{-\frac{9}{2}+n}(c+dx)\,dx + (B\cos^{-n}(c+d$$
$$= \frac{2A(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-7+2n);\frac{1}{4}(-3+2n);\cos^2(c+dx)\right)\sin(c+}{d(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.212964, size = 138, normalized size = 0.85

$$-\frac{2\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^n\left(A(2n-5)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)+B(2n-7)\cos(c}{d(2n-7)(2n-5)\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x]))/Cos[c + d*x]^(9/2),x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(-5 + 2*n)*Hypergeometric2F1[1/2, (-
7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2] + B*(-7 + 2*n)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c +
 d*x]^2])/(d*(-7 + 2*n)*(-5 + 2*n)*Cos[c + d*x]^(7/2))
```

Maple [F] time = 0.391, size = 0, normalized size = 0.

$$\int (b\cos(dx+c))^n (A+B\cos(dx+c))(\cos(dx+c))^{-\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c))/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c))/cos(d*x+c)^(9/2),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

3.927 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=169

$$-\frac{3Ab\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos}}{}$$

```
[Out] (-3*A*b*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2,
(7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[
Sin[c + d*x]^2]) - (3*b*B*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hyper
geometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.099053, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3Ab\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}-\frac{3bB\sin(c+dx)\sqrt[3]{b\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-3*A*b*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2,
(7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[
Sin[c + d*x]^2]) - (3*b*B*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hyper
geometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])
/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}(A+B\cos(c+dx))\,dx = \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt[3]{\cos(c+dx)}}$$
$$= \frac{\left(Ab\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}} + \frac{\left(bB\sqrt[3]{b\cos(c+dx}\right.}{\sqrt[3]{\text{cc}}}$$
$$= -\frac{3Ab\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+\right.}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.475777, size = 140, normalized size = 0.83

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^{4/3}\cos^{m+1}(c+dx)\left(A(3m+10)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)\right.}{d(3m+7)(3m+10)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(B*(7 + 3*m)*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] +
A*(10 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]
^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m))
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (b\cos(dx+c))^{\frac{4}{3}} (A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Bb\cos(dx+c)^2+Ab\cos(dx+c)\right)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos
(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{4}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

3.928 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=167

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+$$

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5
 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeom
etric2F1[1/2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
8 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0945796, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3A\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)(b\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5
 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeom
etric2F1[1/2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
8 + 3*m)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}(A+B\cos(c+dx))\,dx = \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{\cos^{\frac{2}{3}}(c+dx)}$$
$$= \frac{\left(A(b\cos(c+dx))^{2/3}\right)\int \cos^{\frac{2}{3}+m}(c+dx)\,dx}{\cos^{\frac{2}{3}}(c+dx)} + \frac{\left(B(b\cos(c+\right.}{}$$
$$= -\frac{3A\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(\right.}{d(5+3m)\sqrt{\sin^2(c+dx}}$$

Mathematica [A] time = 0.28948, size = 140, normalized size = 0.84

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\left(A(3m+8)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right.\right.}{d(3m+5)(3m+8)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(A*(8 + 3*m)*H
ypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + B*(5 + 3
*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 3*m)*(8 + 3*m))
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (b\cos(dx+c))^{\frac{2}{3}} (A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

3.929 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx$

Optimal. Leaf size=167

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}$$

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4
 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeom
etric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0863866, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3A\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}-\frac{3B\sin(c+dx)\sqrt[3]{b\cos(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```


```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4
 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Si
n[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeom
etric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```


Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}(A+B\cos(c+dx))\,dx = \frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{\frac{1}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt[3]{\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}} + \frac{\left(B\sqrt[3]{b\cos(c+dx)}\right)\int}{\sqrt[3]{\cos(c}}$$
$$= -\frac{3A\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{1}{6}(10+3m\right.}{d(4+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.28472, size = 140, normalized size = 0.84

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\sqrt[3]{b\cos(c+dx)}\cos^{m+1}(c+dx)\left(A(3m+7)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{m}{2}+\frac{5}{3};\cos^2(c+dx)\right)+B(3m\right.}{d(3m+4)(3m+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x]),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(A*(7 + 3*m)*H
ypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + B*(4 + 3*m)
*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x
]^2])*Sqrt[Sin[c + d*x]^2])/(d*(4 + 3*m)*(7 + 3*m))
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m \sqrt[3]{b\cos(dx+c)}\,(A+B\cos(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (B\cos(dx+c)+A)\,(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

3.930 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=167

$$-\frac{3A\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m$$

$$d(3m+5)\sqrt{\sin^2(c+dx}$$

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m)/
6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x]
)^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0874664, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3A\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m$$

$$d(3m+5)\sqrt{\sin^2(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m)/
6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x]
)^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{\sqrt[3]{b\cos(c+dx)}}$$
$$= \frac{\left(A\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int \cos^{\frac{2}{3}+m}(c+d}{\sqrt[3]{b\cos(c+dx)}}$$
$$= -\frac{3A\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)\sin(c+dx}{d(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.286571, size = 140, normalized size = 0.84

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(A(3m+5)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)+B(3m+2)\cos}{d(3m+2)(3m+5)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(5 + 3*m)*Hypergeometric2F1[1/2, (
2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + B*(2 + 3*m)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*
x]^2])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.215, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (A+B\cos(dx+c)) \frac{1}{\sqrt[3]{b\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^{m}}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos(
d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\cos(c+dx))\cos^{m}(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\cos(dx+c)^{m}}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(1/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

3.931 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=167

$$-\frac{3A\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}\right.}{d(3m+4)\sqrt{\sin^2(c+d}}$$

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin
[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/
6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x]
)^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0880054, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$-\frac{3A\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}(b\cos(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}\right.}{d(3m+4)\sqrt{\sin^2(c+d}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6,
 Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin
[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/
6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x]
)^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{(b\cos(c+dx))^{2/3}}$$

$$= \frac{\left(A\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{-\frac{2}{3}+m}(c+dx)\,dx}{(b\cos(c+dx))^{2/3}} + \frac{\left(B\cos^{\frac{2}{3}}(c+dx)\right)\int \cos^{\frac{1}{3}+m}(c+dx)\,d}{(b\cos(c+dx))^{2/3}}$$

$$= -\frac{3A\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(1+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}} -$$

Mathematica [A] time = 0.285737, size = 140, normalized size = 0.84

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(A(3m+4)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)+B(3m+1)\cos(c+\right.}{d(3m+1)(3m+4)(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(4 + 3*m)*Hypergeometric2F1[1/2, (
1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + B*(1 + 3*m)*Cos[c + d*x]*Hyperge
ometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^
2])/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.207, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (A+B\cos(dx+c))(b\cos(dx+c))^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{1}{3}}\cos(dx+c)^m}{b\cos(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos(
d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(A+B\cos(c+dx))\cos^m(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*cos(c + d*x))*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

3.932 $\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=171

$$\frac{3A\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+\right.}{bd(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b}}$$

```
[Out] (3*A*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +
 d*x]^2]) - (3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (
8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(
1/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.0974893, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {20, 2748, 2643}

$$\frac{3A\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}-\frac{3B\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+\right.}{bd(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*A*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c
 + d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c +
 d*x]^2]) - (3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (
8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(
1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+B\cos(c+dx))}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{4}{3}+m}(c+dx)(A+B\cos(c+dx))\,dx}{b\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{\left(A\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{4}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\cos(c+dx)}} + \frac{\left(B\sqrt[3]{\cos(c+dx)}\right)\int \cos^{-\frac{1}{3}+m}(c+}{b\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3A\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\cos^2(c+dx)\right)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}} -$$

Mathematica [A] time = 0.315683, size = 140, normalized size = 0.82

$$-\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(A(3m+2)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)+B(3m-1)\cos\right.}{d(3m-1)(3m+2)(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x]))/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(2 + 3*m)*Hypergeometric2F1[1/2, (
-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + B*(-1 + 3*m)*Cos[c + d*x]*Hyper
geometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d
*x]^2])/(d*(-1 + 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(4/3))
```

Maple [F] time = 0.285, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (A+B\cos(dx+c))(b\cos(dx+c))^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(B\cos(dx+c)+A)(b\cos(dx+c))^{\frac{2}{3}}\cos(dx+c)^m}{b^2\cos(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*co
s(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c))/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B\cos(dx+c)+A)\cos(dx+c)^m}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c))/(b*cos(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```